Decentralized Parameter Estimation and Observation for Cooperative Mobile Manipulation of an Unknown Load using Noisy Measurements

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Abstract— In this paper, a distributed approach for the estimation of kinematic and inertial parameters of an unknown rigid body is presented. The body is manipulated by a pool of ground mobile manipulators. Each robot retrieves a noisy measurement of its velocity and the contact forces applied to the body. Kinematics and dynamics arguments are used to distributively estimate the relative positions of the contact points. Subsequently, distributed estimation filters and nonlinear observers are used to estimate the body mass, the relative position between its geometric center and its center of mass, and its moment of inertia. The manipulation strategy is functional to the estimation process, and is suitably designed to satisfy nonlinear observability conditions that are necessary for the success of the estimation. Numerical results corroborate our theoretical findings.

I. INTRODUCTION

Cooperative manipulation by teams of mobile robots has been one of the hot research topics in the last decades, with applications in the fields of search and rescue, disaster recovering, cooperative transportation, and service robotics. Many aspects of cooperative robotics have been recently studied in great detail, such as optimal control [1], path planning [2], leader-follower control schemes [3]. However, most of the prior work models the payload with a point mass, or with a rigid body with known dynamical parameters [4], [5].

The real-time knowledge of inertial parameters of unknown loads makes collective manipulation tasks more effective and is beneficial in terms of reduction of the control effort. Thus, the benefits provided by on-line estimation techniques are at least twofold: first, more effective control techniques for manipulation, like force control and pose estimation [6], [7], can be applied to achieve better performance with a reduced control effort. Second, time-varying manipulation tasks can be effectively implemented. For example, in transport application it is not rare that the payload is increased by an external cause, or that part of the load is lost during the transportation. Hence, an effective, real-time estimate of the inertial parameters would allow to implement adaptive control techniques, as well as event-driven control algorithms. Finally, the benefits of the

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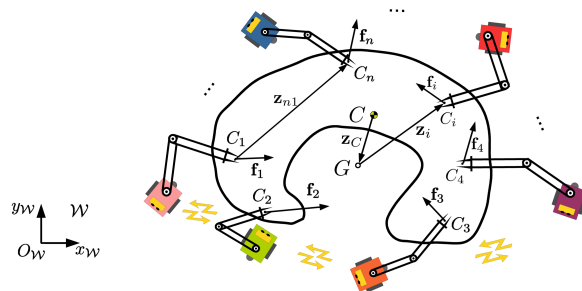
Fig. 1: A group of ground mobile manipulators performing a manipulation task. Each robot exerts a force on the object using a planar manipulator. Robots measure only the end-effector velocity and communicate using peer-to-peer ad-hoc connections. All-to-all communication is not possible. The objective is the distributed observation of the position of the geometric center of all the contact points G w.r.t. the center of mass C and the distributed estimation of the mass, the rotational inertia, and the relative positions of the contact points $(C_1, \ldots, C_n)$ w.r.t. C.

estimation are even improved if they are achieved through a totally distributed approach, which would add flexibility, reduced computational and communication overhead, and fault tolerance [8]. Not much work has been done in the past concerning distributed estimation of inertial parameters. Moreover, the main limitations of the existing research reside in the centralization of the approaches, and in the use of acceleration and absolute positioning measurements [6] [7].

This work seeks to overcome the limitations of the existing approaches, by defining, to the best of our knowledge, the first totally distributed estimation scheme for the inertial parameters of an unknown load manipulated by a pool of Ground Mobile Manipulators (GMMs). The proposed approach relies on the application of contact forces to points which neither accelerations nor relative (and, consequently, absolute) positions are measured, and makes use of only velocity measurements, possibly corrupted by noise, which may deteriorate the estimation performance [10].

The proposed approach is grounded on our preliminary results presented in [9], where noiseless measurements have been considered.

It leverages geometrical and dynamical analysis, and the theory of nonlinear observers to estimate the relative positions of the contact points, which are used to derive the inertial parameters of the unknown payloads. Our algorithm allows the full use of the proposed methodology in generic cooperative manipulation tasks where the force has to be chosen in a closed-loop control fashion relying on the current center-of-mass position estimate.

The paper is structured as follows. In Sec. II, we formalize the distributed estimation problem. In Sec. III, we introduce the distributed estimation procedure and describe the main phases of the algorithm. Then, we first focus on the estimation of the relative positions of the contact points in Sec. IV, and describe in detail the procedure for the estimation of the inertial parameters in Sec. V. The results of the numerical simulations and the accuracy bounds are reported in Sec. VI, and Sec. VII provides the conclusions.

II. PROBLEM STATEMENT

We consider a connected network composed of n mobile robots able to move in a plane, see Fig. 1. The robotic network is represented by an undirected graph $\mathscr{G} = \{\mathscr{I}, \mathscr{E}\}$, where $\mathscr{I} = \{1, \dots, n\}$ is the node set, corresponding to robots, and $\mathscr{E} \subset \mathscr{I} \times \mathscr{I}$ is edge set, corresponding to communication links. Notably, if $(i, j) \in \mathscr{E}$ then robot i can exchange data with j, and vice versa. Furthermore, we indicate with $\mathscr{N}_i = \{j \in \mathscr{I} : (i, j) \in \mathscr{E}\}$ the set of neighbors of robot i. The team objective is to manipulate, in a cooperative fashion, an unknown load B. The position of the center of mass (CoM) C of the load B, with respect to a common reference frame $\mathscr{W} = \{O - \vec{\mathbf{x}}\vec{\mathbf{y}}\}$, is $\mathbf{p}_C \in \mathbb{R}^2$. Using a manipulator, each robot i can exert a force $\mathbf{f}_i$ on the load B in the contact point $C_i \in B$, whose position in $\mathscr{W}$ is $\mathbf{p}_{C_i} \in \mathbb{R}^2$.

