Simultaneous H_∞ vibration control of fluid/plate system via reduced-order controller

Bogdan Robu; Valérie Budinger; Lucie Baudouin; Christophe Prieur and Denis Arzelier

***Abstract*— The problem of active reduction of the structural vibrations induced by the sloshing of large masses of fuel inside partly full tank is considered. The proposed study focuses on an experimental device mimicking an aircraft wing made of an aluminum rectangular plate equipped with piezoelectric patches at the clamped end and with a cylindrical tip-tank, more or less filled with liquid. First, a representative finite dimensional model of the complete system containing the first 5 structural modes of the plate and the first 2 liquid sloshing modes is derived via a model matching procedure. A simultaneous H_∞ control problem associated to the vibration attenuation problem for two different fillings of the tank is stated. Due to the large scale of the synthesis model and to the simultaneous performance requirements, a reduced-order H_∞ controller is computed with HIFOO 2.0 and is compared with individual designs for different filling levels. Experimental results are finally provided illustrating the relevance of the chosen strategy.**

***Index Terms*— Flexible structure, reduced-order control design, H_∞ control, HIFOO.**

I. Introduction

Recent developments in design of commercial transport aircraft mainly focus on lighter and more flexible structures for increased payloads. Coupling between fuel sloshing in partly full tanks, structural deformation and aeroelastic phenomena may therefore not be considered as negligible anymore as it has a prominent influence on flight dynamics and flying qualities [13]. It is well-known that aircraft dynamics may be dramatically affected by propellant motion in partly filled unbaffled tanks causing alteration of damping and loss of lateral or longitudinal stability properties of a free-flying airframe (see [1] and the references therein). Besides, it is worth noting that control/system/structural flexibility/sloshing modes interactions are also of major importance for the stability and performance of space vehicles (see e.g. [6]).

Here, we are mainly concerned by the coupling between the fuel sloshing in a tip-tank and the structural deformation of an aircraft wing. Indeed, for slender structures, the first structural modes are usually in the same range of frequencies that is effective for the sloshing modes as well. Meanwhile, smart materials are now often used for many applications in civil engineering. Thus, flexible structures equipped with piezoelectric patches occupy a major place in the control research area. Their capability of attenuating the vibrations and measuring the deformation is described in [2], [3] among other references. The present paper shows how a piezoelectric device can be useful for the control of vibrations interactions in a specific aeronautical setup.

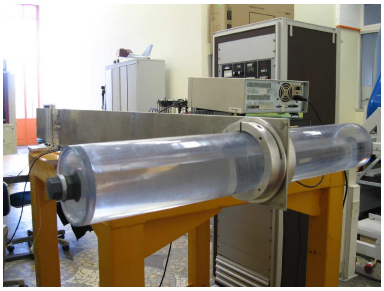

Fig. 1. Plant description: the rectangular plate and the horizontal cylinder

In this objective, the experiment depicted by Figure 1 which is an example of a coupled fluid-flexible structure system may be used as a good benchmark for modern vibrations alleviation control strategies. It has the same first flexible modes as a plane wing [12] and is equipped with a clamped tip-tank that can be filled with liquid or ice up to different levels. If numerous studies have already investigated the control of vibrations of flexible structure via piezoelectric patches (see [5], [9], [19] among others), only a few results are already available in the literature for fluid-structure systems. Reference [10] gives a recent theoretical result and [16] validates the method by means of experimental results. To the best of our knowledge, there are less studies of fluid-structure system dedicated to aerospace applications, except [11] where controllers are designed using a numerical model. In the present paper, in order to take the large numbers of degrees of freedom in the dynamics into account, an infinite-dimensional model is first built and described by partial differential equations (pde). Further on, a finite dimensional model is computed in [14] and is derived from the original distributed parameter model.

The truncated model is then lightly adapted to match with the experimental data obtained from modal analysis of the plant: It is checked on the Bode plots that the model and the experimental setup have the same dynamics. After this model matching, a robust control problem for the fluid-flexible structure system is defined. Its solution aims at rejecting the perturbations that are the source of some undesirable vibrations of the flexible structure and to attenuate the non-desirable sloshing effect. To avoid the possible spillover

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effect when introducing the controller in the loop, a suitable spillover filter (see [9]) is selected. Different H_∞ controllers are then computed and it is shown that given the number of modes to be controlled and the model complexity, reduced-order controllers have to be privileged. To do so, the HIFOO package is used [7]. One of the advantage of this package is to allow to compute simultaneous reduced-order H_∞ controllers (a unique controller for several fillings of the tank) leading to a more robust design for the controller. Finally some experiments on the real setup are realized, and the relevance of the stabilizing controller to suppress vibrations of the fluid-structure system is illustrated.

