Part I
Diagnosis

1 Theory of Reiter

Reiter Diagnosis

Definition 1. A Reiter Diagnosis for an observed system \((SD, COMP, OBS)\) is a minimal set \(\Delta \subset COMP\) such that:

\[
SD, OBS, \{\neg Ab(c), c \in COMP \setminus \Delta\}, \{Ab(c), c \in \Delta\}
\]

is satisfiable.

Theorem 2. A Reiter Diagnosis is equivalent to a Minimal Diagnosis.

An R-diagnosis is seen as a set of components and not a logical sentence. The representation are equivalent.

Reiter Diagnosis: example

Example 3. Davis circuit

If \(OBS = In1(m1, 3), In2(m1, 2), \ldots, Out(a2, 12)\) there are 4 R-diagnoses,

\[\{m1\}; \{a1\}; \{m2, m3\}; \{m2, a2\}\]

The R-diagnosis \(\{m1\}\) is equivalent to the minimal diagnosis

\(\neg Ab(a1) \land \neg Ab(a2) \land Ab(m1) \land \neg Ab(m2) \land \neg Ab(m3)\).

A new example: an additionner

Example 4.
A new example: an additionner

Example 5. SD (behavioural model):

- \( \text{AND}(x) \land \neg \text{Ab}(x) \Rightarrow \text{Out}(x) = \text{and}(\text{In}1(x), \text{In}2(x)) \)
- \( \text{OR}(x) \land \neg \text{Ab}(x) \Rightarrow \text{Out}(x) = \text{or}(\text{In}1(x), \text{In}2(x)) \)
- \( \text{XOR}(x) \land \neg \text{Ab}(x) \Rightarrow \text{Out}(x) = \text{xor}(\text{In}1(x), \text{In}2(x)) \)
- \( \text{AND}(A1); \text{AND}(A2), \text{OR}(O1), \text{XOR}(X1); \text{XOR}(X2) \)

SD (structural model):

- \( \text{Out}(X1) = \text{In}2(A2) \ldots \)

Observations:

- \( \text{In}1(X1) = 1; \text{In}2(X1) = 0; \text{In}1(A2) = 1; \text{Out}(X2) = 1; \text{Out}(O1) = 0. \)

R-diagnoses:

- \{X1\}; \{X2, O1\}; \{X2, A2\}

Properties of R-Diagnoses

**Theorem 6.** \( \emptyset \) is the only R-diagnosis for \((SD, COMP, OBS)\) iff

\[ SD, OBS, \{\neg \text{Ab}(c), c \in COMP\} \]

is satisfiable.

**Theorem 7.** \( \Delta \subseteq COMP \) is a R-diagnosis iff it is a minimal set such that:

\[ SD, OBS, \{\neg \text{Ab}(c), c \in COMP \setminus \Delta\} \]

is satisfiable.

**How to compute R-diagnoses?**

2 Diagnosis computation

**R-conflicts**

**Definition 8.** An R-conflict \( C \) is a set \( \{c_1, c_2, \ldots, c_k\} \) with \( c_i \in COMP \) such that:

\[ SD, OBS, \{\neg \text{Ab}(c), c_i \in C\} \]

is not satisfiable.

An R-conflict is a set of components \( C \subseteq COMP \) which cannot be together in a normal state.

**Definition 9.** An R-conflict is minimal iff there is no strict subset which is also an R-conflict.
R-conflicts: Example 1

Example 10.
There are 2 minimal R-conflicts:

1. ?
2. ?

R-conflicts: Example 2

Example 11.
There are 2 minimal R-conflicts:

1. ?
2. ?

R-conflict and R-Diagnosis

Theorem 12. $\Delta \subseteq COMP$ is an R-diagnosis for $(SD, COMP, OBS)$ iff $\Delta$ is a minimal set such that $COMP \setminus \Delta$ is not an R-conflict.

This theorem is the basis of the algorithm DIAGNOSE from Reiter: it is a lattice exploration.