Indicating with $m \in \mathbb{R}_{>0}$ and $J \in \mathbb{R}_{>0}$, respectively, the mass and the moment of inertia of B, the dynamics of B, subject to forces $\mathbf{f}_1, \dots, \mathbf{f}_n$, reads

$$\ddot{\mathbf{p}}_C = \frac{1}{m} \sum_{i=1}^{n} \mathbf{f}_i \tag{1}$$

$$\dot{\omega} = \frac{1}{J} \sum_{i=1}^{n} (\mathbf{p}_{C_i} - \mathbf{p}_C)^{\perp T} \mathbf{f}_i, \tag{2}$$

where $\omega \in \mathbb{R}$ is the rotational rate of B. The linear operator $(\cdot)^{\perp}$, applied to a vector $\mathbf{v} \in \mathbb{R}^2$, rotates it of $\pi/2$, as

$$\mathbf{v}^{\perp} = Q\mathbf{v} = \underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{=Q} \begin{pmatrix} v^x \\ v^y \end{pmatrix} = \begin{pmatrix} -v^y \\ v^x \end{pmatrix}. \tag{3}$$

We assume that each robot i performs noisy measurements $\tilde{\mathbf{p}}_{C_i} \in \mathbb{R}^2$ of the velocity $\dot{\mathbf{p}}_{C_i}$ of the contact point C_i, i.e.,

$$\tilde{\dot{\mathbf{p}}}_{C_i} = \dot{\mathbf{p}}_{C_i} + v_i, \tag{4}$$

where $v_i \sim N(\mathbf{0}, \Sigma_i)$ is a white Gaussian noise with zero mean and covariance matrix $\Sigma_i \in \mathbb{R}^{2 \times 2}$. Finally, we assume that control inputs and communications are not affected by noise.

Problem 1 (Distributed Estimation of Inertial Parameters). Given a network of n robots moving on a plane and manipulating a load B. Design a distributed strategy such that each robot i, for all $i \in \mathscr{I}$, is able to estimate

1) the mass m of B,
2) the moment of inertia J of B, and
3) the relative positions of the contact point with respect to the load's center of mass, that is, $\mathbf{p}_{C_i} - \mathbf{p}_C \in \mathbb{R}^2$. We remark that this quantity varies in time.

Each robot i can only

1) locally measure the velocity $\tilde{\dot{\mathbf{p}}}_{C_i}$ of the contact point C_i;
2) locally control the applied force $\mathbf{f}_i$ acting on C_i;
3) communicate only with its one-hop neighbors, belonging to the set $\mathscr{N}_i$.

Henceforth, variables with the superimposed symbol $\tilde{}$ refer to noisy measurements, while variables with the superimposed symbol $\hat{}$ refer to estimates.

III. ESTIMATION ALGORITHM OVERVIEW

We indicate with $\mathbf{p}_G = \frac{1}{n} \sum_{i=1}^{n} \mathbf{p}_{C_i}$ the geometric center of all the contact points, and we use the following compact notation for vector differences: $\mathbf{z}_{ij} = \mathbf{p}_{C_i} - \mathbf{p}_{C_j}$, $\mathbf{z}_i = \mathbf{p}_{C_i} - \mathbf{p}_G$, and $\mathbf{z}_C = \mathbf{p}_G - \mathbf{p}_C$. Thus, $\mathbf{p}_{C_i} - \mathbf{p}_C = \mathbf{z}_i + \mathbf{z}_C$.

The following fact holds:

Fact 1. *Assuming that $\mathbf{z}_1$ and $\mathbf{z}_2$ are the relative positions between two points of B expressed in $\mathscr{W}$, and considering two time instants t' and t'', the rigid body constraint can be used to compute $\mathbf{z}_2(t'')$ from $\mathbf{z}_1(t')$, $\mathbf{z}_2(t')$, and $\mathbf{z}_1(t'')$:*

$$\mathbf{z}_2(t'') = \Gamma(\mathbf{z}_1(t'), \mathbf{z}_2(t'), \mathbf{z}_1(t'')) = \tag{5}$$
$$= \frac{\left(\mathbf{z}_2(t')^T \mathbf{z}_1(t')\right) \mathbf{z}_1(t'') + \left(\mathbf{z}_2(t')^T \mathbf{z}_1^{\perp}(t')\right) \mathbf{z}_1^{\perp}(t'')}{\|\mathbf{z}_1(t')\|}.$$

The proposed algorithm consists of four steps that lead the estimates performed by each robot to converge to the real value of the inertial parameters. Each of the four steps, described in the following, converges in a given time interval. We indicate with t_0 the starting time of the algorithm and with t_k, $k = 1, \dots, 4$, the end time of each step. We remark that the steps are sequentially executed, i.e., no information obtained at any step has to be fed back to previous steps at any time.

Step 1: *an estimation $\widehat{\|\mathbf{z}_{ij}\|}$ of $\|\mathbf{z}_{ij}\|$ becomes known after t_1.* Each robot i applies an arbitrary force $\mathbf{f}_i(t)$ to the body and uses $\tilde{\dot{\mathbf{p}}}_{C_i}$ and $\tilde{\dot{\mathbf{p}}}_{C_j}$, with $j \in \mathscr{N}_i$, to estimate $\widehat{\|\mathbf{z}_{ij}\|}$. This is achieved through a least squares estimation that converges at time t_1 and is detailed in Sec. IV.

Step 2: *an estimation $\hat{J}$ of J becomes known after t_2.* For $t \geq t_1$, four concurrent substeps are executed to produce an estimate $\hat{J}$ of the moment of inertia J.

