



Modelling of Human Faces

Andrei Doncescu

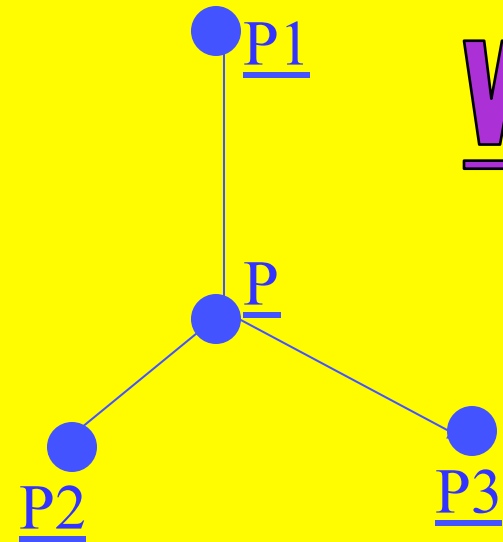
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University of La Rochelle
FRANCE*

Research Interest :

**The construction of a completely 3D
system of subdivision- compression
applied to the points which are
3-connected**

Why 3-connected ?



F.E.M

uses 1-D finite elements
3-connected



The modal analysis gives a number of rigid modes equal to the number of 1-D finite elements assembled to define the face

We avoid the extraordinary points

These organisation of the points allows us an algorithm to obtain the numbering of the vertices at each level of subdivision

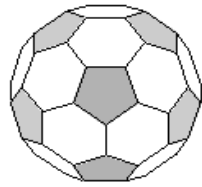
1. *L'analyse modale donne un nombre de modes rigides égal au nombre d'éléments assemblés pour définir une surface.*

Les modes rigides combinés avec le premier mode non-rigide conduisent à l'obtention d'un modèle moyen appelé PORTRAIT ROBOT

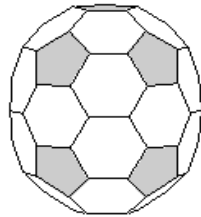
2. *Si la connexité uniforme n'est pas respectée, dans le voisinage des points singuliers, le calcul de déformation n'est pas homogène et un CALCUL RECURSIF DE SUBDIVISION est DIFFICILE.*

Example of surfaces 3-connected : The Fullerenes

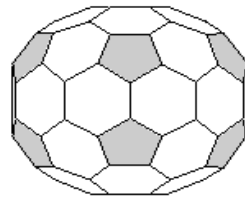
C60



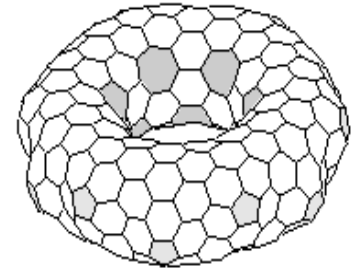
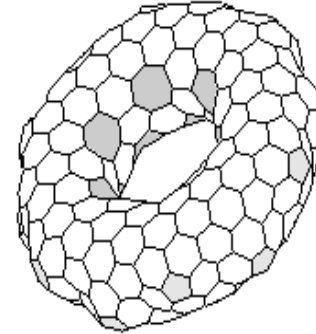
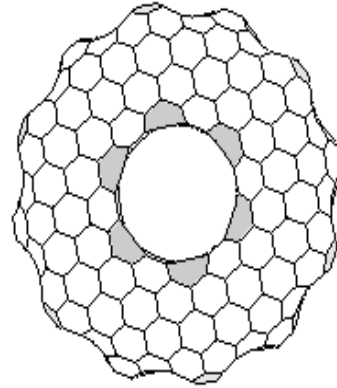
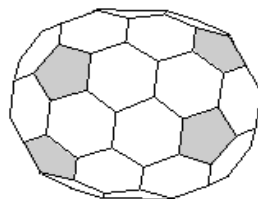
C70



C78



C82

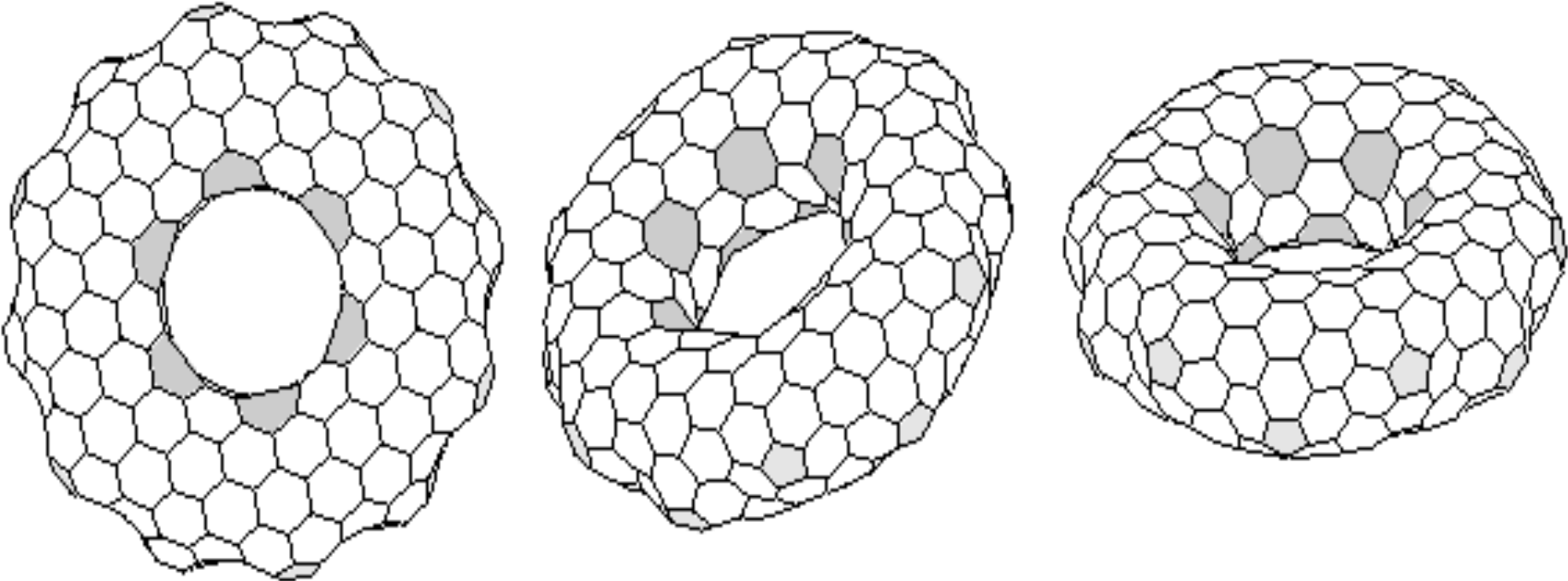


C432



Star-Shaped

Maillage 3-connexe sur un tore



Plan

- Présentation de la transformée en ondelettes
- Subdivision par les ondelettes géométriques
- Compression des surfaces maillées en 3-connexe
- Applications

Subdivision

Wavelets



Resume



Detail

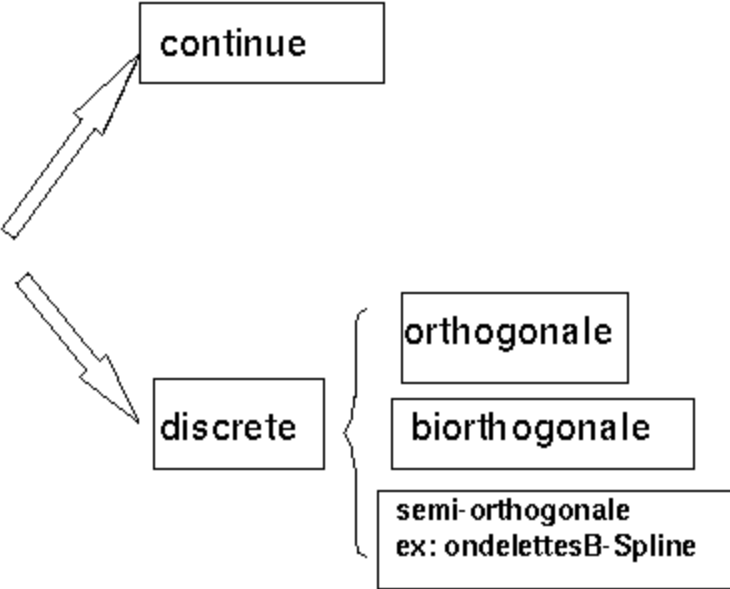
to have a reversible transform

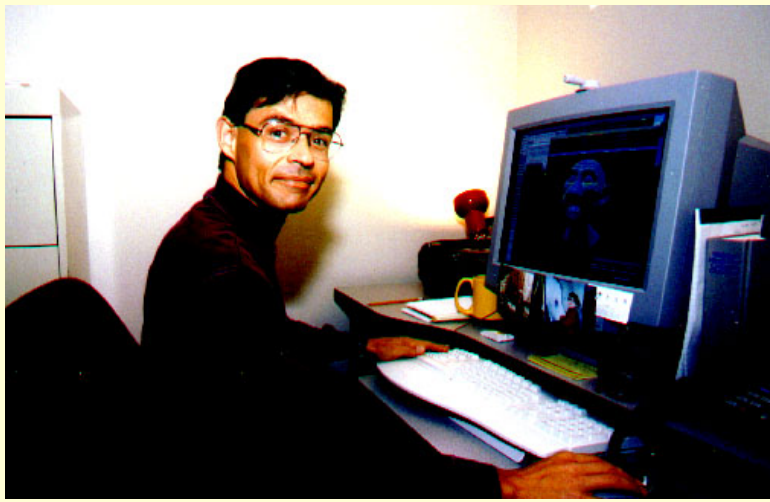
**To modify the detail allowing us
to obtain new objects**

LA TRANSFORMATION D 'INFORMATION

- **La transformation de Fourier** *(fréquence)*
- **La transformation de Fourier à fenêtre** *(temps - fréquence)*
- **La transformation en ondelettes** *(temps - échelle)*
- **Les ondelettes de Malvar** *(temps - fréquence - échelle)*
- **Les paquets d 'ondelettes** *(temps - fréquence)*
- **Matching Pursuits** *(temps - fréquence - échelle)*

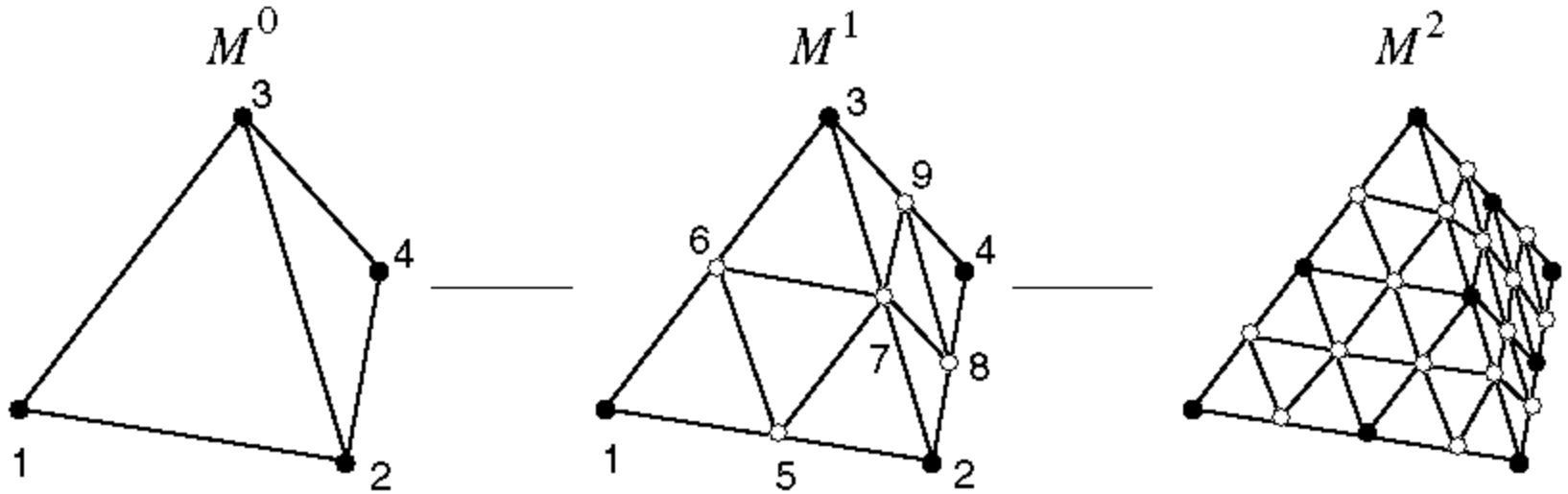
La transformation en ondelettes



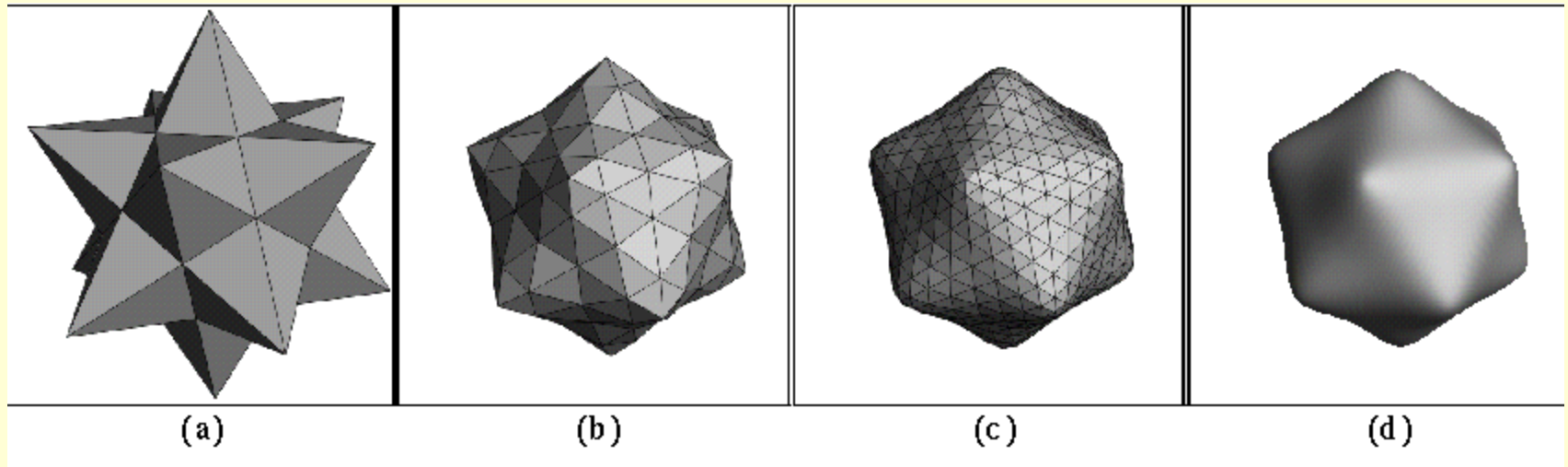


La construction de T. DeRose

Subdivision



- La deuxième génération des ondelettes obtenue par l'algorithme de lifting dans le domaine spatial
- L'applications des ondelettes pour la subdivision et la compression 3D en utilisant les bancs des filtres



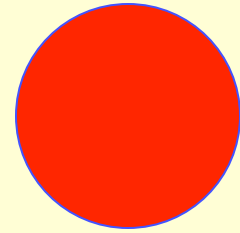


La subdivision de Sweldens et Schroder

- Construction de l'algorithme « lifting »
- Construction des ondelettes bio-orthogonales « lazy »
- Application sur une sphère

Subdivision

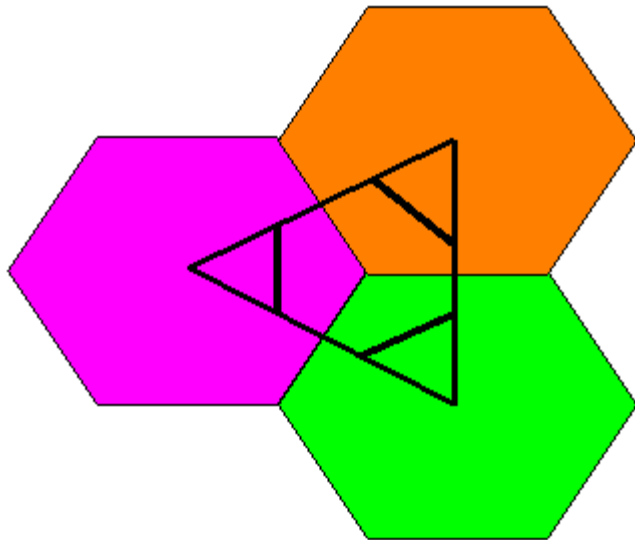
La sphère ?



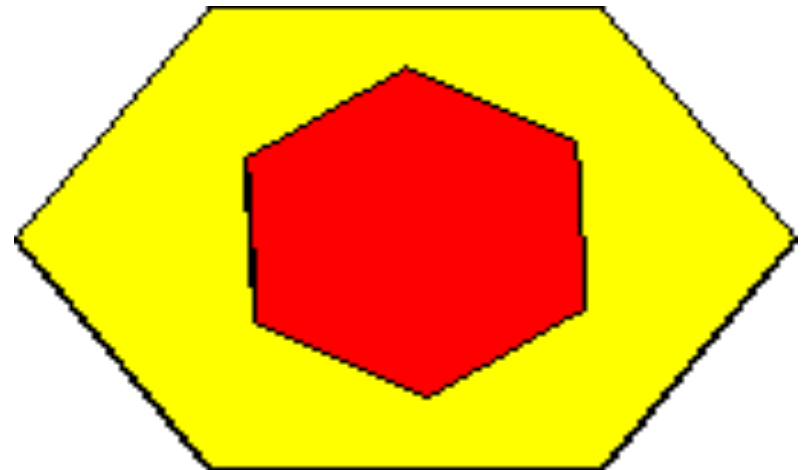
- représente un solide avec l'énergie zéro de déformation
- utilisée dans les algorithmes d'éclairage
- utilisée pour la projection radiale

Les différentes méthodes de subdivision

- Khamlichi-Gourret



- Doncescu-Gourret

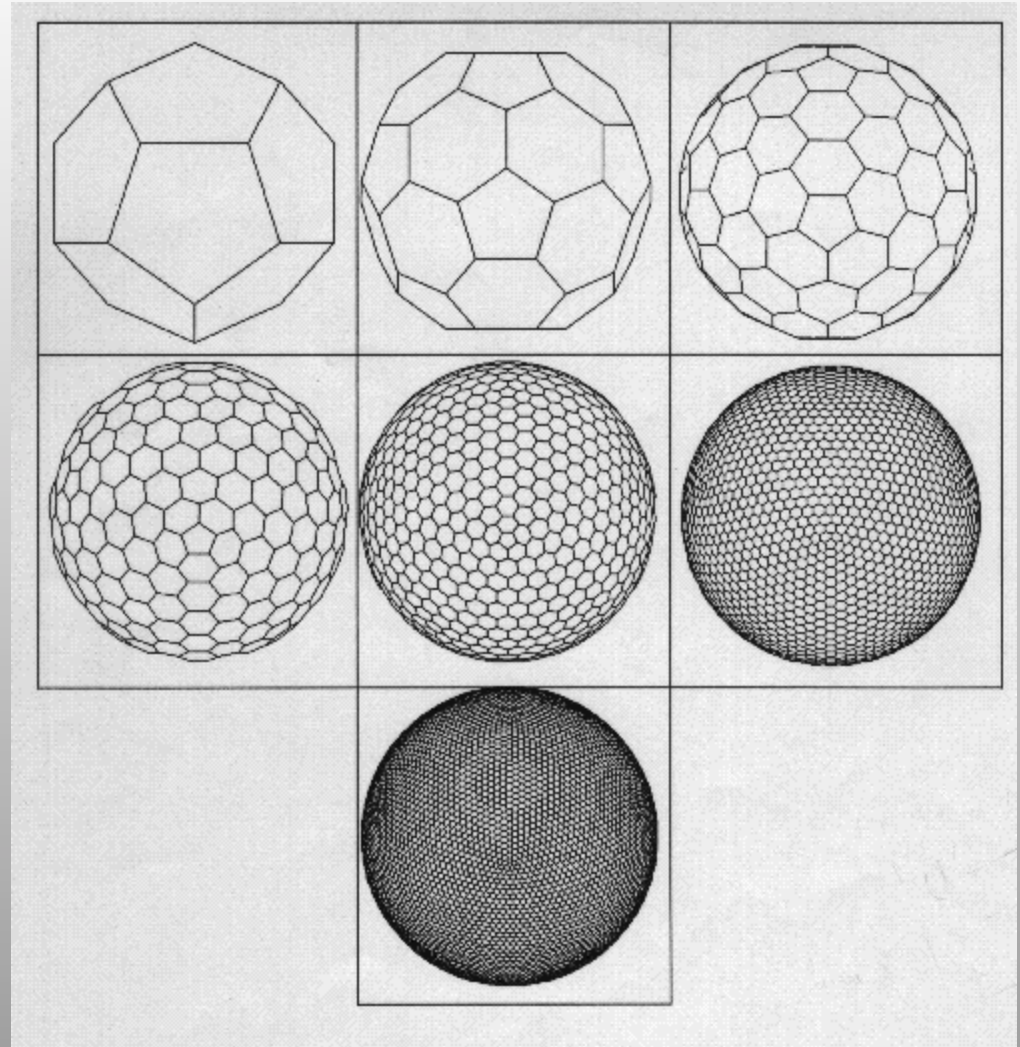


Avantage :

côtés
approximativement
égaux

Inconvénient :

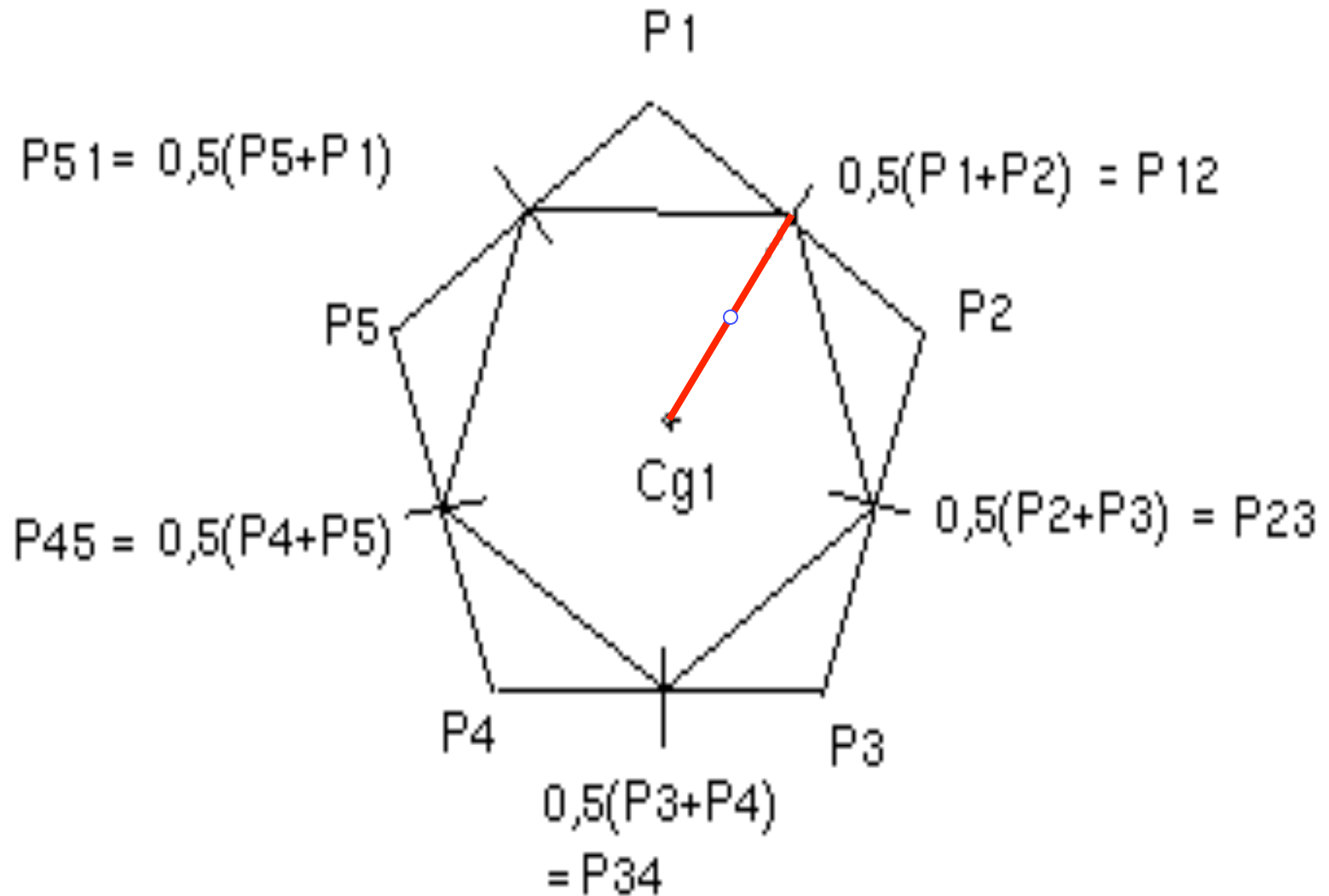
les barycentres ne sont
pas conservés



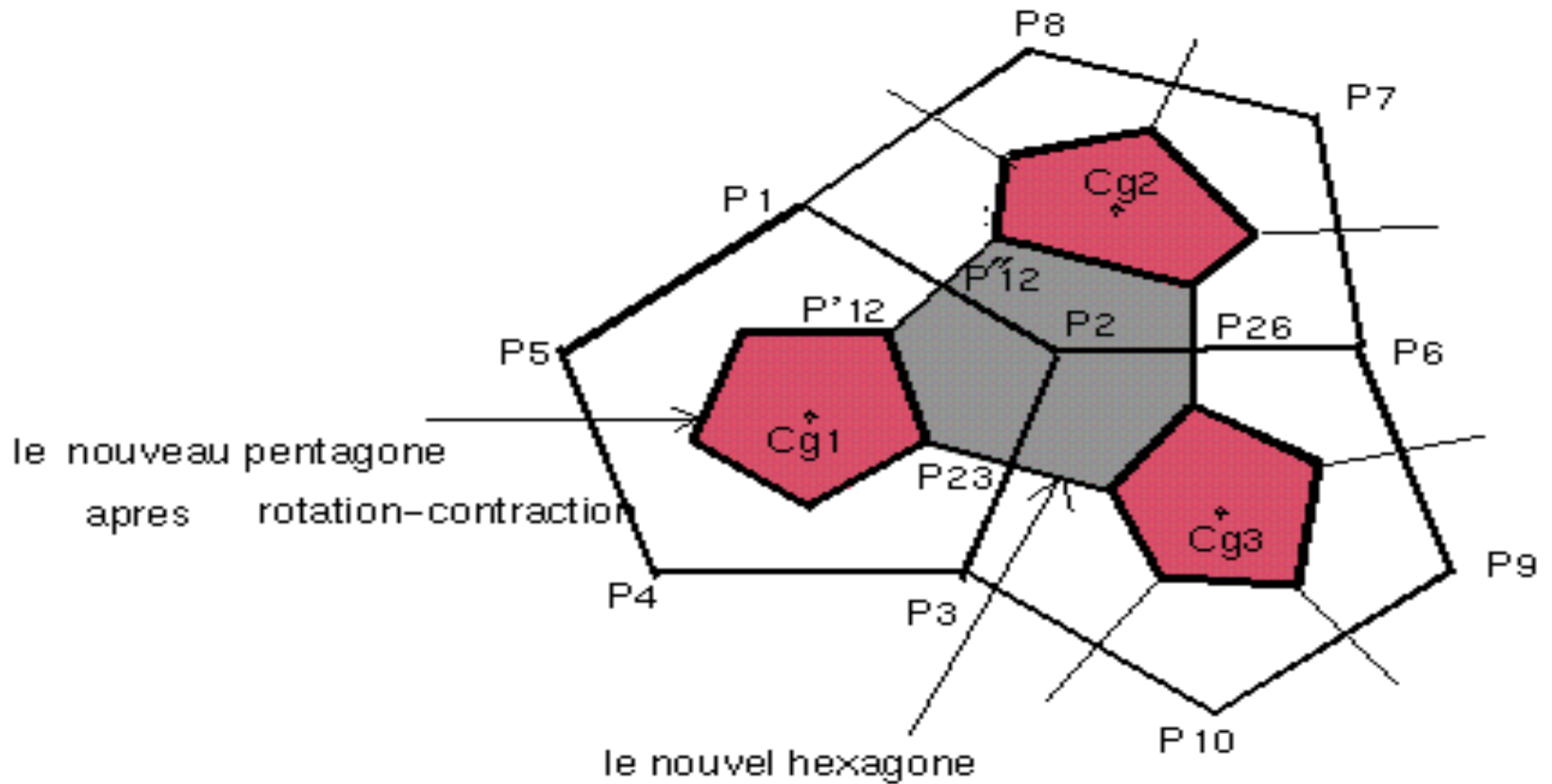
La subdivision par insertion des barycentres

