Comp3620 Artificial Intelligence
Tutorial 2: Logic

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Exercise 1 (Propositional logic)

Decide which of the following formulas are unsatisfiable, satisfiable, or tautoligies, using truth tables:

- $((B \Rightarrow A) \Leftrightarrow A) \Rightarrow B$
- $A \Rightarrow (B \Rightarrow A)$
- $(A \vee (\neg B \wedge \neg A)) \wedge (\neg A \wedge B)$

Exercise 2 (Entailement in propositional logic)

Check by means of truth table method whether the following entailments are valid.

- $P \vee Q, \neg P \models Q$
- $P \Rightarrow Q, Q \Rightarrow R \models P \Rightarrow R$
- $P \Rightarrow Q, Q \models P$
- $P \Rightarrow Q \models \neg(Q \Rightarrow P)$

Exercise 3 (Logical equivalence in propositional logic)

Using algebraic identities, show that $((a \vee c) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a))$ is equivalent to $((b \Rightarrow c) \wedge a)$.

This is an algebraic hand-evaluation: a series of formulas joined by $\equiv$. Don't write just portions of previous formulas and mysteriously re-introduce the dropped parts later. For each step, mention which identity you used.

Exercise 4 (Representation with first-order logic)

Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

- Some students took French in spring 2001.
- Every student who takes French passes it.
- Only one student took Greek in spring 2001.
- The best score in Greek is always higher than the best score in French.
- Every person who buys a policy is smart.
- No person buys an expensive policy.
- There is an agent who sells policies only to people wha are not insured.
- There is a barber who shaves all men in town who do not shave themselves
- A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.
- A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by birth, is a UK citizen by descent.
- Plolitícians can fool some of the people all the time, and they can fool all of the people some of the time, but they can't fool all the people all the time.

Exercise 5 (FOL: Reasoning with equivalence)

Simplify the following formula, so that the body of each quantifier contains only a single atomic formula involving that quantified variable. (Recall, an atomic formula is one with no connectives.) Provide reasoning for each step of your simplification.

$$\forall x, \forall y, \exists z((A(x) \land B(y)) \Rightarrow C(z))$$

Exercise 6 (FOL: Use of inference rules)

Describe the problem in FOL and solve it using inference rules.

1. No ducks are willing to waltz
2. No officers ever decline to waltz
3. All my poultry are ducks
4. My poultry are not officers
5. Does 4 follow from 1, 2, 3?