Sub-step 2.1 each robot i computes $\hat{\mathbf{z}}_{ij}(t)$ and $\widehat{\text{sign}(\omega)}$, using $\tilde{\dot{\mathbf{p}}}_{C_i}$, $\tilde{\dot{\mathbf{p}}}_{C_j}$ (with $j \in \mathscr{N}_i$), and $\widehat{\|\mathbf{z}_{ij}\|}$ (see Sec. IV).

Sub-step 2.2 each robot i computes $\hat{\mathbf{z}}_i(t)$, using $\hat{\mathbf{z}}_{ij}(t)$ and the algorithm in [11].

Sub-step 2.3 each robot i applies a force $\mathbf{f}_i(t) = k_z \hat{\mathbf{z}}_i^{\perp}(t)$. While the body moves, each robot i locally estimates the rotational rate of B as

$$\hat{\omega}_i(t) = \widehat{\text{sign}(\omega)} \frac{\|\tilde{\dot{\mathbf{z}}}_{ij}\|}{\|\mathbf{z}_{ij}\|}, \tag{6}$$

where j is any neighbor in $\mathscr{N}_i$. Then, all robots update their local estimates $\hat{\omega}_i$ using a dynamic consensus algorithm [12] to attain a common estimate $\hat{\omega}$.

Sub-step 2.4 each robot performs an estimate of J, denoted with $\hat{J}_i$, as detailed in Sec. V-A. Then, an average consensus

algorithm [13] is used to reach an agreement on a common estimate $\hat{J}$.

Step 3: *an estimation $\widehat{\mathbf{p}_{C_i} - \mathbf{p}_C}$ of $\mathbf{p}_{C_i} - \mathbf{p}_C$ becomes known after t_3.* Each robot applies an arbitrary nonzero force $\mathbf{f}_i(t)$ and produces an estimate $\hat{\omega}(t)$ of the rotational rate $\omega(t)$. Then, each robot computes $\hat{\mathbf{z}}_C$, relying on the estimate $\hat{J}$ previously performed, and using the nonlinear observer detailed in Sec. V-B. The applied forces must verify the weak condition $\frac{1}{n}\sum_{i=1}^n \mathbf{f}_i(t) \neq \mathbf{0}$, as stated in Proposition V.1. After the convergence time of the observer, indicated with t_3, the unknown vector $\mathbf{p}_{C_i} - \mathbf{p}_C$ is estimated using the formula

$$(\widehat{\mathbf{p}_{C_i}(t) - \mathbf{p}_C(t)}) = \hat{\mathbf{z}}_i(t) + \hat{\mathbf{z}}_C(t) = \tag{7}$$
$$= \hat{\mathbf{z}}_i(t) + \Gamma(\hat{\mathbf{z}}_C(t_3), \hat{\mathbf{z}}_{ij}(t_3), \hat{\mathbf{z}}_{ij}(t)),$$

where Γ is defined in (5) and $j \in \mathcal{N}_i$.

Step 4: *an estimation $\hat{m}$ of m becomes available after t_4.* Each robot i estimates $\hat{\mathbf{p}}_C(t)$, by applying an arbitrary nonzero force $\mathbf{f}_i(t)$ and estimates $\hat{\mathbf{p}}_C(t)$ as

$$\hat{\mathbf{p}}_C(t) = \tilde{\mathbf{p}}_{C_i}(t) - \hat{\omega}(t)(\widehat{\mathbf{p}_{C_i}(t) - \mathbf{p}_C(t)})^{\perp}. \tag{8}$$

We observe that all the quantities in the right hand side of (8) are computed during the previous steps and are locally available after $t \geq t_3$. Based on $\hat{\mathbf{p}}_C$, each robot computes an estimate $\hat{m}_i$, as detailed in Sec. V-C. An average consensus algorithm is then used to converge to a common estimate of the mass $\hat{m}$, which, for $t \geq t_4$, will be known by all the robots.

IV. ESTIMATION OF THE RELATIVE POSITIONS OF THE CONTACT POINTS

Contact points C_i, $i \in \mathscr{I}$, belong to the rigid body B; Thus, the following relation holds for all $(i,j) \in \mathscr{E}$:

$$\mathbf{z}_{ij}^T \mathbf{z}_{ij} = \text{const.} \tag{9}$$

Differentiating both sides of (9), we obtain

$$\dot{\mathbf{z}}_{ij}^T \mathbf{z}_{ij} = 0, \tag{10}$$

which implies that $\dot{\mathbf{z}}_{ij}$ is perpendicular to $\mathbf{z}_{ij}$. Therefore, we have

$$\frac{\mathbf{z}_{ij}}{\|\mathbf{z}_{ij}\|} = \text{sign}(\omega) \frac{\dot{\mathbf{z}}_{ij}^{\perp}}{\|\dot{\mathbf{z}}_{ij}^{\perp}\|} \quad , \text{ i.e., } \quad \mathbf{z}_{ij} = \text{sign}(\omega)\|\mathbf{z}_{ij}\| \ \mathbf{y}_{ij}, \tag{11}$$

where we let $\mathbf{y}_{ij} = \frac{\dot{\mathbf{z}}_{ij}^{\perp}}{\|\dot{\mathbf{z}}_{ij}^{\perp}\|}$. Thus, robot i computes

$$\hat{\mathbf{z}}_{ij} = \widehat{\|\mathbf{z}_{ij}\|}\hat{\mathbf{y}}_{ij}, \tag{12}$$

where $\tilde{\mathbf{y}}_{ij} = \frac{(\tilde{\mathbf{p}}_{C_i} - \tilde{\mathbf{p}}_{C_j})^{\perp}}{\|\tilde{\mathbf{p}}_{C_i} - \tilde{\mathbf{p}}_{C_j}\|}$ and $\widehat{\|\mathbf{z}_{ij}\|}$ can be computed using the strategy detailed in the following.

