From Aerial Vehicles to Aerial Robots through the lens of Tethering and Full Actuation

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LAAS-CNRS, Toulouse, France, Europe, ...

Monday, July 18th, 2016

funded by AEROARMS

For more information about the control methods presented in this talk you can check:

Tethered platforms:

- M. Tognon, S. S. Dash, and A. Franchi, "Observer-based control of position and tension for an aerial robot tethered to a moving platform", *IEEE Robotics and Automation Letters*, vol. 1, no. 2, pp. 732–737, 2016
- M. Tognon, A. Testa, E. Rossi, and A. Franchi, "Exploiting a passive tether for robust takeoff and landing on slopes: Methodology and experiments", in *2016 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Daejeon, South Korea, 2016

Fully-actuated platforms:

- A. Franchi, R. Carli, D. Bicego, and M. Ryll, "Full-pose geometric tracking control on SE(3) for laterally bounded fully-actuated aerial vehicles", in *ArXiv:1605.06645*, 2016. [Online]. Available: `http://arxiv.org/abs/1605.06645`
- M. Ryll, D. Bicego, and A. Franchi, "Modeling and control of FAST-Hex: A fully-actuated by synchronized-tilting hexarotor", in *2016 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Daejeon, South Korea, 2016

For more information about our activity on similar topics, refer to:
`http://homepages.laas.fr/afranchi/robotics/`

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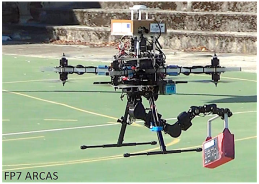
Motivation and Background

Aerial robots for **physical interaction**

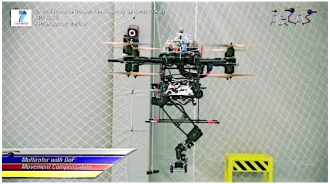
- applications: inspection, maintenance, transportation, manipulation. . .

Some examples in recently EU-funded projects:



Seville Univ. (ARCAS)

DLR (ARCAS)

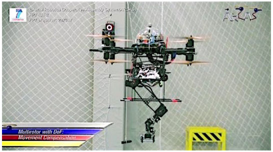


CATEC (ARCAS)

AEROWORKS concept

Floating base

- **active reaction wrench** provided by the thrusters (grounded manipulators have 'passive' ground reaction)
- **inaccurate** positioning (because of noisy sensing and external disturbances)
- **dynamic coupling**

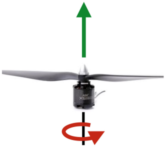


(CATEC/USE, ARCAS)

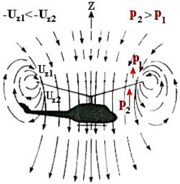
Actuators of the base

- additional **aerodynamic layer**

$$\text{motor torque} \sim \text{propeller acceleration}$$
$$\Downarrow$$
$$\text{propeller speed} \sim \text{thrust force}$$



- **unmodeled aerodynamics**

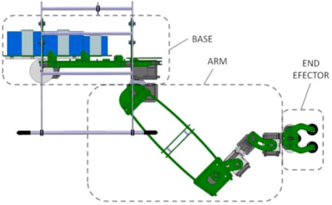


Need for a **lightweight payload**

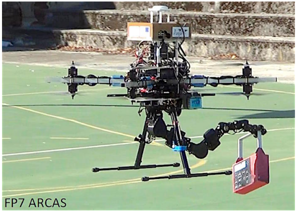
- arms with **weaker motors**
- **minimal number of sensors**
- **flexibility** $\Rightarrow$ vibrations

Need to **save energy**

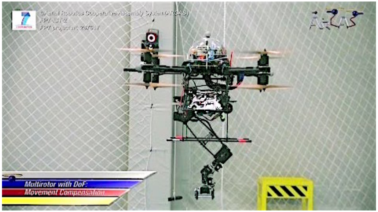
- **underactuated** configurations (i.e., **coplanar** propellers)



CATEC/USE (ARCAS)



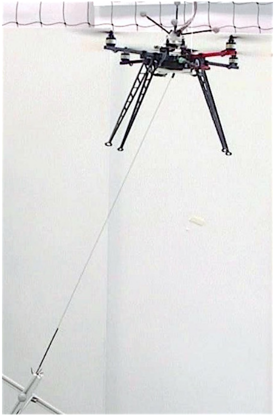
USE (ARCAS)



CATEC (ARCAS)

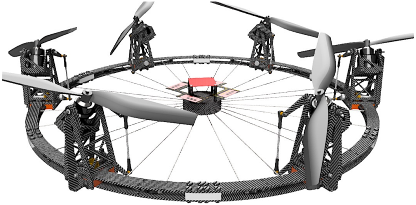
Tethered aerial robots:

- Cable/Bar: **physical connection**
- Modeling, Control, Observation



Fully Actuated Aerial Robots:

- **Full-wrench** exertion
- Mech. Design, Modeling, Control

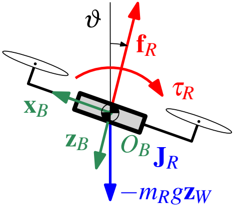


Tethered Aerial Robots

Frames:

- $\mathscr{F}_W = O_W - \{\mathbf{x}_W, \mathbf{y}_W, \mathbf{z}_W\}$ (World frame)
- $\mathscr{F}_B = O_B - \{\mathbf{x}_B, \mathbf{y}_B, \mathbf{z}_B\}$ (Vehicle frame)

$O_B \equiv$ vehicle center of mass (CoM)

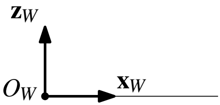


Parameters

- m_R vehicle mass
- J_R vehicle rotational inertia

Configuration and inputs

- ϑ vehicle **attitude (pitch)**
- f_R **intensity** of the **thrust force** $\mathbf{f}_R = -f_R \mathbf{z}_B$
- τ_R **intensity** of the **torque** $\tau_R \mathbf{y}_B$



Available sensors

- $\mathbf{a}$ onboard **accelerometer** (see later for definition)
- ω onboard **gyroscope** ($\equiv \dot{\vartheta}$)

Link can be a **bar**, a **taut tie**, or a **compressed strut**

Passive rotational **joints** at

- O_W ground fixed point
- O_B vehicle CoM

Parameters and assumptions

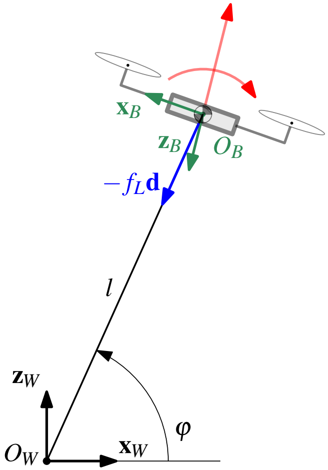
- l link length (constant)
- **negligible** mass and inertia w.r.t. m_R and J_R
- **negligible** deformation and elasticity

Configuration

- φ **link elevation**
- f_L **link internal force**
  - $f_L > 0$ **tension** (bar or tie)
  - $f_L < 0$ **compression** (bar or strut)

Available sensors at link-side

- **none**



Why an aerial vehicle **linked/tethered** to the ground?