The paper is organized as follows. In Section II the plant under consideration, equipped with piezoelectric patches (sensors and actuators) is presented and the control problem under consideration in this paper is introduced. In Section III, the model matching problem is solved. Then a reduced-order robust controller is computed and its effectiveness is checked by experiments in Section III. Finally, a first order simultaneous controller is computed and tested on the experimental setup.

II. Problem statement and control objectives

A. Plant description

The plant to be controlled is located at ISAE-ENSICA, Toulouse, France.

The device is composed of an aluminium rectangular plate and a plexiglas horizontal cylindrical tip-tank filled with liquid (see Figures 1 and 2).

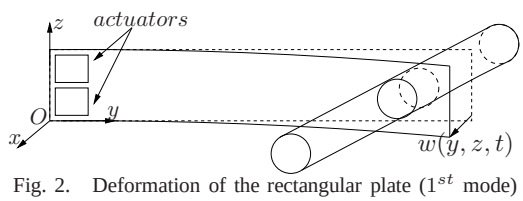

Fig. 2. Deformation of the rectangular plate (1st mode)

The length of the plate is along the horizontal axis and the width along the vertical one. The plate is clamped on one end and free on the three other sides. The characteristics of the aluminium plate are given in Table I.

Plate length	L	1.36 m
Plate width	l	0.16 m
Plate thickness	h	0.005 m
Plate density	ρ	2970 kg m^{-3}
Plate Young modulus	Y	75 GPa
Plate Poisson coefficient	ν	0.33

TABLE I

PLATE CHARACTERISTICS

The two piezoelectric actuators made from PZT (Lead zirconate titanate) are bonded next to the plate clamped side. Two sensors (made from PVDF - Polyvinylidene fluoride), are located on the opposite side of the plate with respect to the actuators. The characteristics of the collocated sensors and actuators are given in Table II. The optimal placement of the actuators and sensors is still an open problem. This is not our problem though in this paper since the position of the actuators and sensors can not be changed on the experimental setup.

Actuator length	0.14 m
Actuator width	0.075 m
Actuator thickness	0.0005 m
Sensor length	0.015 m
Sensor width	0.025 m
Sensor thickness	0.0005 m
Actuator/Sensor density	7800 kg m^{-3}
Actuator/Sensor Young modulus	67 GPa
Actuator piezoelectric coefficient	$-210e^{-12}$ m V^{-1}
Sensor piezoelectric coefficient	-9.6 N (Vm)$^{-1}$
Actuator/Sensor Poisson coefficient	0.3

TABLE II

CHARACTERISTICS OF THE PIEZOELECTRIC PATCHES

The tank is centered at 1.28 m from the plate clamped side and is symmetrically spread along the horizontal axis. Due to the configuration of the whole system (see Figure 2), the tank undergoes a longitudinal movement when the plate has a flexion movement and a pitch movement if the plate has a torsion movement. It has the dimensions given in Table III and it can be filled with water or ice up to an arbitrary level. When the tank is filled with water up to a level close to 0 or close to the cylinder diameter (near empty tank empty or near full tank), there is no sloshing behavior, and it can be easily modeled by a steady.

Tank exterior diameter	0.11 m
Tank interior diameter	0.105 m
Tank length	0.5 m
Tank density	1180 kg m^{-3}
Tank young modulus	4.5 GPa

TABLE III

CHARACTERISTICS OF THE CYLINDRICAL TANK

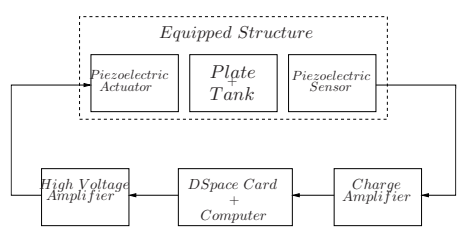

Fig. 3. Equipped experimental setup

The experimental setup is depicted in Figure 3. The system (plate + tank) is connected to the computer via a high voltage amplifier delivering ± 100 V and a charge amplifier 2635 from Brüer & Kjaer. Thus, the controller is implemented in a DSpace© board.