It is clear that $\|\mathbf{z}_{ij}\|$ cannot be estimated if $\omega \equiv 0$, $\forall t$, since this implies $\dot{\mathbf{z}}_{ij} \equiv \mathbf{0}$. Thus, considering any time interval in which $\omega \neq 0$ we differentiate with respect to time both sides of (11) obtaining

$$\dot{\mathbf{z}}_{ij} = \text{sign}(\omega)\|\mathbf{z}_{ij}\| \ \dot{\mathbf{y}}_{ij}. \tag{13}$$

Since $\dot{\mathbf{y}}_{ij}$ is not directly known, we apply a first-order low-pass filter to both sides of (13), denoting with $\dot{\mathbf{z}}_{ij}^f$ and $\dot{\mathbf{y}}_{ij}^f$ the filtered versions of $\dot{\mathbf{z}}_{ij}$ and $\dot{\mathbf{y}}_{ij}$, respectively (refer to the Appendix of [9] for a detailed explanation). Taking the squared norm of both sides after filtering, we obtain

$$\|\dot{\mathbf{z}}_{ij}^f\|^2 = \|\mathbf{z}_{ij}\|^2 \ k_f^2 \|\mathbf{y}_{ij} - \mathbf{y}_{ij}^f\|^2, \tag{14}$$

where k_f is the gain of the low pass filter. Finally, $\widehat{\|\mathbf{z}_{ij}\|}^2$ can be obtained by solving the linear least squares problem

$$\begin{bmatrix} \|\tilde{\mathbf{z}}_{ij}^f(t_1)\|^2 \\ \vdots \\ \|\tilde{\mathbf{z}}_{ij}^f(t_q)\|^2 \end{bmatrix} = \widehat{\|\mathbf{z}_{ij}\|}^2 \begin{bmatrix} k_f^2 \|\tilde{\mathbf{y}}_{ij}(t_1) - \tilde{\mathbf{y}}_{ij}^f(t_1)\|^2 \\ \vdots \\ k_f^2 \|\tilde{\mathbf{y}}_{ij}(t_q) - \tilde{\mathbf{y}}_{ij}^f(t_q)\|^2 \end{bmatrix}, \tag{15}$$

where $t_1 \ldots t_q$ are the $q > 0$ acquisition times of the noisy measurements.

Furthermore, an estimate of $\text{sign}(\omega)$ at time t is given by

$$\widehat{\text{sign}(\omega)} = \text{sign}\left[\left(\tilde{\mathbf{z}}_{ij}^f(t)\right)^T \left(\tilde{\mathbf{y}}_{ij}(t) - \tilde{\mathbf{y}}_{ij}^f(t)\right) \right]. \tag{16}$$

V. ESTIMATION OF INERTIAL PARAMETERS

In this section, we describe three algorithms for estimating, in the presented order: (*i*) the moment of inertia J, (*ii*) the time-varying vector $\mathbf{z}_C$, *i.e.*, the position of the center of mass C relative to the geometric center $\mathbf{p}_G$, and (*iii*) the mass m.

A. Estimation of the Moment of Inertia J

Noting that $\mathbf{p}_{C_i} - \mathbf{p}_C = \mathbf{z}_i + \mathbf{z}_C$, we can rewrite (2) as

$$\dot{\omega} = \frac{1}{J}\sum_{i=1}^n \mathbf{z}_i^{\perp T} \mathbf{f}_i + \frac{1}{J}\mathbf{z}_C^{\perp T}\sum_{i=1}^n \mathbf{f}_i, \tag{17}$$

where $\mathbf{z}_i$ can be retrieved locally by each robot during Step 2. Thus, the only unknown quantities in (17) are J and $\mathbf{z}_C$.

If each robot i applies a force $\mathbf{f}_i = k_z \mathbf{z}_i^{\perp}$ at each time t, being k_z any constant, then the second sum in (17) vanishes. Consider, in fact

$$\sum_{i=1}^n \mathbf{f}_i = k_z \sum_{i=1}^n \mathbf{z}_i^{\perp} = k_z \sum_{i=1}^n (\mathbf{p}_{C_i} - \mathbf{p}_G)^{\perp} = k_z Q \sum_{i=1}^n (\mathbf{p}_{C_i} - \mathbf{p}_G)$$
$$= k_z Q \left(\sum_{i=1}^n \mathbf{p}_{C_i} - \sum_{i=1}^n \mathbf{p}_G \right) = k_z Q (n\mathbf{p}_G - n\mathbf{p}_G) = 0.$$

Thus, (17) simplifies as

$$\dot{\omega} = \frac{k_z}{J}\sum_{i=1}^n \mathbf{z}_i^{\perp T}\mathbf{z}_i^{\perp} = \frac{k_z}{J}\sum_{i=1}^n \|\mathbf{z}_i\|^2. \tag{18}$$

We observe that quantity $k_z\sum_{i=1}^n \|\mathbf{z}_i\|^2$ in (18) is constant over time. Therefore, this yields a constant angular acceleration, which allows the distributed identification of J by means of the following distributed algorithm:

1) Before applying any force, each robots distributively computes the constant value $w = k_z\sum_{i=1}^n \|\hat{\mathbf{z}}_i\|^2$. This can be done in a distributed way by means of a standard average consensus algorithm;

2) Each robot applies a constant force $\mathbf{f}_i = k_z\hat{\mathbf{z}}_i^{\perp}$. This can be done only based on local information;

3) Thanks to the local estimate $\hat{\omega}$ of ω, each robot computes a local estimate $\hat{J}_i$ of J using the approach presented in the Appendix of [9];

4) When the local estimates converge, the robots run a further consensus phase, attaining an agreement on a common estimate $\hat{J}$. This operation averages out the uncertainty in the estimates $\hat{J}_i$ caused by the noise in $\hat{\omega}$ and $\hat{\mathbf{z}}_i$.