Physically interactive **uses**

- **pull/pushing**
- resist strong wind
- landing/take-off from/on
  - a **sloped** surface
  - a **moving** platform (e.g., ship)

Other **uses**

- enduring **power**
- **high-bandwidth communication** channel

Some **application** fields

- transportation/manipulation
- inspection and surveillance
- communication relay

EC-SafeMobil (CATEC)

System dynamics:

$$\ddot{\varphi} = -\frac{g}{l}\cos\varphi + \frac{\cos(\varphi+\vartheta)}{m_R l} f_R$$

$$\ddot{\vartheta} = \frac{1}{J_R}\tau_R$$

$$f_L = -m_R g \sin\varphi + m_R l \dot{\varphi}^2 + \sin(\varphi+\vartheta) f_R$$

- $(\varphi, \dot{\varphi}, \vartheta, \dot{\vartheta})$ system state
- (f_R, τ_R) control inputs
- (φ^d, f_L^d) desired outputs
- $(\mathbf{a}, \omega)$ onboard measurements

System dynamics:

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Control Problem

Design a control law for the inputs (f_R, τ_R) in order to

- **asymptotically steer** (φ, f_L) along a sufficiently smooth **desired trajectory** (φ^d, f_L^d)

Using **only**

- onboard **accelerometer** and onboard **gyroscope**

examples

System dynamics:

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examples

Delocalization of Measurements/Desired Output

Measurements $\rightarrow$ **onboard** & proprioceptive
Desired **outputs** $\rightarrow$ "**off-board**" & "exteroceptive"

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ a_1 \cos x_1 \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_2 \cos(x_1 + x_3) & 0 \\ 0 & 0 \\ 0 & a_3 \end{bmatrix} \mathbf{u} \tag{1}$$

$$\mathbf{y} = \begin{bmatrix} x_1 \\ \frac{1}{a_2} x_2^2 + \frac{a_1}{a_2} \sin x_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \sin(x_1 + x_3) & 0 \end{bmatrix} \mathbf{u} \tag{2}$$

where

$$\mathbf{x} = \begin{bmatrix} \varphi \\ \dot{\varphi} \\ \vartheta \\ \dot{\vartheta} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} f_R \\ \tau_R \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \varphi \\ f_L \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \to \mathbf{y}^d(t) = \begin{bmatrix} \varphi^d \\ f_L^d \end{bmatrix} = \begin{bmatrix} y_1^d \\ y_2^d \end{bmatrix}$$

and $a_1 = -g/l, \quad a_2 = 1/(m_R l), \quad a_3 = 1/J_R$

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and $a_1 = -g/l, \quad a_2 = 1/(m_R l), \quad a_3 = 1/J_R$

Temporary assumption **(will be relaxed with the observer)**

Temporarily assume that $\mathbf{x}$ is **fully measurable**

System is not input-output linearizable with static feedback (s-fl) $\rightarrow$ Redefine a new input as $\bar{\mathbf{u}} = [\ddot{u}_1 \ u_2]^T = [\bar{u}_1 \ \bar{u}_2]^T$
New system state $\bar{\mathbf{x}} = \begin{bmatrix}\varphi & \dot{\varphi} & \vartheta & \dot{\vartheta} & u_1 & \dot{u}_1\end{bmatrix}^T$

[1] A. Isidori, *Nonlinear Control Systems, 3rd edition*. Springer, 1995, ISBN: 3540199160.

System is not input-output linearizable with static feedback (s-fl) → Redefine a new input as $\bar{\mathbf{u}} = [\ddot{u}_1 \ u_2]^T = [\bar{u}_1 \ \bar{u}_2]^T$

New system state $\bar{\mathbf{x}} = \left[\varphi \ \dot{\varphi} \ \vartheta \ \dot{\vartheta} \ u_1 \ \dot{u}_1\right]^T$

Need for further differentiation to see the new input $\bar{\mathbf{u}}$ appear in both output channels

$$\begin{bmatrix} y_1^{(4)} \\ y_2^{(2)} \end{bmatrix} = \mathbf{b}(\bar{\mathbf{x}}) + \underbrace{\begin{bmatrix} a_2 \cos(x_1 + x_3) & -a_2 a_3 \sin(x_1 + x_3) u_1 \\ \sin(x_1 + x_3) & a_3 \cos(x_1 + x_3) u_1 \end{bmatrix}}_{\bar{\mathbf{E}}(\bar{\mathbf{x}})} \bar{\mathbf{u}}, \qquad (3)$$

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$\det\left(\bar{\mathbf{E}}(\bar{\mathbf{x}})\right) = \dfrac{u_1}{l m_R J_R}$, as long as $u_1 \neq 0$, the control law $\bar{\mathbf{u}} = \mathbf{E}^{-1}(\bar{\mathbf{x}})\left[-\mathbf{b}(\bar{\mathbf{x}}) + \mathbf{v}\right]$, brings the system in the form $\begin{bmatrix} y_1^{(4)} \\ y_2^{(2)} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{v}$

The system is input-output linearizable with dynamic feedback iff $u_1 \neq 0$

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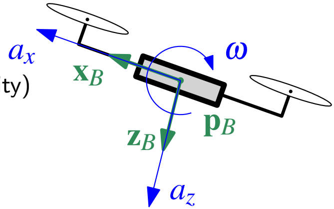
The system is input-output linearizable with dynamic feedback iff $u_1 \neq 0$

Exact feedback linearization

Total relative degree = dimension of $\bar{\mathbf{x}}$
$\Rightarrow$ The controlled closed-loop system has no internal dynamics [1]

[1] A. Isidori, *Nonlinear Control Systems, 3rd edition*. Springer, 1995, ISBN: 3540199160.