In order to obtain the Bode plots, tests are made using a sampling time of 0.004 seconds on the DSpace© card and a Dynamic Signal Analyzer.

B. Control objectives

The main control objective is to attenuate the vibrations of the plate and the sloshing modes in the tank. The control input is the voltage applied to one of the two piezoelectric actuators. The output of the system is the voltage measured with a piezoelectric sensor collocated with the piezoelectric actuator used for control. The to-be-controlled output is composed of the output of the system and the control input (this will be the z-variable in Figure 6 below). The flexible structure is subject to a disturbance created by a voltage of frequencies between $0\,\text{Hz}$ and $50\,\text{Hz}$ applied to the other piezoelectric actuator (see Figure 2 where the two actuators are presented). This perturbation is a source of vibrations of the fluid-structure system and is modeled by a low-pass filter of order 1 with a bandwidth of $50\,\text{Hz}$. The filter is located before the piezoelectric actuator used as a disturbance actuator and has the transfer function given by $H_1(s) = \frac{100\pi}{s+100\pi}$. The objective is to minimize the maxima of the frequency response of the transfer function between the control input and this output. This specification is formulated as minimizing the H_∞ norm of the transfer between the input disturbances and the output.

The computation of the dynamical model is based on two coupled partial differential equations (one for the flexible structure and one for the sloshing liquid). For the numerical computing, it is necessary to consider a finite-dimensional approximation. Only the first five modes of the deformation of the plate and the first two sloshing modes will be considered. The controller also has to ensure the rejection of the neglected high frequency modes. To avoid this spill-over phenomenon, a filter must be added to the output. This filter is a high-pass filter to attenuate the modes that have not been taken into account in the computation of the finite-dimensional model.

One should also be careful that the control signal applied to the piezoelectric actuators must be in the range $\pm 100\,\text{V}$ due to physical constraint in the experimental setup. It is also interesting to consider different fillings of the tank in the design methodology since the fuel quantity is varying during the flight. Therefore, the influence of the tank filling on the performance should be taken into account. Thus, two different fillings of the tank 70% and 90% will be considered in the synthesis and in the experiments as well. In the sequel, the filling of the tank is denoted by e ($e = 0.7$ means that the tank is 70% filled).

The controller synthesis is detailed in Section III-C. Thus, in Section III-C.1, a reduced order H_∞ controller will be designed using HIFOO algorithm.

III. Controller synthesis

A. Finite dimensional model

The state space representation of the experimental setup is given by the set of equations (1), where the state space vector X is a combination of the state vector of the plate X_p (which physically represent the kinematic parameters of the plate modes) and the state vector of the liquid X_l (which represents the kinematic parameters of the liquid sloshing approximation):

$$X = \begin{pmatrix} X_p \\ X_l \end{pmatrix},$$

$$\begin{cases} \dot{X} = \begin{pmatrix} A_p & A_{lp} \\ A_{pl} & A_l \end{pmatrix} X + \begin{pmatrix} B_p \\ B_{pl} \end{pmatrix} u + \begin{pmatrix} B_w \\ \mathbf{0} \end{pmatrix} w \\ z = \begin{pmatrix} C_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} X + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y = \begin{pmatrix} C_p & \mathbf{0} \end{pmatrix} X. \end{cases} \tag{1}$$

The plate deformation is described by a classical plate equation with Neumann homogeneous boundary conditions while the liquid sloshing is described by a Laplace equation and an unstationary Bernoulli equation with classical boundary conditions related to tank speed. A more detailed description of the matrices is given in [14] and is not presented here.

In (1) the subscripts p and l differentiate the matrices related to the plate (p) from the ones related to the liquid sloshing (l). In the same time the subscript lp denotes the influence of the plate on the liquid and pl vice-versa. Therefore, A_p and A_l are the dynamic matrices of the plate and liquid. B_p and C_p are the control and output matrices of the plate. A_{pl} expresses the influence of the liquid sloshing on the plate movement while A_{lp} expresses the influence of the plate bending on the liquid sloshing modes. B_w is the perturbation matrix similar in construction with B_p and $\mathbf{0}$ denotes null matrices of appropriate dimensions.

Moreover in (1), y stands for the measured output, i.e. the voltage measured through the piezolectric sensor, u is the control input i.e. the voltage applied to the piezoelectric actuator, z stands for the output to be controlled and is composed of y and u. Finally, w is the disturbance input, generated by the voltage applied to the piezoelectric actuator used as source of disturbances.

