Design of indicators for the detection of time shift failures in $(max, +)$-linear systems

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Motivations

Time shift failure detection...

- Time shift failures: unexpected delays in an assembly line, slowing down of a conveyor belt in a luggage conveyor...
- Detection: is the flow of timed observations resulting from a normal or an abnormal behavior of the system?

...in $(max, +)$-linear systems[1]

- Discrete Event Systems characterized by *delay* and synchronization phenomena
- Graphical representation by Timed Event Graph (TEG, subclass of Timed Petri Net for which each place has exactly one upstream and one downstream transition)
- Linear modelling using idempotent semiring theory

[1] F. Baccelli *and al.* : Synchronization and Linearity. Wiley and sons, 1992.

Outline

Idempotent semiring

Definition. Idempotent semiring

Set $\mathcal{D}$ endowed with two inner operations

- $\oplus \rightarrow$ associative, commutative, idempotent ($a \oplus a = a$)
  neutral element ε
- $\otimes \rightarrow$ associative, distributes over the sum
  neutral element e

Kleene star operator: $\forall a \in \mathcal{D}, a^* = \bigoplus_{i \geq 0} a^i$ with $a^0 = e$.

Example. Idempotent semiring $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

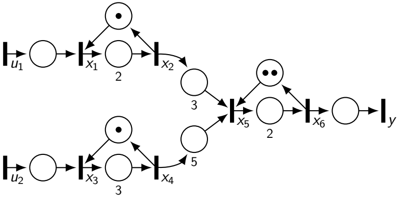
Set of formal series with two commutative variables γ and δ, Boolean coefficients in $\mathbb{B}$, exponents in $\mathbb{Z}$, and for which each element is an equivalence class modulo $\gamma^* \otimes (\delta^{-1})^*$
$\rightarrow$ Used to model $(max, +)$-linear systems

Models of $(max, +)$-linear systems

$(max, +)$-linear system on $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

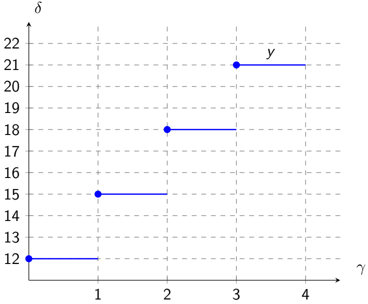
- A series s of $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$: cumulative flow of events γ over time δ
- In $\gamma^n\delta^t \in s$, the n^{th} event occurs at earliest at time t

A TEG of an assembly line



$$u = \begin{pmatrix} \gamma^0\delta^2 \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^5 \oplus \gamma^4\delta^{+\infty} \\ \gamma^0\delta^2 \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^5 \oplus \gamma^4\delta^{+\infty} \end{pmatrix}$$

$$y = h \otimes u =$$
$$\gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$$



Problem statement

In a $(max, +)$-linear system, from observations of the system and with the knowledge of its behavior, are we able to know if a time shift failure happens in the system?

- Assume that the flow of inputs u is observed
- Assume that the functional mode of the system is known by the transfer function h
- The expected flow of output $\tilde{y}$ is computed ($\tilde{y} = h \otimes u$)
- Assume that the real flow of output y is observed

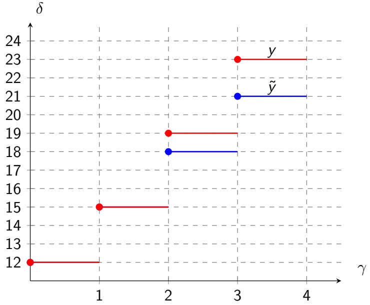
By the use of mathematical operators of $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, make a comparison between the expected flow of output $\tilde{y}$ and the real flow of output y