Definition 13. A lattice is (roughly) a non-empty partial order set $(S, \subseteq)$ such that every element $a, b$ have an infimum $\inf(a, b)$ (a “lower bound” element) and a supremum $\sup(a, b)$ (an “upper bound” element).
Search space for R-diagnoses

Example 14.
The search space is a lattice.

**DIAGNOSE algorithm**

Breadth-first search on the lattice from the empty set $\emptyset$

1. let $X$ the current node in the search
2. Call a theorem prover and ask:
   \[
   \text{Is } COMP \setminus X \text{ an R-conflict?}
   \]
3. if yes, eliminate the nodes $X'$ such that $X' \cap (COMP \setminus X) = \emptyset$
   - $X'$ cannot be a minimal diagnosis.
4. if no, $X$ is a minimal diagnosis, eliminate the descendants

**DIAGNOSE algorithm: example**

Example 15.
Sets in brackets are R-conflicts. Three minimal diagnoses: $\{X1\} ; \{X2, O1\} ; \{X2, A2\}$
Another way to solve the problem

The intersection between a diagnosis and any R-conflicts is not empty ⇒ Hitting set

**Theorem 16.** \( \Delta \subseteq \text{COMP} \) is an R-diagnosis for \((SD, \text{COMP}, \text{OBS})\) iff \( \Delta \) is a minimal hitting set for the set of minimal conflicts of \((SD, \text{COMP}, \text{OBS})\)

General diagnosis engine (GDE) from de Kleer.

**R-diagnosis: a minimal hitting set problem**

**Definition 17.** Let \( S = \{S_1, \ldots, S_n\} \) be a set of sets, \( H \) is a hitting set of \( S \) iff

\[
H \subseteq \bigcup_{S_i \in S} S_i
\]

and

\[
\forall S_i \in S, H \cap S_i \neq \emptyset
\]

**Example 18.** \( S = \{\{a, b\}, \{c, b\}, \{e, f\}\} \) The following sets are hitting sets of \( S \):

- \( H = \{a, b, c, e\} \)
- \( H = \{b, e\} \) (\( H \) is minimal)
- \( H = \{a, c, f\} \) (\( H \) is minimal)

The following sets are not hitting sets of \( S \):

- \( H = \{a, b\} \)
- \( H = \{b, e, g\} \)

**GDE algorithm**

1. Computation of all the minimal R-conflicts.
   
   - Use of an ATMS (Assumption Truth Maintenance System)
   - Update of beliefs about assumptions by retractation of knowledge and declaration of new ones

2. Computation of the minimal hitting set on the obtained R-conflicts

**R-conflict and R-Diagnosis: examples**

**Example 19.** **Additionner:**

The 2 minimal R-conflicts \( \{X1, X2\} \) and \( \{X1, A2, O1\} \) correspond to the 3 minimal diagnoses: \( \{X1\} : \{X2, O1\} \); \( \{X2, A2\} \)

**Davis circuit:**

The 2 minimal R-conflicts: \( \{a1, m1, m2\} \) and \( \{a1, a2, m1, m3\} \) correspond to the 4 minimal diagnoses: \( \{m1\} : \{a1\} \); \( \{a2, m2\} : \{m2, m3\} \)
3 Incremental Diagnosis

Incremental diagnosis

GDE or DIAGNOSE solve the diagnosis problem in a off-line way.

- The observation set is supposed to be complete

In some systems, an observation is the result of a test, an action, a measurement from the environment to the system.

Definition 20. The incremental diagnosis problem is to:

- compute a diagnosis based on a partial set of observations
- choose what could be the next measurement to perform in the system: prediction

Predicted observations

Definition 21. An R-diagnosis $\Delta$ predicts $O$ iff

$$SD, OBS, \{\neg Ab(c), c \in COMP \setminus \Delta\}, \{Ab(c), c \in \Delta\} \models O$$

Given the system SD, the current set of observations OBS and the current diagnosis $\Delta$, the system should produce the observation $O$.

Predicted observations: example

- $\Delta_1 : \{m1\}$ predicts $Out(m2) = 6$
- $\Delta_2 : \{m2, m3\}$ predicts $Out(m2) = 4$ and $Out(m3) = 6$. 
Updating an R-Diagnosis

**Theorem 23.** Confirmation: A R-diagnosis for \((SD, COMP, OBS)\) which predicts \(O\) is a R-diagnosis for \((SD, COMP, OBS \land O)\).