The system model (1) is established for the first N modes of the plate and for the first $M \neq N$ modes of the liquid sloshing. As it is observed in Section III-B below, this choice allows to consider the flexional deformation of the plate but also the first torsional modes of the plate. Besides, it is shown in [8] that the first modes contain the main part of the energy of the deformation of the flexible structure. Moreover, using the energy approach from [17], it is possible to check that the first five modes contain almost all the energy of the structure. Since the objective is to control the flexional deformation but also the torsional modes of the plate, the number of modes considered is also chosen so that torsion modes are included. Therefore the controller will be computed with $N = 5$ and $M = 2$. So X_p in $\mathbb{R}^{2N}$ and X_l in $\mathbb{R}^{2M}$ and the total state space vector X belongs to $\mathbb{R}^{14}$.

B. Model matching

The mathematical model was validated in a previous paper [14] by a comparison of a time-response for a given initial deformation of the plate. However, in order to obtain a model which provides a good match of the measured frequency

response, some adjustments are required by considering the Bode plots. This adjustment is done following a trial-and-error method (first the frequencies are matched and then is the damping). Other methods are possible for flexible structures (see e.g., [15] and references therein).

This model matching is necessary since some mechanical elements are not well known and have not been taken into account in the modeling in [14]. These elements include the circular ring that is used to attach the tank on the plate, the dynamical behavior of the piezoelectric patches, the non-homogeneity of the plate and the weight of the tank.

As a first step of the model matching, the frequencies of the plate are adjusted. A bigger amount of liquid will be sensed by the plate as an increase of the total mass of the plate and this will lead to (see [4, Chapter 11]) a shift towards zero of all the mode frequencies. A second step of the model matching is the adding of a static gain that corresponds to the high frequency modes neglected during the model reduction static correction. This allows to get a more realistic model at low frequencies. The comparison of the bode plots for $e = 0.7$ and $e = 0.9$ on Figures 4 and 5 shows that the model, for $e = 0.7$ is quite accurate with respect to the real data while there is some discrepancy in the amplitude of the first sloshing mode for $e = 0.9$.

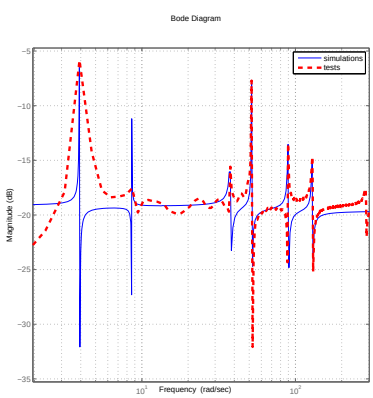

Fig. 4. Frequency matching for the tank filling level $e = 0.9$ (numerical model - plain line and experimental setup - dotted line)

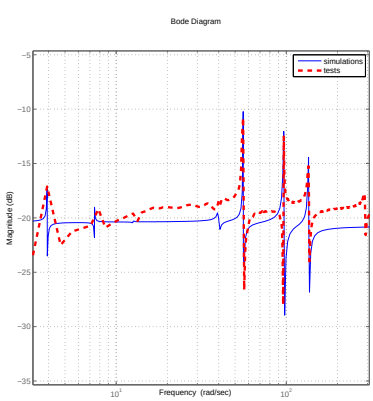

Fig. 5. Frequency matching for the tank filling level $e = 0.7$ (numerical model - plain line and experimental setup - dotted line)

In the considered figures, the first peak corresponds to the first flexional mode of the plate (0.625 Hz) and the second peak to the first sloshing mode (1.25 Hz for $e = 0.7$ and 1.415 Hz for $e = 0.9$) in the tank. The next four peaks are respectively representing: the first torsional mode (the third peak) (6.38 Hz) and the second (8.75 Hz), third (14.45 Hz) and forth (21.50 Hz) flexional modes of the plate. The second mode of the liquid sloshing (1.99 Hz for $e = 0.7$ and 2.15 Hz for $e = 0.9$) cannot be identified on the Bode diagrams due to its very small amplitude.

C. Robust H_∞ control problem

In this section a robust controller is computed and some experiments are performed.

