Exercice 1

$$y(k+2) - 3\,y(k+1) + 2\,y(k) = u(k) \qquad u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & \text{sinon} \end{cases}$$

$$\left(z^2 - 3z + 2\right) Y(z) = \frac{z}{z-1}$$

$$Y(z) = \frac{1}{(z-1)^2(z-2)} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z-2}$$

$$A = \frac{1}{11}$$

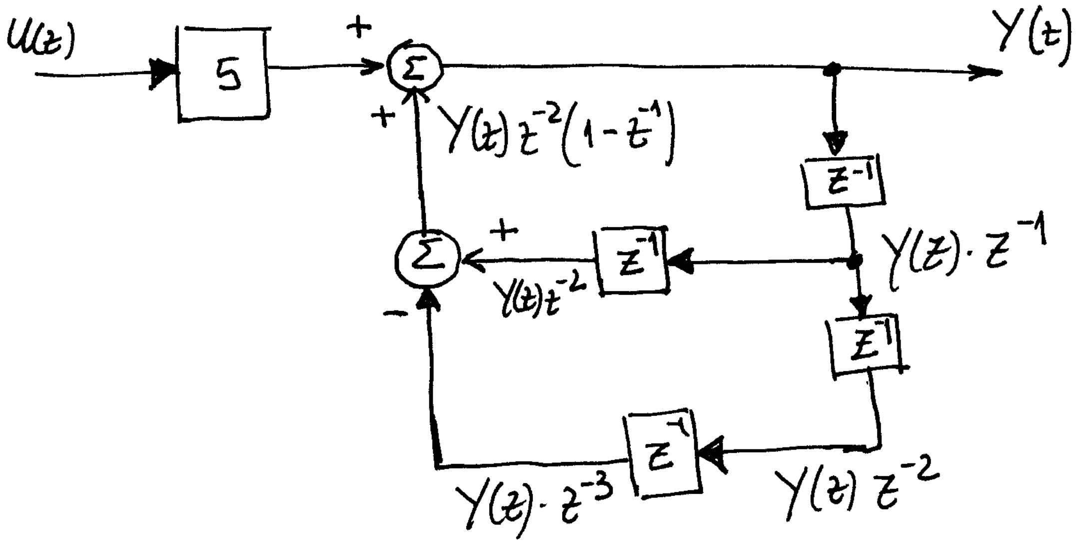
$$B = -\frac{5}{11} \qquad \Rightarrow \quad Y(z) = \frac{1}{11}\left(\frac{z}{z-1} - \frac{5}{(z-1)^2} - \frac{z}{z-2}\right)$$

$$C = -\frac{1}{11}$$

$$\downarrow$$

$$y(k) = \frac{1}{11}\left(u(k) - 5\,k\,u(k) + 2^k u(k)\right)$$

Exercice 2



$$Y(z) = 5\,U(z) + Y(z)\,z^{-2}\left(1 - z^{-1}\right)$$

$$\frac{Y(z)}{U(z)} = \frac{5}{1 - z^{-2} + z^{-3}}$$

Exercice 3

$$G(p) = \frac{k}{(p+1)(p+3)} \qquad k>0 \qquad T_S = 0{,}25$$

Dérivation

1) $$p \leftrightarrow \frac{1-z^{-1}}{T_S}$$

$$G_1(z) = \frac{k}{\left(\frac{1-z^{-1}}{T_S}+1\right)\left(\frac{1-z^{-1}}{T_S}+3\right)} = \frac{k T_S^2 z^2}{(1{,}2z-1)(1{,}6z-1)} = \frac{0.04 k z^2}{(1{,}2z-1)(1{,}6z-1)}$$

Intégration

$$p \leftrightarrow \frac{1}{T_S(1-z^{-1})}$$

$$G_2(z) = \frac{k}{\left(\frac{2}{T_S(z-1)}+1\right)\left(\frac{z}{T_S(z-1)}+3\right)} = \frac{0{,}04k(z-1)^2}{(1{,}2z-0{,}2)(1{,}6z-0{,}6)}$$

$$G_3(p) = \frac{k}{(p+1)(p+3)} \rightarrow \frac{k}{2} \times \frac{1-e^{-T_S}}{z-e^{-T_S}} - \frac{k}{2}\,\frac{1-e^{-3T_S}}{3(z-e^{-3T_S})}$$

$$G_3(z) = \frac{k}{6} \times \frac{0{,}0926z + 0{,}620}{(z-0{,}819)(z-0{,}549)}$$