IMU

- Gyroscope: angular rate (angular velocity intensity)

$$\omega = \dot{\vartheta}$$

- Accelerometer: specific acceleration in the body frame

$$\mathbf{a} = R_W^B(\ddot{\mathbf{p}}_B + g\mathbf{z}_W) = [a_x,\ 0,\ a_z]^T$$

$$a_x = \cos(\varphi+\vartheta)\left[l\dot{\varphi}^2 - g\sin\varphi + \frac{f_R}{m_R}\sin(\varphi+\vartheta)\right]$$

$$a_z = \sin(\varphi+\vartheta)\left[l\dot{\varphi}^2 - g\sin\varphi + \frac{f_R}{m_R}\sin(\varphi+\vartheta)\right] - \frac{f_R}{m_R}$$

Go to aerial vehicle model

Observer: State Reduction using Gyroscope

Full model
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ a_1 \cos x_1 + a_2 \cos(x_1 + x_3) u_1 \\ 0 \\ a_3 u_2 \end{bmatrix}$$

State reduction

$u_3 = \omega$ Gyroscope measurement:

$x_4 = u_3$ one state becomes an input

Reduced model
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ a_1 \cos x_1 + a_2 \cos(x_1 + x_3) u_1 \\ u_3 \end{bmatrix}$$

Nonlinear Observer: High Gain Observer (HGO)[2]

- provides a state estimation $\hat{\mathbf{x}}$ that converges to the actual state value $\mathbf{x}$: $\lim_{t\to\infty}\hat{\mathbf{x}} = \mathbf{x}$
- requires: system in the triangular form

Desired (Triangular) form

$$\dot{\underline{\mathbf{x}}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underline{\mathbf{x}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \phi(\underline{\mathbf{x}}, \underline{\mathbf{u}}) + \boldsymbol{\lambda}(\underline{\mathbf{u}})$$

$$w = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \underline{\mathbf{x}}$$

- $\underline{\mathbf{x}}$: state vector
- $\underline{\mathbf{u}}$: control input
- w: measurement
- $\phi, \boldsymbol{\lambda}$: any nonlinear functions

Actual form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ a_1 \cos x_1 + a_2 \cos(x_1 + x_3) u_1 \\ u_3 \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} \cos(x_1 + x_3)\left(l x_2^2 - g \sin x_1 + \frac{1}{m_R} \sin(x_1 + x_3) u_1\right) \\ \sin(x_1 + x_3)\left(l x_2^2 - g \sin x_1 + \frac{1}{m_R} \sin(x_1 + x_3) u_1\right) - \frac{u_1}{m_R} \end{bmatrix}$$

- $\mathbf{x}$: state vector
- $\mathbf{u}$: control input
- $\mathbf{a}$: measurements

State and Measurements Transformation needed to get from the actual to the desired form

2 H. K. Khalil, *Nonlinear Systems*, 3rd. Prentice Hall, 2001, ISBN: 978-0130673893.

Reduced model

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ a_1\cos x_1 + a_2\cos(x_1+x_3)u_1\\ u_3\end{bmatrix}$$

$$a_x = \cos(x_1+x_3)\left(lx_2^2 - g\sin x_1 + \frac{1}{m_R}\sin(x_1+x_3)u_1\right)$$
$$a_z = \sin(x_1+x_3)\left(lx_2^2 - g\sin x_1 + \frac{1}{m_R}\sin(x_1+x_3)u_1\right) - \frac{u_1}{m_R}$$

State transformation

$$\begin{aligned} z_1 &= x_1 + x_3\\ z_2 &= x_2\\ z_3 &= a_1\cos x_1 + a_2\cos(x_1+x_3)u_1 \ (= \dot{x}_2 = \dot{z}_2)\end{aligned}$$

Transformed model (I)

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}u_3\\ 0\\ a_1z_2\sin x_1 + a_2(\cos z_1)\dot{u}_1 - a_2(\sin z_1)(z_2+u_3)u_1\end{bmatrix}$$

$$a_x = \cos z_1\left(lz_2^2 - g\sin x_1 + \frac{1}{m_R}\sin z_1 u_1\right)$$
$$a_z = \sin z_1\left(lz_2^2 - g\sin x_1 + \frac{1}{m_R}\sin z_1 u_1\right) - \frac{u_1}{m_R}$$

Transformed model (I)

$$\begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}u_3\\ 0\\ a_1 z_2 \sin x_1 + a_2(\cos z_1)\dot u_1 - a_2(\sin z_1)(z_2+u_3)u_1\end{bmatrix}$$

$$a_x = \cos z_1\left(l z_2^2 - g\sin x_1 + \frac{1}{m_R}\sin z_1 u_1\right)$$

$$a_z = \sin z_1\left(l z_2^2 - g\sin x_1 + \frac{1}{m_R}\sin z_1 u_1\right) - \frac{u_1}{m_R}$$

From the accelerometer

$$\bar w_1 = \mathrm{atan2}\left(\pm\frac{a_x}{\eta}, \pm\frac{a_z + u_1/m_R}{\eta}\right) = z_1 + k\pi$$

$$\sin x_1 = \frac{1}{g}\left(\pm\eta + l z_2^2 + \frac{1}{m_R}\sin z_1 u_1\right)$$

$$;\ \eta = \sqrt{a_x^2 + \left(a_z + \frac{u_1}{m_R}\right)^2} \neq 0$$

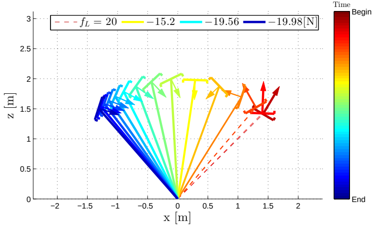
Transformed model (II)

$$\begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\sigma(z_1, z_2, z_3, u_1, \dot u_1, \eta) + \begin{bmatrix}u_3\\ 0\\ 0\end{bmatrix}$$

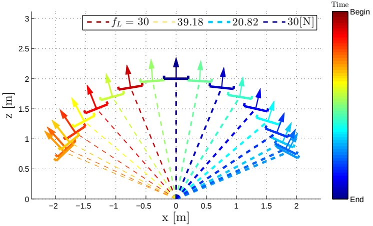
is in **triangular form!**

$$\bar w_1 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + k\pi$$

From tension to compression



Sinusoidal trajectories



M. Tognon and A. Franchi, "Nonlinear observer-based tracking control of link stress and elevation for a tethered aerial robot using inertial-only measurements", in *2015 IEEE Int. Conf. on Robotics and Automation*, Seattle, WA, 2015, pp. 3994–3999

M. Tognon, S. S. Dash, and A. Franchi, "Observer-based control of position and tension for an aerial robot tethered to a moving platform", *IEEE Robotics and Automation Letters*, vol. 1, no. 2, pp. 732–737, 2016