Time comparison between flows

Definition. Time shift function between $y, \tilde{y} \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

$$\forall n \in \mathbb{Z}, \quad \mathcal{T}_{y,\tilde{y}}(n) = \mathcal{D}_y(n) - \mathcal{D}_{\tilde{y}}(n)$$

where $y = \bigoplus_{n\in\mathbb{Z}} \gamma^n \delta^{\mathcal{D}_y(n)}$ and $\tilde{y} = \bigoplus_{n\in\mathbb{Z}} \gamma^n \delta^{\mathcal{D}_{\tilde{y}}(n)}$



$$y = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{+\infty}$$

$$\tilde{y} = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$$

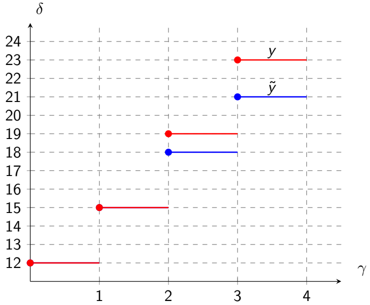
$$\mathcal{T}_{y,\tilde{y}}(0) = 12 - 12 = 0$$
$$\mathcal{T}_{y,\tilde{y}}(1) = 15 - 15 = 0$$
$$\mathcal{T}_{y,\tilde{y}}(2) = 19 - 18 = 1$$
$$\mathcal{T}_{y,\tilde{y}}(3) = 23 - 21 = 2$$

Time comparison between flows

Theorem [MaxPlus, 1991][2] Bounds on time shift function

$$\forall n \in \mathbb{Z}, \quad \mathcal{D}_{y \phi \tilde{y}}(0) \leq \mathcal{T}_{y,\tilde{y}}(n) \leq -\mathcal{D}_{\tilde{y} \phi y}(0)$$

where operator ϕ comes from the residuation theory ($y \phi \tilde{y}$ is the optimal solution to inequality $x \otimes \tilde{y} \preceq y$)



$$y \phi \tilde{y} = \gamma^0 \delta^{\mathbf{0}} \oplus \gamma^1 \delta^3 \oplus \gamma^2 \delta^7 \oplus \gamma^3 \delta^{11} \oplus \gamma^4 \delta^{+\infty}$$

$$\tilde{y} \phi y = \gamma^0 \delta^{\mathbf{-2}} \oplus \gamma^1 \delta^2 \oplus \gamma^2 \delta^6 \oplus \gamma^3 \delta^9 \oplus \gamma^4 \delta^{+\infty}$$

So $\mathcal{D}_{y \phi \tilde{y}}(0) = 0$
and $-\mathcal{D}_{\tilde{y} \phi y}(0) = 2$

[2] MaxPlus: Second order theory of min-plus linear systems and its application to discrete event systems, CDC, 1991

Single Output Indicator

Definition. Indicator of an observed MISO[3] (max, +)-linear system

Let $h \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{1 \times p}$ be the transfer function of the system
Let $u \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p}$ and $y \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ be its observable input and output flows

$$I_{SO}(u, y) = \begin{cases} \textit{false} \text{ if for } \tilde{y} = hu,\ \Sigma_\tau(y, \tilde{y}) = [0; 0] \\ \textit{true} \text{ otherwise} \end{cases}$$

with $\Sigma_\tau(y, \tilde{y}) = [\mathcal{D}_{y \phi \tilde{y}}(0); -\mathcal{D}_{\tilde{y} \phi y}(0)]$

[3] Multiple Inputs Single Output.

Single Output Indicator

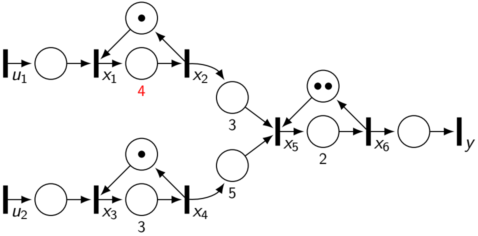
Proposition. Correctness of the I_{SO} indicator

The indicator $I_{\mathrm{SO}}(u, y)$ is correct, that is it returns *true* only when the system is failing.