- Linéarisation de la rotation
- Rotation

Première étape : rotation-contraction



Deuxième étape : contraction



Construction d'un hexagone devant chaque noeud

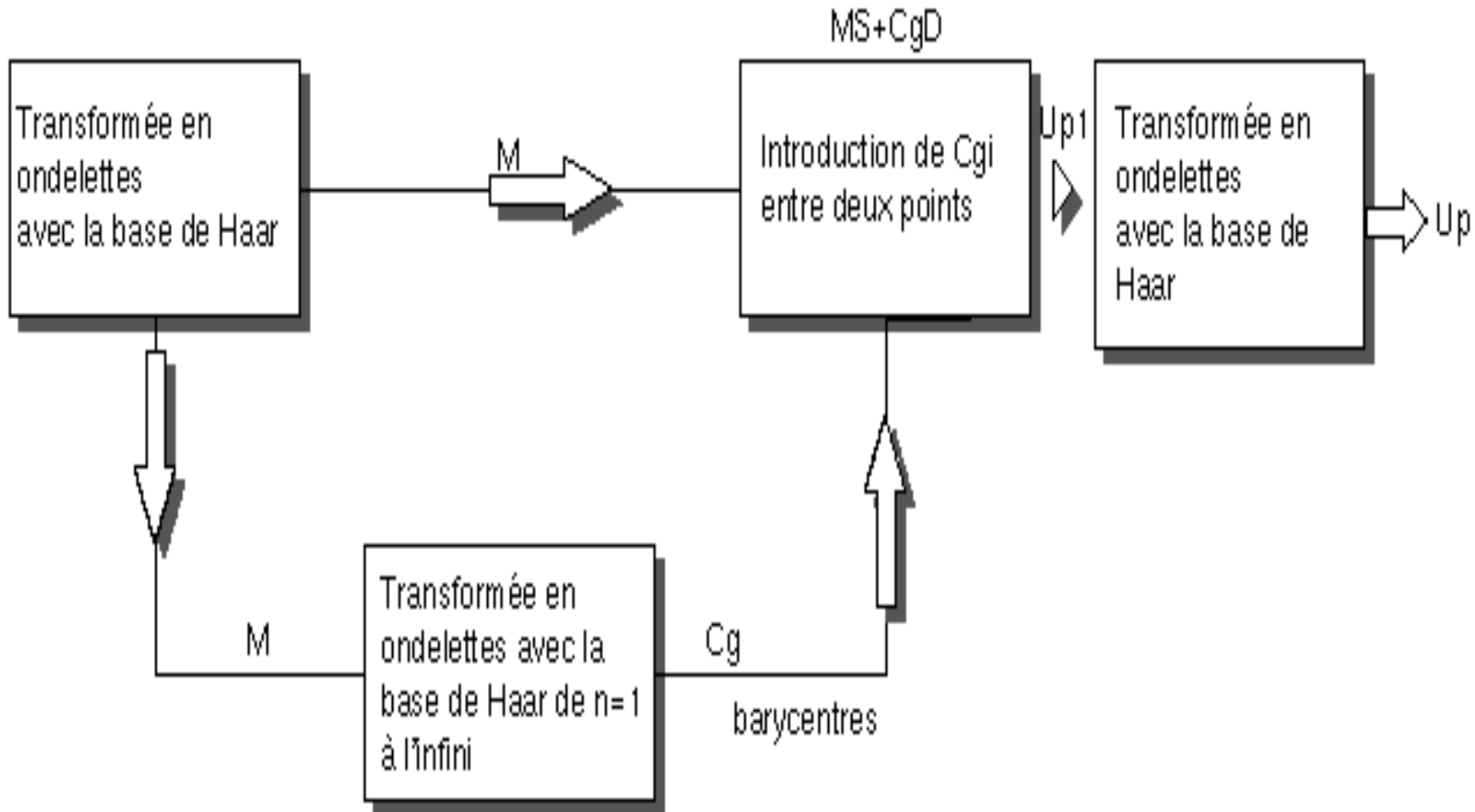
La base de Haar

$$\begin{aligned} \phi(x) &= 1 \text{ pour } x \in [0,1] \\ h(n) &= \int 0.5 \phi(x/2)\phi(x-n)dx \end{aligned} \Rightarrow \begin{cases} h(0)=h(1)=0.5 \text{ et } h(n)=0 \\ g(1)=-g(0)=0.5 \text{ et } g(n)=0 \end{cases}$$

$$H(\omega) = \sum \exp(-i\omega/2)\cos(\omega/2)$$

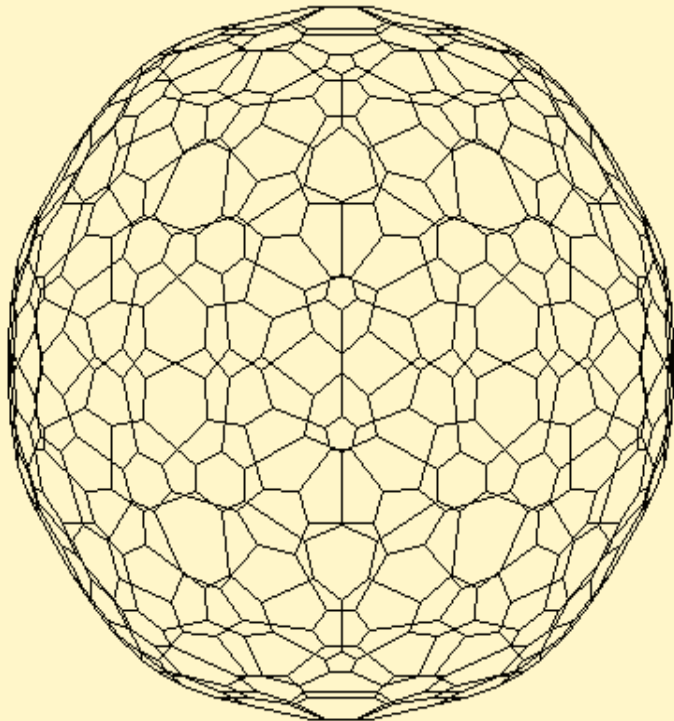
$$G(\omega) = \sum -i \exp(-i\omega/2)\sin(\omega/2)$$

Subdivision des surfaces 3-connexe en utilisant la base de Haar

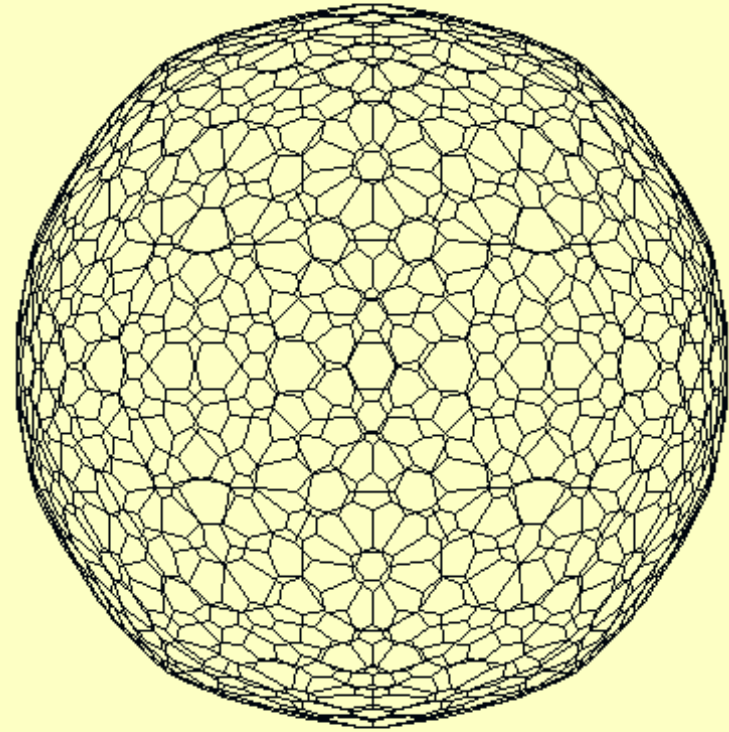


Le résultat de la subdivision par la base de Haar

Sphère : niveau 3 de subdivision



Sphère : niveau 4 de subdivision

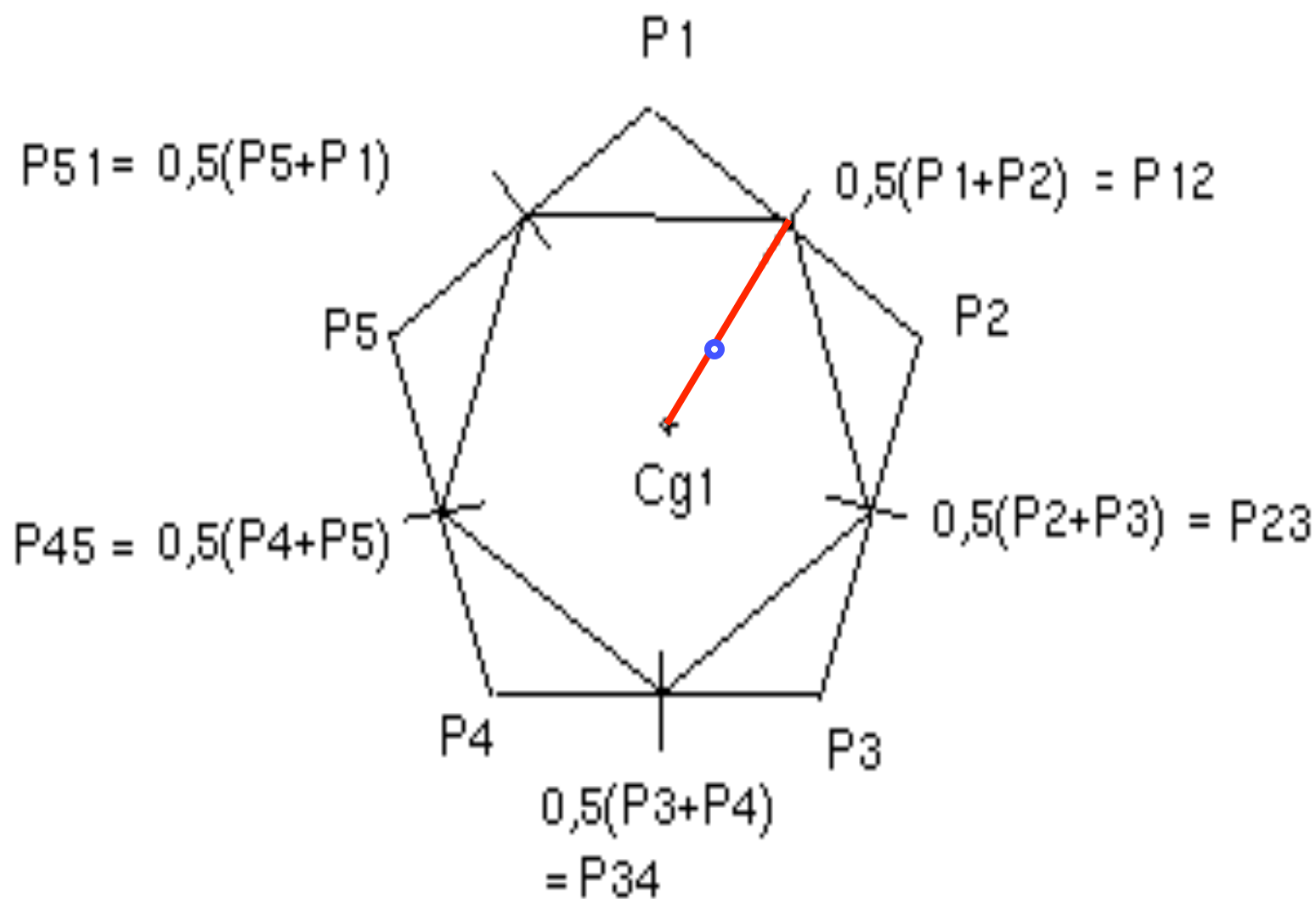


Construction des filtres
associés à une base
d'ondelettes

Un filtre $G(\omega)$ est associé à une base d'ondelettes, si la condition de reconstruction exacte (1) est satisfaite, lorsque le filtre est de type passe-haut (2).

$$|G(\omega)| \times |G(\omega)| + |G(\omega+\pi)| \times |G(\omega+\pi)| = 1 \quad (1)$$

$$G(\pi) = 1 \quad \text{et} \quad G(0) = 0 \quad (2)$$



Application des ondelettes pour la division

$$P_3 = tP_1 + (1-t)P_2$$

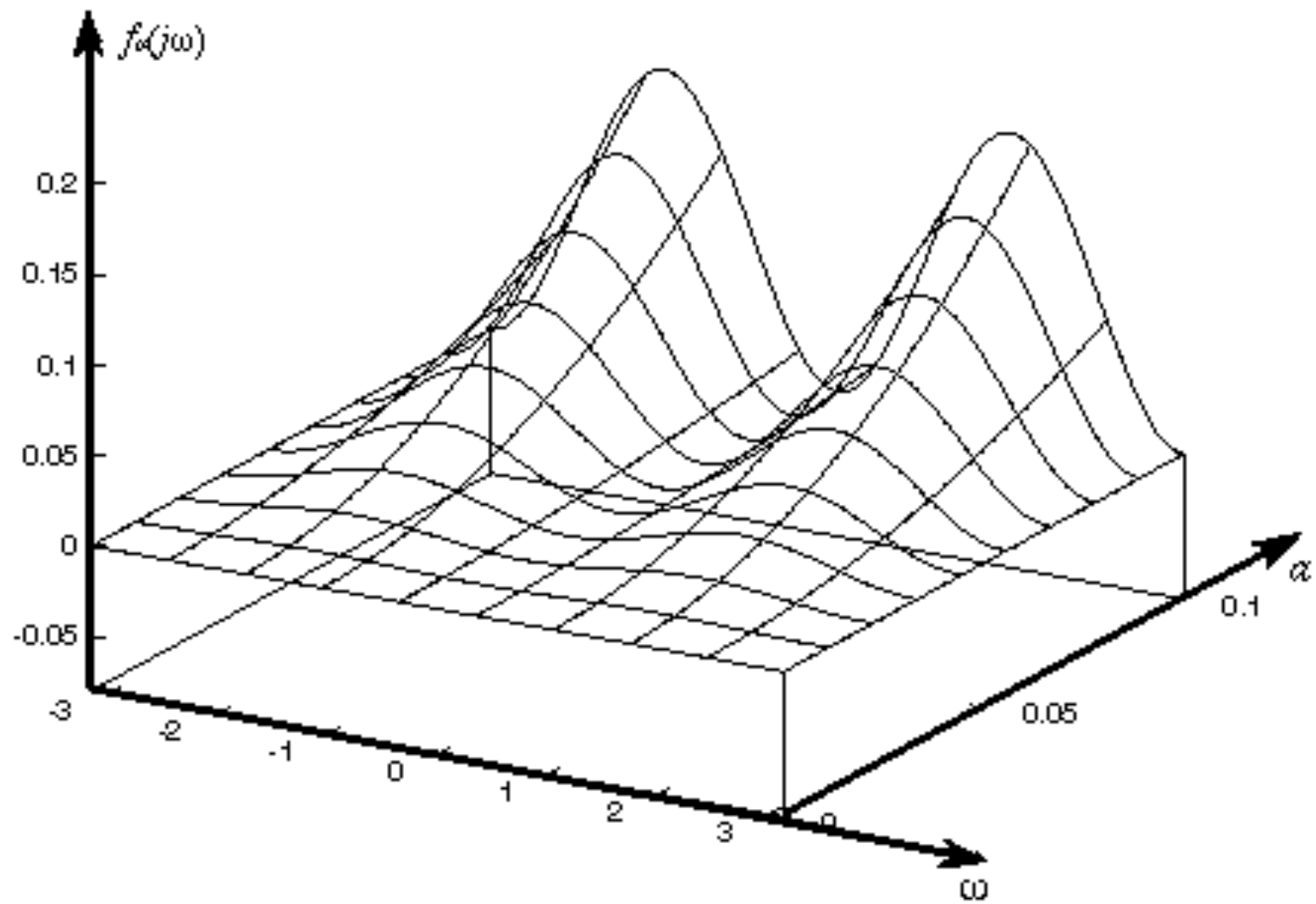
$$P_3 = h(0)P_1 + h(1)P_2$$

$$\begin{aligned}g(0) &= 0.5 - a \\g(1) &= -0.5 - a \\g(2) &= a \\g(3) &= a\end{aligned}$$

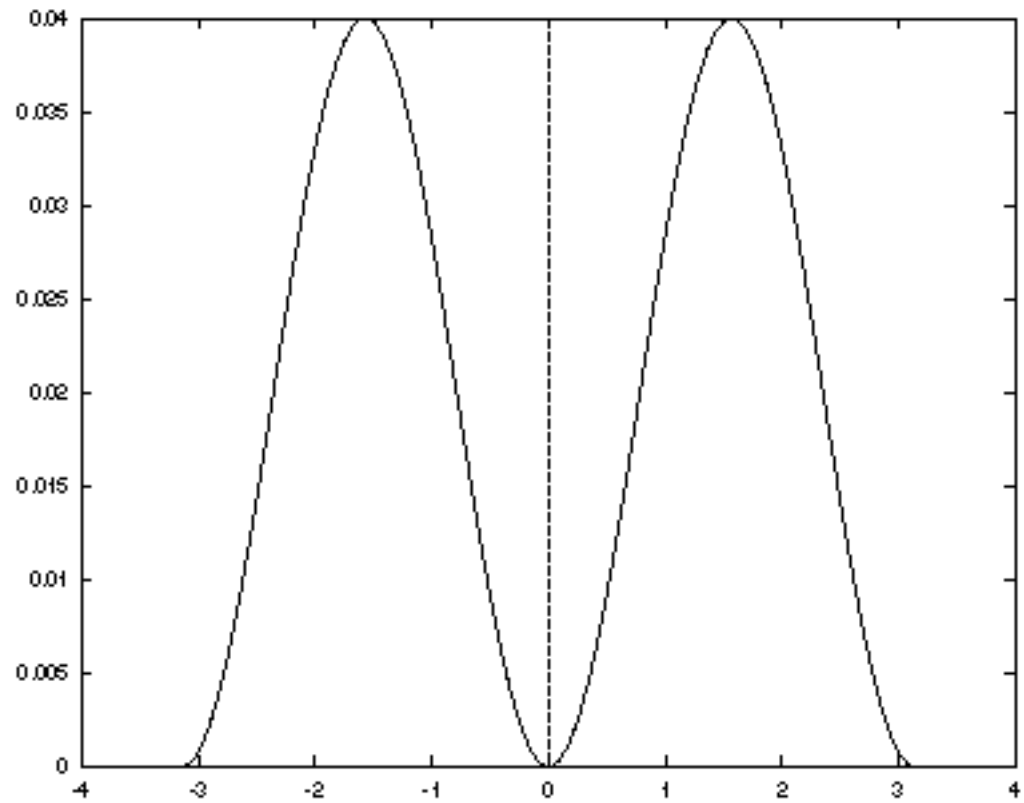


$$\begin{aligned}g(0) &= 0.483 \\g(1) &= -0.517 \\g(2) &= 0.017 \\g(3) &= 0.017\end{aligned}$$

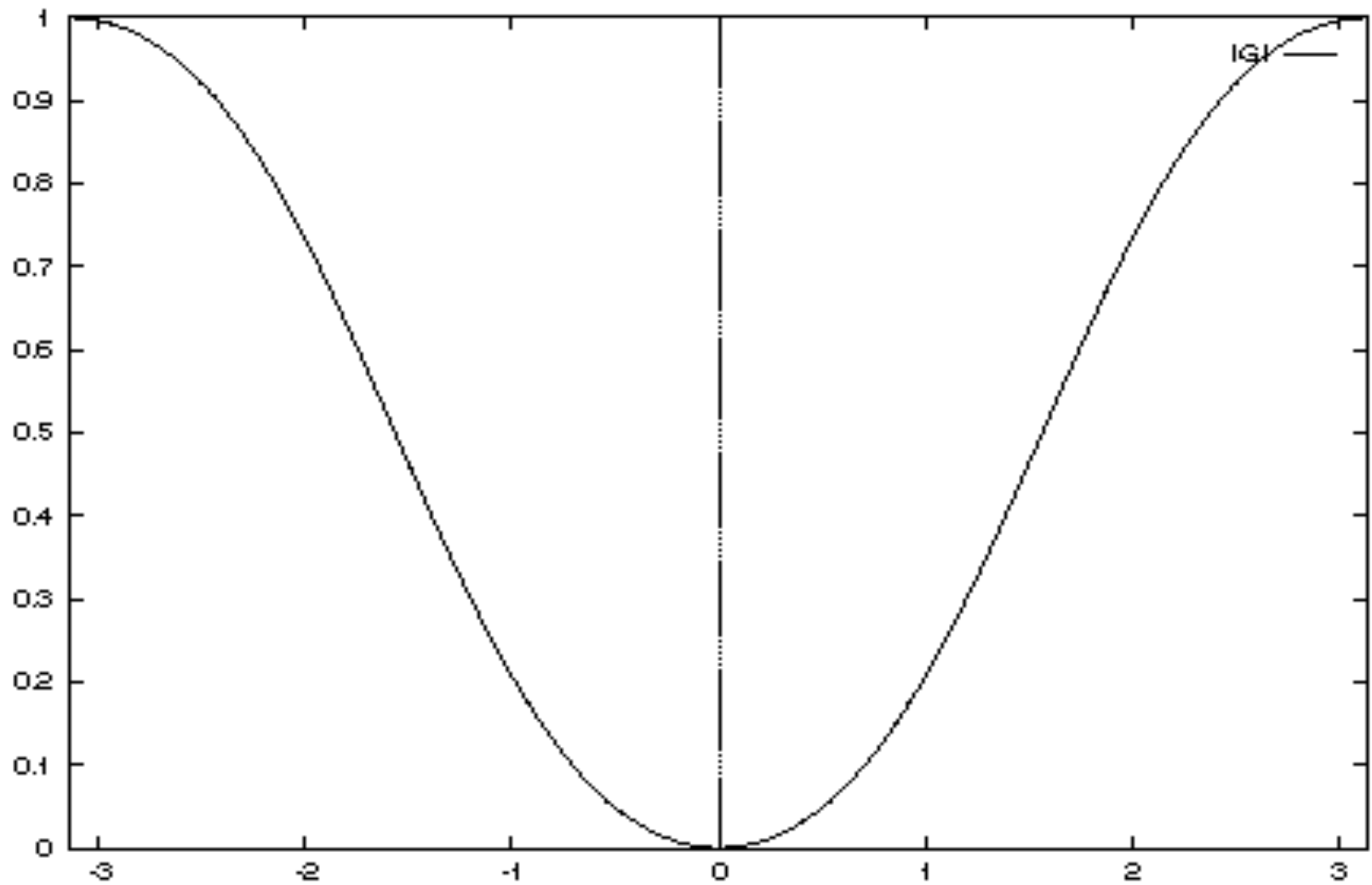
$$\begin{aligned}h(0) &= -0.017 \\h(1) &= 0.017 \\h(2) &= 0.517 \\h(3) &= 0.483\end{aligned}$$



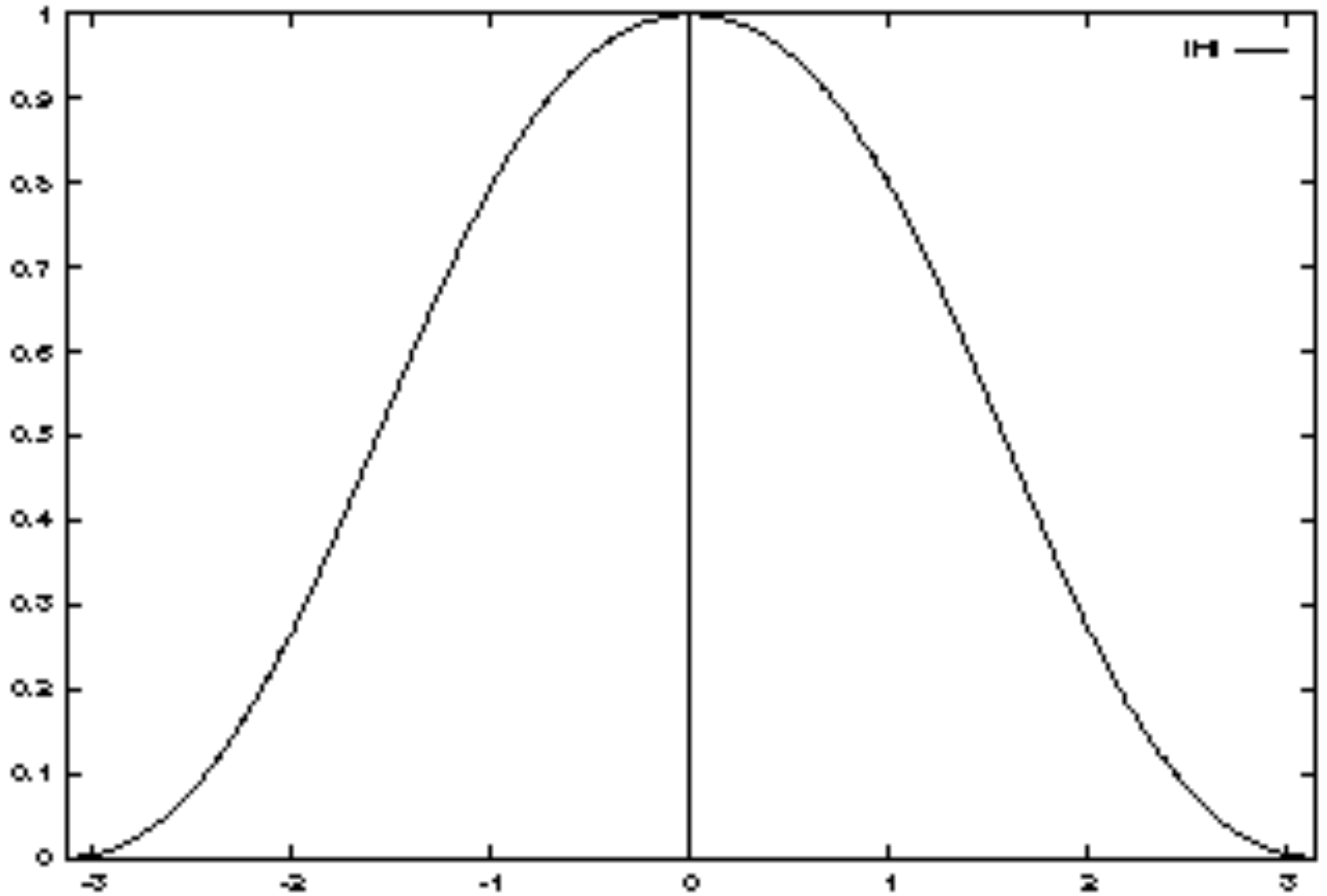
L'erreur de reconstruction



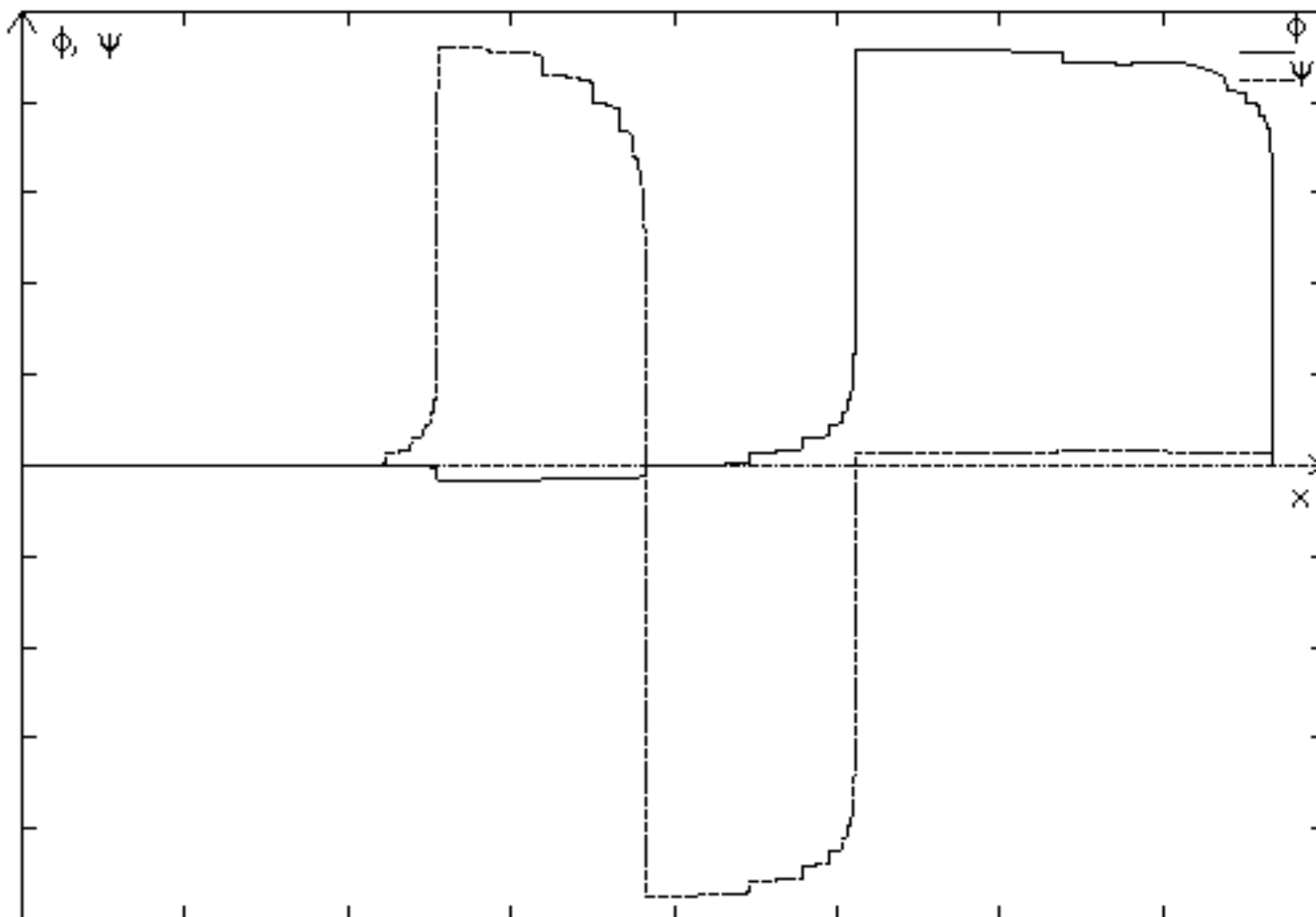
L'erreur de reconstruction pour $a=0.017$