We note that no perfect time synchronization on the start time of the application of forces $\mathbf{f}_i$ is needed, since each robot will eventually apply the force $\mathbf{f}_i = k_z \hat{\mathbf{z}}_i^\perp$.

We observe that design or technical constraints, such as the necessity of keeping the angular or linear velocity of the body bounded, may hold. In this case, the forces required in this phase of the estimation could be applied only for a limited time interval. In this case, the movement can be easily stopped when needed using a pure damping force based on a local velocity feedback. However, should the time be not enough for estimation purposes, the process can be repeated several times after each stop, to ensure the acquisition of the measurements necessary to identify J.

B. Observer for the Relative Position $\mathbf{z}_C$ of the CoM

Assume that during the manipulation task each robot applies an arbitrary force $\mathbf{f}_i(t)$. It is straightforward to verify that $\mathbf{f}_i(t)$ can be expressed as

$$\mathbf{f}_i(t) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{f}_i(t) + \Delta \mathbf{f}_i(t) = \mathbf{f}_{\text{mean}}(t) + \Delta \mathbf{f}_i(t). \tag{19}$$

Then, (17) becomes

$$\dot{\omega} = \frac{1}{J} \left(\sum_{i=1}^{n} \mathbf{z}_i^{\perp T} \right) \mathbf{f}_{\text{mean}}(t) + \frac{n}{J} \mathbf{z}_C^{\perp T} \mathbf{f}_{\text{mean}}(t) +$$
$$\frac{1}{J} \sum_{i=1}^{n} \mathbf{z}_i^{\perp T} \Delta \mathbf{f}_i + \frac{1}{J} \mathbf{z}_C^{\perp T} \sum_{i=1}^{n} \Delta \mathbf{f}_i = \tag{20}$$
$$\frac{n}{J} \mathbf{z}_C^{\perp T} \mathbf{f}_{\text{mean}}(t) + \frac{1}{J} \sum_{i=1}^{n} \mathbf{z}_i^{\perp T} \Delta \mathbf{f}_i,$$

given that $\sum_{i=1}^{n} \mathbf{z}_i^{\perp T} = 0$ and $\sum_{i=1}^{n} \Delta \mathbf{f}_i = 0$. Thus, (20) can be written as

$$\dot{\omega} = \mathbf{z}_C^{\perp T} \bar{\mathbf{f}} + \eta, \tag{21}$$

where we let $\bar{\mathbf{f}} = \frac{n}{J} \mathbf{f}_{\text{mean}}$ and $\eta = \frac{1}{J} \sum_{i=1}^{n} \mathbf{z}_i^{\perp T} \Delta \mathbf{f}_i$.

By means of standard dynamic consensus algorithms, it is possible to reach an agreement on $\bar{\mathbf{f}}$. To this purpose, each robot i needs to exchange only the local quantity $\mathbf{f}_i(t)$ with its neighbors. This implies that robot i can locally compute $\Delta \mathbf{f}_i = \mathbf{f}_i - \bar{\mathbf{f}}$. By exchanging the local quantity $\mathbf{z}_i^{\perp T} \Delta \mathbf{f}_i$ with its neighbors and applying again the dynamic consensus algorithm, also η can be locally computed in a distributed way. Therefore, in the following we can safely assume that both $\bar{\mathbf{f}}$ and η are locally known to each robot.

Since $\mathbf{z}_C$ is a vector with constant norm rigidly attached to the object, we have

$$\dot{\mathbf{z}}_C^\perp = -\mathbf{z}_C \omega. \tag{22}$$

Combining (21) and (22), we obtain the following autonomous nonlinear system

$$\begin{cases} \dot{x}_1 = -x_2 x_3 \\ \dot{x}_2 = x_1 x_3 \\ \dot{x}_3 = x_1 \bar{f}_y - x_2 \bar{f}_x + \eta \\ y = x_3, \end{cases} \tag{23}$$

where we let $z_C^x = x_1$, $z_C^y = x_2$, $\omega = x_3$, and $\bar{\mathbf{f}} = (\bar{f}_x \ \bar{f}_y)^T$. The system output is assumed to be $y = x_3 = \omega$, since the rotational rate is locally estimated by each robot using (6). Therefore, estimating $\mathbf{z}_C$ is equivalent to observe the state of the nonlinear system (23) with output $y = x_3 = \omega$, and where $\bar{f}_y$, $\bar{f}_x$, and η are known inputs.

Before designing a suitable nonlinear observer, the observability of system (23) is studied.

Proposition V.1. If $x_3 \not\equiv 0$ and $\begin{bmatrix} \bar{f}_x(t) & \bar{f}_y(t) \end{bmatrix}^T \not\equiv \mathbf{0}^T$, then system (23) is locally observable in the sense of [14].

Proof. The observability matrix [14] is

$$O = \begin{pmatrix} 0 & 0 & 1 \\ \bar{f}_y & -\bar{f}_x & 0 \\ -\bar{f}_x x_3 & -\bar{f}_y x_3 & -\bar{f}_x x_1 - \bar{f}_y x_2 \end{pmatrix}, \tag{24}$$

whose determinant is $\det(O) = -x_3(\bar{f}_x^2 + \bar{f}_y^2)$. System (23) is locally observable in the sense of [14] iff O is invertible, from which the thesis follows. □

Thus, vector $\mathbf{z}_C$ is observable from local velocity measurements if and only if the rotational rate of the object and the average vector of the applied forces are not identically zero. The nonlinear observer is designed as follows.