M. Tognon, A. Testa, E. Rossi, and A. Franchi, "Exploiting a passive tether for robust takeoff and landing on slopes: Methodology and experiments", in *2016 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Daejeon, South Korea, 2016

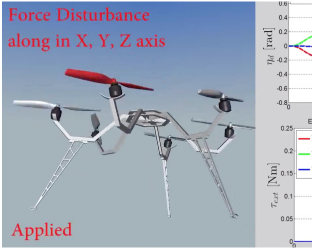
Fully-actuated Aerial Robots

Underactuated

- − **position-only** control (coupled position and orientation
- − **force-only** control in interaction
- \+ only (low) internal drag (**efficient**)
- \+ **lower** complexity

Fully-actuated

- \+ **full-pose** control (independent control of position and orientation)
- \+ **full-wrench** control in interaction
- − internal wrench (**wasted** energy)
- − **higher** complexity



Multi-rotor aerial platforms are essentially made of **two elements**:

- a **rigid body** → rigid body **dynamics**

$$\begin{bmatrix} m\ddot{\mathbf{p}}_B^W \\ \mathbf{J}\dot{\boldsymbol{\omega}}_B^W \end{bmatrix} = -\begin{bmatrix} mg\mathbf{e}_3 \\ \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{f}^W \\ \boldsymbol{\tau}^B \end{bmatrix}}_{\text{total input wrench}} \qquad \text{where } \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

- a **set of propellers** attached to the body → **total input wrench**

Wrenches of the **single propellers**:

$$\mathbf{f}_i^B = \mathbf{R}_{S_i}^B \begin{bmatrix} 0 \\ 0 \\ c_f \end{bmatrix} \underbrace{w_i|w_i|}_{u_i}, \quad i = 1, \ldots, n$$

$$\boldsymbol{\tau}_i^B = \mathbf{R}_{S_i}^B \begin{bmatrix} 0 \\ 0 \\ \pm c_\tau \end{bmatrix} \underbrace{w_i|w_i|}_{u_i}, \quad i = 1, \ldots, n$$

Total **input wrench**:

$$\mathbf{f}^W = \mathbf{R}_B^W \sum_{i=1}^{n} \mathbf{f}_i^B = \mathbf{R}_B^W \mathbf{F}_1 \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = \mathbf{R}_B^W \mathbf{F}_1 \mathbf{u} \tag{5}$$

$$\boldsymbol{\tau}^B = \sum_{i=1}^{n} \mathbf{p}_{B,S_i}^B \times \mathbf{f}_i^B + \sum_{i=1}^{n} \boldsymbol{\tau}_i^B = \mathbf{F}_2 \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix} = \mathbf{F}_2 \mathbf{u} \tag{6}$$

Putting (5) and (6) in (4):

$$\begin{bmatrix} m\ddot{\mathbf{p}}_B^W \\ \mathbf{J}\dot{\boldsymbol{\omega}}_B^W \end{bmatrix} = -\begin{bmatrix} mg\mathbf{e}_3 \\ \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} + \begin{bmatrix} \mathbf{R}_B^W & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix} \mathbf{u}, \qquad \text{where } \mathbf{u} = \begin{bmatrix} w_1|w_1| \\ \vdots \\ w_n|w_n| \end{bmatrix}$$

Putting (5) and (6) in (4):

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- all propellers are **coplanar** $\Rightarrow \mathbf{F}_1$ is **rank deficient**

$$\mathbf{F}_1 = c_f \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 1 & \cdots & 1 \end{bmatrix} = c_f \begin{bmatrix} \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{1}^T \end{bmatrix}$$

- the control force is

$$c_f \mathbf{R}_B^W \begin{bmatrix} 0 \\ 0 \\ \mathbf{1}^T\mathbf{u} \end{bmatrix}$$

- it can be arbitrarily oriented **only changing the whole-body orientation $\mathbf{R}_B^W$**
- the propeller speeds $\mathbf{u}$ control **only the amplitude** of the force

Putting (5) and (6) in (4):

$$\begin{bmatrix} m\ddot{\mathbf{p}}_B^W \\ \mathbf{J}\dot{\boldsymbol{\omega}}_B^W \end{bmatrix} = -\begin{bmatrix} mg\mathbf{e}_3 \\ \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} + \begin{bmatrix} \mathbf{R}_B^W & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix} \mathbf{u}, \qquad \text{where } \mathbf{u} = \begin{bmatrix} w_1|w_1| \\ \vdots \\ w_n|w_n| \end{bmatrix}$$

- If **coplanarity assumption is relaxed** then $\Rightarrow \mathbf{F}_1$ can be made **full-rank**

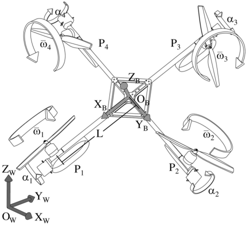
$$\mathbf{F}_1 = c_f \begin{bmatrix} \star & \star & \cdots & \star \\ \star & \star & \cdots & \star \\ \star & \star & \cdots & \star \end{bmatrix}$$

- the **control force** is

$$\mathbf{R}_B^W \mathbf{F}_1 \mathbf{u} = c_f \mathbf{R}_B^W \begin{bmatrix} \star & \star & \cdots & \star \\ \star & \star & \cdots & \star \\ \star & \star & \cdots & \star \end{bmatrix} \mathbf{u}$$

- using $\mathbf{u}$, **both orientation and amplitude** of the force can be decided **independently** of the whole-body orientation $\mathbf{R}_B^W$

quadrotor + tilting propellers[3]



[3] M. Ryll, H. H. Bülthoff, and P. Robuffo Giordano, "A novel overactuated quadrotor unmanned aerial vehicle: Modeling, control, and experimental validation", *IEEE Trans. on Control Systems Technology*, vol. 23, no. 2, pp. 540–556, 2015.

[4] S. Rajappa, M. Ryll, H. H. Bülthoff, and A. Franchi, "Modeling, control and design optimization for a fully-actuated hexarotor aerial vehicle with tilted propellers", in *2015 IEEE Int. Conf. on Robotics and Automation*, Seattle, WA, 2015, pp. 4006–4013.