Le module du filtre passe-haut G pour $a=0.017$

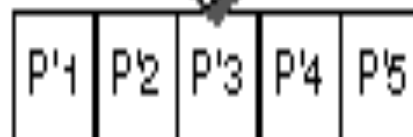
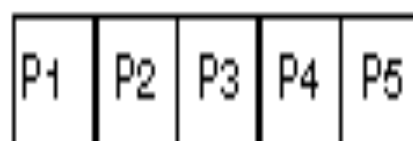
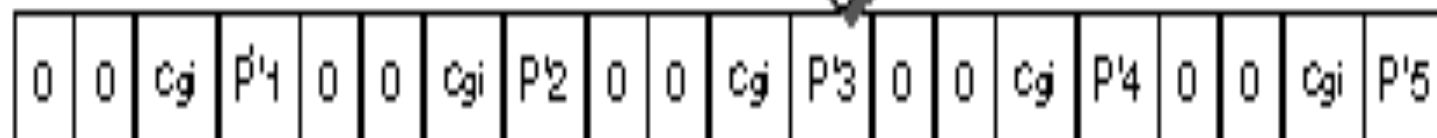
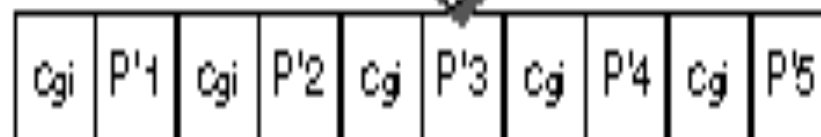
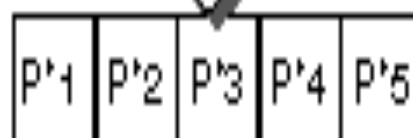


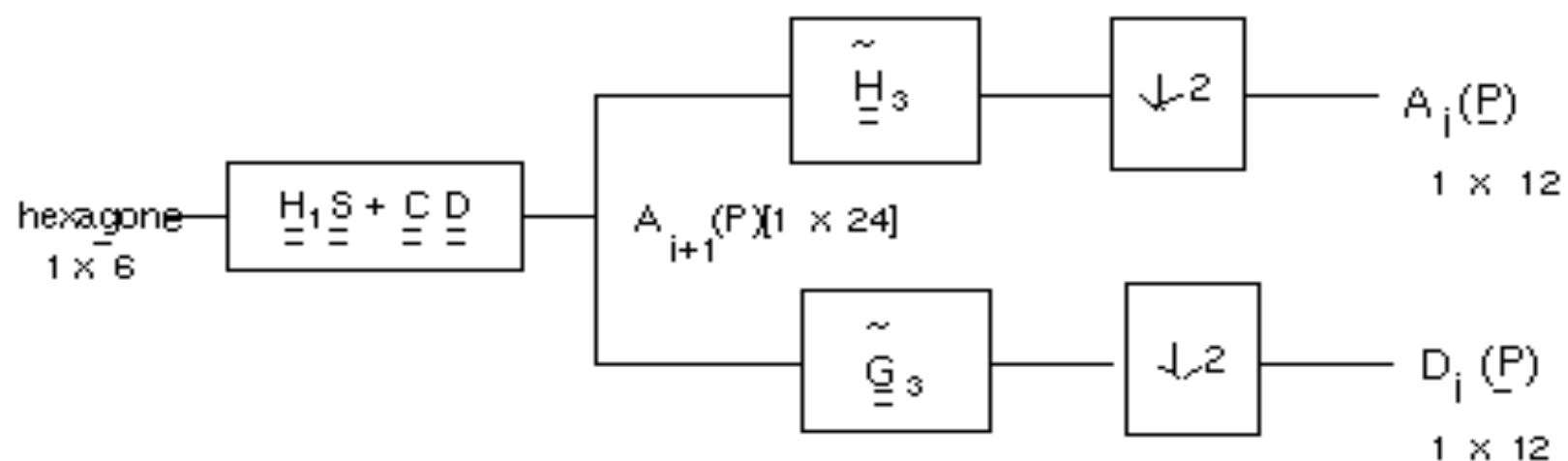
Le module du filtre passe-bas H pour $a=0.017$



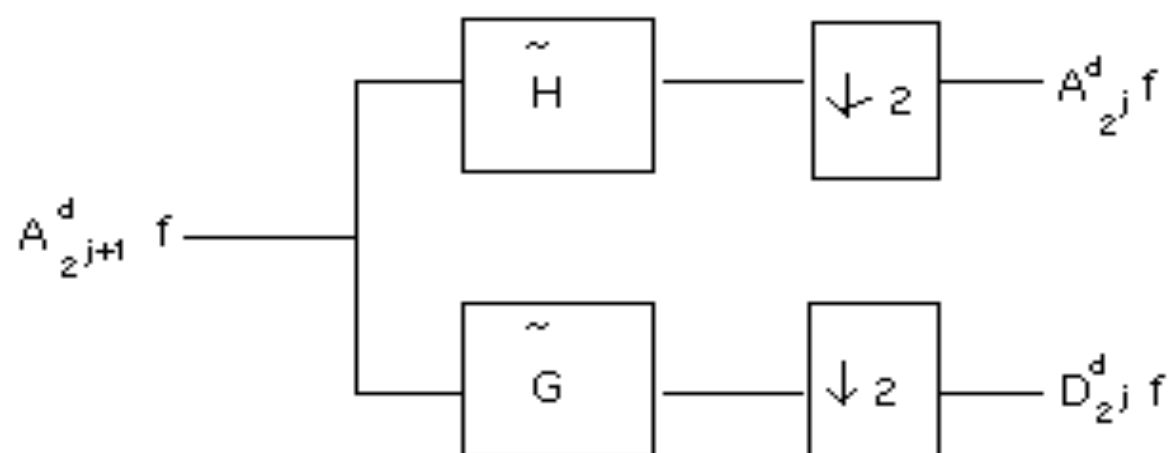
Les fonctions φ et ψ obtenues par l'algorithme en cascade

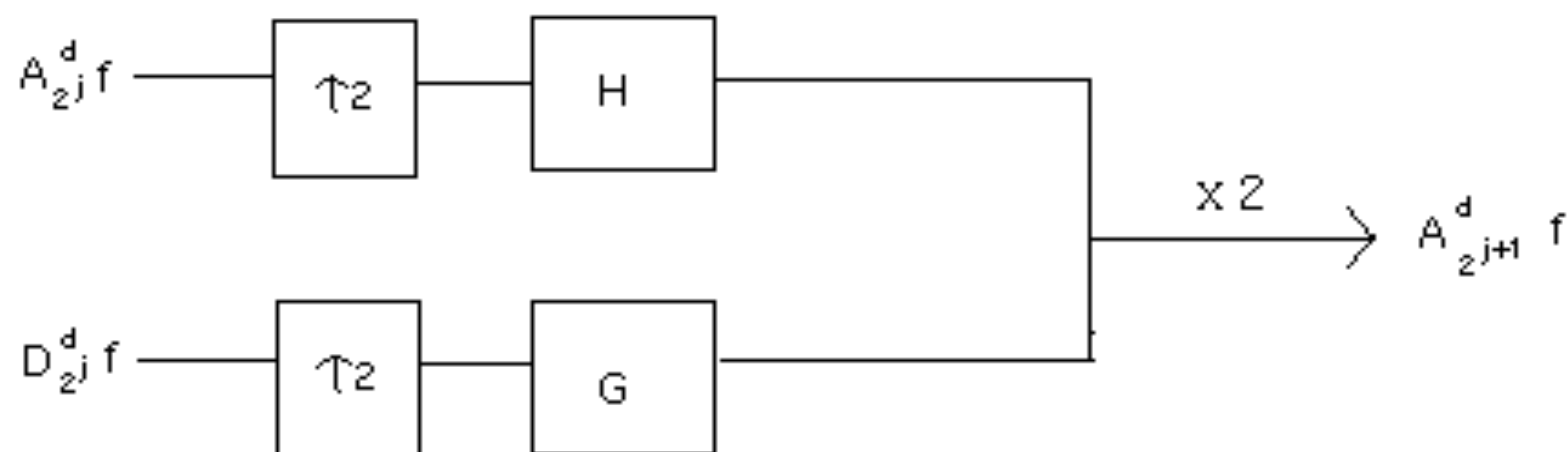
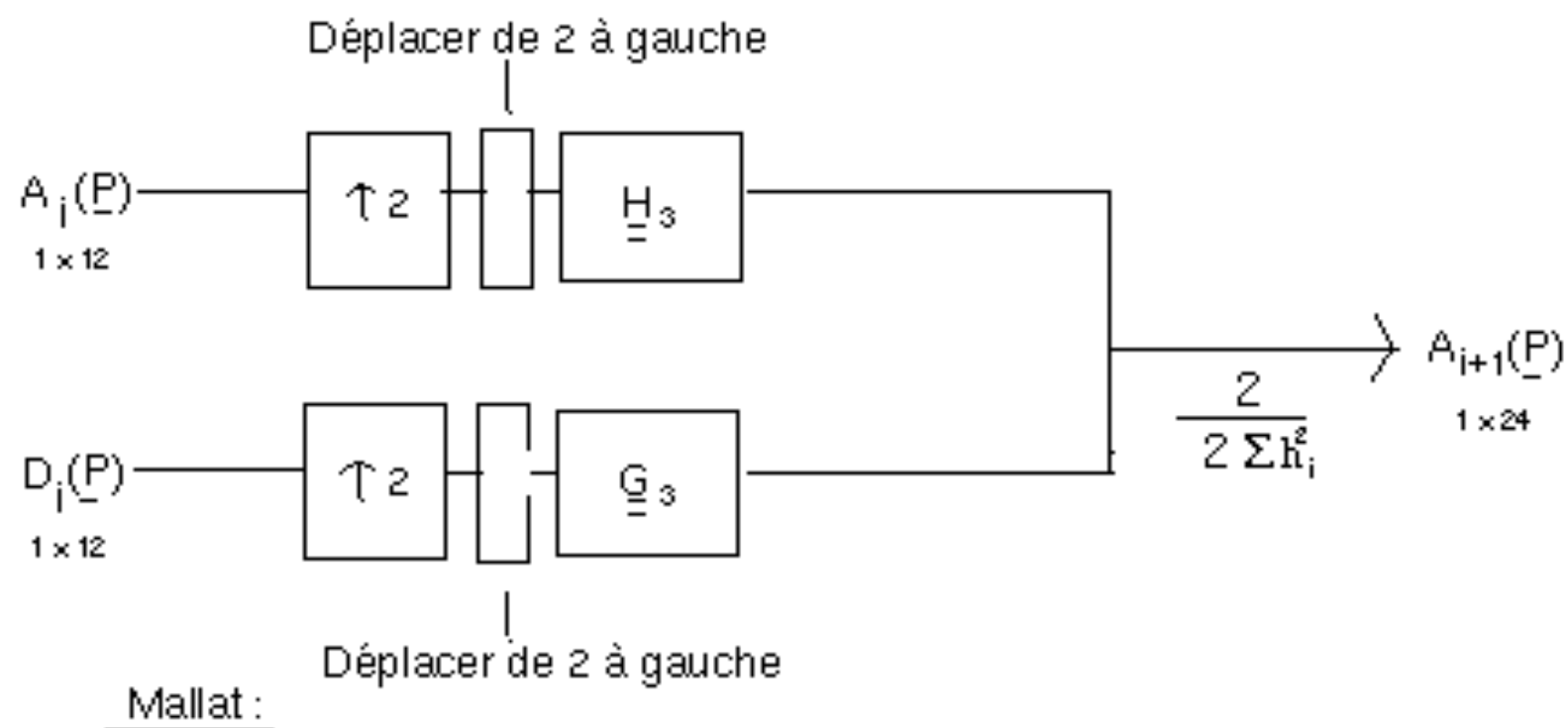
Implémentation

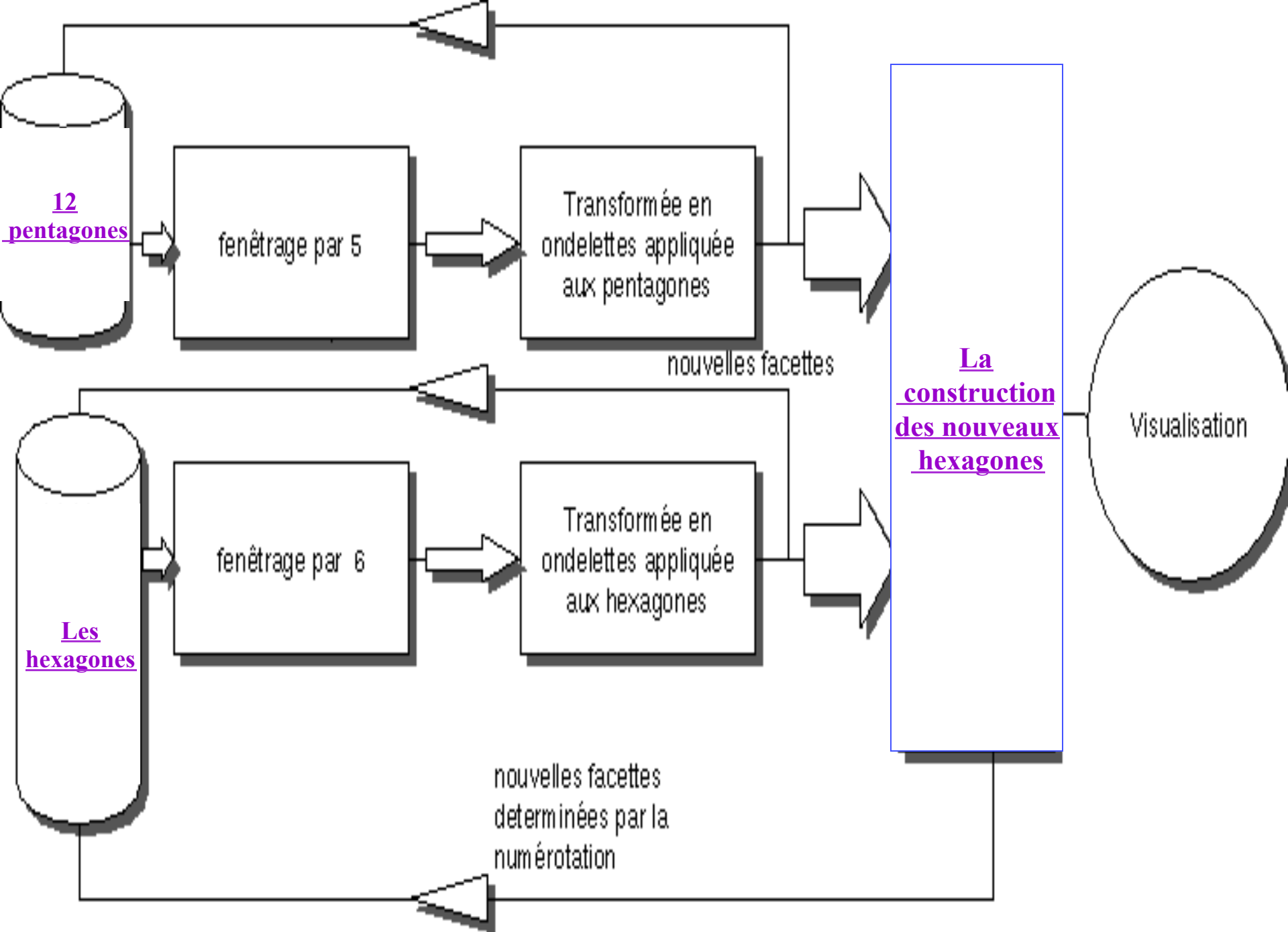
$\underline{A}_i(P)$  $\underline{A}_i(P) (\underline{H}_i \underline{S} + \underline{C} \underline{D})$  $\underline{A}_{i+1}(P)$ 



Mallat :







12 pentagones

fenêtrage par 5

Transformée en ondelettes appliquée aux pentagones

nouvelles facettes

La construction des nouveaux hexagones

Visualisation

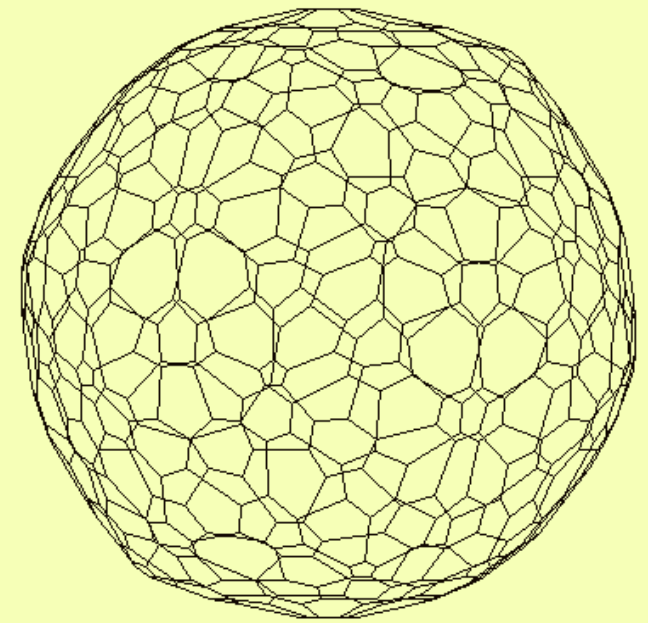
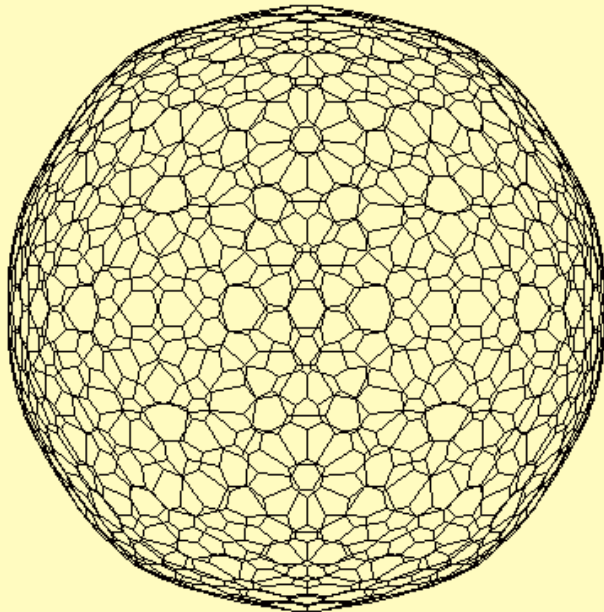
Les hexagones

fenêtrage par 6

Transformée en ondelettes appliquée aux hexagones

nouvelles facettes déterminées par la numérotation

Le niveau 3 de subdivision



Le niveau 4 de subdivision

Subdivision par les ondelettes
rotationnelles

$$y = Hx$$

$$z = Gx$$

où

$$H = \begin{pmatrix} h_0 & h_1 & h_2 & h_3 & 0 & 0 & 0 & 0 \\ \dots & 0 & 0 & h_0 & h_1 & h_2 & h_3 & 0 & 0 \\ \dots & 0 & 0 & 0 & 0 & h_0 & h_1 & h_2 & h_3 \\ \dots & & & & & & & & \end{pmatrix}$$

La reconstruction du signal x_i est obtenue :

$$x_i = \sum_k h_{i-2k} y_k + g_{i-2k} z_k$$

sous la forme matricielle, elle devient :

$$x = H^T y + G^T z$$

où H^T et G^T sont les matrices transposées.

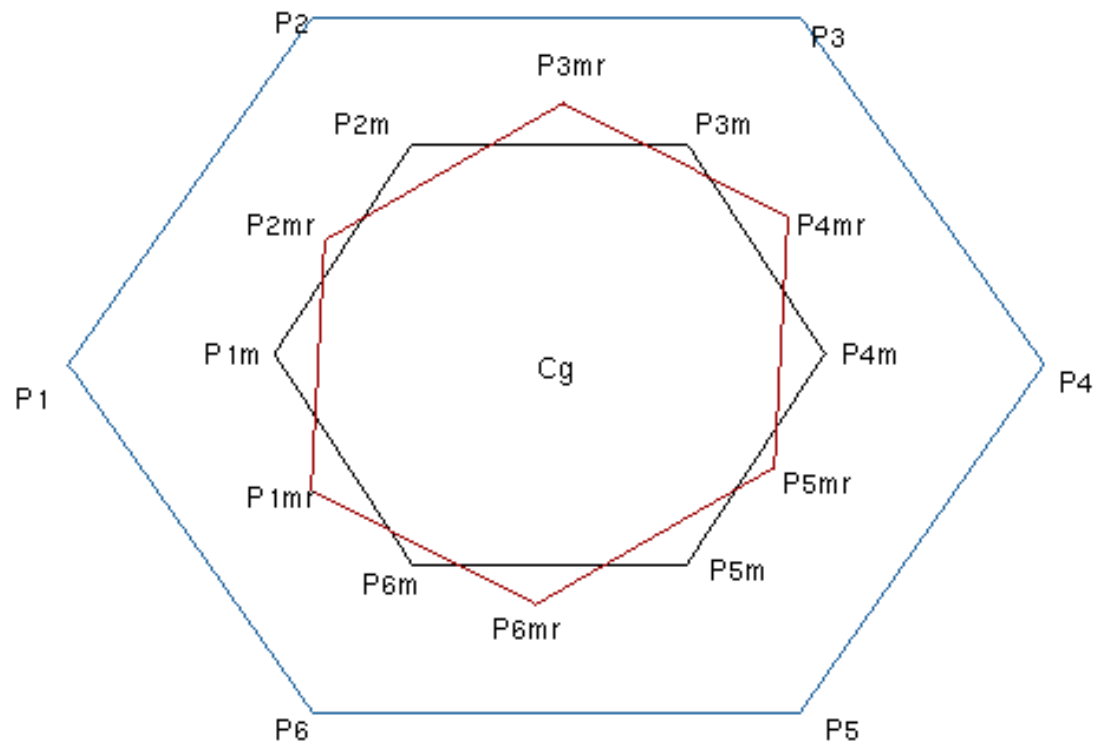
Les conditions sur les filtres peuvent s'écrire sur la forme matricielle :

- $H^T H + G^T G = I$

- $GH^T = HG^T = 0$

- $HH^T = GG^T = I$

- $H^T R^T R H + G^T R^T R G = I$



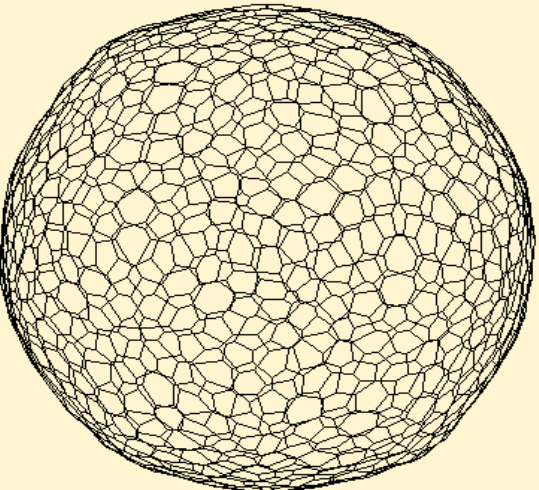
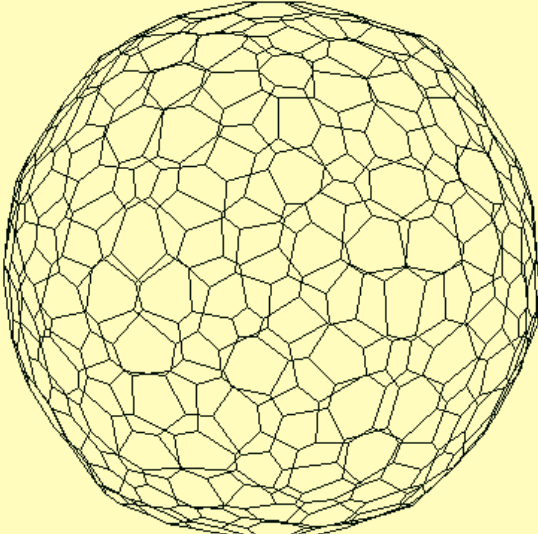
$$P1m = h0Cg + h1P1$$

$$h0 = 0.5$$

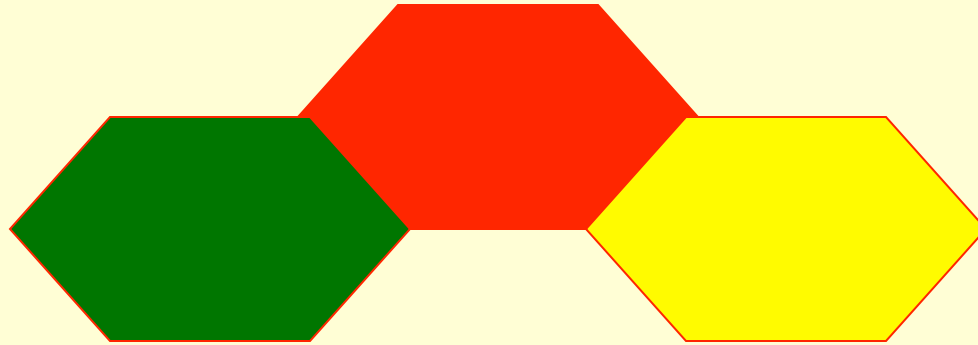
$$h1 = 0.5$$

Subdivision using rotational wavelets

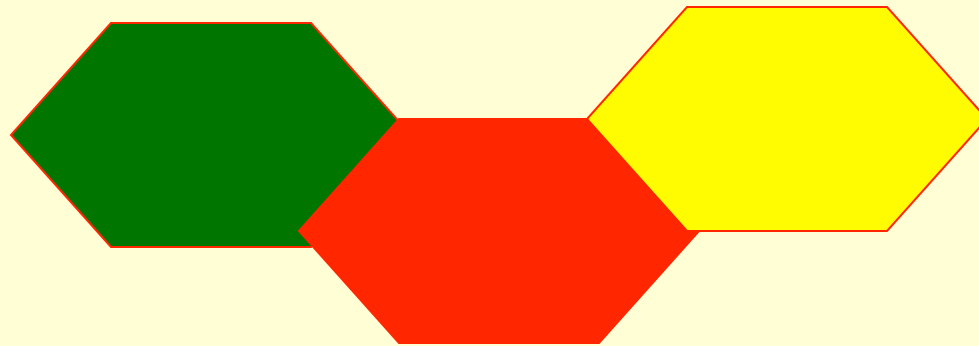
Level 3 of subdivision →



← Level 4 of subdivision



Construction de la base de données



Geometrical Parameters

Vertices

- $V_0 = 20$ (level 0)
- $V_1 = 60$ (level 1)
- $V_i = 3 * V_{i-1} = 20 * 3^i$ (level i)

Pentagons

- $P_i = 12$ (level i)

Hexagons

- $H_0 = 0$ (level 0)
- $H_1 = 20$ (level 1)
- $H_i = H_{i-1} + V_{i-1} = 10 (3^i - 1)$ (level i)

Edges

- $E_i = 10 * 3^{i+1}$ (level i)

Noeuds:

$V_0 = 20$ nombre de noeuds pour le dodécaèdre (niveau 0)

$V_1 = 60$ nombre de noeuds pour l'icosaèdre tronqué (niveau 1)

$V_i = 6V_{i-2} + V_{i-1} = 3V_{i-1}$ (niveau $i \geq 2$)

$V_i = 20 \times 3^i$.