Sketch of proof

- Let y_1 and y_2 be two identical series
  $x_1 = y_1 \phi y_2 = y_2 \phi y_1 = y_1 \phi y_1$
- Because of properties of operators * and ϕ
  $x_1 = (x_1)^* = \mathbf{e} \oplus \dots = \gamma^{\mathbf{0}}\delta^{\mathbf{0}} \oplus \dots$
- Therefore $\Sigma_\tau(y_1, y_2) = [0; 0]$

Single Output Indicator



Time failure from transition x_1 to transition x_2: delay of 2 time units

$$y \not\phi \tilde{y} = \gamma^0 \delta^{\mathbf{0}} \oplus \gamma^1 \delta^3 \oplus \gamma^2 \delta^7 \oplus \gamma^3 \delta^{11} \oplus \gamma^4 \delta^{+\infty}$$

$$\tilde{y} \not\phi y = \gamma^0 \delta^{\mathbf{-2}} \oplus \gamma^1 \delta^2 \oplus \gamma^2 \delta^6 \oplus \gamma^3 \delta^9 \oplus \gamma^4 \delta^{+\infty}$$

$$\Sigma_\tau(y, \tilde{y}) = [\mathcal{D}_{y \not\phi \tilde{y}}(0); -\mathcal{D}_{\tilde{y} \not\phi y}(0)] = [0; 2]$$

$$I_{SO}(u, y) = true \quad \Rightarrow \quad \text{The time shift failure is detected!}$$

Multiple Outputs Indicator

Definition. Indicator of an observed MIMO[4] $(\max, +)$-linear system

Let $h \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q\times p}$ be the transfer function the system
Let $u \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p}$ and $y \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q}$ be its observable input and output flows

$$I_{\mathrm{MO}}(u, y) = \bigvee_{i=1}^{q} I_{\mathrm{SO}}(u, y_i)$$

where $I_{\mathrm{SO}}(u, y_i)$ is a modified version of the single output indicator:

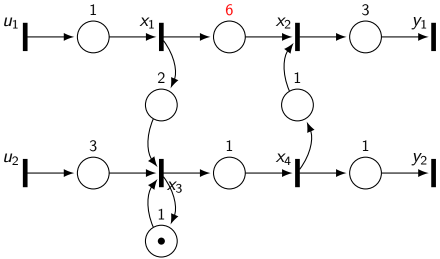
$$I_{\mathrm{SO}}(u, y_i) = \begin{cases} \textit{false} \text{ if for } \tilde{y} = hu,\ \Sigma_\tau(y_i, \tilde{y}_i) = [0; 0] \\ \textit{true} \text{ otherwise} \end{cases}$$

Proposition. Correctness of the I_{MO} indicator

By extension, the indicator $I_{\mathrm{MO}}(u, y)$ is correct.

[4]Multiple Inputs Multiple Outputs.

Multiple Outputs Indicator



Time failure from transition x_1 to transition x_2: delay of 4 time units

$$\Sigma_\tau(y_1, \tilde{y}_1) = [0;1] \quad \Rightarrow \quad I_{SO}(u, y_1) = true$$

$$\Sigma_\tau(y_2, \tilde{y}_2) = [0;0] \quad \Rightarrow \quad I_{SO}(u, y_2) = false$$

$$I_{MO}(u, y) = I_{SO}(u, y_1) \vee I_{SO}(u, y_2) = true \vee false = true$$

$\Rightarrow$ The time shift failure is detected!

What we have done

- Propose indicator for the detection of time shift failures in $(\max, +)$-linear systems
- For MISO and MIMO systems
- Implemented with the `minmaxgd`[5] C++ library

What is next?

- Perform fault localization and identification
- Extend these results to intervals of timed process

[5] B. Cottenceau *and al.* : Data processing tool for calculation in diod, WODES, 2000

Thank you for your attention!