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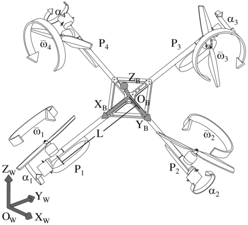
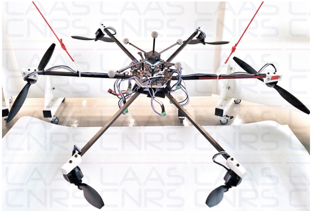
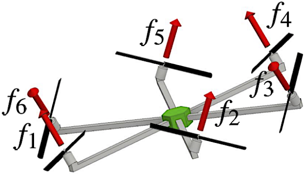


planar hexarotor with tilted propellers[4]





[3] M. Ryll, H. H. Bülthoff, and P. Robuffo Giordano, "A novel overactuated quadrotor unmanned aerial vehicle: Modeling, control, and experimental validation", *IEEE Trans. on Control Systems Technology*, vol. 23, no. 2, pp. 540–556, 2015.

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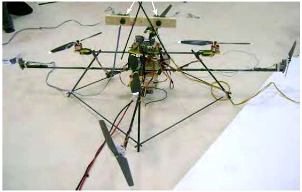
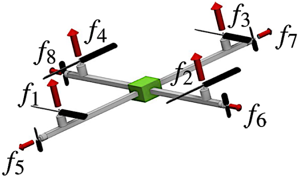
4+4 orthogonal rotors[5]

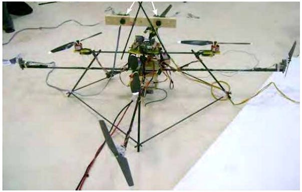


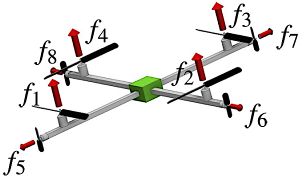


[5] H. Romero, S. Salazar, A. Sanchez, and R. Lozano, "A new UAV configuration having eight rotors: Dynamical model and real-time control", in *46th IEEE Conf. on Decision and Control*, New Orleans, LA, 2007, pp. 6418–6423.

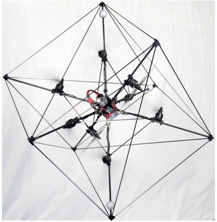
[6] D. Brescianini and R. D'Andea, "Design, modeling and control of an omni-directional aerial vehicle", in *2016 IEEE Int. Conf. on Robotics and Automation*, Stockholm, Sweden, 2015.

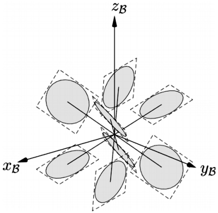
4+4 orthogonal rotors[5]





cubic octorotor[6]





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[6] D. Brescianini and R. D'Andea, "Design, modeling and control of an omni-directional aerial vehicle", in *2016 IEEE Int. Conf. on Robotics and Automation*, Stockholm, Sweden, 2015.

Given a **reference pose (6D) trajectory**:

- $\mathbf{p}_{Br}^{W}(t)$ (position of the CoM)
- $\mathbf{R}_{Br}^{W}(t)$ (orientation of the main body)

Dynamics:

$$\begin{bmatrix} m\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{p}}_B^W \\ \dot{\boldsymbol{\omega}}_B^W \end{bmatrix} = -\begin{bmatrix} mg\mathbf{e}_3 \\ \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{R}_B^W & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}}_{6\times 6} \underbrace{\begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix}}_{6\times n} \mathbf{u}, \qquad \text{where } \mathbf{u} = \begin{bmatrix} w_1|w_1| \\ \vdots \\ w_n|w_n| \end{bmatrix}$$

Inverse dynamics:

$$\mathbf{u} = \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix}^{+} \begin{bmatrix} \mathbf{R}_W^B & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \left(\begin{bmatrix} m\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \left(\begin{bmatrix} \ddot{\mathbf{p}}_{Br}^W \\ \dot{\boldsymbol{\omega}}_{Br}^W \end{bmatrix} + \mathbf{v} \right) + \begin{bmatrix} mg\mathbf{e}_3 \\ \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} \right)$$

Exactly linearized error system $\begin{bmatrix} \ddot{\mathbf{p}}_B^W - \ddot{\mathbf{p}}_{Br}^W \\ \dot{\boldsymbol{\omega}}_B^W - \dot{\boldsymbol{\omega}}_{Br}^W \end{bmatrix} = \mathbf{v}$

then use any linear-systems control law for $\mathbf{v}$ to steer $\mathbf{p}_B^W \to \mathbf{p}_{Br}^W(t)$ and $\mathbf{R}_B^W \to \mathbf{R}_{Br}^W(t)$

Given a **reference pose (6D) trajectory**:

- $\mathbf{p}_{Br}^{W}(t)$ (position of the CoM)
- $\mathbf{R}_{Br}^{W}(t)$ (orientation of the main body)

Dynamics:

$$\begin{bmatrix} m\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{p}}_B^W \\ \dot{\boldsymbol{\omega}}_B^W \end{bmatrix} = - \begin{bmatrix} mg\mathbf{e}_3 \\ \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{R}_B^W & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}}_{6\times 6} \underbrace{\begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix}}_{6\times n} \mathbf{u}, \qquad \text{where } \mathbf{u} = \begin{bmatrix} w_1|w_1| \\ \vdots \\ w_n|w_n| \end{bmatrix}$$

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Exactly linearized error system $\begin{bmatrix} \ddot{\mathbf{p}}_B^W - \ddot{\mathbf{p}}_{Br}^W \\ \dot{\boldsymbol{\omega}}_B^W - \dot{\boldsymbol{\omega}}_{Br}^W \end{bmatrix} = \mathbf{v}$

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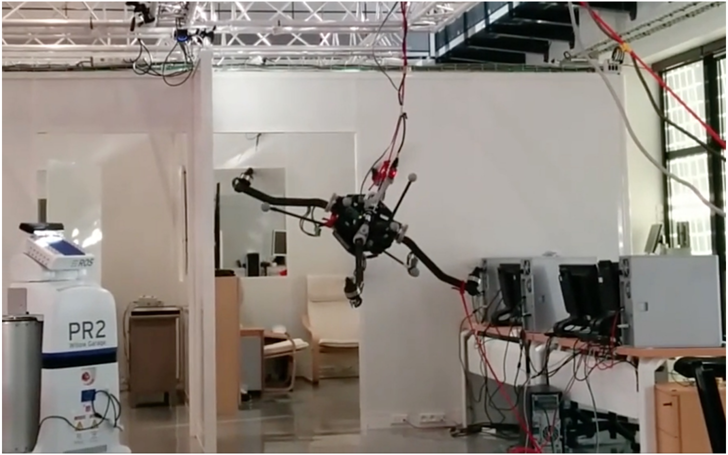
quadrotor + tilting propellers