Facettes:

Pentagones

$P_i = 12$ niveau $i = 0, 1, 2, \dots$

Hexagones

$H_0 = 0$ (niveau 0)

$H_1 = 20$ (niveau 1)

$H_i = 60 + 6H_{i-2} + H_{i-1}$ (niveau $i \geq 2$)

$H_i = V_{i-1} + H_{i-1} = 20 \cdot 3^{i-1} + H_{i-1}$

$H_i = 20(3^{i-1} + 3^{i-2} + \dots + 1) = 20 \frac{3^i - 1}{3 - 1} = 10(3^i - 1)$

côtés:

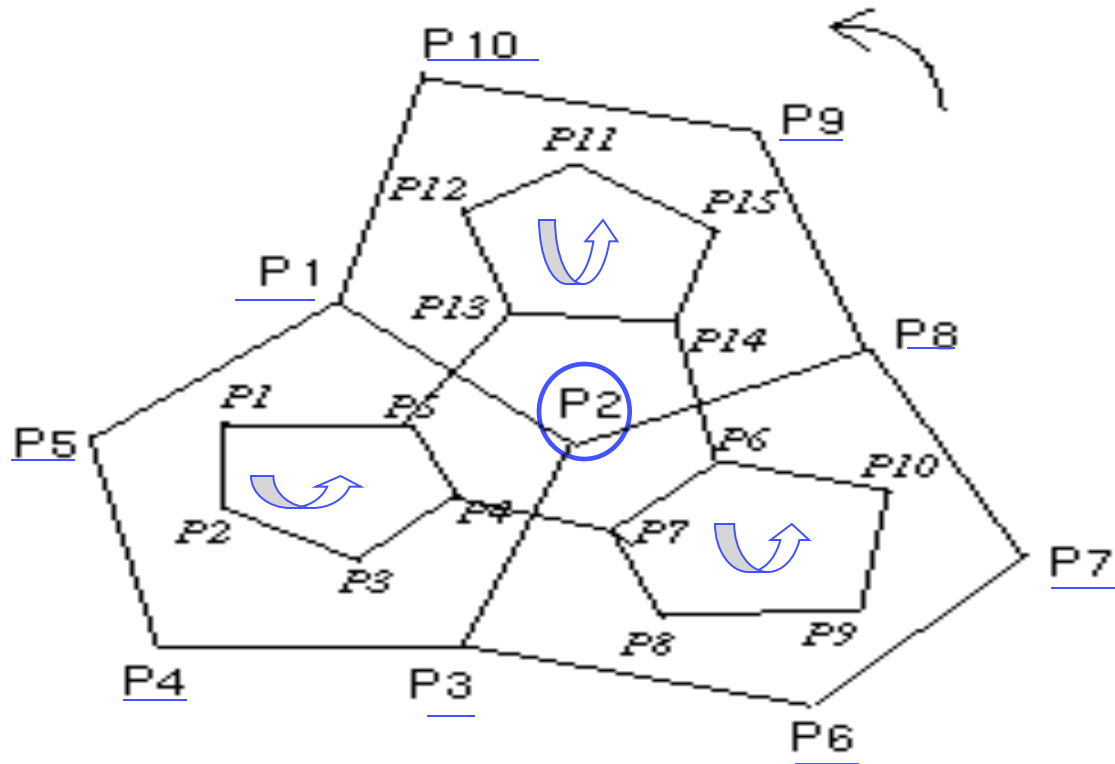
générées par la relation d'Euler (utilisable par la sphère qui est une surface de genre 0):

$V_i - E_i + F_i = 2$

qui produit $E_i = V_i + F_i - 2$ avec $F_i = P_i + H_i$

$E_i = 20 \cdot 3^i + 12 + 10(3^i - 1) - 2 = 10 \times 3^{i+1}$

Vertices Numbering



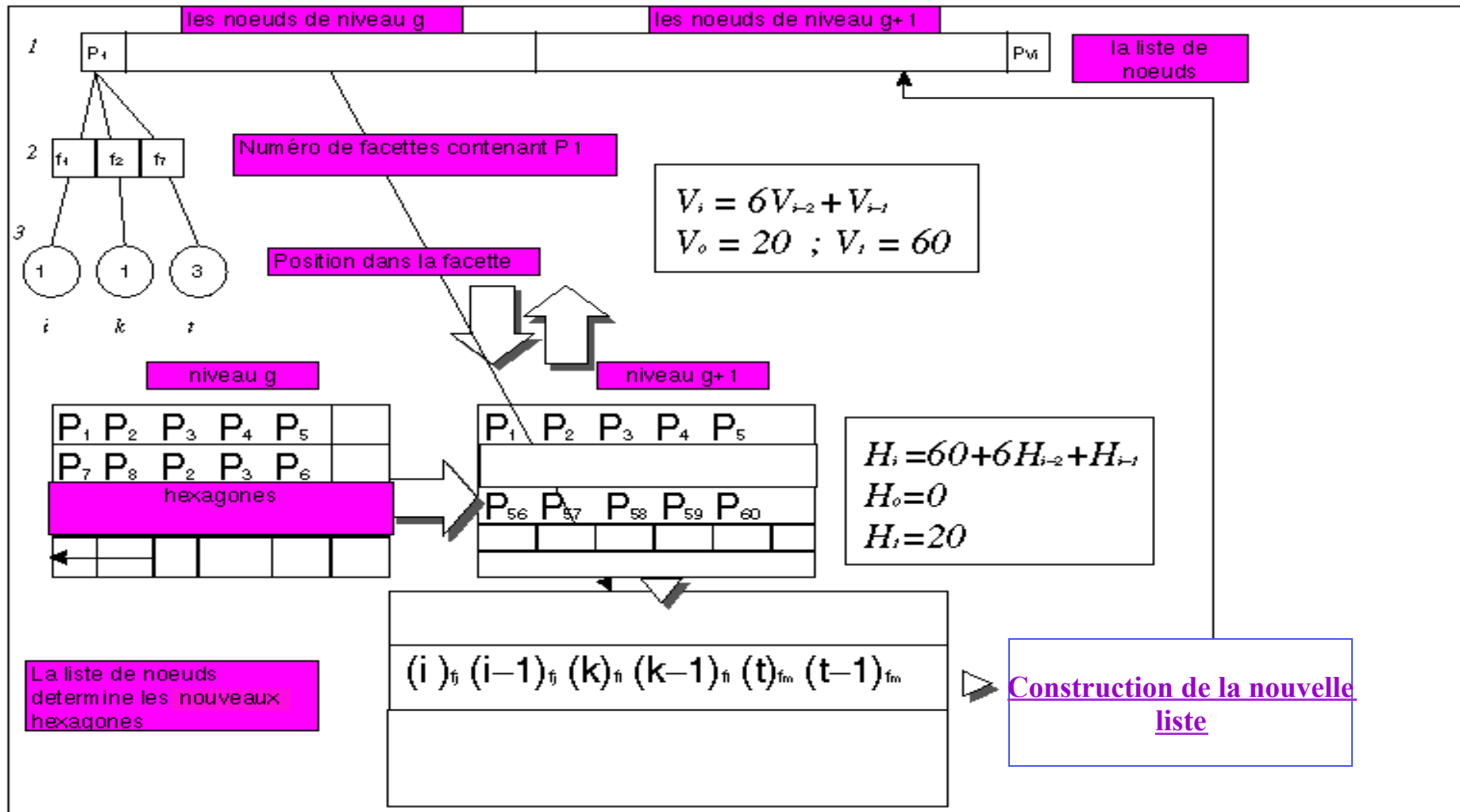
P1	P5	P4	P3	P2
P8	P2	P3	P6	P7
P9	P10	P1	P2	P8



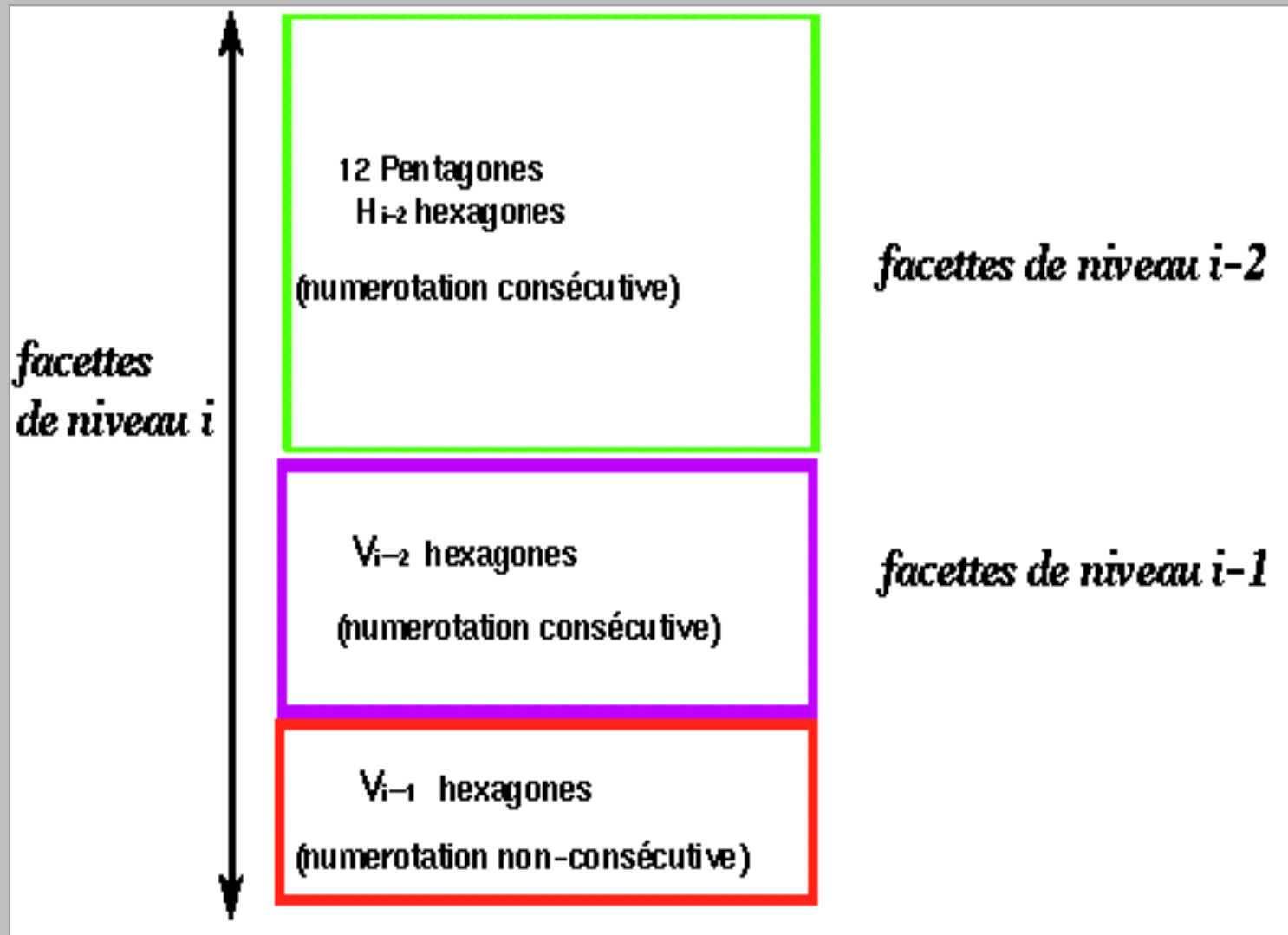
P1	P2	P3	P4	P5
P6	P7	P8	P9	P10
P11	P12	P13	P14	P15

P5 P4 P7 P6 P14 P13

Construction de la numérotation des points

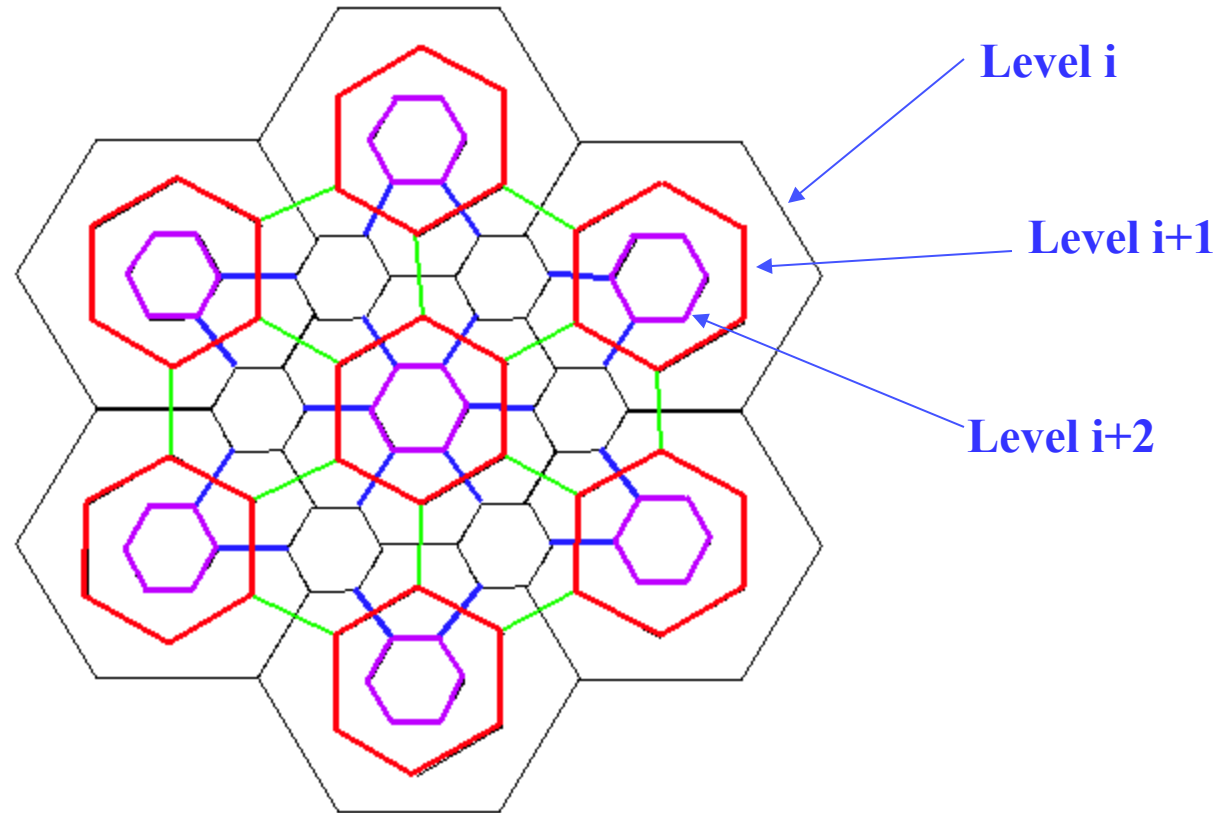


L'organisation du fichier contenant les points



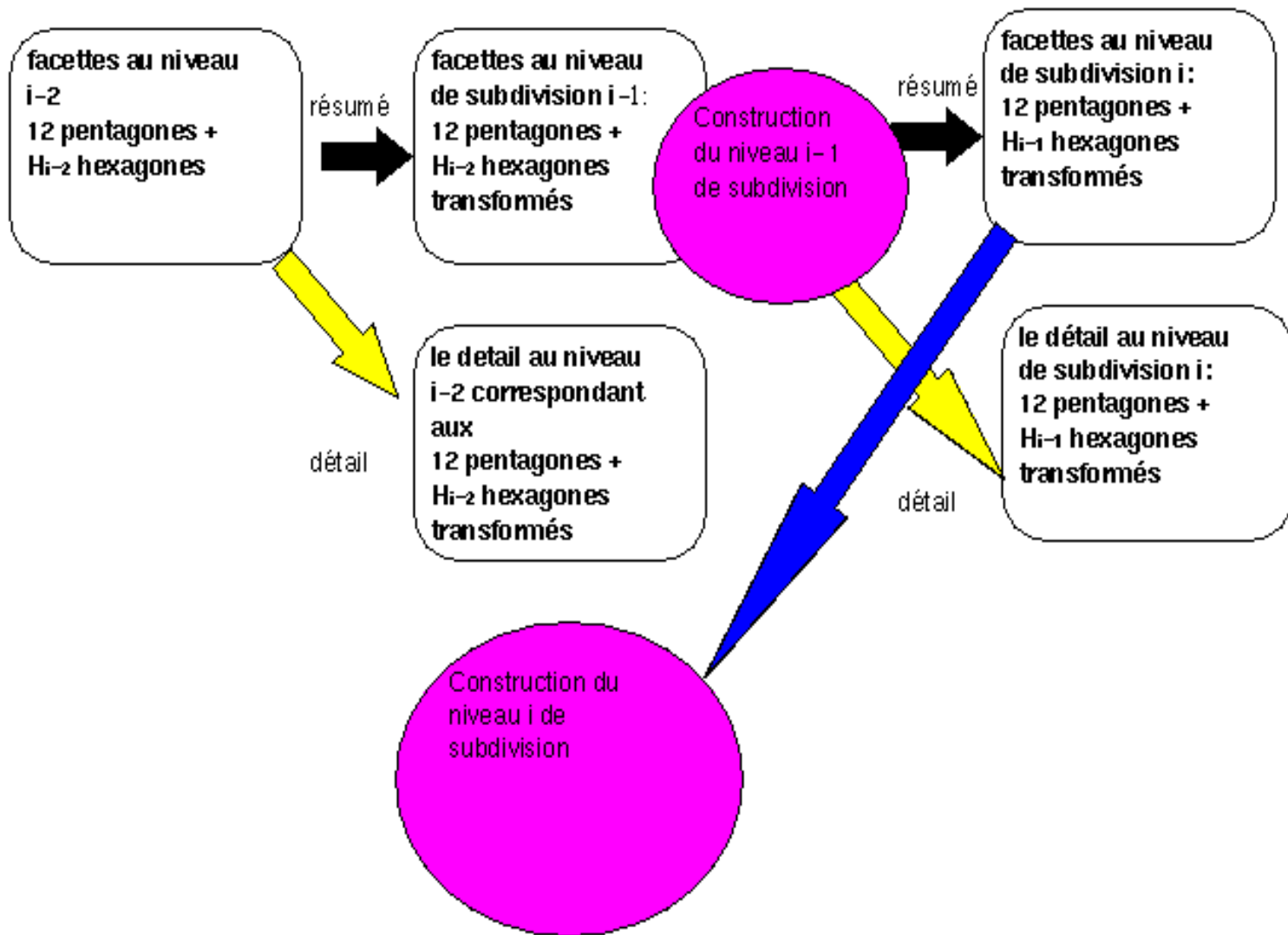
La transformée inverse

The Inverse Transform



Originality : 3 LEVELS OF SUBDIVISION

**Transformée
en
ondelettes
directe**



résumé

facettes au niveau
de subdivision i
12 pentagones +
 H_{i-1} hexagones
transformés



Découpage du
fichier résumé:
12 pentagones
+
 H_{i-2} hexagones



**Transformée
en
ondelettes
inverse**

détail

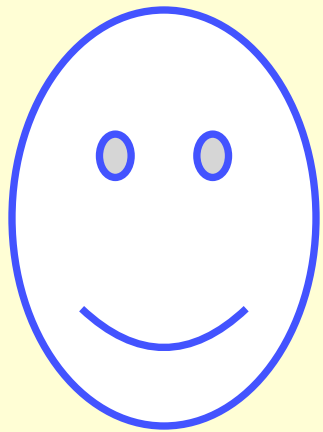
au niveau de
subdivision i :
12 pentagones +
 H_{i-1} hexagones
transformés



Découpage du
fichier détail:
12 pentagones
+
 H_{i-2} hexagones



Construction
du niveau $i-1$
de subdivision



Modélisation du visage par le maillage 3-connexe



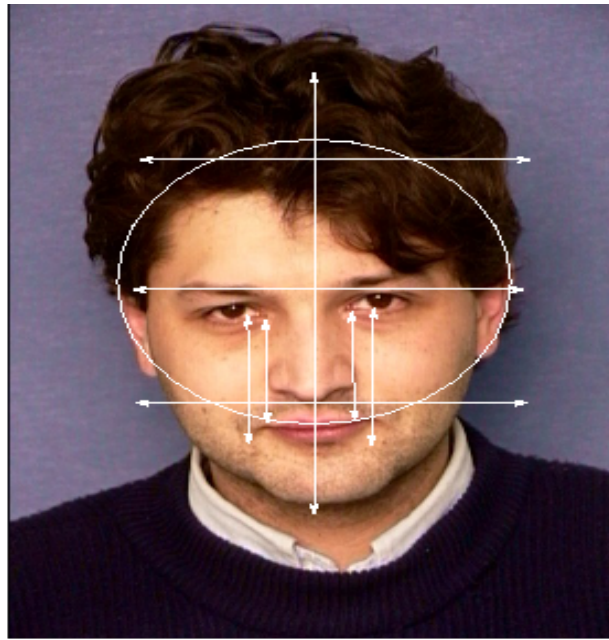
Sphere
Convexe Radiale

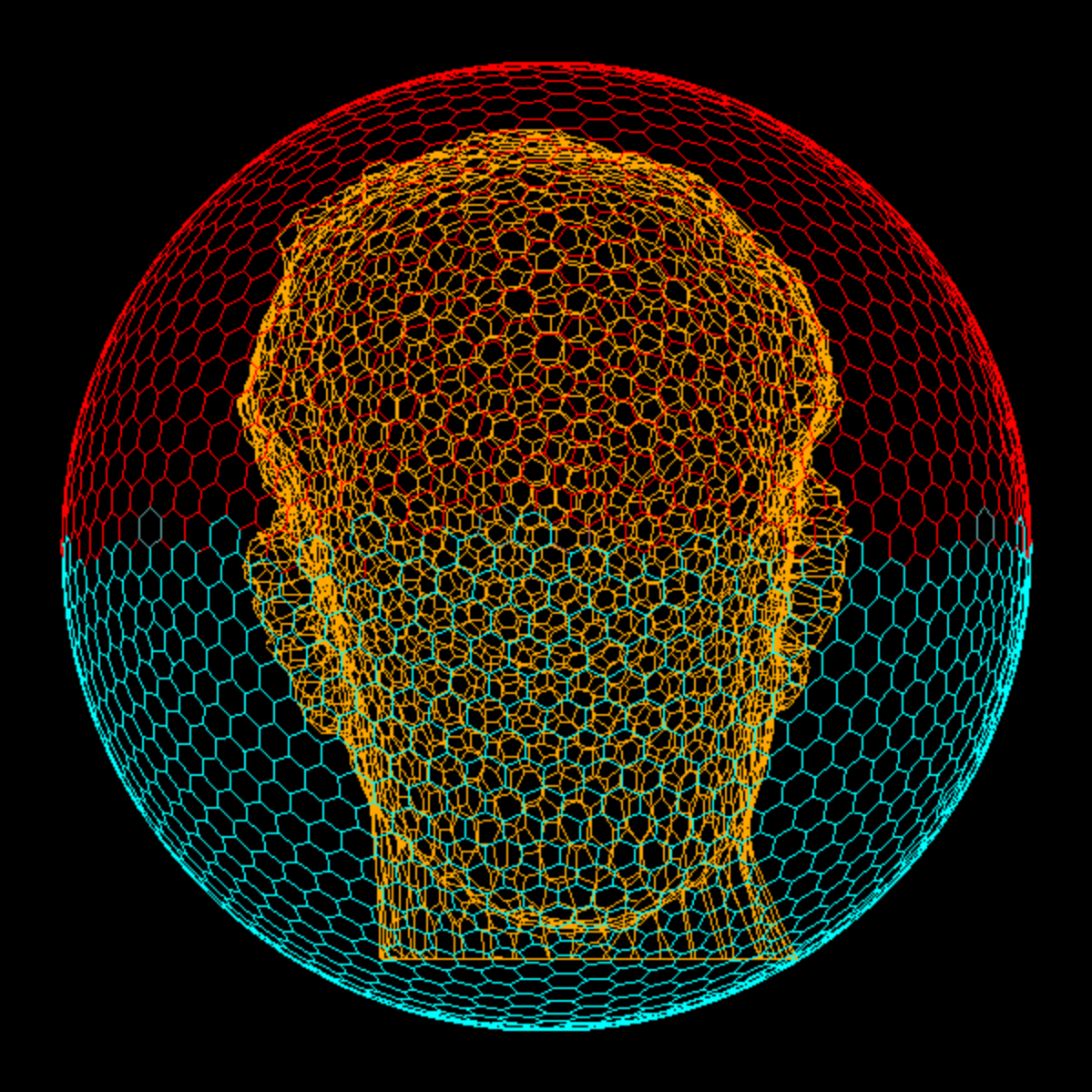


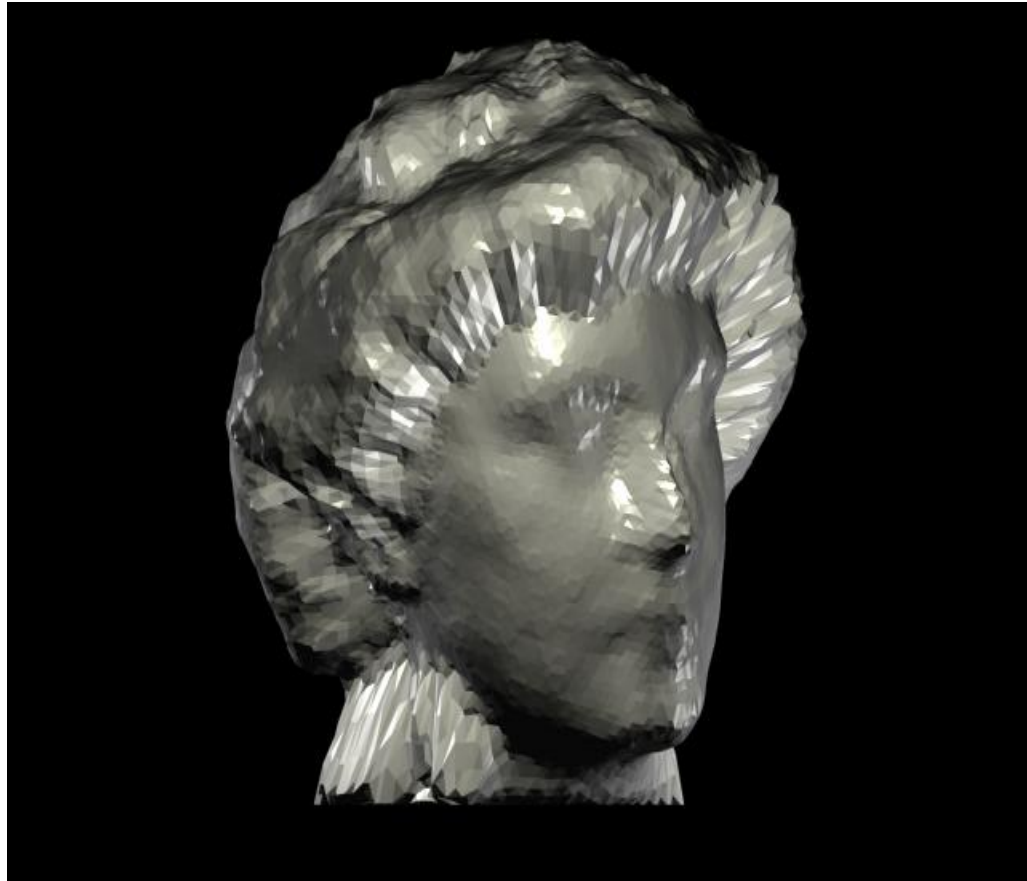
Non Convexe
Radiale



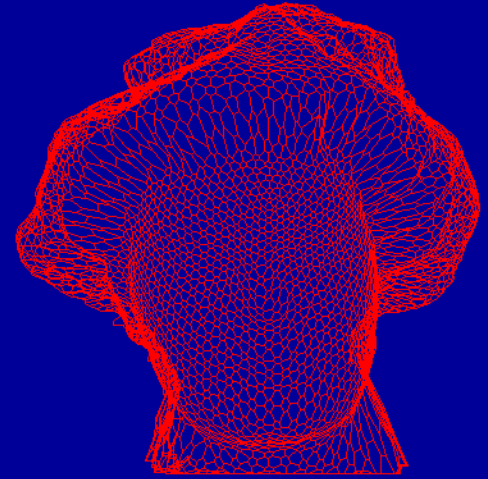
Tore
Non Convexe
Non Radiale



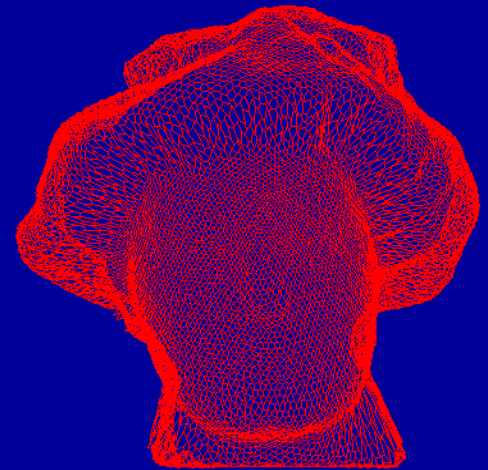


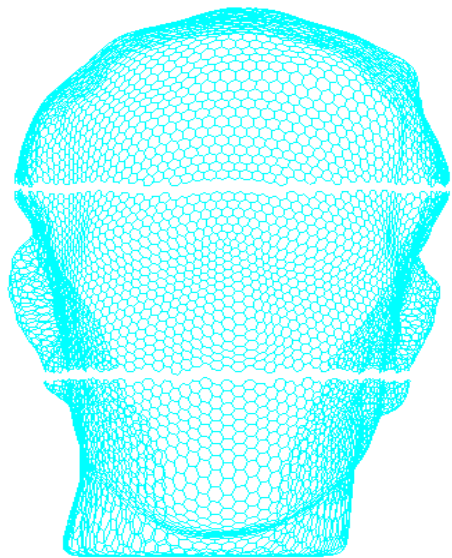


N=6

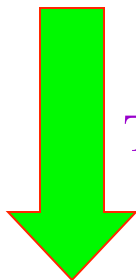


N=7

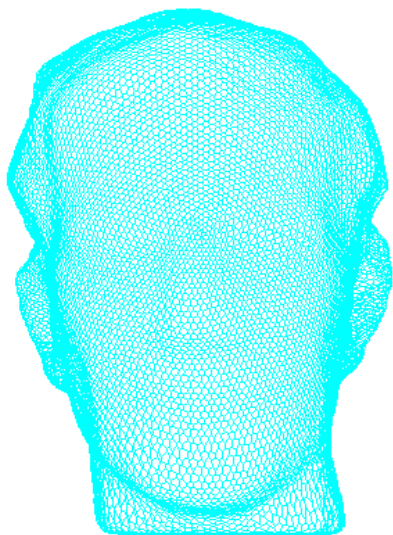




$N_s=6$



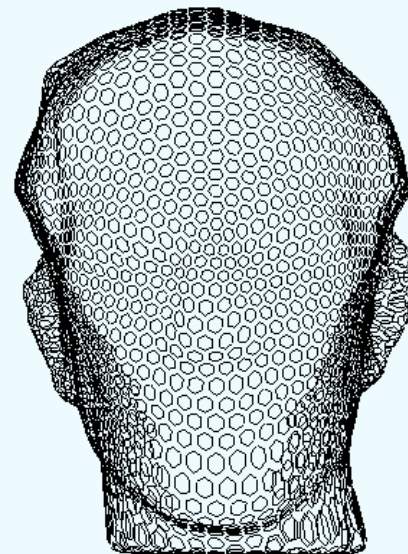
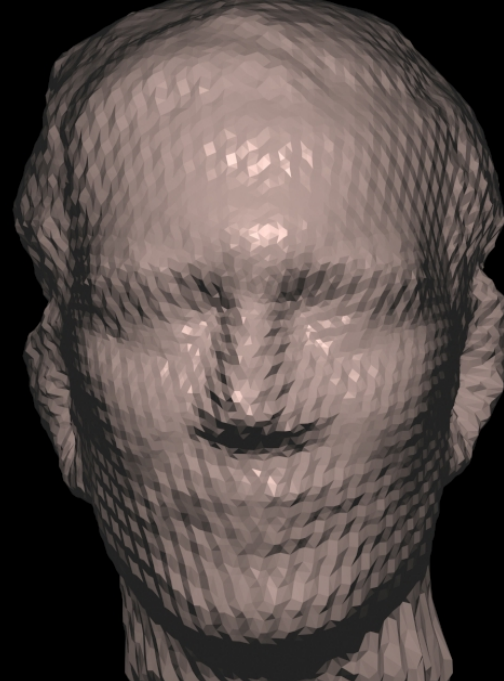
TD