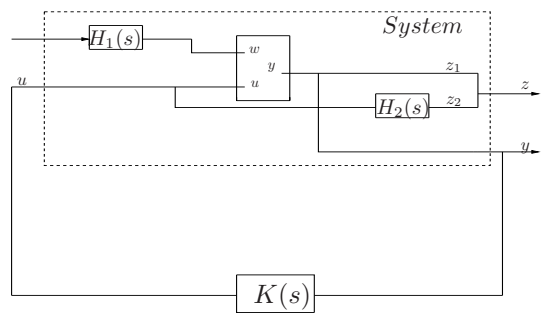

Fig. 6. Standard H_∞ problem

The controller is calculated using the standard H_∞ problem given in Figure 6. In order to take the disturbances on the system into account, the low-pass filter $H_1(s)$ is included in the design scheme.

The residual modes divergence describing the spillover phenomenon is a common problem when working with a truncation of an infinite-dimensional system. In order to avoid this non desirable effect, a high-pass filter $H_2(s)$ is added on the controlled output (see [18]). This filter has a transfer function

$$H_2(s) = \frac{(1 + \frac{s}{2\pi 25})^3}{(1 + \frac{s}{2\pi 160})^3}$$

that allows to get a 60 dB attenuation above the cut-off frequency of 25 Hz. The cut-off frequency is slightly greater than the frequency of the last considered mode in the controller synthesis.

The H_∞ controller is designed and is first tested through simulations and on the experimental setup afterwards. The simulation model is a system of larger dimension, with the first six modes of the plate and the first three modes of the liquid sloshing, in order to check the existence of the spillover effect. Two different levels of tank filling are considered ($e = 0.7$ and $e = 0.9$).

When looking at the synthesis plant model augmented by the filters H_1 and H_2 and keeping in mind the number of modes retained for design, it is clear that a full-order H_∞ controller given by the Matlab© Robust Control Toolbox will have severe restrictions for its numerical implementation on the experimental set-up. Moreover such a solution cannot directly tackle the problem of different filling levels in the

tip-tank. It is therefore reasonable to resort to algorithms for the synthesis of reduced-order controllers for the simultaneous H_∞ control problem. This is the case of the HIFOO package based on non smooth optimization from [7].

1) Order of the reduced-order Hifoo controllers: At first, one specific controller is considered for each tank filling level. In order to choose the suitable order of these HIFOO controllers, H_∞ controllers of different orders for a fixed tank filling of 0.7 are computed using the diagram of Figure 6. The computations show that controllers of order 4 and 1, have almost the same H_∞ norm: 4.28 for a 4th order and 4.24 for a 1st order. Consequently, a 4th order controller and a 1st order controller for the same tank filling $e = 0.7$ are tested on the plant. The idea behind this is to see if greater order controllers are really more efficient than a very simple first-order controller.

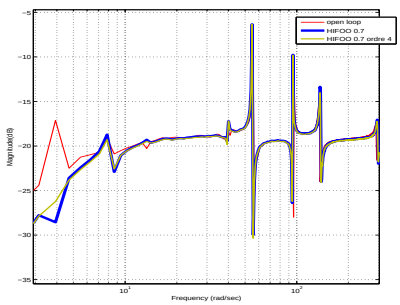

Fig. 7. Comparison between a 4th and a 1st order HIFOO controller - $e = 0.7$ (the Bode plots are between the input w and the output z)

The experimental results are plotted in Figure 7. The Bode plots show the closed-loop attenuation in the case of a 4th order and a 1st order controller computed using HIFOO. One can notice a slightly better attenuation for the first sloshing mode in case of the 4th order controller and a better attenuation (approx. 4 dB) for the first flexion mode in the case of the first-order controller.

It can be seen that the complexity of a 4th order controller is not justified. Therefore, from now on, only first-order controllers will be computed with HIFOO.

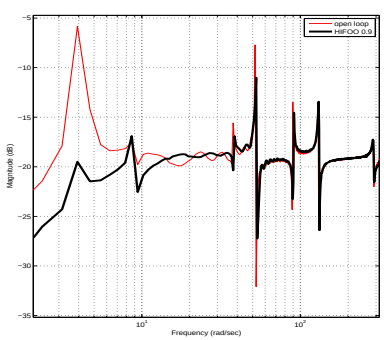

Fig. 8. Open loop and closed loop using Hifoo controller - $e = 0.9$ (the Bode plots are between the input w and the output z of the system)

Experimental results are given in Figures 8 and 9. It may be observed that the first peak is well attenuated for the

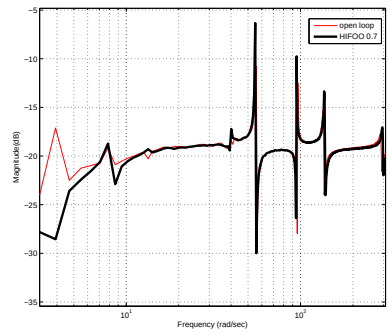

Fig. 9. Open loop and closed loop using Hifoo controller - $e = 0.7$ (the Bode plots are between the input w and the output z of the system)

different considered tank fillings. An attenuation of 14 dB is measured when $e = 0.9$ and of 11.7 dB when $e = 0.7$.