Theorem V.1. Consider the following dynamical system

$$\begin{aligned} \dot{\hat{x}}_1 &= -\hat{x}_2 x_3 + \bar{f}_y(x_3 - \hat{x}_3) \\ \dot{\hat{x}}_2 &= \hat{x}_1 x_3 - \bar{f}_x(x_3 - \hat{x}_3) \\ \dot{\hat{x}}_3 &= \hat{x}_1 \bar{f}_y - \hat{x}_2 \bar{f}_x + k_e(x_3 - \hat{x}_3) + \eta, \end{aligned} \tag{25}$$

where $k_e > 0$. If $x_3 \not\equiv 0$ and $\begin{bmatrix} \bar{f}_x(t) & \bar{f}_y(t) \end{bmatrix}^T \not\equiv \mathbf{0}$, then system (25) is an asymptotic observer for system (23), *i.e.*, defining $\hat{\mathbf{x}} = (\hat{x}_1 \ \hat{x}_2 \ \hat{x}_3)^T$ and $\mathbf{x} = (x_1 \ x_2 \ x_3)^T$, $\hat{\mathbf{x}} \to \mathbf{x}$ asymptotically.

Proof. Define the error vector as $\mathbf{e} = (e_1 \ e_2 \ e_3)^T = ((x_1 - \hat{x}_1) \ (x_2 - \hat{x}_2) \ (x_3 - \hat{x}_3))^T$; the error dynamics is given by

$$\dot{\mathbf{e}} = \begin{pmatrix} 0 & -x_3 & -\bar{f}_y \\ x_3 & 0 & \bar{f}_x \\ \bar{f}_y & -\bar{f}_x & -k_e \end{pmatrix} \mathbf{e}. \tag{26}$$

Define the following candidate Lyapunov function:

$$V(\mathbf{e}) = \frac{1}{2} \left(e_1^2 + e_2^2 + e_3^2 \right), \tag{27}$$

whose time derivative along the system trajectories is

$$\begin{aligned} \dot{V}(\mathbf{e}) =&\, e_1(-e_2 x_3 - \bar{f}_y e_3) + \\ &\, e_2(x_3 e_1 + \bar{f}_x e_3) + e_3(\bar{f}_y e_1 - \bar{f}_x e_2 - k_e e_3) \\ =&\, -e_1 e_2 x_3 - e_1 \bar{f}_y e_3 + e_2 x_3 e_1 + \\ &\, e_2 \bar{f}_x e_3 + e_3 \bar{f}_y e_1 - e_3 \bar{f}_x e_2 - k_e e_3^2 \\ =&\, -k_e e_3^2, \end{aligned} \tag{28}$$

which is negative semidefinite. Considering the set $\mathcal{V} = \left\{ \mathbf{e} \text{ s.t. } \dot{V}(\mathbf{e}) = 0 \right\} = \left\{ \mathbf{e} \text{ s.t. } e_3 = 0 \right\}$, and generic vector $\bar{\mathbf{e}} = (\bar{e}_1 \ \bar{e}_2 \ 0)^T \in \mathcal{V}$, it is easy to verify by means of (26) that the first and second time derivatives of e_3 along a trajectory containing $\bar{\mathbf{e}}$ are given by

$$\left. \frac{de_3}{dt} \right|_{\bar{\mathbf{e}}} = \bar{f}_y \bar{e}_1 - \bar{f}_x \bar{e}_2 \quad \left. \frac{d^2 e_3}{dt^2} \right|_{\bar{\mathbf{e}}} = \left. \frac{de_1}{dt} \right|_{\bar{\mathbf{e}}} = -x_3 (\bar{f}_y \bar{e}_2 + \bar{f}_x \bar{e}_1). \tag{29}$$

Therefore, if x_3 is not vanishing, then the largest invariant set $M \subset \mathcal{V}$ consists of the only equilibrium point $(0 \ 0 \ 0)^T$. Thus, the thesis holds due to the invariance Krasovskii–LaSalle principle [15]. ∎

We remark that observer (25) can be implemented in a distributed fashion by resorting only to local information.

The estimation error of the proposed observer vanishes asymptotically in the ideal case of absence of noise. Actually, each robot sets the force $\bar{\mathbf{f}}_a = \frac{n}{\hat{J}} \mathbf{f}_{\text{mean}}$ as input for the local observer, which is affected by noise, due to the presence of $\hat{J}$. Furthermore, the observer relies on the noisy estimate $\hat{\omega}$, computed using (6). Due to its asymptotic stability, we expect that in presence of noise the estimation error will remain bounded around the actual value of the parameter. The definition of a formal proof of the estimation error boundedness will be given as future work. However, in Sec. VI, we will numerically characterize the bound of the estimation error with respect to noise terms.

C. Estimation of the Mass

Assume that each robot applies an arbitrary force $\mathbf{f}_i(t)$ and that $\mathbf{f}_{\text{mean}}(t)$, defined as in (19), is not zero. Thus, (1) becomes

$$\ddot{\mathbf{p}}_C = \frac{n}{m} \mathbf{f}_{\text{mean}}. \tag{30}$$

We remind that each robot can distributively compute $\mathbf{f}_{\text{mean}}$. Furthermore, each robot is able to estimate the velocity of the center of mass as

$$\hat{\dot{\mathbf{p}}}_C(t) = \widetilde{\dot{\mathbf{p}}}_{C_i}(t) + \hat{\omega}(t) \left(\hat{\mathbf{z}}_i(t) + \hat{\mathbf{z}}_C(t) \right)^{\perp}. \tag{31}$$

Therefore, similarly to $\hat{J}_i$, each robot can locally compute an estimate $\hat{m}_i$ using the approach in the Appendix of [9]. Finally, the robots agree on a global estimation $\hat{m}$ using an average consensus algorithm in order to average out the noise of each local mass estimator.