2) $$FTBF(z) = \frac{0{,}04z^2}{(0{,}04k+1{,}92)z^2 - 2{,}8z + 1}$$

pour la stabilité $z = \frac{w+1}{w-1}$

$$1 + G(z) \rightarrow (0{,}04k+0{,}12)w^2 + (0{,}08k+1{,}84)w + (5{,}72+0{,}04k) = 0$$

$\Downarrow$

Système stable.

idem pour $G_2(z)$

$$1 + G_3(z) = 0 \rightarrow$$

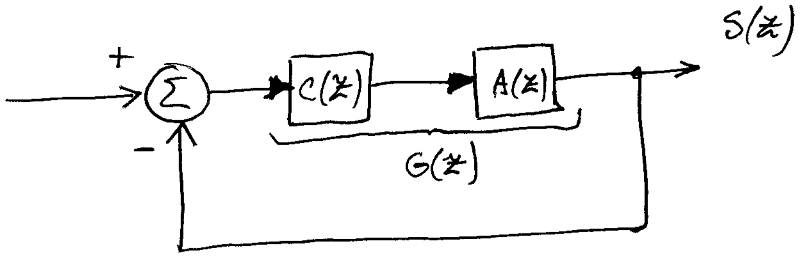
$$\left(\frac{0{,}713}{6}k + 0{,}032\right)w^2 + \left(\frac{-0{,}620k}{3} + 1{,}1\right)w + \left(\frac{0{,}527}{6}k + 2{,}82\right) = 0$$

pour que le système soit stable

$$1{,}1 - \frac{0{,}62}{3}k > 0 \Rightarrow 0 < k < 5{,}32$$

Exercice 4

$$A(z) = \frac{z + 0.7}{z - 0.7}$$



$T_S = 0.2s$

On veut :

$$G(z) = \frac{K\left(1 + e^{-2\xi\omega_n T_S} - 2e^{-\xi\omega_n T_S}\cos T_S\omega_n\sqrt{1-\xi^2}\right)}{z^2 - 2ze^{-\xi\omega_n T_S}\cos\omega_n T_S\sqrt{1-\xi^2} + 2e^{-\xi\omega_n T_S}}$$

$$G(z) = C(z)\,A(z)$$

$$G(z) = \frac{a}{(z-1)(z-b)} \quad \text{pour une } \varepsilon_p = 0$$

FTBF $\quad H(z) = \dfrac{a}{z^2 - (1+b)z + a + b}$

ainsi

$$H(z) = \frac{K_{BF}\left(1 + e^{-2\xi_{BF}\omega_{nBF}T_S} - 2e^{-\xi_{BF}\omega_{nBF}T_S}\cos\omega_{nBF}T_S\sqrt{1-\xi_{BF}^2}\right)}{z^2 - 2ze^{-\xi_{BF}\omega_{nBF}T_S}\cos\omega_{nBF}T_S\sqrt{1-\xi_{BF}^2} + 2e^{-\xi_{BF}\omega_{nBF}T_S}}$$

avec $\xi_{BF} = 0.6 \qquad \omega_{nBF} = \dfrac{3}{t_m} = 7.5$ rad/s

$$\Rightarrow \quad H(z) = \frac{0.87\,K_{BF}}{z^2 - 0.29z + 0{,}81}$$

par identification $\begin{cases} a = 1{,}52 \\ b = -0.71 \end{cases}$

$$G(z) = \frac{1{,}52}{(z-1)(z+0{,}71)}$$

Gain statique égal à 1 ($\varepsilon_p = 0$)

$$C(z) = \frac{1{,}52(z-0{,}7)}{(z-1)(z+0{,}71)(z+0{,}7)}$$

Exercice 5

Soit $K(p) = K_p \left(1 + \frac{1}{T_i p} + T_p p \right)$

$$K(z) = K_p \left(1 + \frac{T_i}{z-1} + T_p \left(1 - z^{-1}\right) \right)$$

$$K(z) = K_p \left(1 + \frac{T_i}{z-1} + T_p \frac{z-1}{z} \right)$$

$$1 + K(z) = 1 + K_p \left(1 + \frac{T_i}{z-1} + T_p \frac{z-1}{z} \right)$$

$$\Rightarrow \left(1 + T_p\right) K_p z^2 - \left(T_p + 1\right) K_p z - T_i K_p + 1 = 0$$

$$z_{12} = \frac{(T_p+1) K_p \pm \sqrt{(1+T_i)^2 K_p^2 + 4 (1+T_p) K_p (T_i K_p + 1}}{2}$$

$|z_{12}| > 1$

$K(p)$ n'est pas numérisable.