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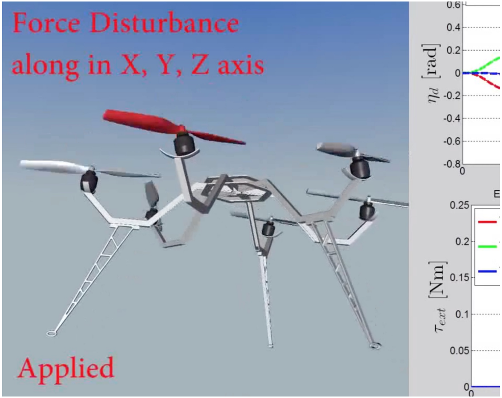
planar hexarotor with tilted propellers



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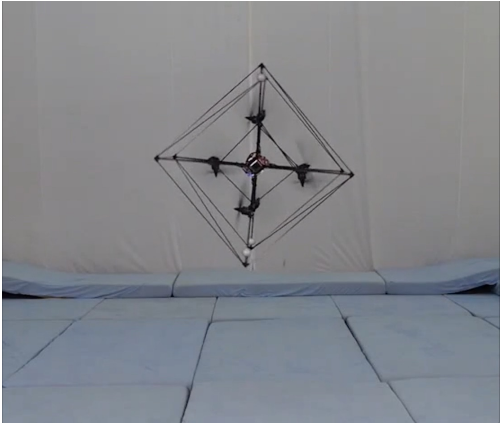
cubic octorotor



D. Brescianini and R. D'Andea, "Design, modeling and control of an omni-directional aerial vehicle", in *2016 IEEE Int. Conf. on Robotics and Automation*, Stockholm, Sweden, 2015

The wrench exerted by the propellers has **several limitations**

- **maximum speed** $\sim \frac{\text{maximum motor torque}}{\text{propeller drag}}$
  (**considered** in this talk)
- **only positive speeds** due to non-symmetric propeller shape
  (**considered** in this talk)
- **maximum/minimum speed rate** $\sim \frac{\text{maximum/minimum motor torque}}{\text{motor/propeller inertia}}$
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Inverse dynamics:

- desired wrench obtained by **matrix (pseudo)inversion**
- set of feasible forces not considered
- the **smaller** the **cant angles** the **larger** the input **forces**

Inverse dynamics approach may lead to **unfeasible propeller speeds** ($>> 0$ or < 0)

How to **overcome the drawbacks** of the previous approach?

Using a novel method presented here[7] [8]

Let's look at the dynamics while following any trajectory $\mathbf{p}_B^W(t)$ with $\mathbf{R}_B^W(t)$

$$\begin{bmatrix} \ddot{\mathbf{p}}_B^W + mg\mathbf{e}_3 \\ \mathbf{J}\dot{\boldsymbol{\omega}}_B^W + \boldsymbol{\omega}_B^W \times \mathbf{J}\boldsymbol{\omega}_B^W \end{bmatrix} = \begin{bmatrix} \mathbf{R}_B^W \mathbf{F}_1 \\ \mathbf{F}_2 \end{bmatrix} \mathbf{u}, \qquad \text{where } \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u} \in \mathscr{U} \text{ (admissible inputs)}$$

It is interesting to **analyze** the **set of admissible input forces** when

- the input torque is **constrained**, i.e., $\mathbf{F}_2\mathbf{u} = \boldsymbol{\tau}$ for a given $\boldsymbol{\tau}$
- the **propeller speeds** are **feasible**, i.e., $\mathbf{u} \in \mathscr{U}$

$$\mathscr{U}_1(\boldsymbol{\tau}) = \{\mathbf{u}_1 = \mathbf{F}_1\mathbf{u} \quad \text{s.t.} \quad \mathbf{F}_2\mathbf{u} = \boldsymbol{\tau} \quad \text{and} \quad \mathbf{u} \in \mathscr{U}\}$$

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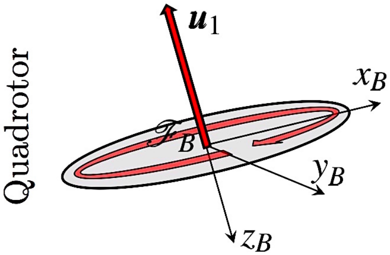
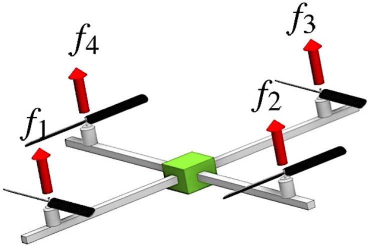
[7] A. Franchi, R. Carli, D. Bicego, and M. Ryll, "Full-pose geometric tracking control on SE(3) for laterally bounded fully-actuated aerial vehicles", in *ArXiv:1605.06645*, 2016. [Online]. Available: http://arxiv.org/abs/1605.06645.

[8] M. Ryll, D. Bicego, and A. Franchi, "Modeling and control of FAST-Hex: A fully-actuated by synchronized-tilting hexarotor", in *2016 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Daejeon, South Korea, 2016.

Set of **admissible input forces** (in body frame)

$$\mathscr{U}_1(\boldsymbol{\tau}) = \{\mathbf{u}_1 = \mathbf{F}_1\mathbf{u} \quad \text{s.t.} \quad \mathbf{F}_2\mathbf{u} = \boldsymbol{\tau} \quad \text{and} \quad \mathbf{u} \in \mathscr{U}\}$$





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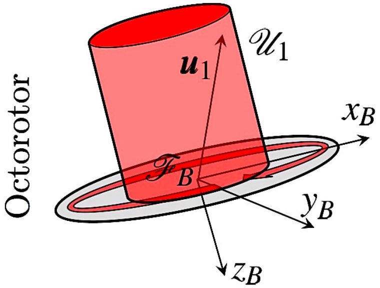
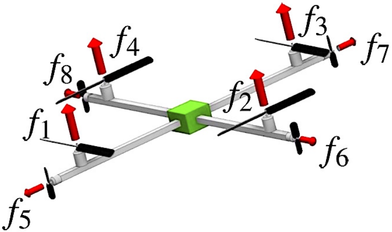
Octorotor for $\boldsymbol{\tau} = \mathbf{0}$ (approximation)