VI. SIMULATIONS AND ACCURACY BOUNDS

We validate the estimation algorithm by running numerical simulations. We assume $n = 4$ robots manipulating a C-shaped planar rigid body with mass $m = 5$ kg and moment of inertia $J = 8.6891$ kg m^2. Communications among robots occur through the links $\mathcal{E} = \{(1,2),(2,3),(3,4)\}$. We assume that each robot is able to measure the velocity of the contact point, and that the measurement is affected by a Gaussian noise with zero mean and covariance matrix $\Sigma_i = \sigma^2 \mathbf{I}$, with $\sigma = 0.2$ m/s, and where $\mathbf{I} \in \mathbb{R}^{2 \times 2}$ is the identity matrix.
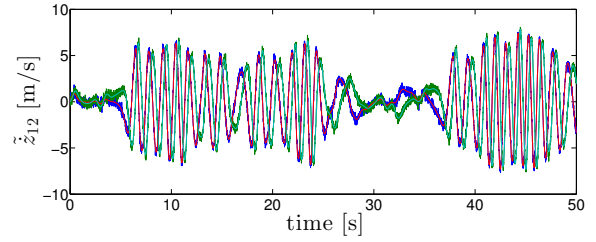

Fig. 2: The measured velocity difference $\tilde{\mathbf{z}}_{12}$. The trends of $\tilde{\mathbf{z}}_{23}$ and $\tilde{\mathbf{z}}_{34}$ are very similar and not reported here due to space constraints.
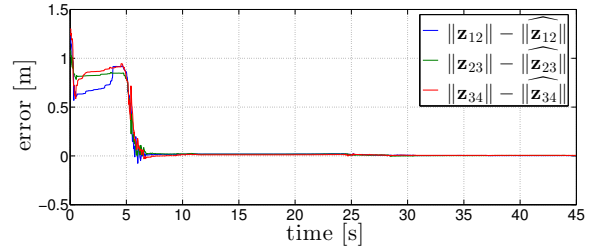

Fig. 3: Estimation errors of the distances between neighbors.

The algorithm starts with the estimation of the relative distance between neighboring contact points: each robot sets an arbitrary force and executes Step 1. We observe that the estimation must stop when the measured noisy signals $\tilde{\mathbf{z}}_{ij}$ have a level such that the signal-to-noise ratio is too low to perform an estimate. In this case, the estimation stops when $\|\tilde{\mathbf{z}}_{ij}\| \leq 1$ m/s (the measured $\tilde{\mathbf{z}}_{12}$ is illustrated in Fig. 2, as an example). Figure 3 illustrates the errors in the estimation of the distances using the strategy presented in Sec. IV. Handling noisy measurements involves a certain inaccuracy in the estimation process, whose standard deviation is quantified in the following. Suppose to perform a least squares estimation using τ observations (υ_t, ψ_t), $t = 1, \ldots, \tau$, of the model $\psi = \theta \upsilon$. The estimate $\hat{\theta} = (\Upsilon^T \Upsilon)^{-1} \Upsilon^T \Psi$, where $\Upsilon = [\upsilon_1^T \ldots \upsilon_\tau^T]^T$ and $\Psi = [\psi_1^T \ldots \psi_\tau^T]^T$. The standard deviation $\sigma_{\hat{\theta}}$ of the estimates $\hat{\theta}$ is

$$\sigma_{\hat{\theta}} = \sigma_\psi \frac{\tau}{\tau \sum_{t=1}^{\tau} \upsilon_t^2 - (\sum_{t=1}^{\tau} \upsilon_t)^2}, \tag{32}$$

where σ_ψ is the standard deviation of the observations ψ. Thus, the uncertainty of the estimation of the constant parameters $\|\mathbf{z}_{ij}\|$, m, and J has the form of (32) [16].

Subsequently, Step 2 is executed toward the local estimation of the moment of inertia $\hat{J}_i$. The estimation process starts as soon as robot i has locally collected a sufficient number of samples, and yields the local estimates $\hat{J}_1 = 8.7053 \pm 0.0035$ kg m^2, $\hat{J}_2 = 8.7208 \pm 0.0035$ kg m^2, $\hat{J}_3 = 8.7320 \pm 0.0032$ kg m^2, and $\hat{J}_4 = 8.7151 \pm 0.0035$ kg m^2. Each robot checks the convergence of the least squares estimation evaluating the variance of the estimator [16]. Then, the local estimates $\hat{J}_i$ are exchanged over the network and an average consensus is run to agree on the same estimate, that is, $\hat{J} = 8.7183 \pm 0.0004$ kg m^2. The result of such estimation is reported in Fig. 4. The estimate of the moment of inertia $\hat{J}$ and the observation of $\hat{\omega}(t)$, known thanks to the previous step, are used for the
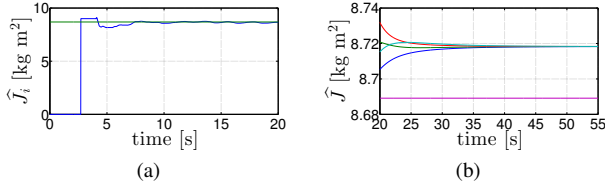
Fig. 4: Estimation of the moment of inertia J of B. (a) Least squares estimation of J: the estimate converges as soon as the number of samples is sufficient. (b) After the convergence of the estimator, the network runs an average consensus to reach an agreement on $\hat{J}$.
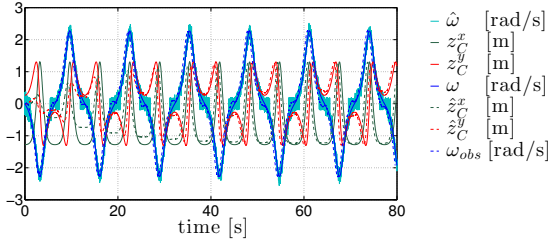

Fig. 5: Observation of the vector $\mathbf{z}_C$ and of the angular rate ω: dashed lines refer to observed values, while continuous lines refer to real values. For the angular rate, the measure $\hat{\omega}$ (continuos light blue line) in input to the observer is also plotted.

observation of $\mathbf{z}_C$. During this step, each robot applies the same constant force $\mathbf{f}_i = \mathbf{f}$, for all $i = 1, \dots, 4$. In Fig. 5, the observations $\hat{\mathbf{z}}_C$ and ω_{obs} are illustrated.