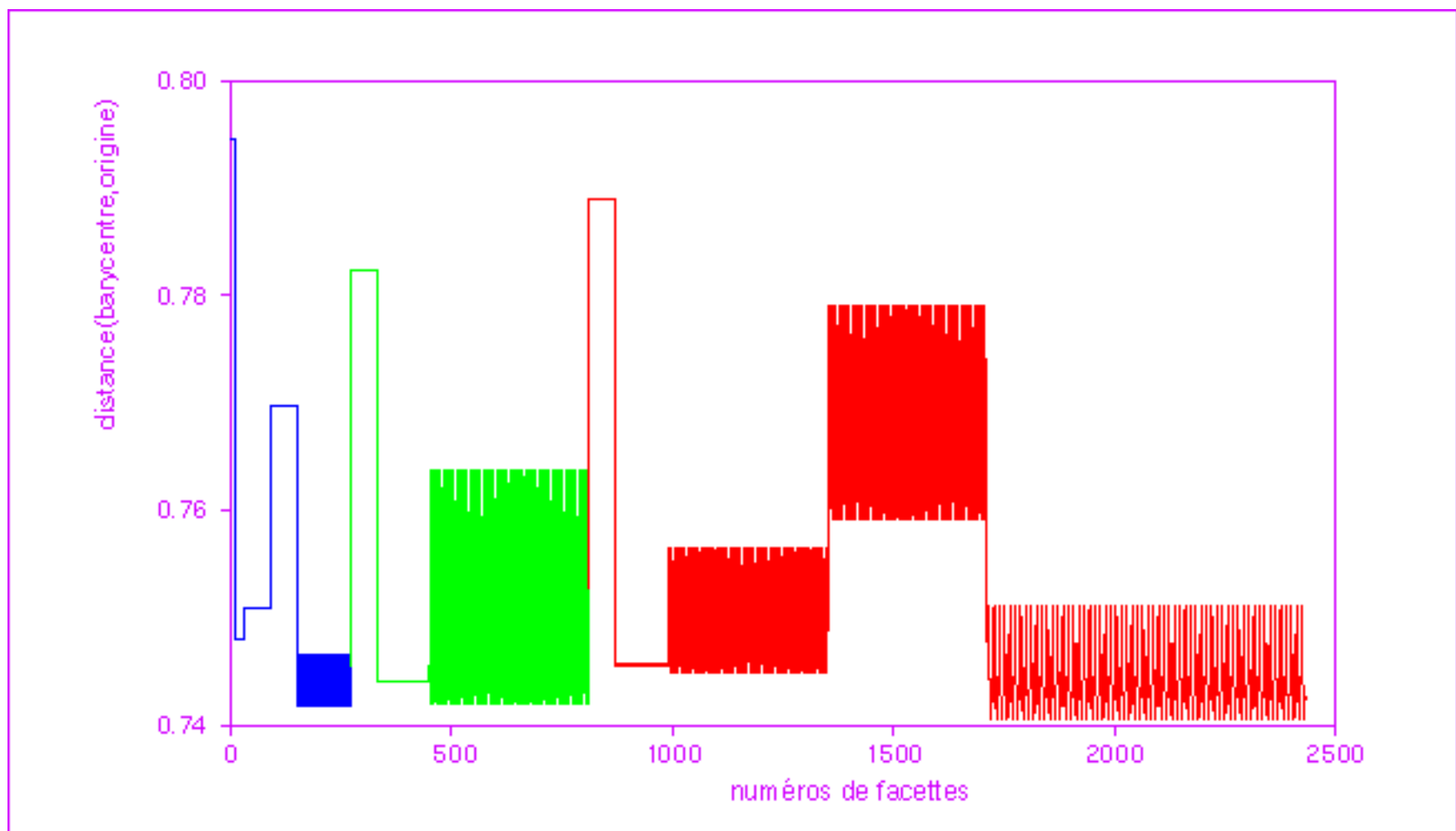


TI



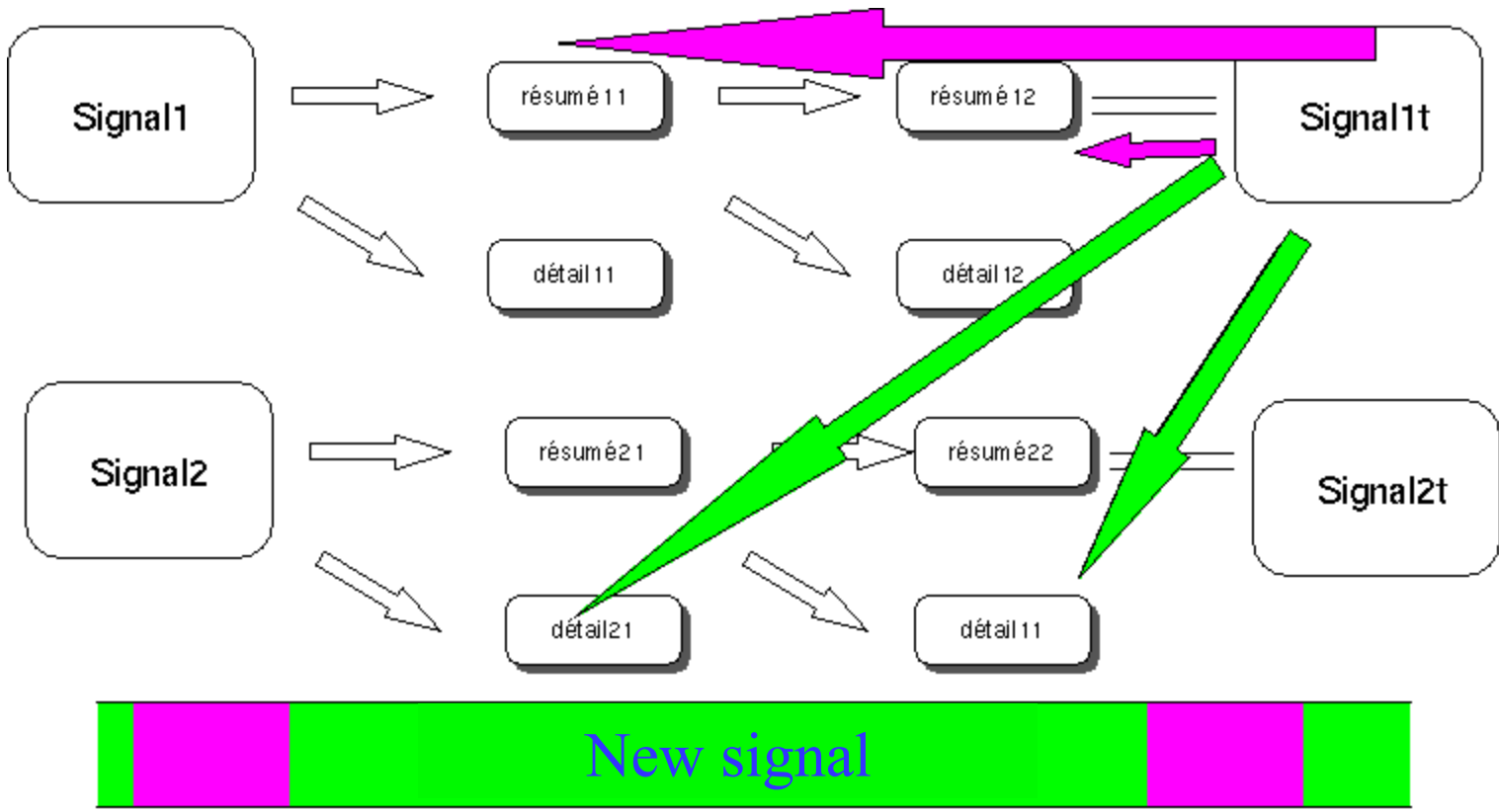
$N_s=7$

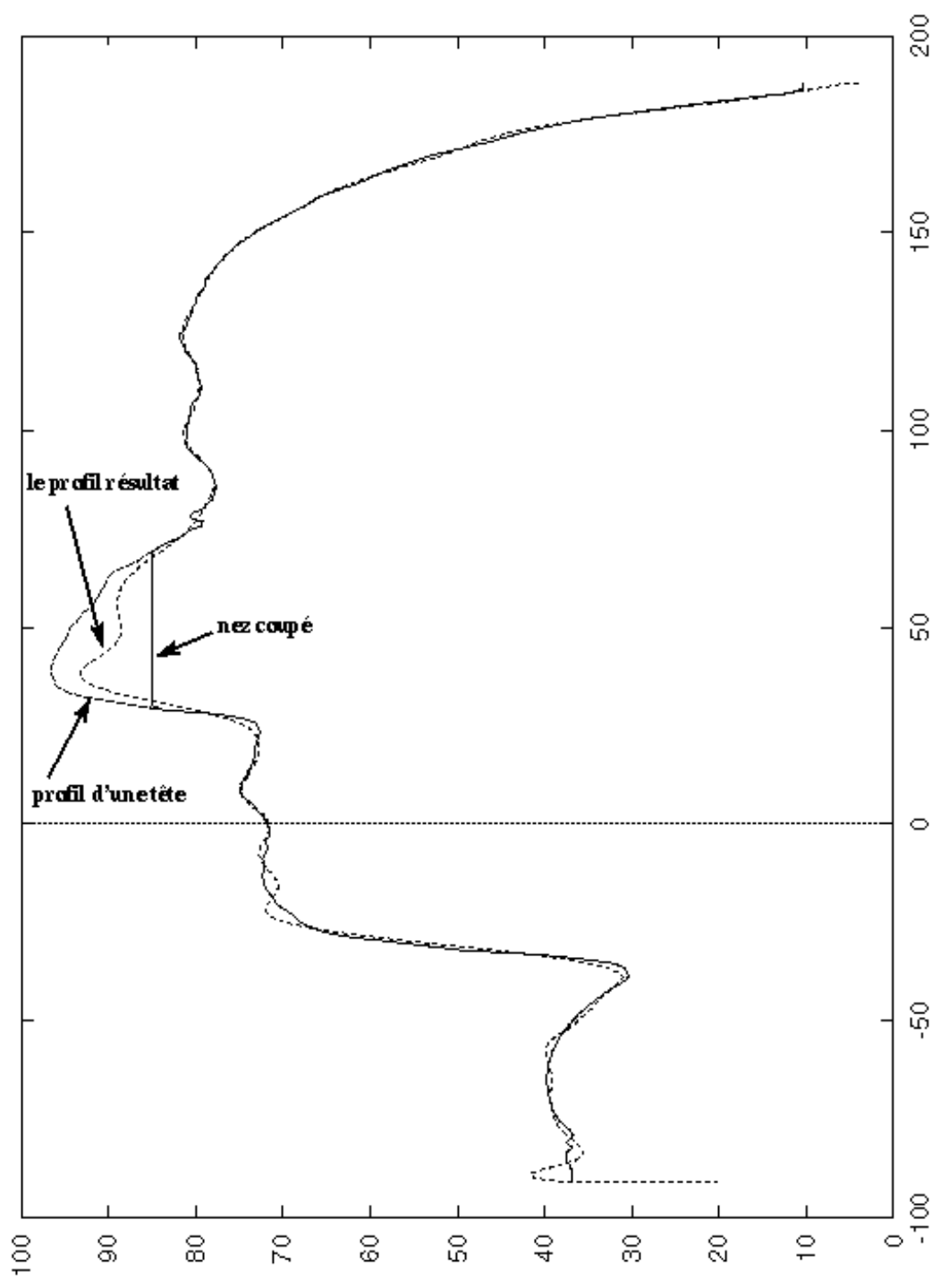




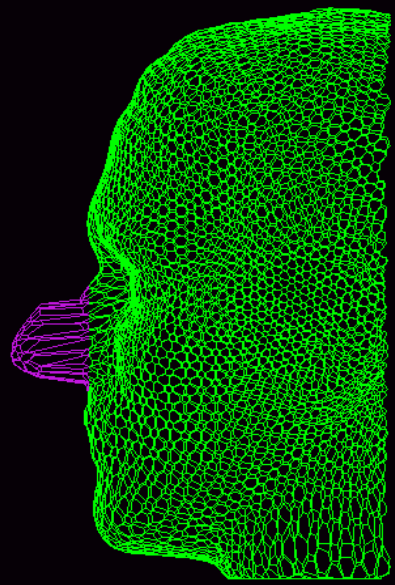
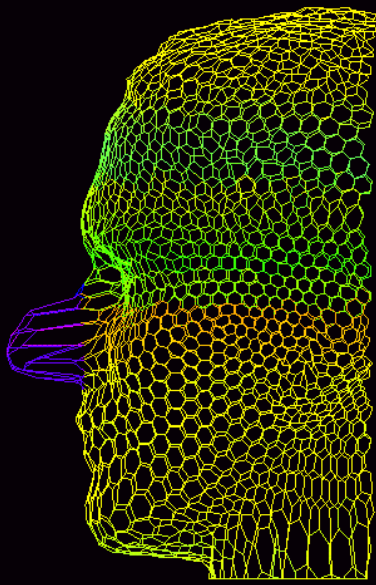
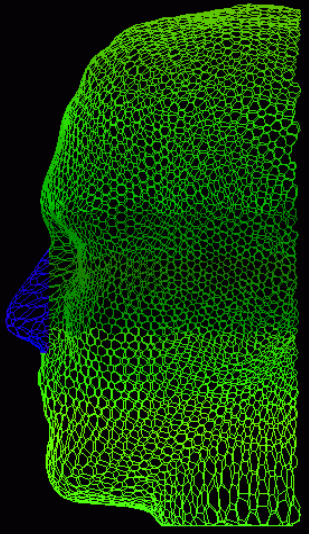
Modification du détail pour des **objets 3D**

**Signal obtained by combination
of resumes from Signal1 with
details of Signal2**

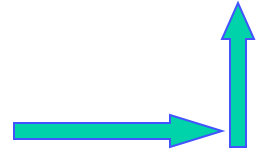
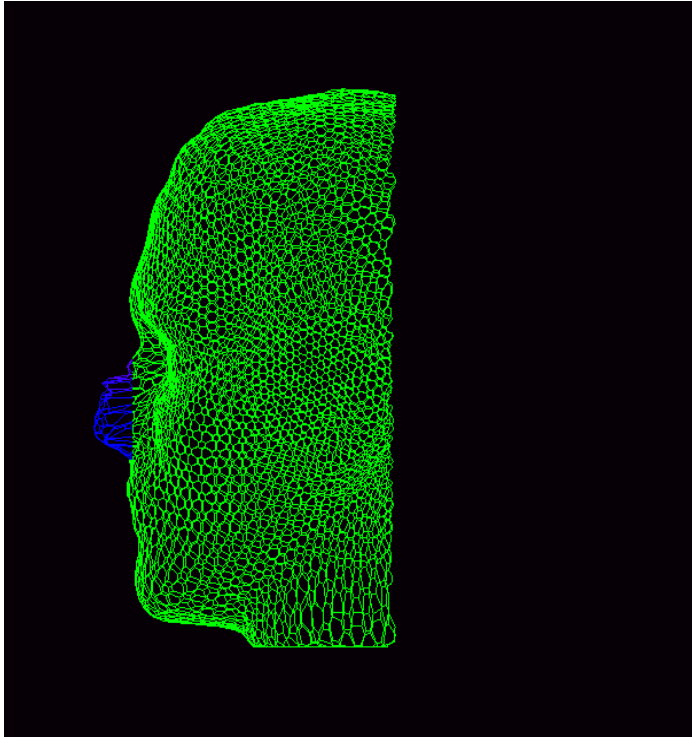
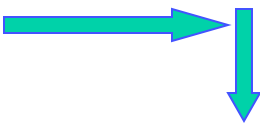




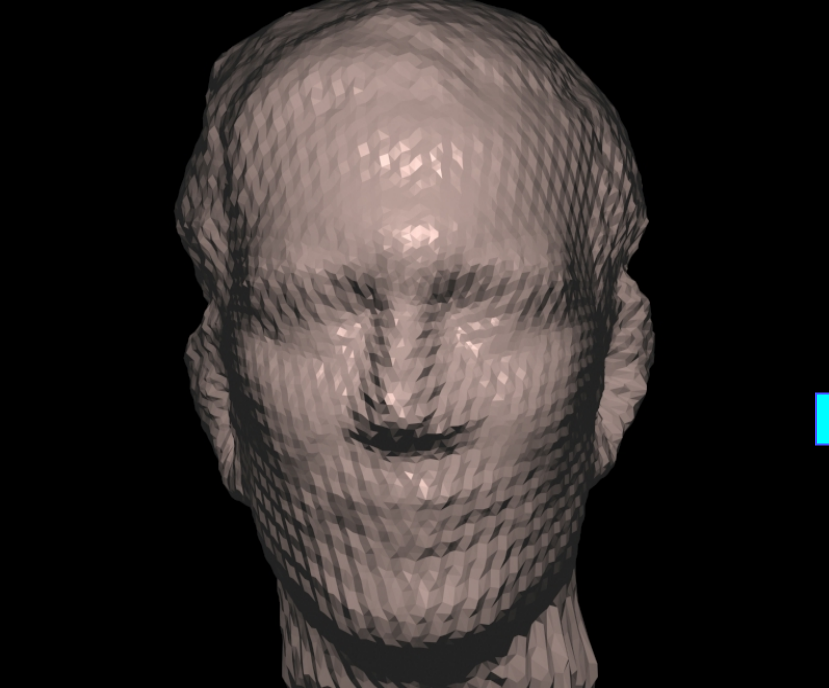
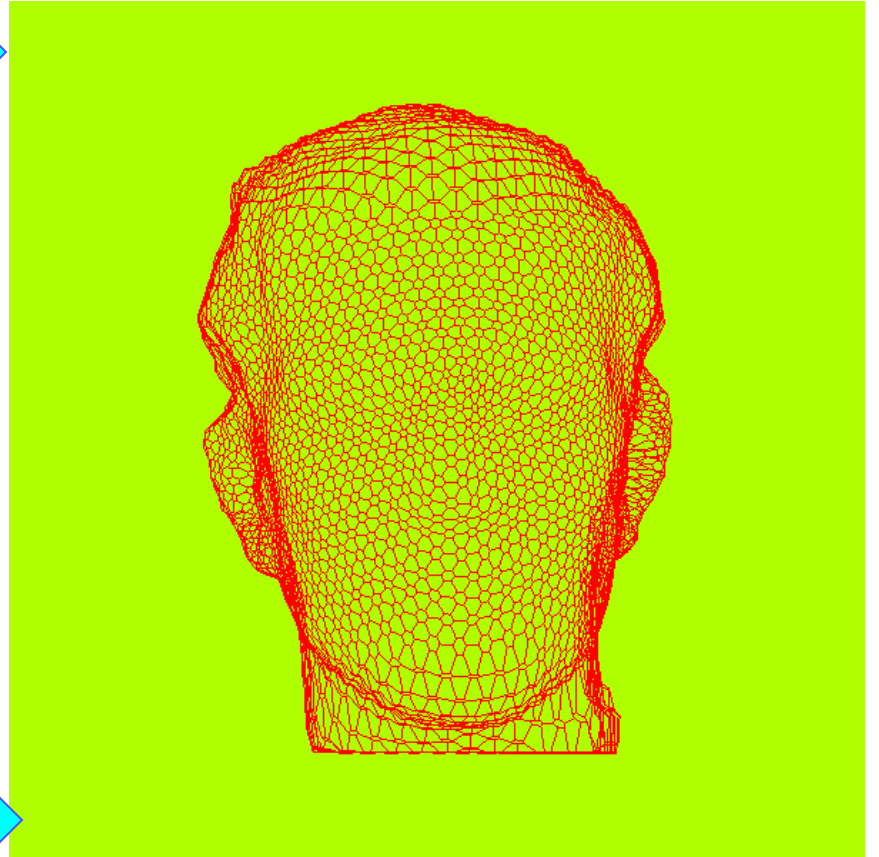
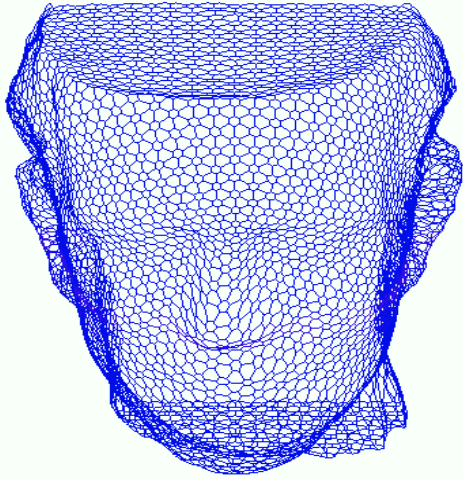
Detail Modification



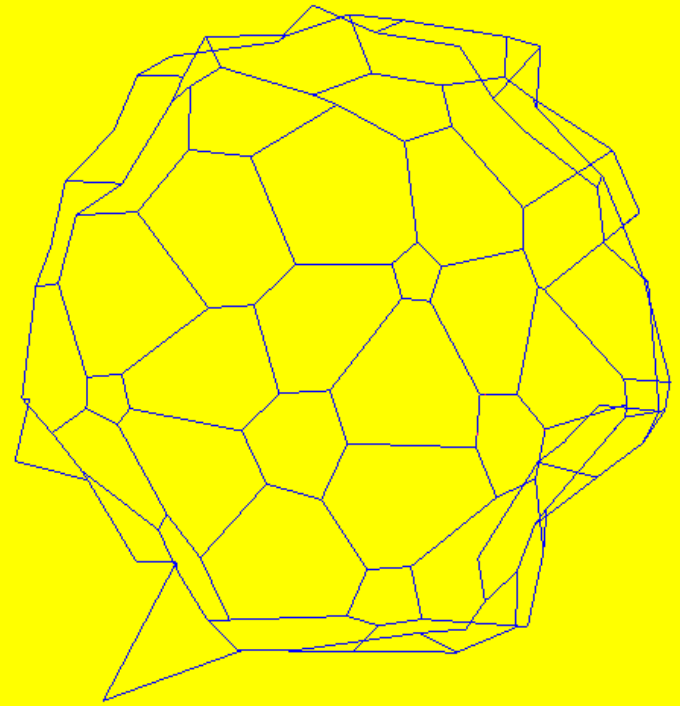
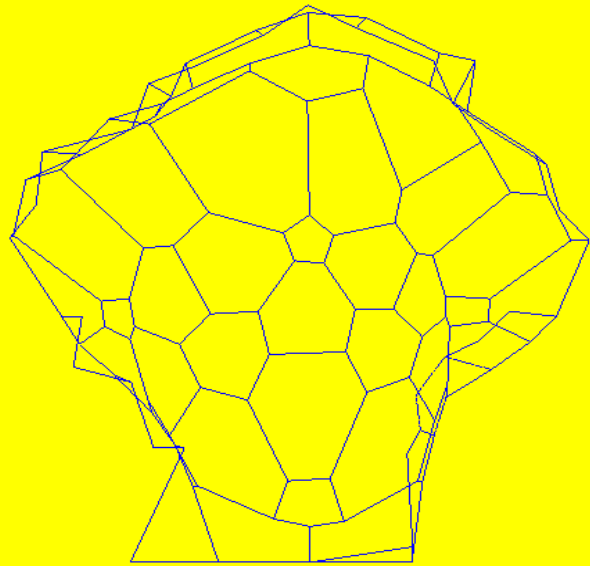
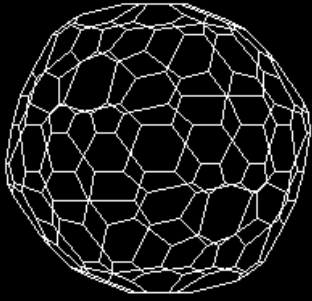
resume



details



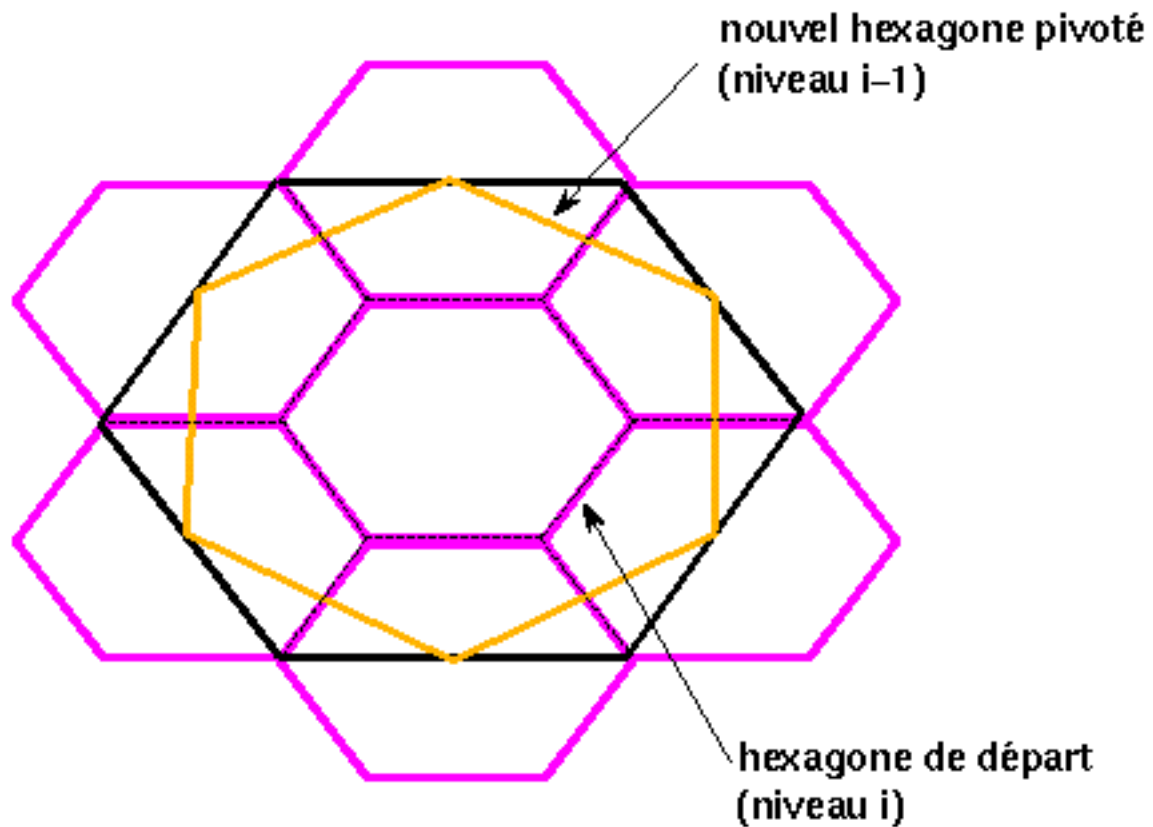
N2 → N8
↓
N2

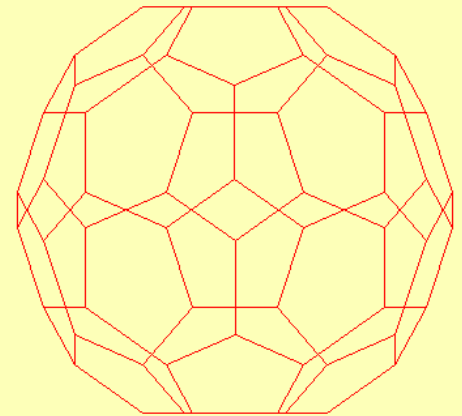
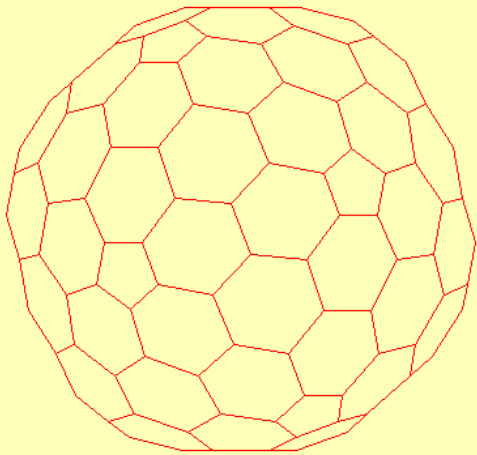
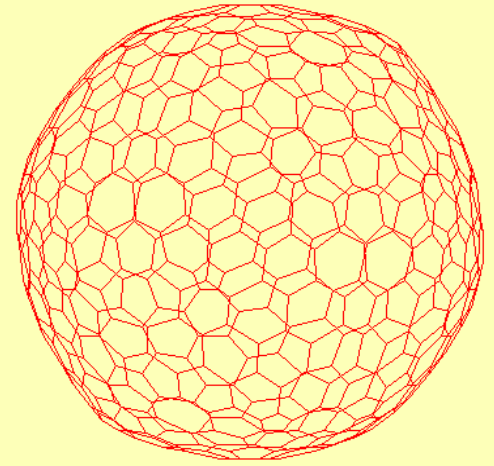
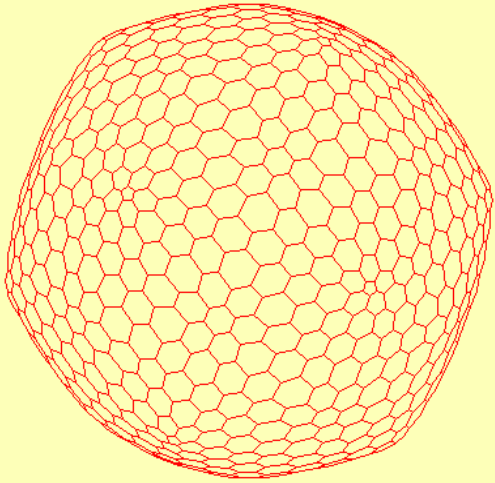