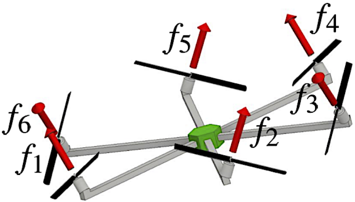


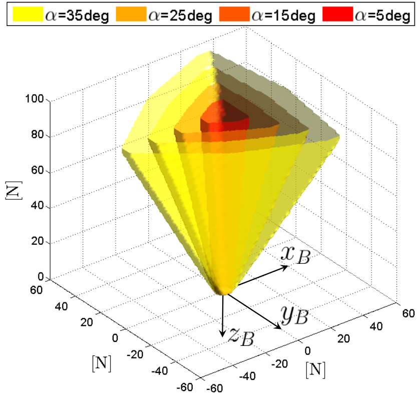


Set of **admissible input forces** (in body frame)

$$\mathscr{U}_1(\boldsymbol{\tau}) = \{\mathbf{u}_1 = \mathbf{F}_1\mathbf{u} \quad \text{s.t.} \quad \mathbf{F}_2\mathbf{u} = \boldsymbol{\tau} \quad \text{and} \quad \mathbf{u} \in \mathscr{U}\}$$

Hexarotor for $\boldsymbol{\tau} = \mathbf{0}$ for different cant angles α





Position **tracking errors**

$$\mathbf{e}_p = \mathbf{p}_B - \mathbf{p}_r, \quad \text{and} \quad \mathbf{e}_v = \dot{\mathbf{p}}_B - \dot{\mathbf{p}}_r. \tag{7}$$

Reference force vector

$$\mathbf{f}_r = m\ddot{\mathbf{p}}_r + mg\mathbf{e}_3 - \mathbf{K}_p\mathbf{e}_p - \mathbf{K}_v\mathbf{e}_v, \tag{8}$$

where $\mathbf{K}_p$ and $\mathbf{K}_v$ are positive diagonal gain matrixes

Remark

If $\mathbf{u}$ could always be chosen such that:

$$\mathbf{R}_B^W\mathbf{F}_1\mathbf{u} = \mathbf{f}_r = m\ddot{\mathbf{p}}_r + mg\mathbf{e}_3 - \mathbf{K}_p\mathbf{e}_p - \mathbf{K}_v\mathbf{e}_v,$$

then $\mathbf{e}_p \to \mathbf{0}$ and $\mathbf{e}_v \to \mathbf{0}$ exponentially.
However, this is not always possible, due to the **input saturation**

Idea

Relax the orientation tracking if the position tracking is not possible

$$\mathscr{R}(\mathbf{f}_r) = \{\mathbf{R} \in SO(3) \mid \exists \mathbf{u} \in \mathscr{U},\ \mathbf{R}\mathbf{F}_1\mathbf{u} = \mathbf{f}_r \wedge \mathbf{F}_2\mathbf{u} = \mathbf{0}\} \tag{9}$$

Set of orientations of the main body that **allow to exert $\mathbf{f}_r$** on the CoM while ensuring

- **propeller speeds feasibility**, i.e., $\mathbf{u} \in \mathscr{U}$
- a **given input torque**, e.g., $\mathbf{F}_2\mathbf{u} = \mathbf{0}$

Position–orientation compatibility

Simultaneous tracking of both $\mathbf{p}_r(t)$ and $\mathbf{R}_r(t)$ is possible

$$\Updownarrow$$

$$\mathbf{R}_r(t) \in \mathscr{R}(\mathbf{f}_r(t))$$

Position-Tracking-Compatible Orientations

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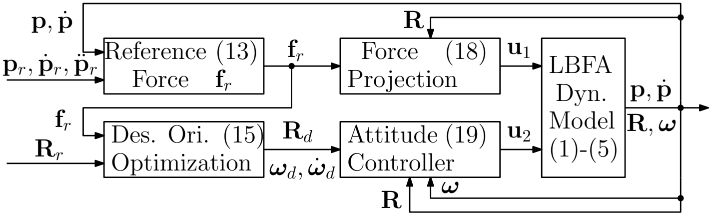
$$\mathbf{R}_r(t) \in \mathscr{R}(\mathbf{f}_r(t))$$

Non-compatibility $\Rightarrow$ **relax** the orientation tracking

- compute a new **desired orientation** $\mathbf{R}_d \in SO(3)$
- **reference control torque** $\boldsymbol{\tau}_r = \boldsymbol{\omega}_B \times \mathbf{J}\boldsymbol{\omega}_B - \mathbf{K}_R\mathbf{e}_R - \mathbf{K}_\omega\boldsymbol{\omega}_B$

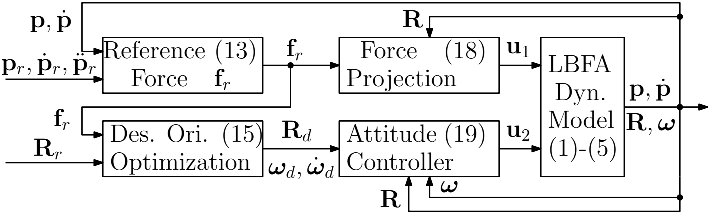
  where $\mathbf{e}_R$ is the **orientation error** in $SO(3)$ defined as $\mathbf{e}_R = \frac{1}{2}(\mathbf{R}_d^T\mathbf{R}_B - \mathbf{R}_B^T\mathbf{R}_d)^\vee$



Control algorithm in pills

At every t, given $\mathbf{p}_r, \mathbf{R}_r$:

1. compute $\mathbf{f}_r = m\ddot{\mathbf{p}}_r + mg\mathbf{e}_3 - \mathbf{K}_p\mathbf{e}_p - \mathbf{K}_v\mathbf{e}_v$



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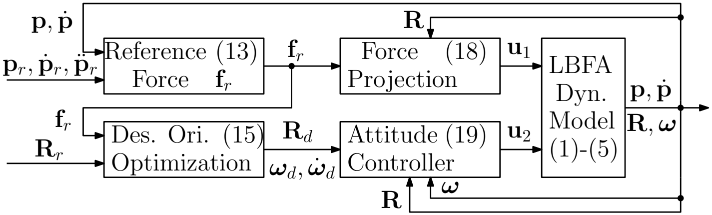
At every t, given $\mathbf{p}_r, \mathbf{R}_r$:

1. compute $\mathbf{f}_r = m\ddot{\mathbf{p}}_r + mg\mathbf{e}_3 - \mathbf{K}_p\mathbf{e}_p - \mathbf{K}_v\mathbf{e}_v$
2. solve $\mathbf{R}_d = \underset{\mathbf{R}\in\mathscr{R}(\mathbf{f}_r)}{\text{argmin}}\ \textbf{dist}(\mathbf{R}, \mathbf{R}_r)$