We now characterize numerically the uncertainty of the nonlinear observer in estimating $\mathbf{z}_C$, by running 1000 independent trials for different values of the variance of the noise on the angular rate, $\sigma^2_{\omega_{\text{obs}}}$. For each trial, a sinusoidal signal with random amplitude, frequency, and phase is used for the components of the force applied by each robot. The simulation yields a an almost constant trend and independent from the value of $\sigma_{\omega_{\text{obs}}}$. Therefore, its mean value can be considered as a good approximation of the standard deviation, *i.e.*, $\sigma_{\mathbf{z}_C^x} = 0.075$ m and $\sigma_{\mathbf{z}_C^y} = 0.033$ m. Once the observer converges, the estimation of $\hat{\mathbf{p}}_C$ and $\hat{m}$ can be executed. The estimation of the mass, illustrated in Fig. 6, is carried out using the estimation of the angular rate computed by the observer, ω_{obs}. First, each robot estimates locally, respectively, $\hat{m}_1 = 4.8367 \pm 0.0451$ kg, $\hat{m}_2 = 4.8491 \pm 0.0455$ kg, $\hat{m}_3 = 4.8824 \pm 0.0453$ kg, and $\hat{m}_4 = 4.8384 \pm 0.0447$ kg and then, after the average consensus, the network agrees on the value $\hat{m} = 4.8517 \pm 0.0113$ kg.

VII. CONCLUSION

In this paper, we have addressed the problem of the distributed estimation of the inertial parameters of an unknown load manipulated by a network of robots. We assume that each robot is able to control the force exerted on the load and to measure only the velocity of the point where the force is exerted. Only local communication between neighboring robots is allowed. In partic-

ular, we have focused on the influence of the measurement noise on the estimate by defining suitable strate-
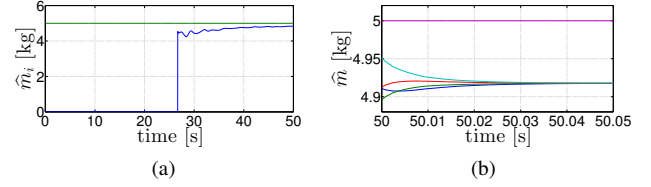

Fig. 6: Estimation of the mass m of B. (a) Least squares estimation of m: the estimate converges as soon as the number of samples is sufficient. (b) After the convergence of the estimator, the network runs an average consensus in order to reach an agreement on $\hat{m}$.

gies. The algorithm has been validated through several numerical simulations.

REFERENCES

[1] D. Sieber, F. Deroo, and S. Hirche, "Formation-based approach for multi-robot cooperative manipulation based on optimal control design," in *2013 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Tokyo, Japan, Nov. 2013, pp. 5227–5233.

[2] A. Yamashita, T. Arai, J. Ota, and H. Asama, "Motion planning of multiple mobile robots for cooperative manipulation and transportation," *IEEE Trans. on Robotics*, vol. 19, no. 2, pp. 223–237, 2003.

[3] A. Yufka, O. Parlaktuna, and M. Ozkan, "Formation-based cooperative transportation by a group of non-holonomic mobile robots," in *2010 IEEE Int. Conf. on Systems, Man, and Cybernetics*, Istanbul, Turkey, Oct 2010, pp. 3300–3307.

[4] D. Lee and M. W. Spong, "Bilateral teleoperation of multiple cooperative robots over delayed communication network: theory," in *2005 IEEE Int. Conf. on Robotics and Automation*, Barcelona, Spain, Apr. 2005, pp. 360–365.

[5] A. C. Satici and M. W. Spong, "Nonholonomic cooperative manipulation of polygonal objects in the plane," in *51st IEEE Conf. on Decision and Control*, Maui, HI, Dec. 2012, pp. 2439–2446.

[6] Y. Yong, T. Arima, and S. Tsujio, "Inertia parameter estimation of planar object in pushing operation," in *2005 IEEE Int. Conf. on Information Acquisition*, Hong Kong and Macau, China, June 2005, pp. 356–361.

[7] D. Kubus, T. Kroger, and F. M. Wahl, "On-line estimation of inertial parameters using a recursive total least-squares approach," in *2008 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, Nice, France, Sep. 2008, pp. 3845–3852.

[8] N. A. Lynch, *Distributed Algorithms*. Morgan Kaufmann, 1997.

[9] A. Franchi, A. Petitti, and A. Rizzo, "Distributed estimation of the inertial parameters of an unknown load via multi-robot manipulation," in *53rd IEEE Conf. on Decision and Control*, Los Angeles, CA, Dec. 2014, pp. 6111–6116.

[10] L. Ljung, *System Identification: Theory for the User, 2nd edition*. Prentice Hall, 1999.

[11] R. Aragues, L. Carlone, C. Sagues, and G. Calafiore, "Distributed centroid estimation from noisy relative measurements," *Systems & Control Letters*, vol. 61, no. 7, pp. 773–779, 2012.

[12] M. Zhu and S. Martinez, "Discrete-time dynamic average consensus," *Automatica*, vol. 46, no. 2, pp. 322–329, 2010.

[13] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[14] R. Hermann and A. J. Krener, "Nonlinear controllability and observability," *tac*, vol. 22, no. 5, pp. 728–740, 1977.

[15] J. J. E. Slotine and W. Li, *Applied nonlinear control*. Prentice Hall, 1991.

[16] S. Van de Geer, "Least squares estimators," ser. Encyclopedia Statistics in The Behavioral Sciences, B. Everitt and D. Howell, Eds. John Wiley, 2005.