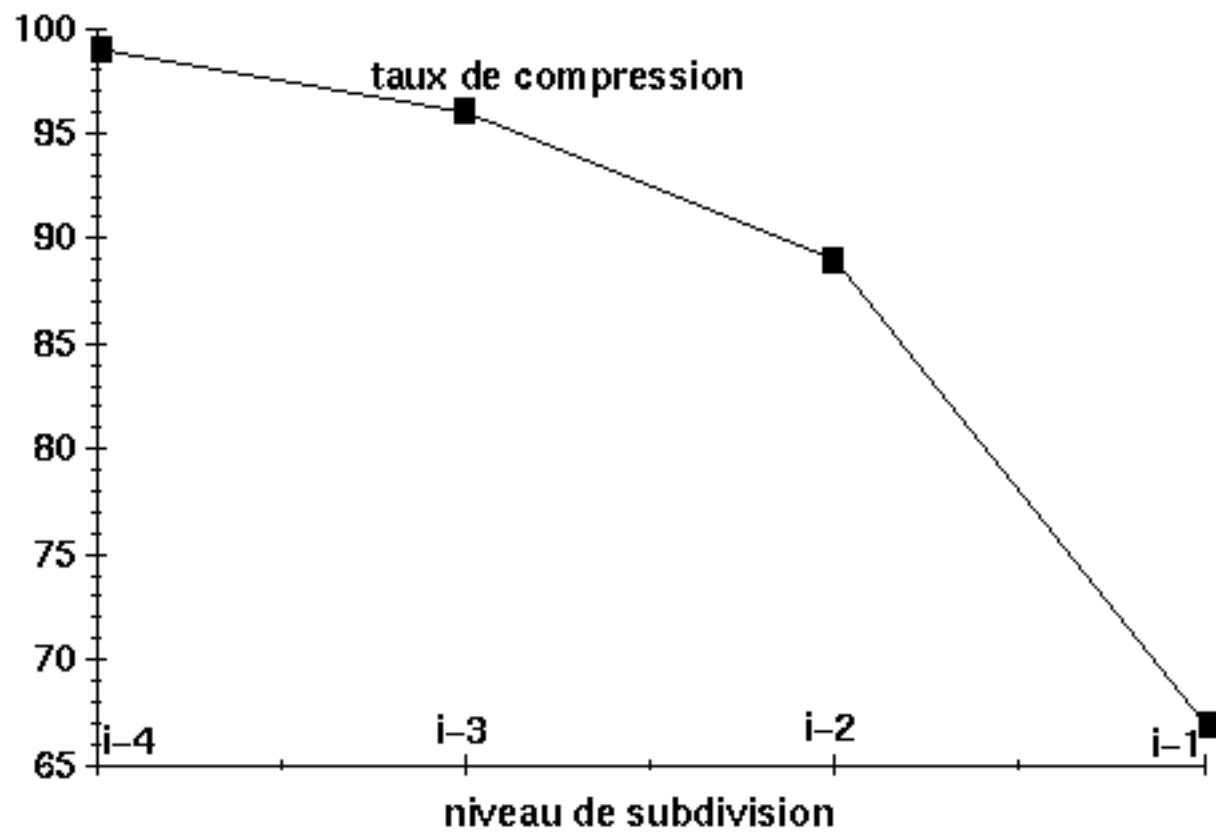
Compression

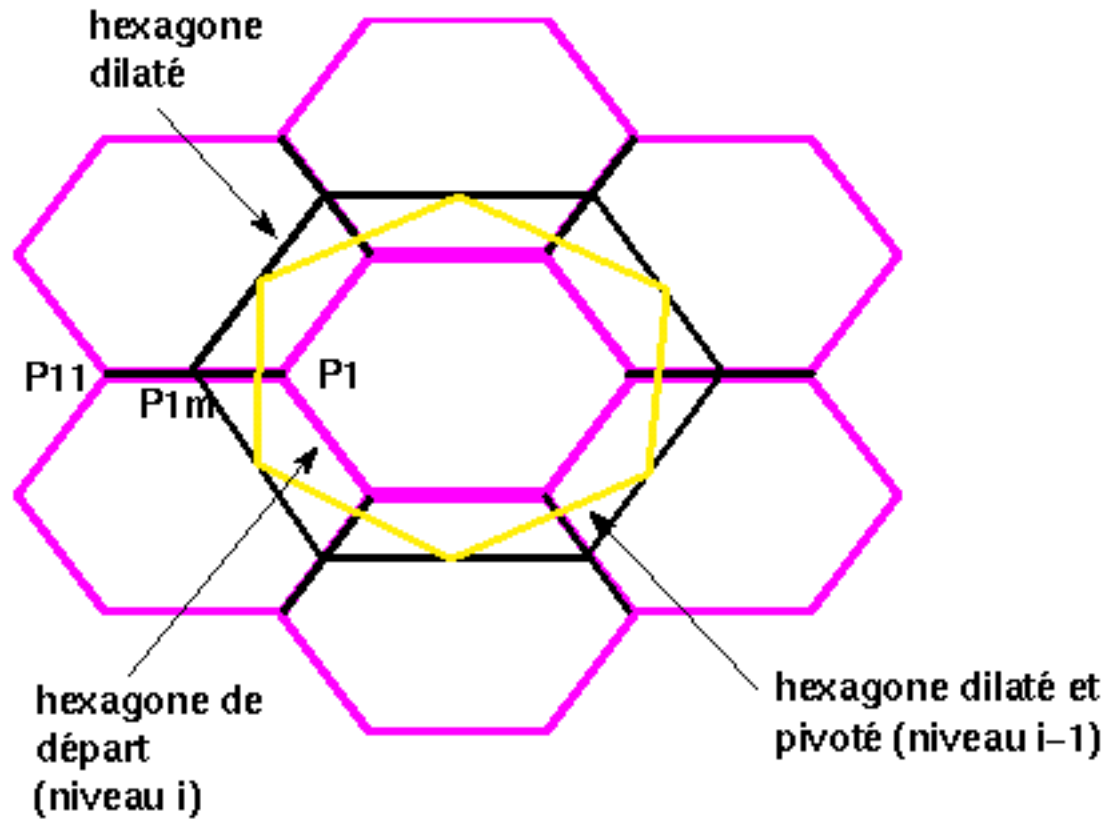
La méthode des milieux extérieurs

MPE









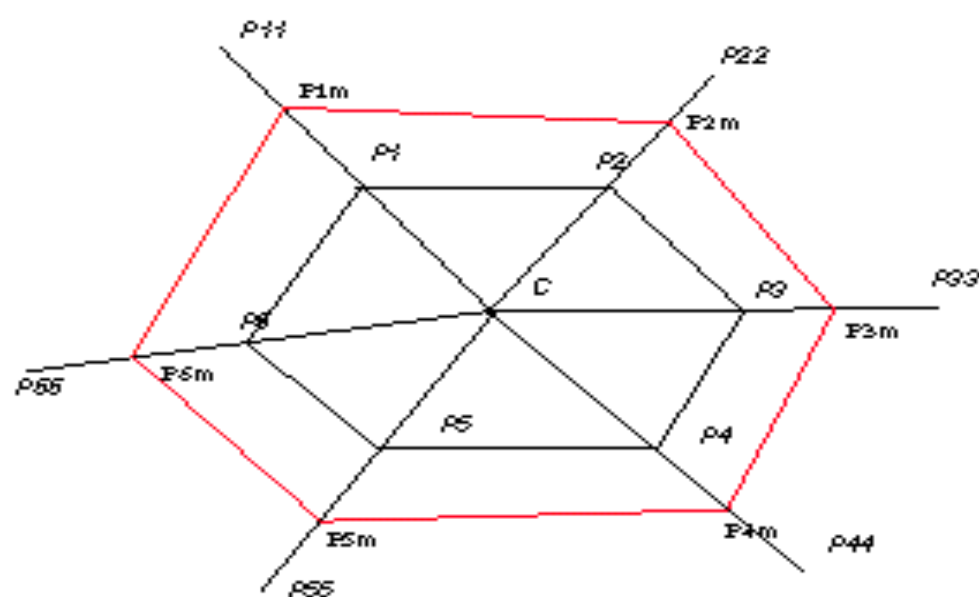
$$[P_1 \ P_{11} \ P_2 \ P_{22} \ P_3 \ P_{33} \ P_4 \ P_{44} \ P_5 \ P_{55} \ P_6 \ P_{66}]$$

$$S = \frac{P_1 + P_{11}}{2} \frac{P_2 + P_{22}}{2} \frac{P_3 + P_{33}}{2} \frac{P_4 + P_{44}}{2} \frac{P_5 + P_{55}}{2} \frac{P_6 + P_{66}}{2}$$

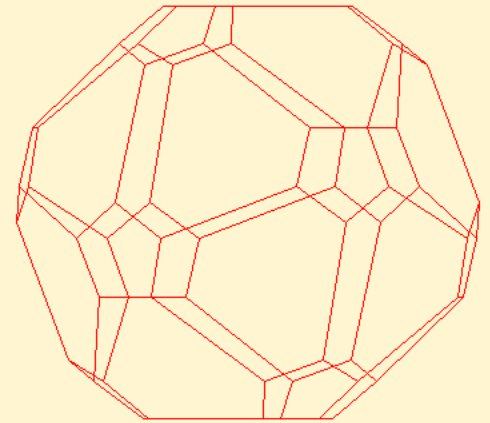
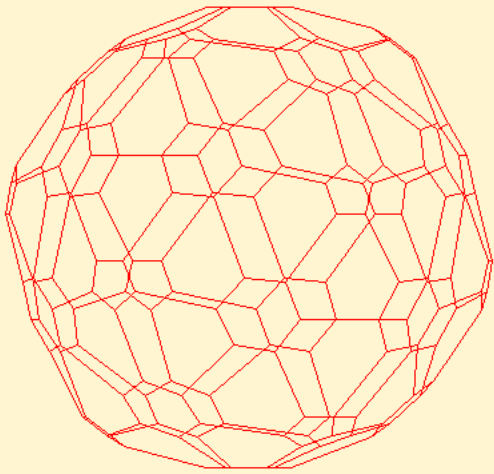
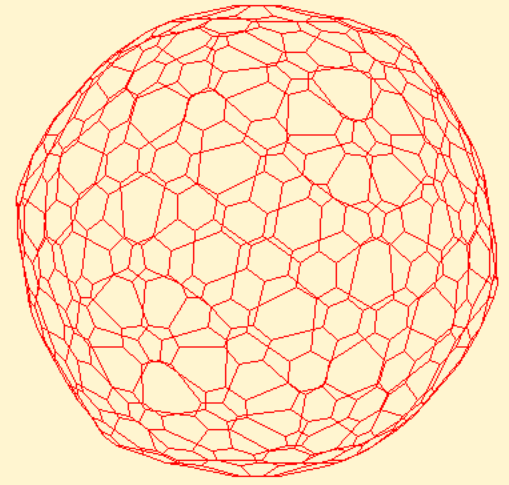
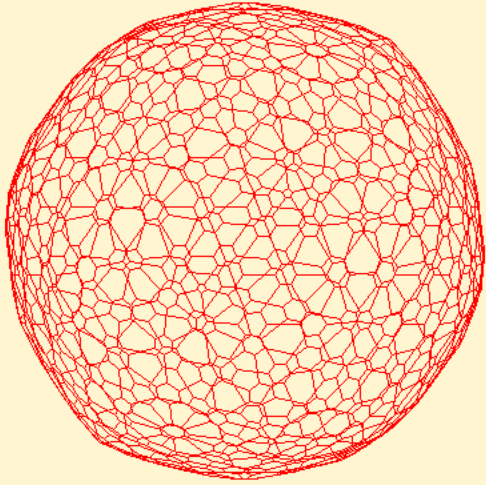
et le détail de l'hexagone dilaté:

$$D = \frac{P_1 - P_{11}}{2} \frac{P_2 - P_{22}}{2} \frac{P_3 - P_{33}}{2} \frac{P_4 - P_{44}}{2} \frac{P_5 - P_{55}}{2} \frac{P_6 - P_{66}}{2}$$

$$S1 = \frac{P_{1m} + P_{2m}}{2} \frac{P_{2m} + P_{3m}}{2} \frac{P_{3m} + P_{4m}}{2} \frac{P_{4m} + P_{5m}}{2} \frac{P_{5m} + P_{6m}}{2} \frac{P_{6m} + P_{1m}}{2}$$

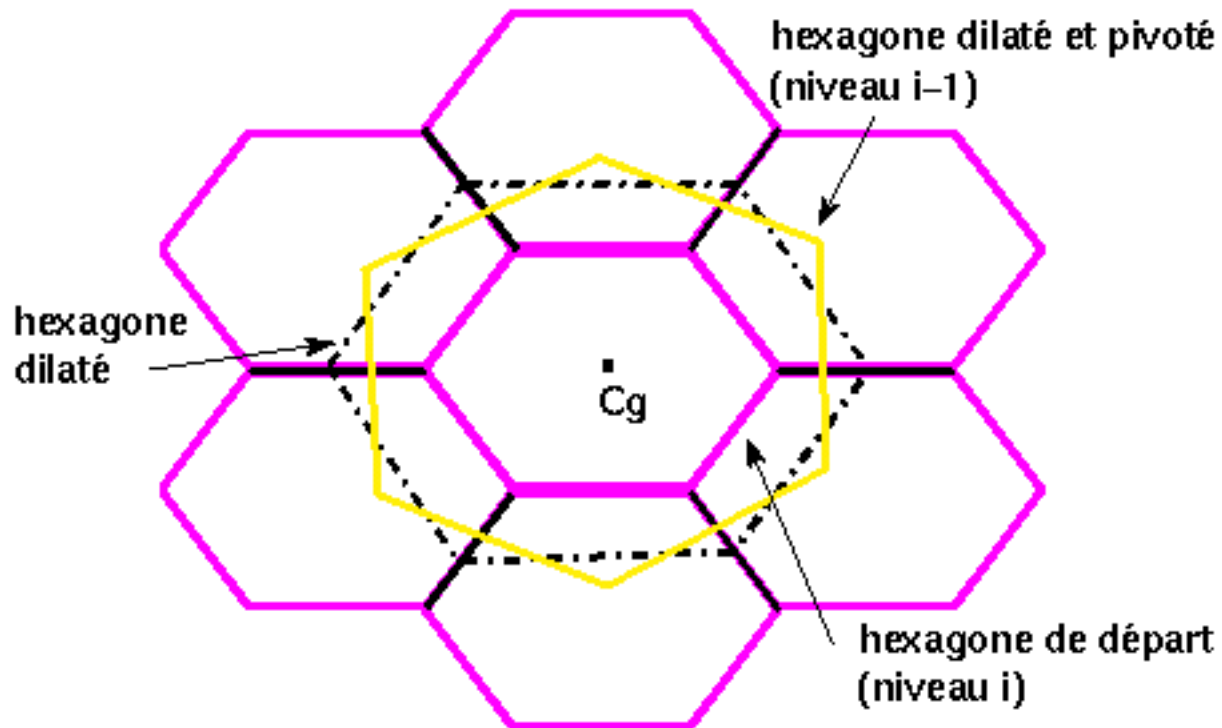


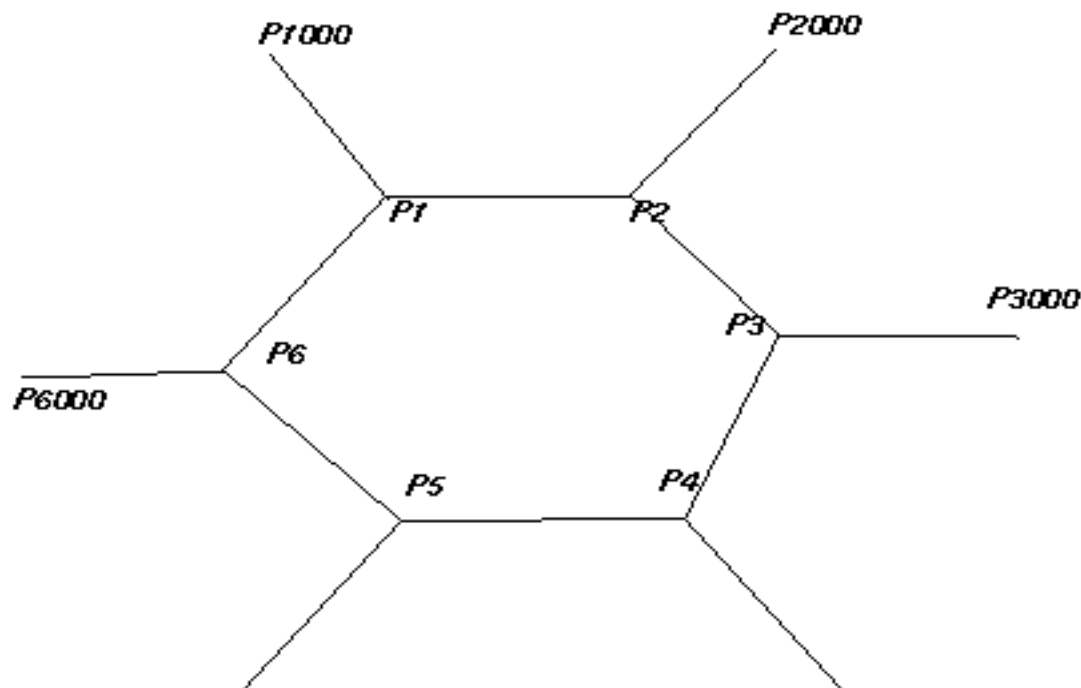
$$D1 = \frac{P_{1m} - P_{2m}}{2} \frac{P_{2m} - P_{3m}}{2} \frac{P_{3m} - P_{4m}}{2} \frac{P_{4m} - P_{5m}}{2} \frac{P_{5m} - P_{6m}}{2} \frac{P_{6m} - P_{1m}}{2}$$



La méthode des milieux rotationnels

MMROT





$[P_1 P_{1000} P_2 P_{2000} P_3 P_{3000} P_4 P_{4000} P_5 P_{5000} P_6 P_{6000}]$

après l'utilisation de la base de Haar, le résumé est :

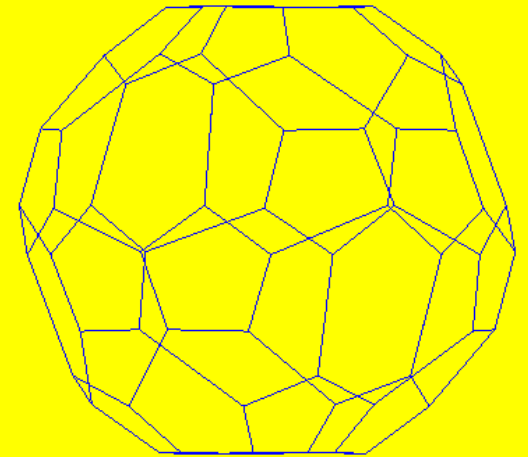
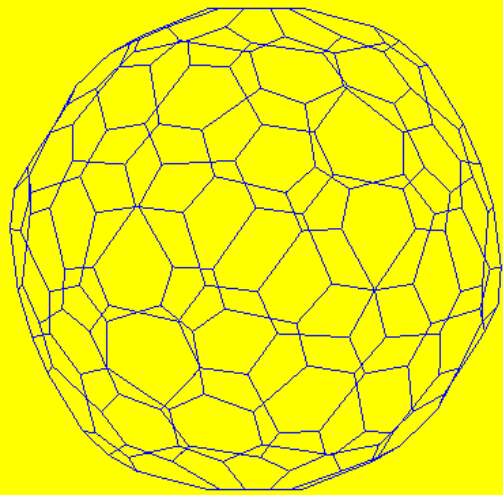
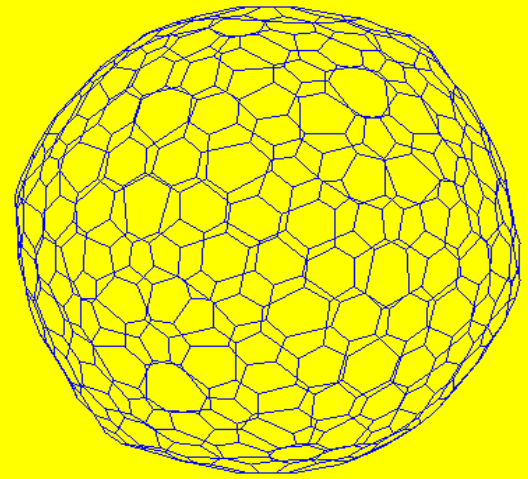
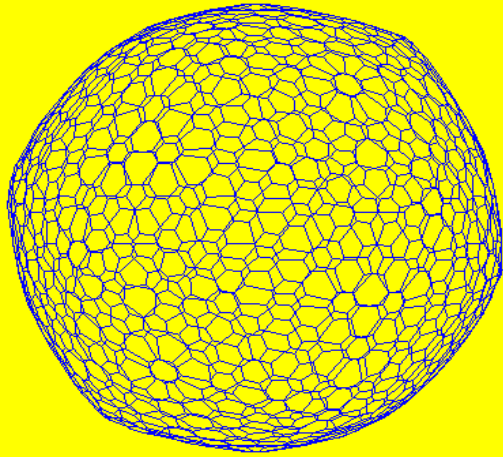
$$S = \frac{P_1 + P_{1000}}{2} \frac{P_2 + P_{2000}}{2} \frac{P_3 + P_{3000}}{2} \frac{P_4 + P_{4000}}{2} \frac{P_5 + P_{5000}}{2} \frac{P_6 + P_{6000}}{2}$$

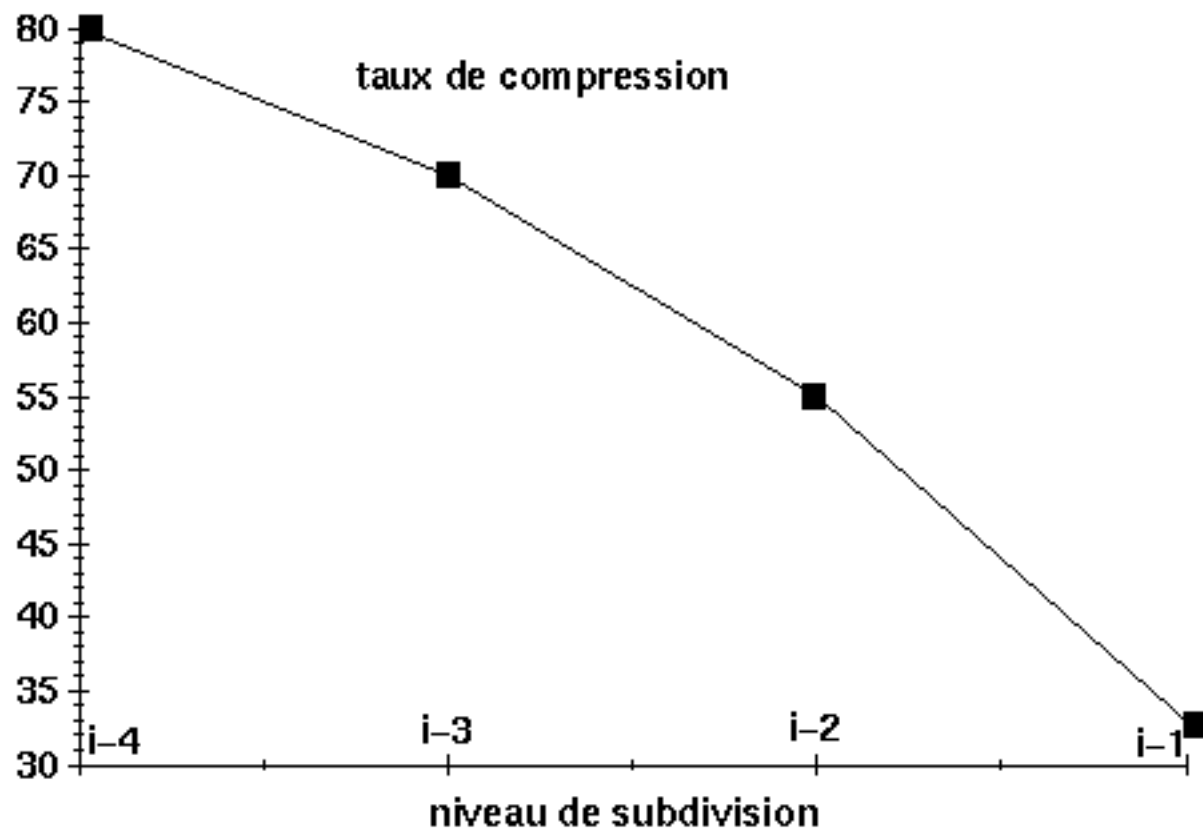
et le détail :

$$D = \frac{P_1 - P_{1000}}{2} \frac{P_2 - P_{2000}}{2} \frac{P_3 - P_{3000}}{2} \frac{P_4 - P_{4000}}{2} \frac{P_5 - P_{5000}}{2} \frac{P_6 - P_{6000}}{2}$$

$$S^T R^T R S + D^T D = I$$

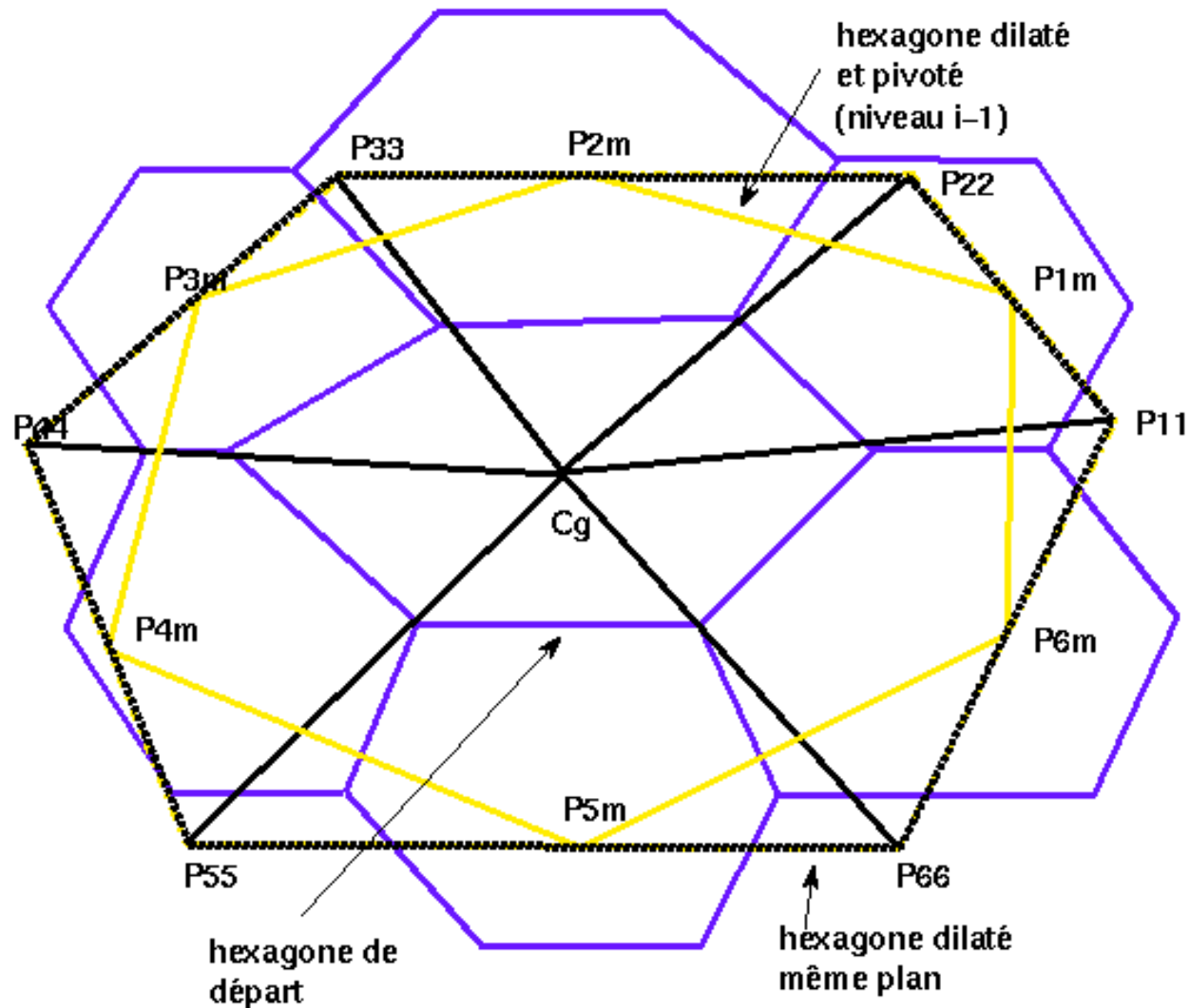






La méthode des barycentres

MB



Partant de la relation :

$$P_1 = h_0 C_g + h_1 P_{11}$$

P_{11} est obtenu :

$$P_{11} = \frac{P_1 - h_0 C_g}{h_1}$$

$$P_{1m} = \frac{P_{11} + P_{22}}{2} = \frac{\frac{P_1 - h_0 C_g}{h_1} + \frac{P_2 - h_0 C_g}{h_1}}{2}$$

en remplaçant h_1 h_2 par 0.5 0.5, nous obtenons :

$$P_{1m} = P_1 - C_g + P_2$$

1. Les filtres d'Adelsson et Simoncelli

[1 2 1] et [-1 2 -1]

Le signal est :

$$P_1, \frac{-C_g}{2}, P_2, \frac{-C_g}{2}, P_3, \frac{-C_g}{2}, P_4, \frac{-C_g}{2}, P_5, \frac{-C_g}{2}, P_6, \frac{-C_g}{2}$$

Le résumé est :

$$P_1 - C_g + P_2, P_2 - C_g + P_3, P_3 - C_g + P_4, P_4 - C_g + P_5, P_5 - C_g + P_6, P_6 - C_g + P_1$$