Control algorithm in pills

At every t, given $\mathbf{p}_r, \mathbf{R}_r$:

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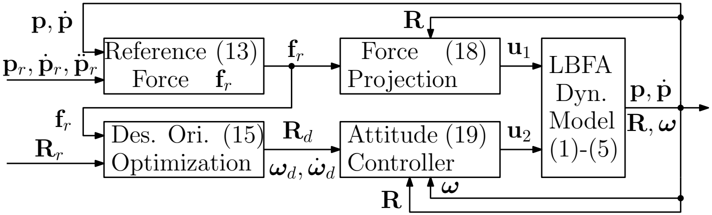
3. compute $\boldsymbol{\tau}_r = \boldsymbol{\omega}_B \times \mathbf{J}\boldsymbol{\omega}_B - \mathbf{K}_R\mathbf{e}_R - \mathbf{K}_\omega\boldsymbol{\omega}_B$, to track $\mathbf{R}_d$



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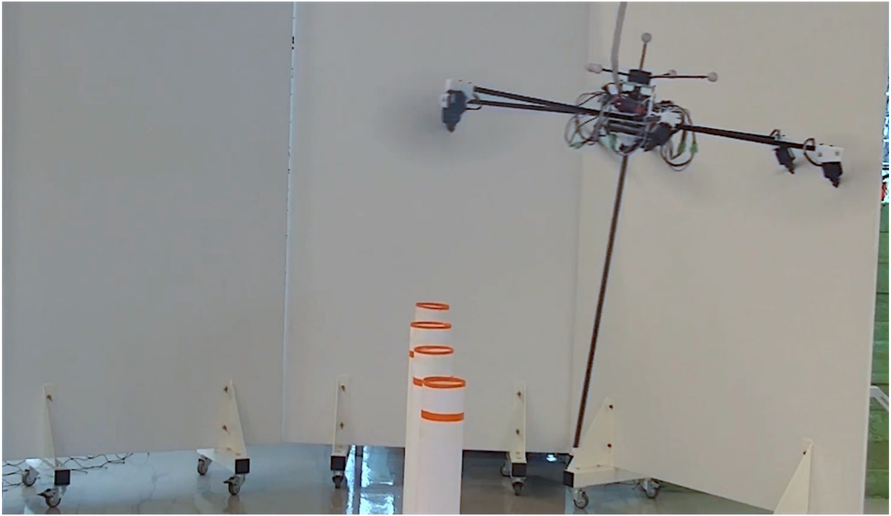
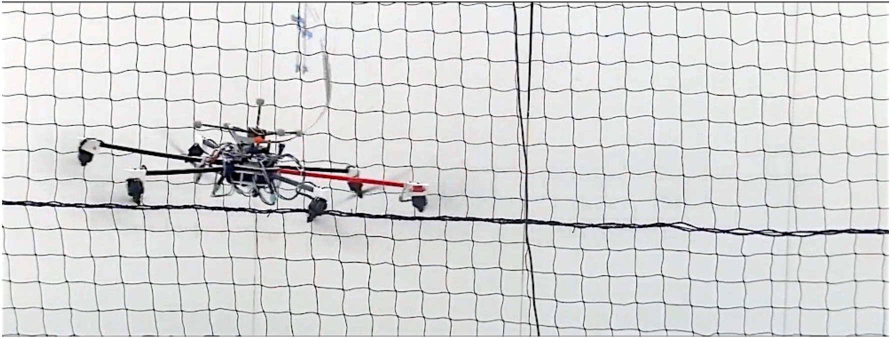
4. compute $\mathbf{u}$ to implement $\boldsymbol{\tau}_r$ and $\mathbf{f}_r$

Experiments: Non-horizontal Hovering

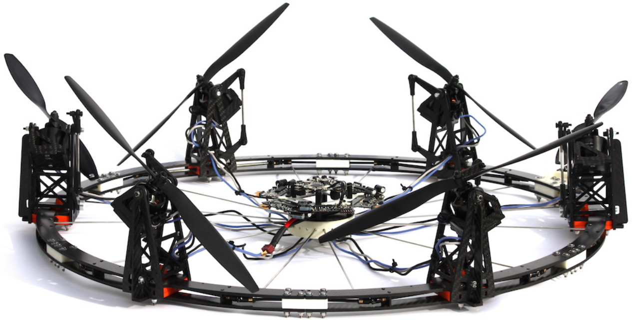


Exploit the 6 DoFs for position and orientation independent regulation



Current and Future Works

FAST-Hex: Hexarotor with Adjustable Cant Angles



- momentum-based external **wrench observer**
- 6D **admittance control** at the **tooltip**

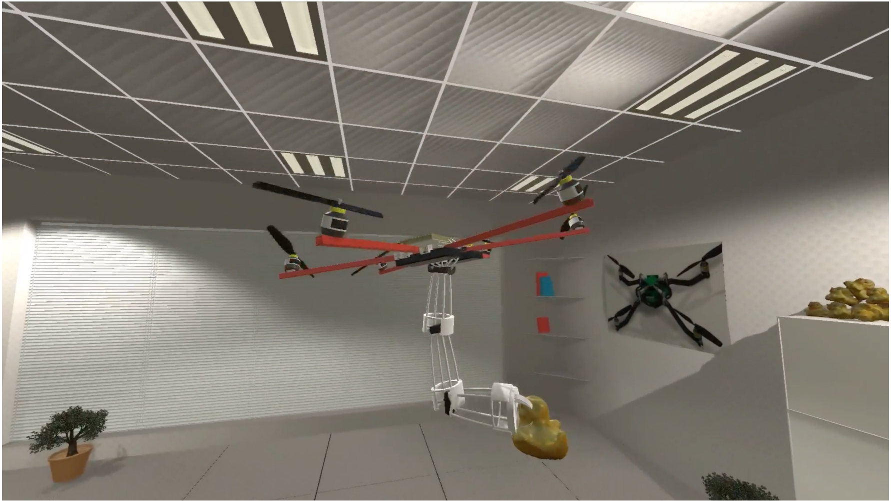
Rope-pulling

- unstable operation for a co-planar multirotor
- (3D orientation dynamics made stiffer than 3D translation one)

Peg-in-hole

- unstable operation for a co-planar multirotor

- aerial **manipulators** with a **fully-actuated base**



preliminary simulation
control of a **'truly' redundant** aerial manipulator

Acknowledgements

For related work, visit `http://homepages.laas.fr/afranchi/robotics/`

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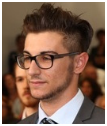

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Questions?

AEROARMS