If the predicted observation \(O\) is real (the measurement gives \(O\)), then the diagnosis is *confirmed* by the observation \(O\).

**Theorem 24.** Invalidation: A R-diagnosis for \((SD, COMP, OBS)\) which predicts \(\neg O\) is not a R-diagnosis for \((SD, COMP, OBS \land O)\).

If a diagnosis predicts something which is not true, it means that the diagnosis becomes a *wrong hypothesis* and is *invalid*.

Updating an R-Diagnosis

1. Input: \((SD, COMP, OBS)\) an observed system, \(O\) a new observation
2. Check if \(\Delta\) predicts \(O\)
3. if yes then \(\Delta\) is confirmed
   
   \(\Delta\) is a diagnosis of \((SD, COMP, OBS \land O)\)
4. Check if \(\Delta\) predicts \(\neg O\)
5. If yes then look at supersets of \(\Delta\)

Updating an R-Diagnosis: example

\begin{itemize}
\item \(\Delta_1 = \{m1\}\) predicts \(Out(m2) = 6\)
\item \(\Delta_2 = \{m2, m3\}\) predicts \(Out(m2) = 4\)
\item \(\Delta_3 = \{a1\}\) predicts \(Out(m2) = 6\)
\item \(\Delta_4 = \{a2, m2\}\) predicts \(Out(m2) = 4\)
\end{itemize}

If \(O\) is \(Out(m2) = 5\), every diagnosis is invalidated. The new ones are supersets: \{\(a2, m1, m2\}\}, \{\(a1, m2, m3\}\}, \{\(a1, a2, m2\}\}, \{\(m1, m2, m3\}\)
Discriminability/ Diagnosability

**Definition 26.** Let $O$ be an observation which confirms $\Delta_1$ and invalidates $\Delta_2$, we say that $O$ *discriminates*.

**Definition 27.** A system $(SD, COMP)$ is *diagnosable* if for any set of possible measurements (any complete set of observations) we have a *unique* diagnosis.

In a diagnosable system, we have enough information (observations) to discriminate between all the diagnoses and to get only one.

Using an incremental diagnosis algorithm on a diagnosable system, we have the guarantee that it *converges* to one diagnosis.

**Diagnosability: example**

![Diagram](image)

*Example 28.*

If we can observe only $A, B, C, D, E, F, G$ then the system is *not diagnosable*. If we observe $A, B, C, D, E, F, G, X, Y, Z$ then the system is *diagnosable*. The observations from $B, C, D, E, G$ do not allow to discriminate between diagnoses involving $m_2, m_3, a_2$.

**Summary**

- Theory of Reiter: notions of *R-Diagnosis, R-conflicts*
- Logic representation $\equiv$ set representations (minimal diagnoses)
- Algorithms:
  - DIAGNOSE: use of a theorem prover, exploration a *lattice*
  - GDE: computation of conflicts and *hitting sets* computation
- *Incremental diagnosis*: update the diagnoses with new measurements
- *Discriminability-Diagnosability* of systems
  - The more information we have, the less numerous are the diagnoses.
4 And the rest

So many things...

- Non-monotonic reasoning
  - Monotonicity: if $KB \models \alpha$ then with a new information $\beta$, we still have $KB \land \beta \models \alpha$
  - The world is full of exceptions: every bird can fly, so the emu does!
  - Nonmonotonic logics: Default logic, Circumscription

- Uncertainty
  - Strong assumption: our knowledge is complete!
  - How to express and make reasoning about ignorance, incompleteness
  - Use of probability theory (Bayesian networks, Markov Decision Process, Fuzzy logic)

So many things...

- Inconsistency
  - Always reasoning with consistency! boring! and bounded! (incompleteness)
  - What about reasoning about inconsistencies: $1 + 1 = 3$ for 1 big enough!
  - Paraconsistent logics...

- I give up, I do not have time...