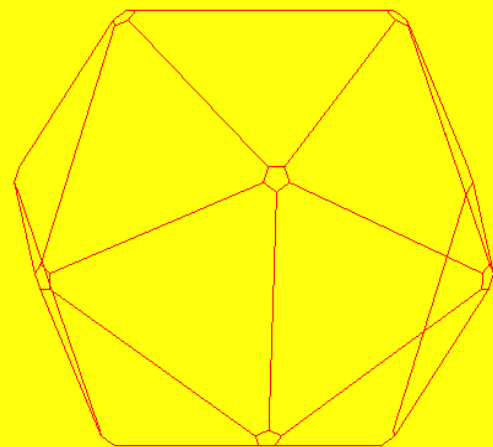
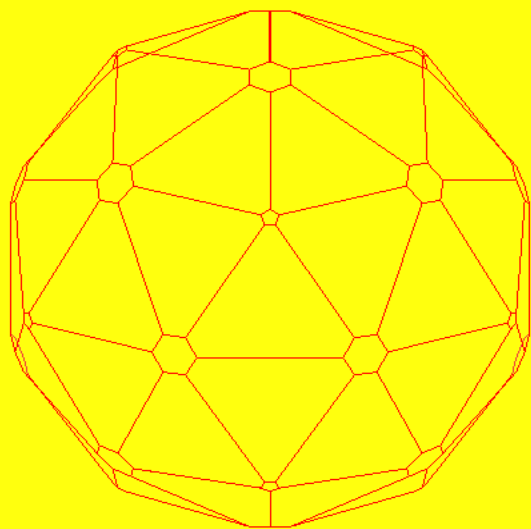
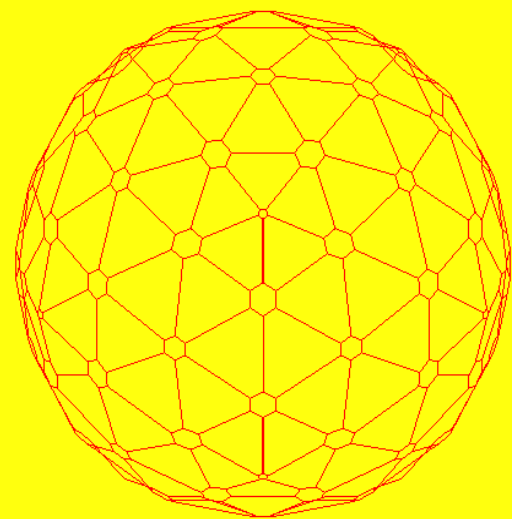
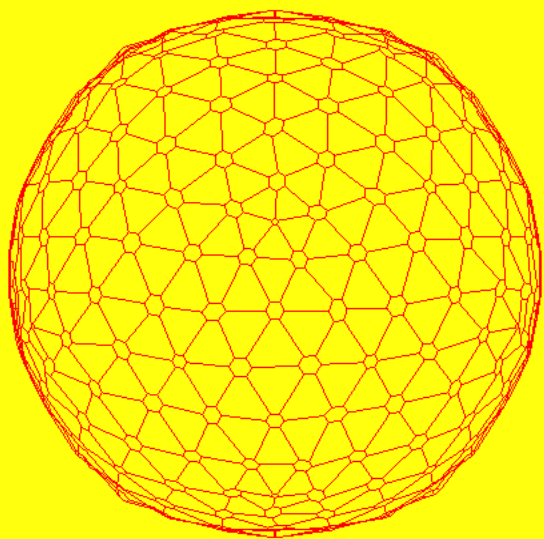
2. Les ondelettes splines d 'ordre trois

Pour la transformée directe, les deux filtres sont [1,2,1] et [-1,-2,-1]

Pour la transformée inverse, les deux filtres sont [-1,2,1] et [-1,2,-1]

Le signal est :

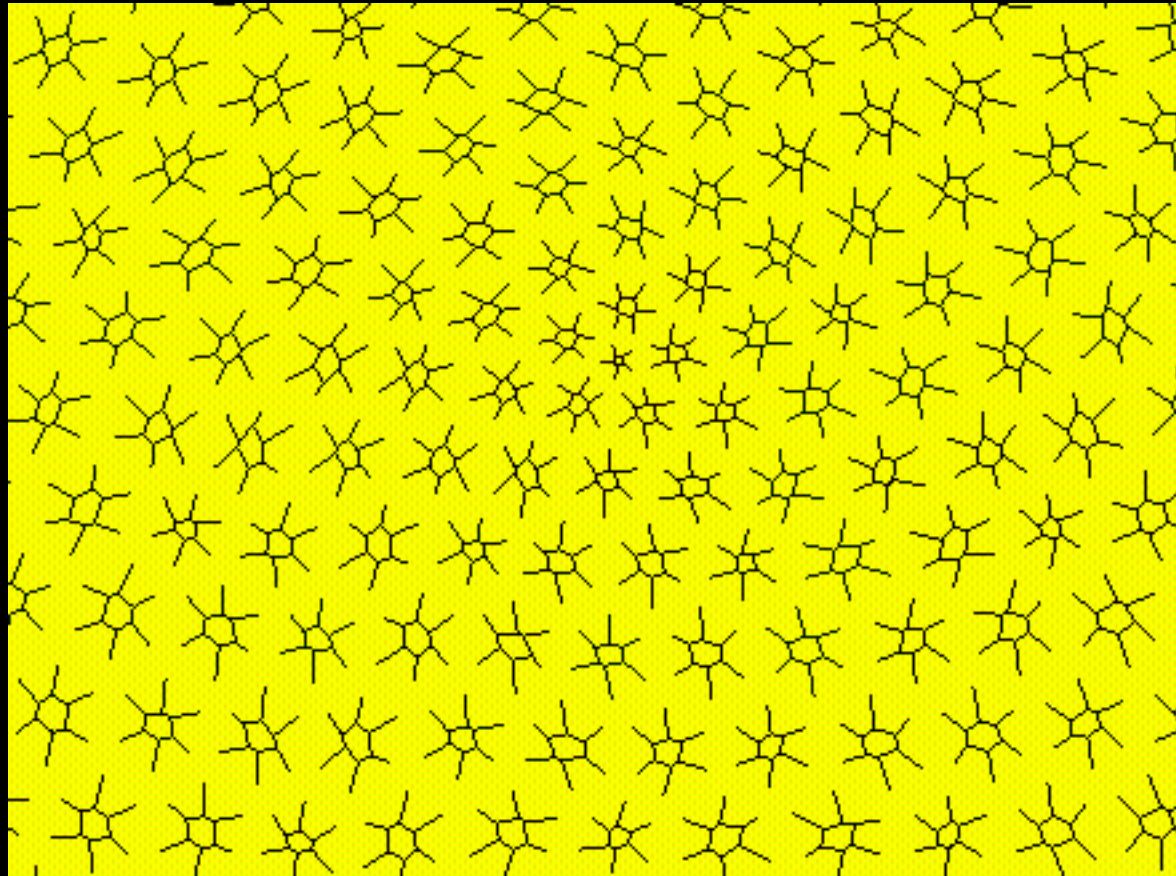
$$P_1, \frac{C_g}{2}, P_2, \frac{C_g}{2}, P_3, \frac{C_g}{2}, P_4, \frac{C_g}{2}, P_5, \frac{C_g}{2}, P_6, \frac{C_g}{2}$$

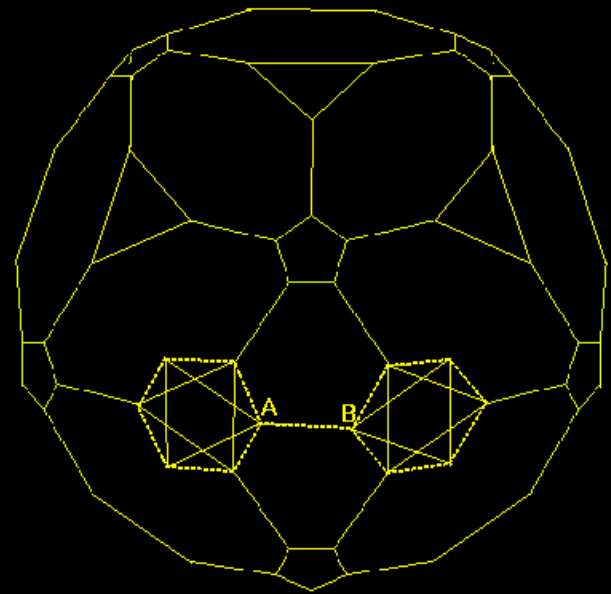
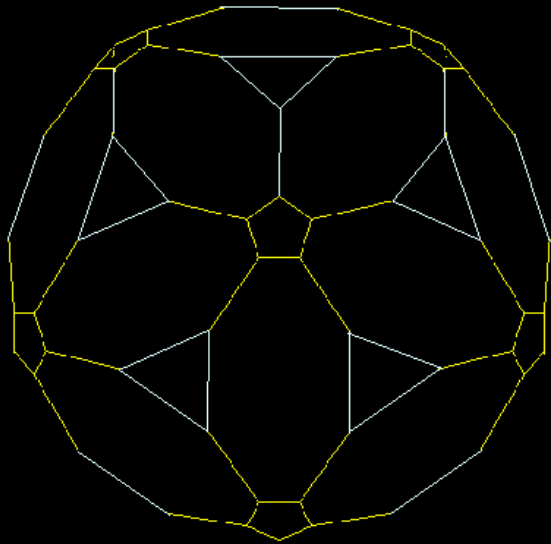


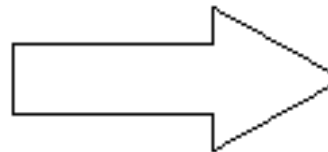
Construction du système de compression

MÉTHODEs	CONSERVATION DU BARYCENTRE	RECONSTRUCTION	POSSIBILITÉ D'UTILISATION DE LA T. O.	LINÉARISATION DE LA ROTATION
M. DES MILIEUX EXTÉRIEURS <i>MPE</i>	N	<i>IMPOSSIBLE</i>	N	O
M. DES MILIEUX DES MILIEUX <i>MMM</i>	N	<i>SEULEMENT POUR LES FACETTES UTILISÉES ET EXACTE</i>	O	O
M. DES MILIEUX ROTATIONNELS <i>MMROT</i>	N	<i>SEULEMENT POUR LES FACETTES UTILISÉES ET EXACTE</i>	O	N
M. DES BARYCENTRES <i>MB</i>	O	<i>SEULEMENT POUR LES FACETTES UTILISÉES ET INEXACTE POUR LES SOMMETS, EXACTES POUR LES BRYCENTRES</i>	O	O

Le maillage obtenu par la transformée inverse





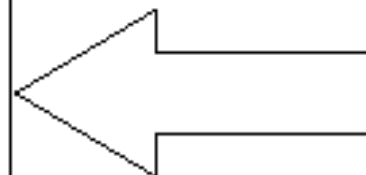
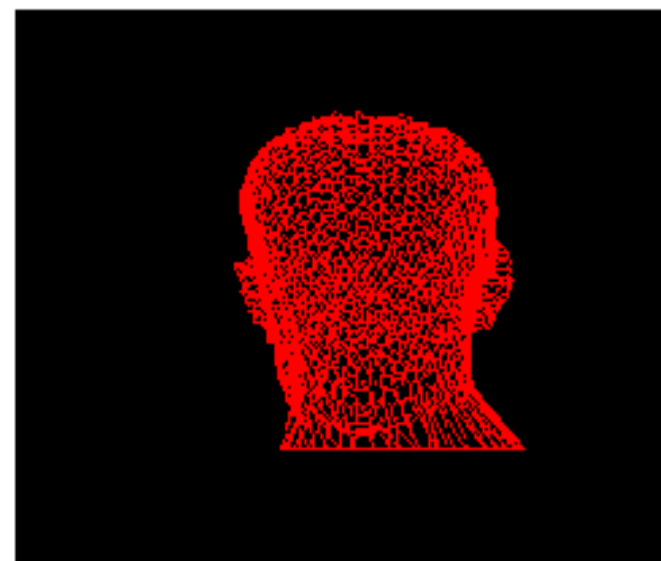
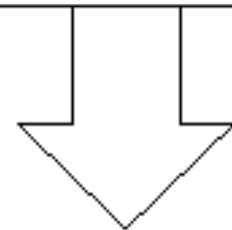


Compression

par la transformée inverse de la subdivision
(reconstruction)

ou

en utilisant une des quatre méthodes de
compression

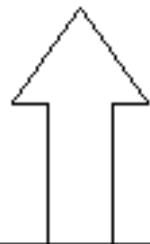


Subdivision

par transformée inverse de la compression
(reconstruction par Mmm , Mmrot , Mb)

ou

en utilisant une des deux méthodes de
subdivision
(liniarisation des rotations ou ondelettes
rotationnelles)



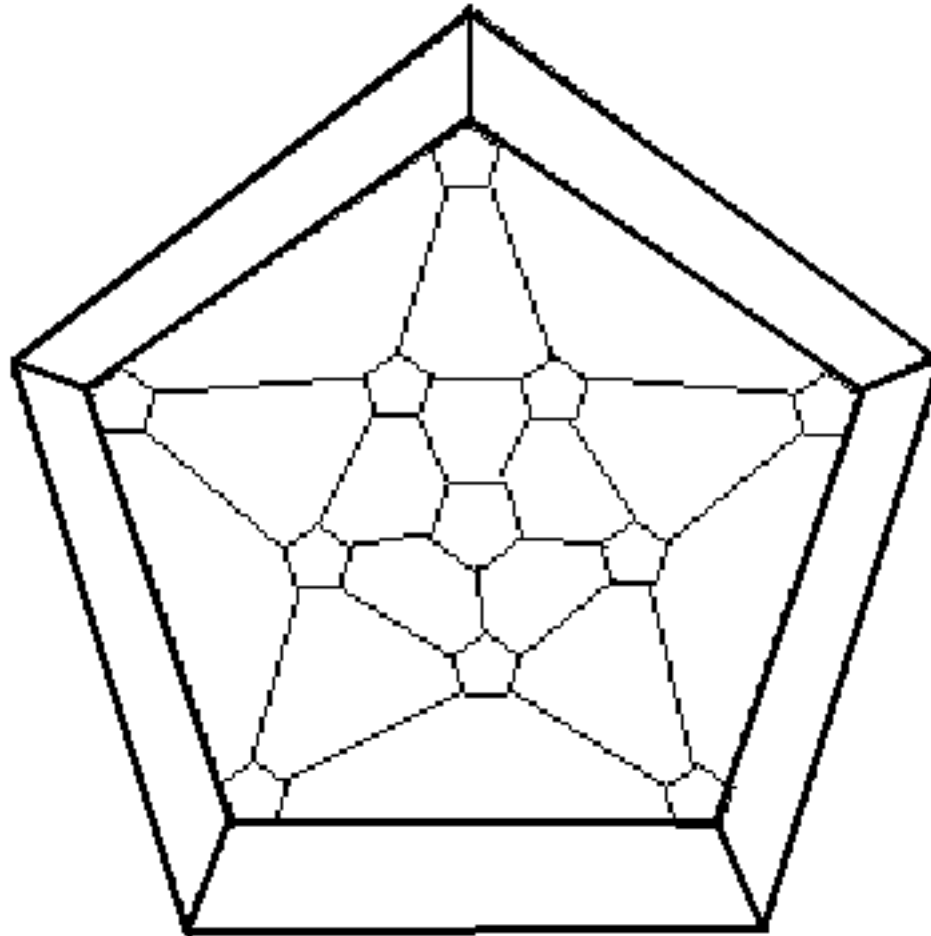
Applications

- *F.E.M : Synthesize a mean model of the face named a photo-fit identikit*
- *Fast and progressive visualisation*
- *Morphing 3D*
- *Multimedia applications*

Transformation of unorganized points



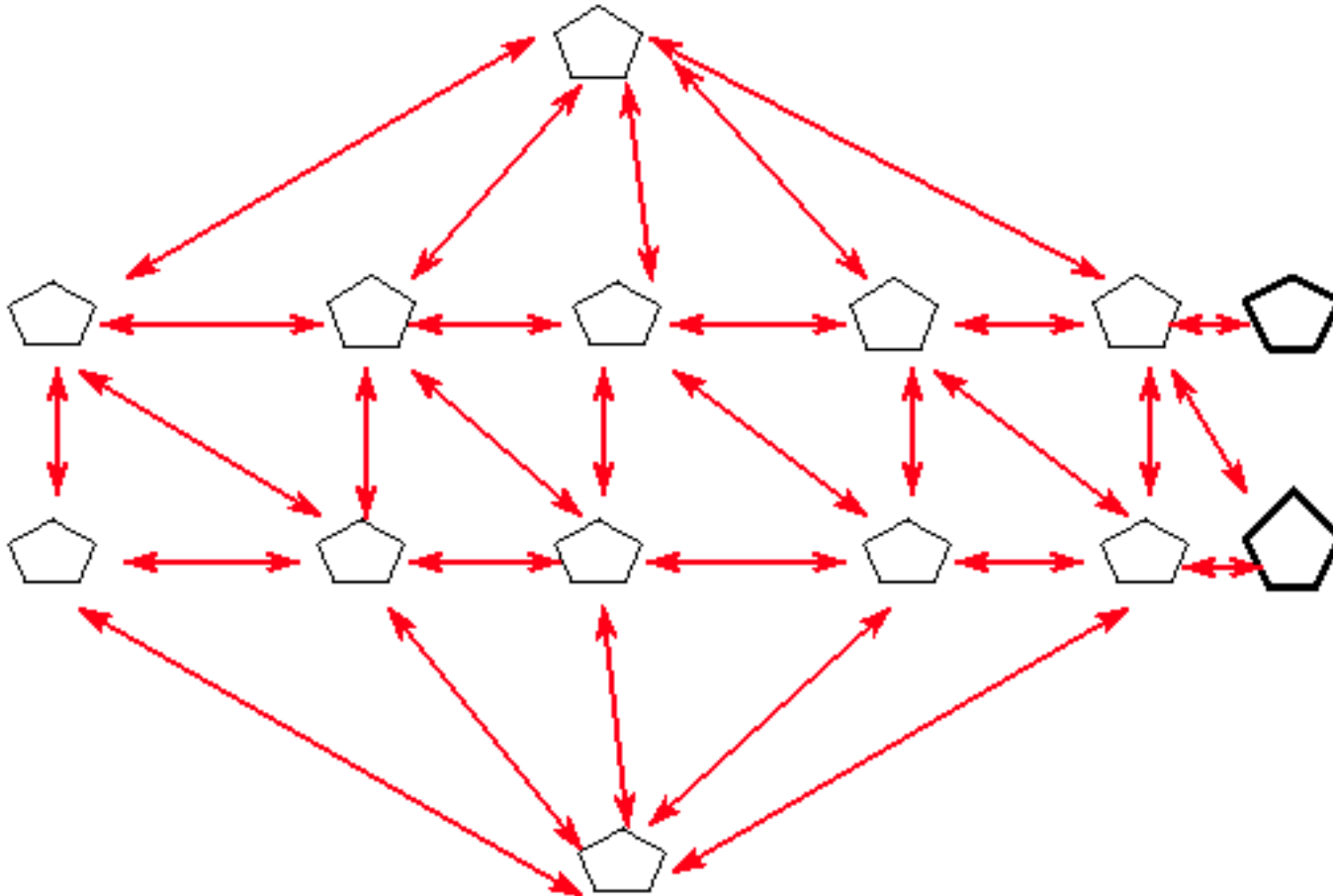
Homogenous mesh with 3-connected vertices



Le diagramme de Schlegel associé au fullerène C60

First step

The construction of 3-connected organized points from unorganized points

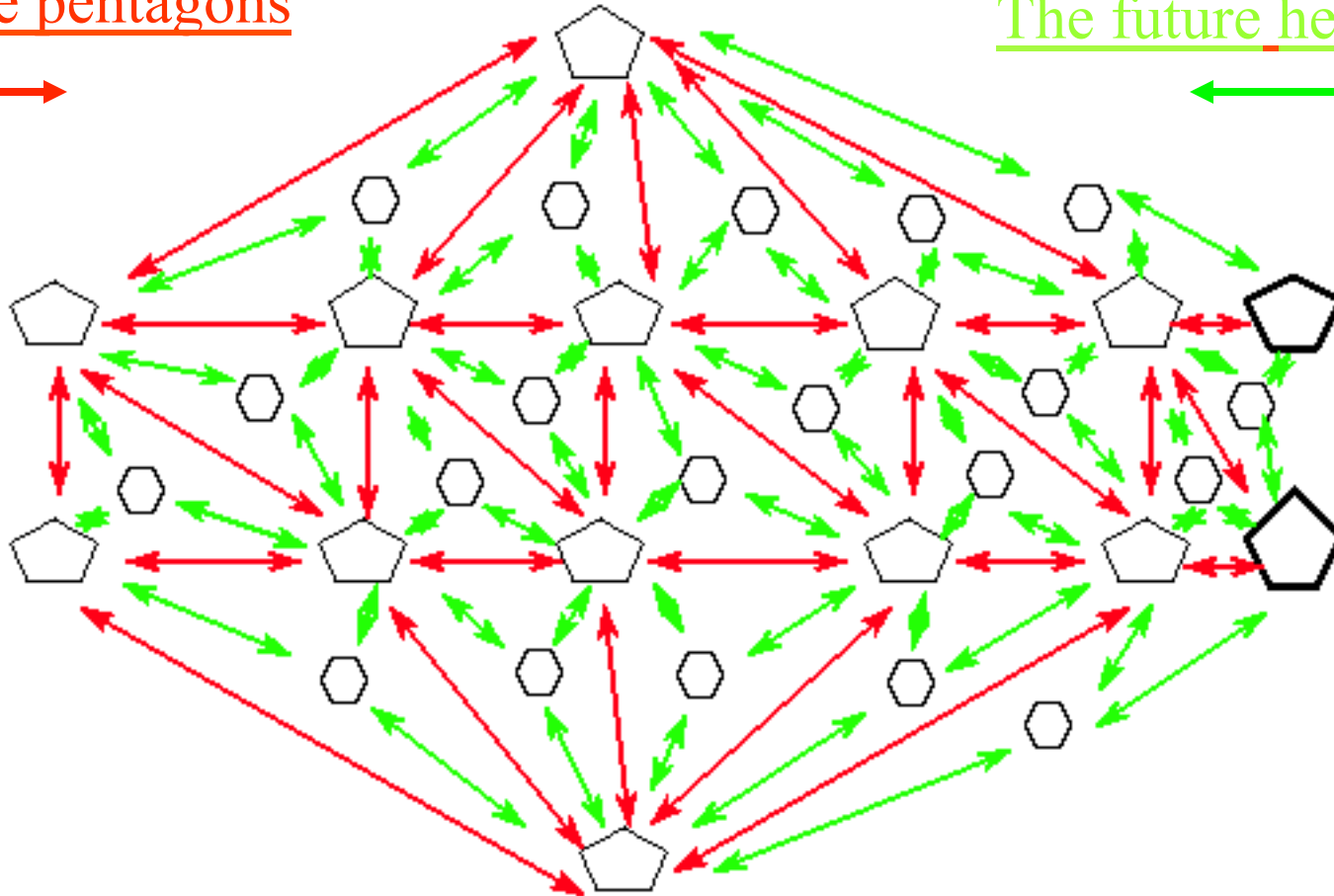


Second step

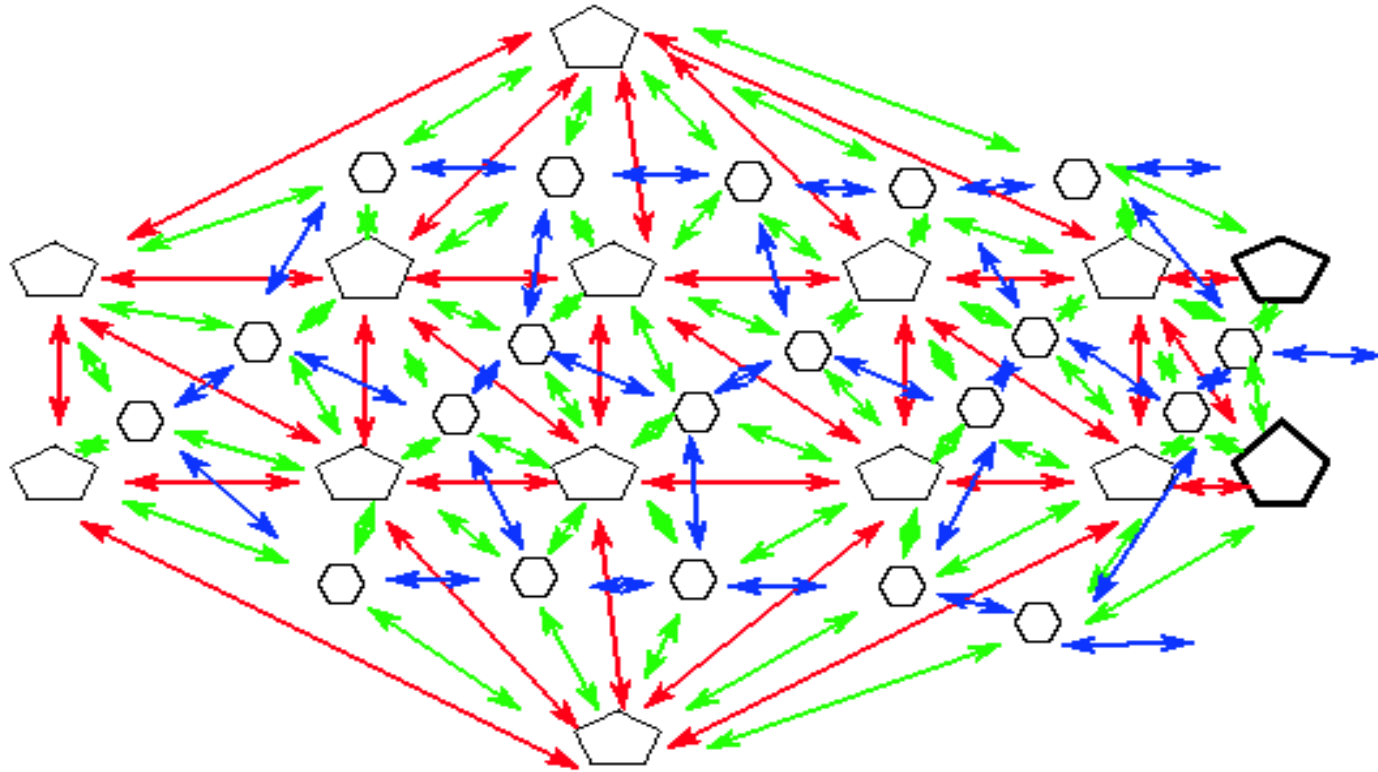
The construction of 3-connected organized points from unorganized points

The future pentagons

The future hexagons



Generalisation

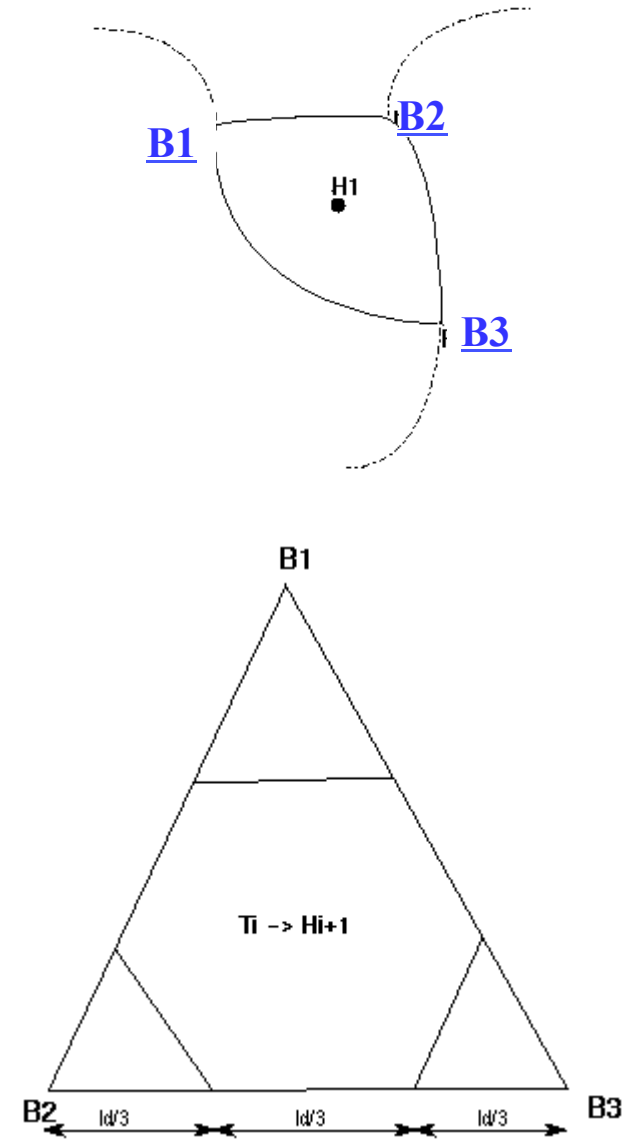
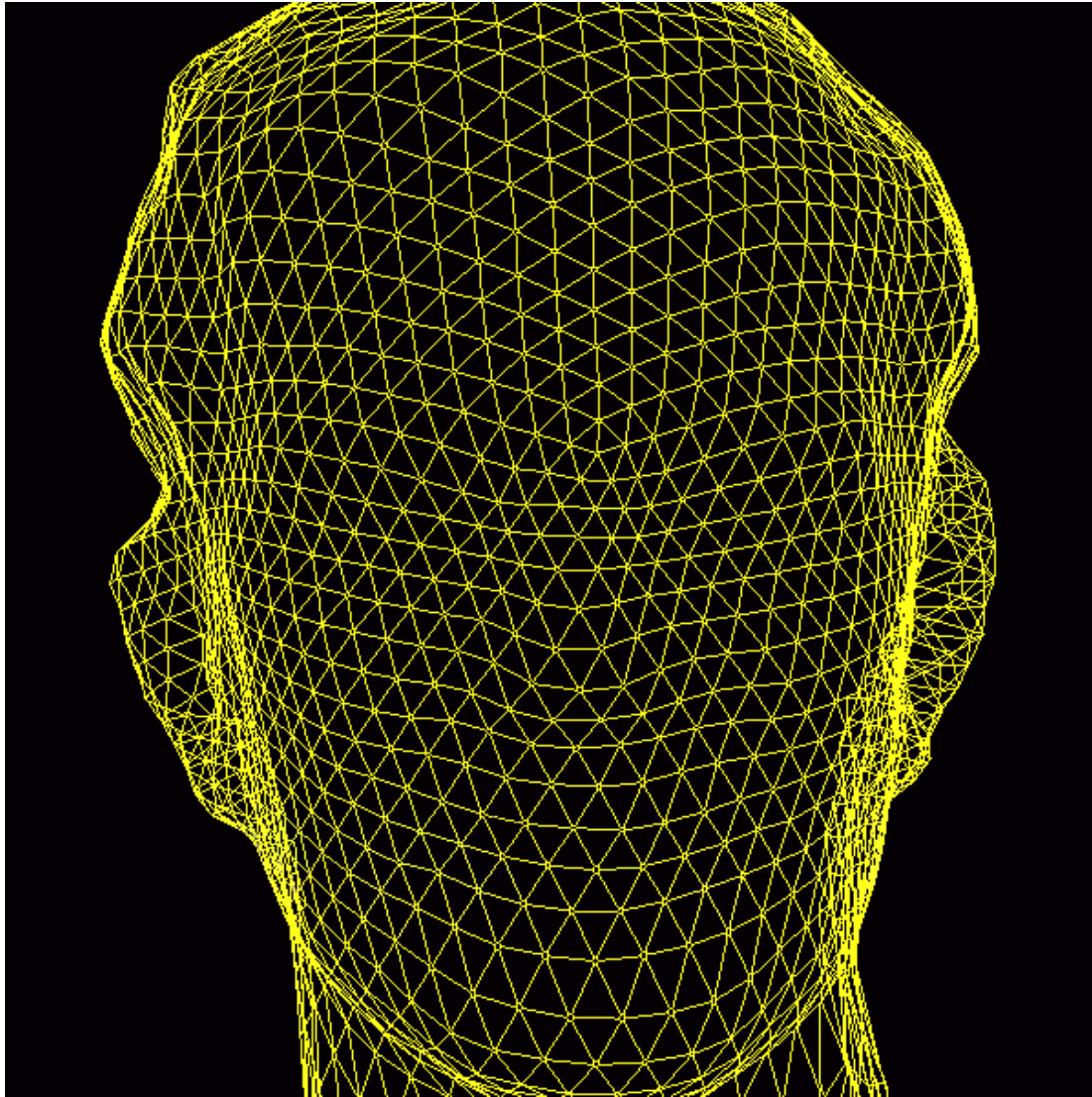


$$B_h = 10(3^i - 1)$$

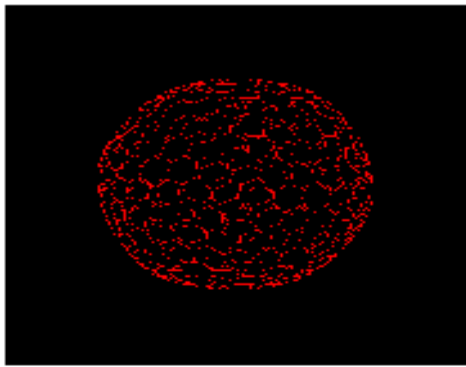
$$T = 20 * 3^i$$

$$H = 10(3^{i+1} - 1)$$

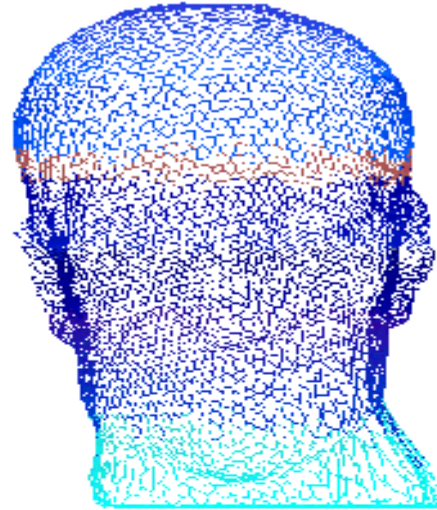
Construction of 3-Connected Surfaces from Unorganized Points



