Exercise 1 Unification

For each pair of atomic sentences, give the most general unifier if it exists.

a. $P(A, B, B), P(x, y, z)$

b. $Q(y, G(A, B)), Q(G(x, x), y)$

c. $Older(Father(y), y), Older(Father(x), John)$

d. $Knows(Father(y), y), Knows(x, x)$

Exercise 2 (Prolog)

The following Prolog code defines a predicate $P$:

\[
P(X, [X|Y]). \]
\[
P(X, [Y|Z]) :- P(X, Z). \]

a. Show proof trees and solutions for the queries $P(A, [1, 2, 3])$ and $P(2, [1, A, 3])$.

b. What standard list operation does $P$ represent?

Exercise 3 (Resolution in FOL)

Consider the following knowledge base of two first-order logic sentences:

A: $\forall x \forall y \ LostATrial(x) \land Client(y, x) \Rightarrow \lnot Happy(y)$

B: $\forall x \ SucceessfulLawyer(x) \Rightarrow \exists y \ Client(y, x) \land Happy(y)$

Use resolution to prove that this knowledge base entails the sentence

C: $\forall x \ SucceessfulLawyer(x) \Rightarrow \lnot LostATrial(x)$
Exercise 4 (Minimal hitting sets)

Let $S = \{\{a, b\}, \{b, c\}, \{d, e\}\}$

1. Draw the search space of the hitting sets of $S$.
2. How many hitting sets are there?
3. How many minimal hitting sets are there?
4. A new set of elements $\{f\}$ is added to $S$, how does the size of the search space evolve?

Exercise 5 (Diagnosis)

We consider the following system.

1. Write the behavioural of the system.
2. Write the structural model of the system.
3. All the inputs and outputs of the system are measured and their value is 1. Write $OBS$.
4. What are the minimal $R$-conflicts?
5. What are the $R$-diagnoses?
6. What are the explanations?
7. Now we suppose that if a component has a normal behaviour then we are sure that its state is normal, answer the questions 1-6.