Concerning the first twisting mode (3rd peak on the Bode plots) the attenuation is very small for $e = 0.7$ and quite good for $e = 0.9$ (1.5 dB). For higher order modes it is seen that the controller for $e = 0.9$ is also quite efficient.

A conclusion of this part is that the first flexion mode, which is the most important in terms of plate displacement from its equilibrium position, is well attenuated for all the considered cases.

2) Simultaneous reduced-order Hifoo controller: In practice the liquid in the plane tanks is varying during flight. Therefore, one controller must be valid for different fillings.

The first-order controller previously calculated for the tank 90% filled is tested on the tank 70% filled. One can notice from Figure 10 that the controller increases the amplitude of the first vibration mode of the plate and does not attenuate the other modes. Therefore a simultaneous first order robust controller is computed for all different levels using again the HIFOO package under Matlab©.

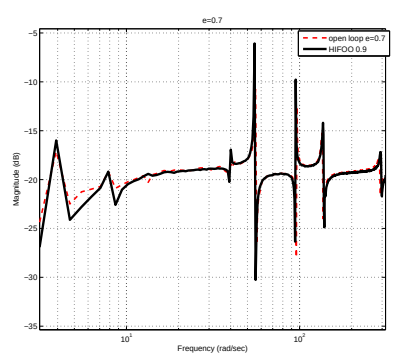

Fig. 10. HIFOO controller calculated for the tank 90% filled and tested on the tank 70% filled

In Figures 11 and 12, the Bode plot of the experimental setup in open-loop and the Bode plot of the experimental setup in closed-loop using the simultaneous HIFOO controller are compared for different tank fillings.

It is observed that the first and most important mode in terms of plate displacement is very well attenuated for $e = 0.9$ (10 dB) and $e = 0.7$ (5.7 dB). Regarding the twisting

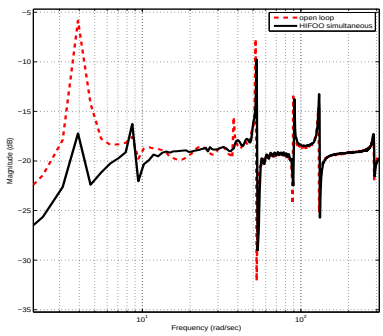

Fig. 11. Open loop and closed loop using simultaneous Hifoo controller - $e = 0.9$ (the Bode plots are between the input w and the output z)

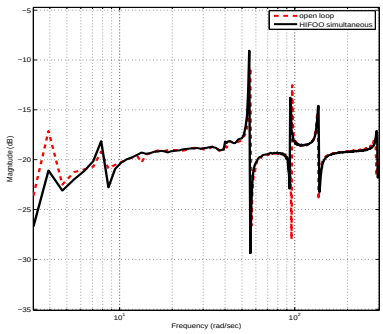

Fig. 12. Open loop and closed loop using simultaneous Hifoo controller - $e = 0.7$ (the Bode plots are between the input w and the output z)

mode and higher order modes they are also well attenuated especially for $e = 0.9$.

IV. Conclusion

After presenting a reduced state-space model built from an infinite-dimensional model for a flexible plant with sloshing phenomena, a model matching is performed to have a better similarity in terms of amplitude and frequency between the model and the plant. Then, two controllers are calculated for the model with 5 modes for the plate and 2 modes for liquid sloshing using HIFOO package. The Robust Control Toolbox from Matlab© is not used since is not well adapted to the system under study. Then, tests on the experimental setup are achieved. With a simple first-order controller computed using HIFOO, some vibrations alleviation may be obtained for the first modes on one tested tank filling but performance is not so good when applying this controller for another tank filling demonstrating some lack of robustness. This last issue may be greatly improved by designing a simultaneous first-order controller that shows much better performance on both tip-tank fillings. Some works are in progress to better identify the damping term of the model and to compute a controller that is robust to the filling of the tank.

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