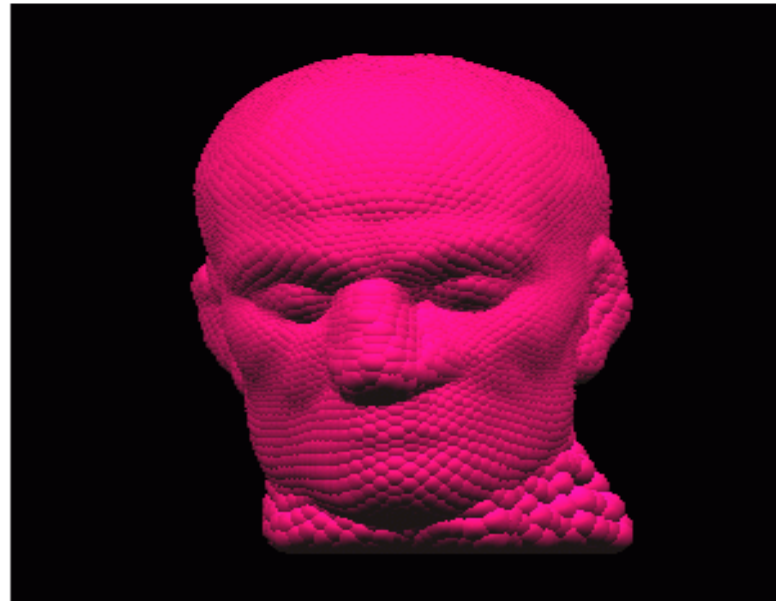
Interpolation by spheres



Radial
Projection

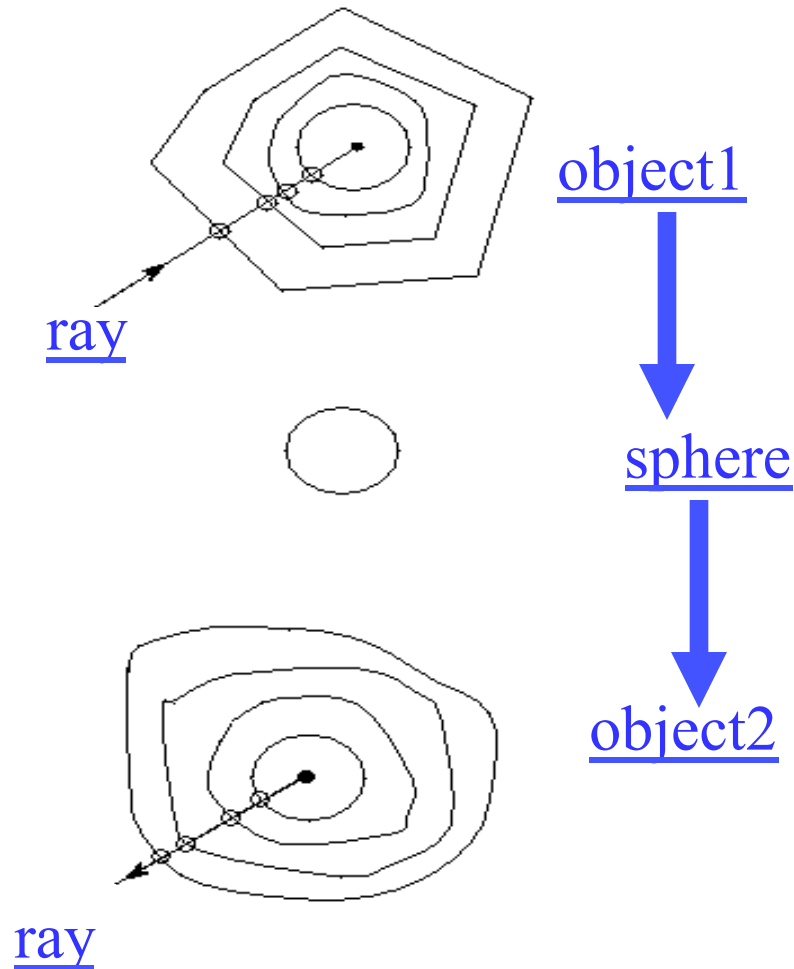


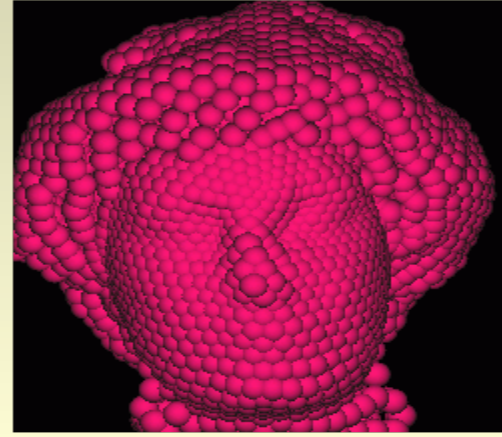
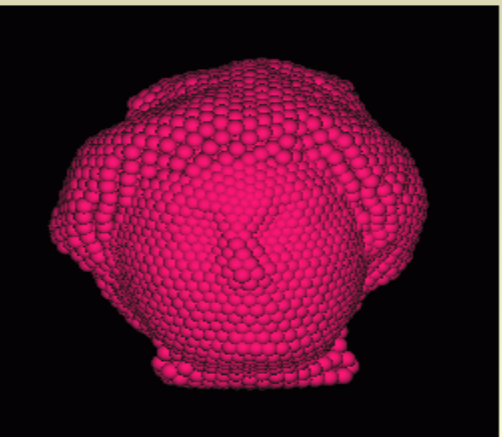
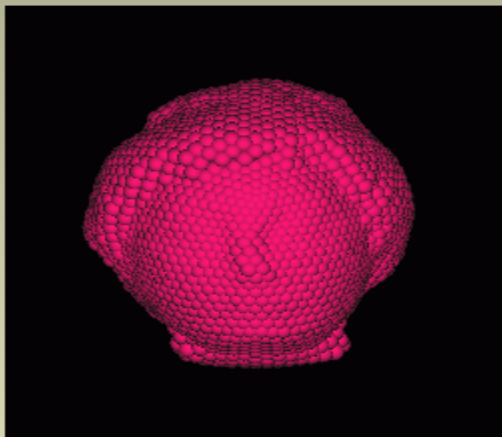
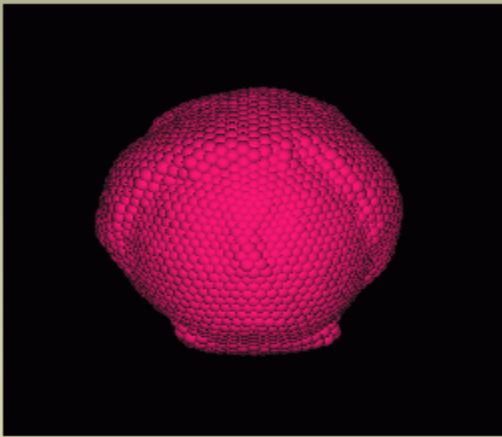
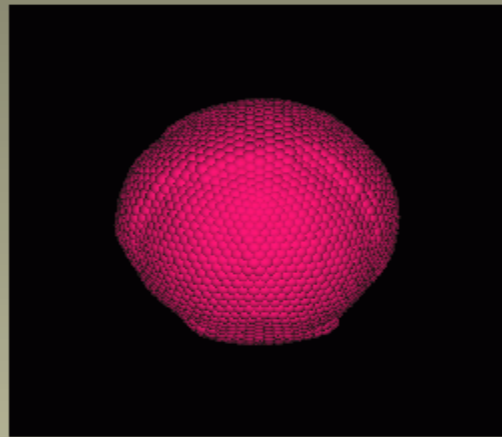
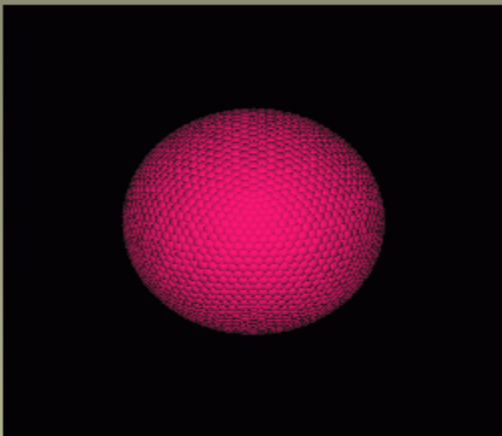
Interpolation
by spheres



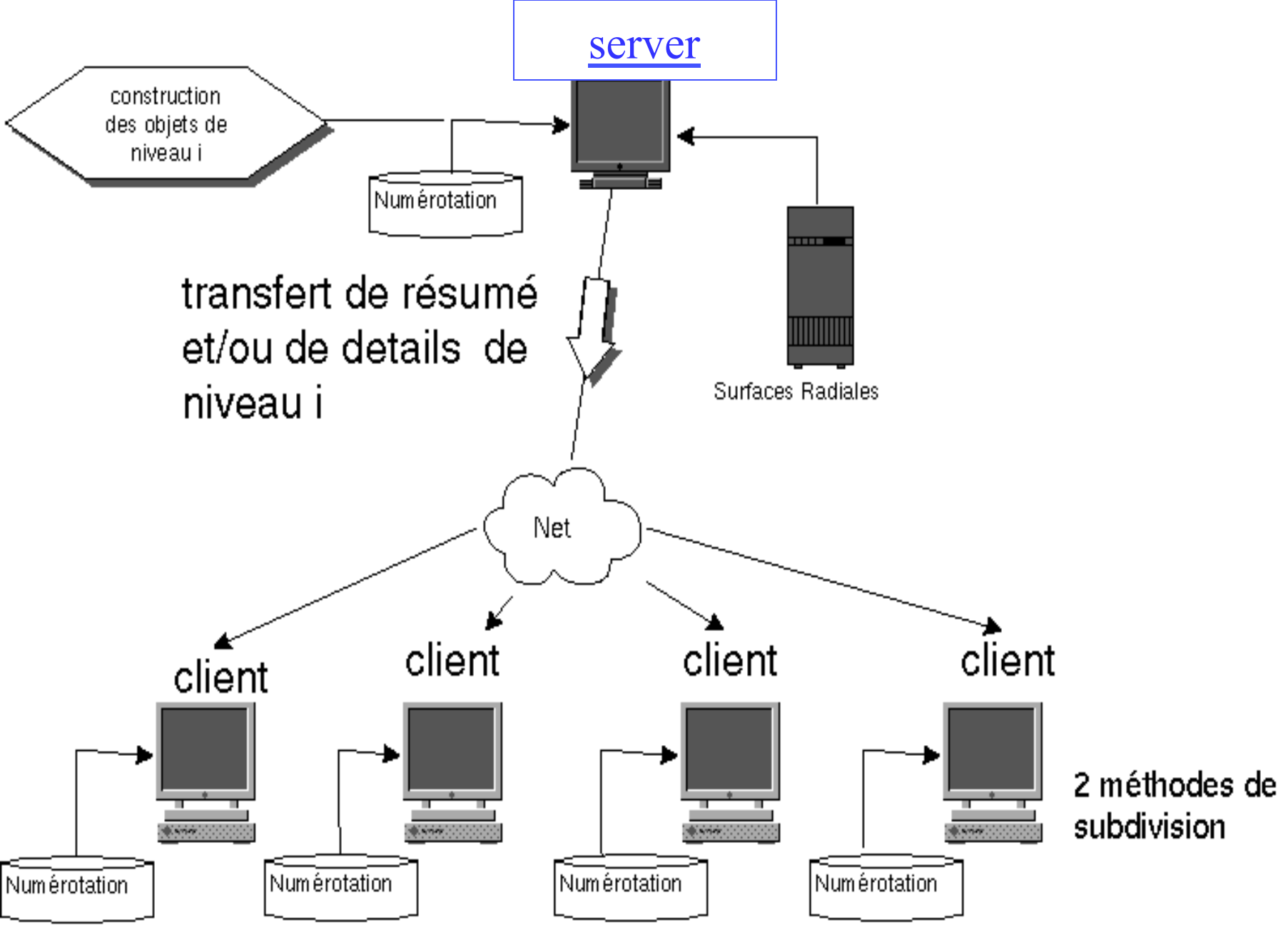
Morphing

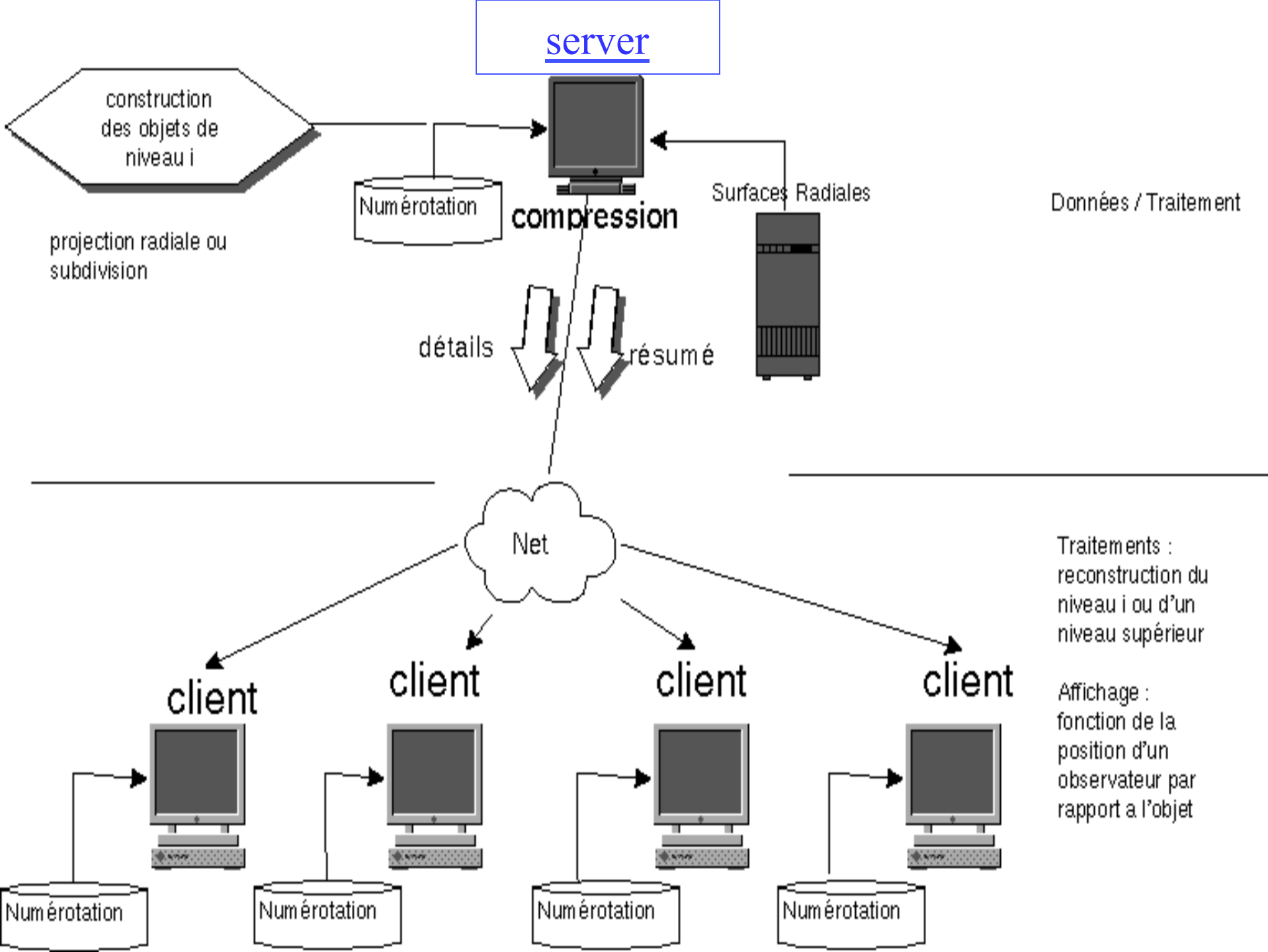
Morphing 3D applied to the objects with homogeneous mesh and 3-connected vertices



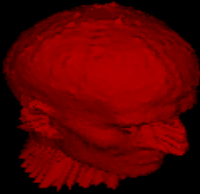


Applications multimedia





Level 5



Level 6

Parallélisation de l'algorithme

Fichier contenant les points et les facettes au niveau de subdivision i

1

2

3

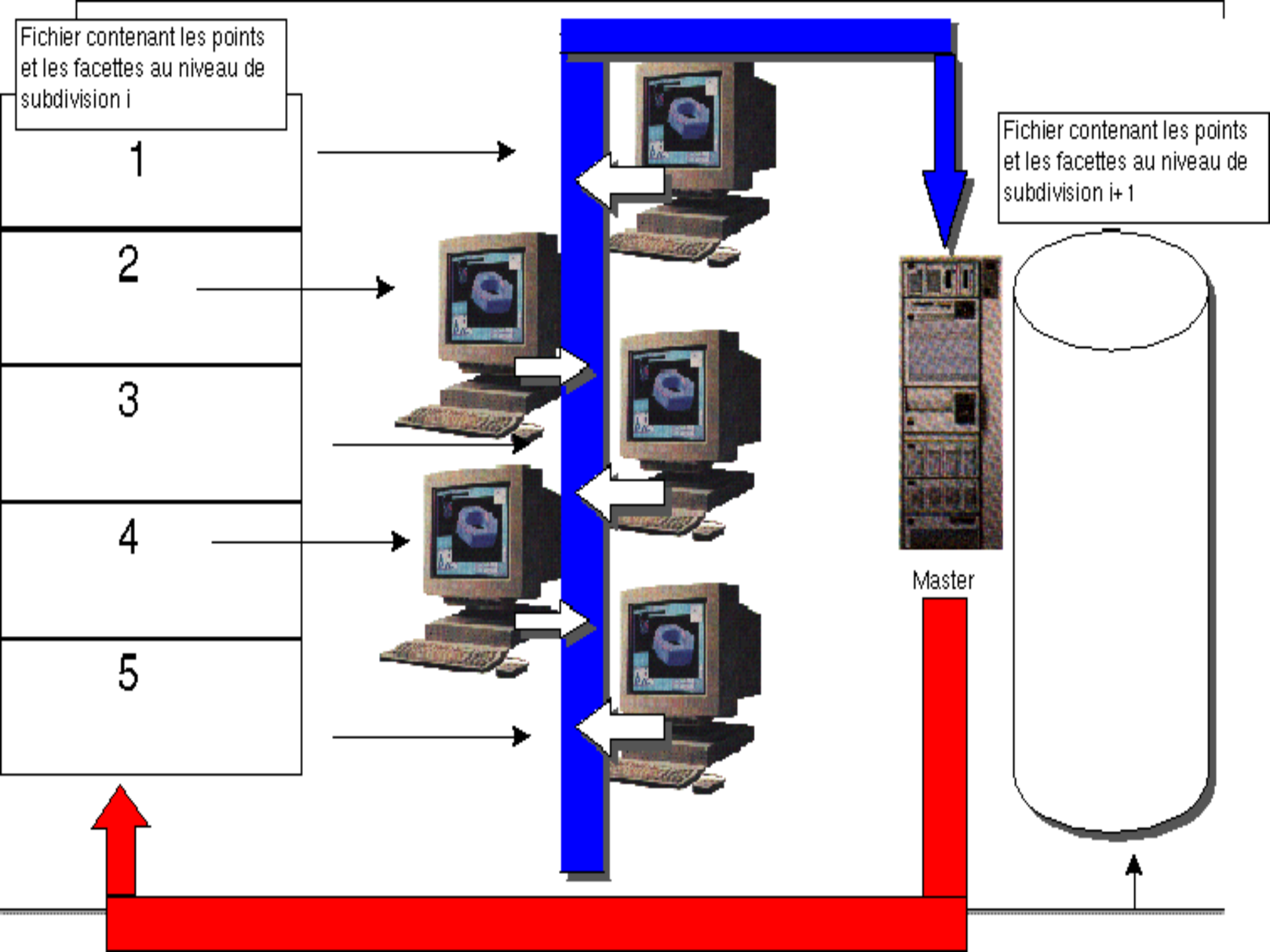
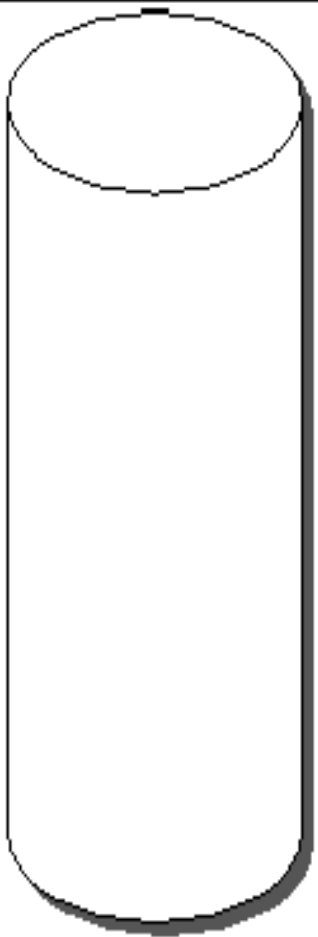
4

5

Fichier contenant les points et les facettes au niveau de subdivision $i+1$



Master



SEC

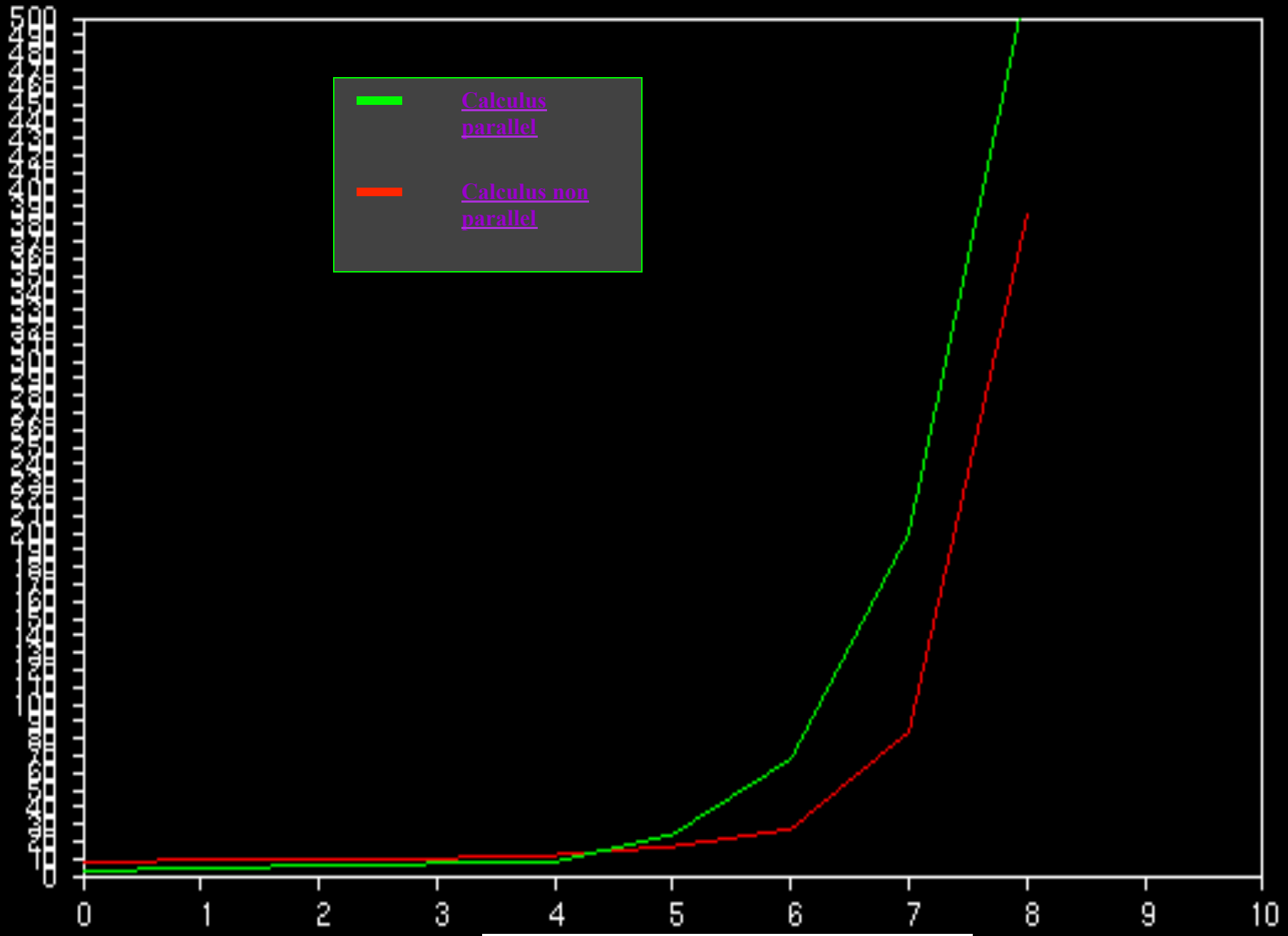
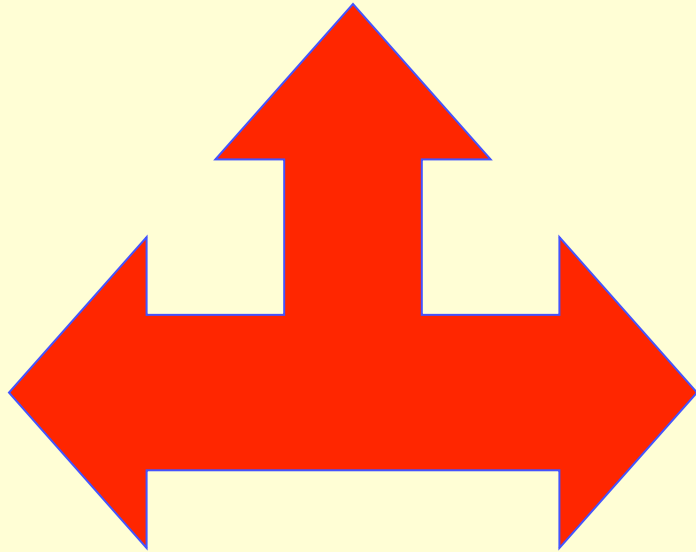


Figure 1: Calculus



Conclusion

Perspective de recherche



- Subdivision locale 3-connexes
 - ◆ Mise au point d'un nouveau système de numérotation pour la connexion de sphères
 - ◆ Construction de différentes pièces mécaniques maillées en 3-connexes
- Comparaison entre les surfaces obtenues par les ondelettes sur des surfaces 3-connexes et les nurbs

