Another Point of View on Diagnosability

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Abstract: This paper provides a new definition of diagnosability, that allows one to check the diagnosability of any set of system states, and by extension of properties that depend on the system state. The existing definitions and approaches for checking diagnosability apply to faults or sets of faults, and comparison shows that the new definition generalizes the existing ones. This new definition is applied to repair preconditions, and an example shows how this brings complementary information compared to classical fault diagnosability.

Keywords: Diagnosis, Design, Fault diagnosis, System diagnosis, Systems design, Computer-aided diagnosis, Computer-aided design.

1. INTRODUCTION

Complex and critical systems require close supervision when running, and the model-based community has produced a lot of work in this area. In particular, model-based diagnosis is an increasingly active research domain, and the problem of diagnosability analysis has been addressed many times Sampath et al. [1995], Travé-Massuyès et al. [2006a], Bayoudh et al. [2008]. In a diagnosable system, although it is impossible to know the exact state of the system, the supervisor is aware of which anticipated faults have happened and which have not. However, the information needed by a supervisor in such systems is not limited to fault presence, and fault diagnosability in a system does not guarantee that this system is easy to supervise. A system designer hence needs to verify the diagnosability of more than just the faults.

This paper presents a definition for diagnosability that can be applied to any set of states in a system. The model used in this paper is state-based, and most properties can be mapped to a set of system states. This provides the possibility to check the diagnosability of state dependent properties. When a property is diagnosable, the system supervisor is always able to assess whether the current system state verifies this property. Fault presence or absence are examples of such state dependent properties. The definition can also be used to perform diagnosability analysis for faults, in the same way as existing approaches do. But its extension to any set of states also allows one to check the diagnosability of many other properties, like for example repair preconditions, which is illustrated in this paper.

2. RELATED WORK

Diagnosis has been an active research topic for many years and numerous approaches have been proposed to cope with on-line as well as post mortem diagnosis Blanke et al. [2006] Hanscher et al. [1992]. More recently a significant trend has moved the activities of the diagnosis community towards the analysis of the properties related to diagnosis. Several pieces of work deal with defining and checking diagnosability Krysander and Nyberg [2005], Travé-Massuyès et al. [2006b], Bayoudh et al. [2008] and a unified characterization bridging state based and event based modeled systems has been proposed Travé-Massuyès et al. [2006a]. Diagnosability guarantees that all the anticipated faulty situations of a system are discriminable one from the other, although the state of the system is partially observed. This property is quite important because it indicates that the instrumentation providing the observations about the system is well designed and sufficient to provide an explanation of what is going on. However, nowadays systems are required to run more and more autonomously and they are expected to cope with unanticipated situations by themselves, in particular when faults occur. Hence, diagnosability has been more recently addressed together with the requirements for repairability in order to provide a formal definition for self-healability Cordier et al. [2007].

3. EXISTING DIAGNOSABILITY APPROACHES

This section presents diagnosability approaches existing in the literature, in order to compare them to the new approach defined in section 4. All approaches rely on a formal description of the system behaviour, in the absence and in the presence of faults.

3.1 System representation

The system is assumed to be described by a proposition \( sd \) which can be expressed in propositional logic. The set of models of this proposition is denoted \( SD \), it contains all variable tuples satisfying \( sd \) and describes the set of all the system states, faulty or non faulty. The set of variables is denoted \( V \). Some of the variables characterize the presence or absence of faults, these are called mode variables. \( O \) denotes the set of observable variables. Generally, mode variables are not observable. The set \( OBS \) contains all the possible system observations. In other words, it contains the models of the restriction of \( sd \) to the variables in \( O \).
3.2 Faults and fault modes

Various faults may occur in the system, modifying its behaviour and possibly making it unable to fulfill its function. Several faults may be present at the same time.

A fault mode characterizes the behaviour of the system under a given combination of faults. It assesses the presence of some faults as well as the absence of the other faults. The normal mode is one of the many fault modes, it assesses the absence of all faults. The occurrence of a permanent fault changes the fault mode of the system.

A fault is characterized by one mode variable, whose value indicates whether the fault is present or not. A fault mode is identified by a value for the tuple of all mode variables. \(SD_f\) is the description of the fault mode \(f\), i.e. the set of states in which the fault mode is verified. As any state belongs to exactly one fault mode, the set of all \(SD_f\) is a partition of \(SD\), as illustrated in Fig. 1.

3.3 Projection on observable variables

An operation called projection on observable variables and noted \(P_{OBS}\) is used. It takes as input a system state expressed as a variable value tuple, and outputs the tuple of observable values obtained in this state. For example, if \(V\) contains 5 variables, and if the first and third are observable, then:

\[
P_{OBS} : SD \rightarrow OBS \quad (v_1, v_2, v_3, v_4, v_5) \rightarrow (v_1, v_3)
\]

The inverse projection \(P_{OBS}^{-1}\) is defined from \(OBS\) to \(2^{SD}\) as follows, \(s\) being a state and \(\sigma\) an observation:

\[
P_{OBS}^{-1}(\sigma) = \{s \in SD, P_{OBS}(s) = \sigma\}
\]

The projection \(P_{OBS}\) associates a system state to the observation that is received under this state. The inverse projection associates an observation to the set of all states that may have originated this observation. When applied to fault modes, this projection is the base of the classical diagnosability analysis approaches.

3.4 Fault mode diagnosability

The classical definition for fault signature and diagnosability is provided now. The definition of diagnosability states that the system cannot produce a common observation under two different fault modes Trévé-Massuyès et al. [2006a].

Definition 1. (Fault mode Signature and Diagnosability).

The signature of a fault mode \(f\) is the set of all possible observations when the system state belongs to the mode \(f\).

\[
Sig(f) = \{P_{OBS}(s), s \in SD_f\}
\]

A system is diagnosable if and only if, \(f_1\) and \(f_2\) being fault modes:

\[
\forall f_1, f_2 \quad f_1 \neq f_2 \Rightarrow \text{Sig}(f_1) \cap \text{Sig}(f_2) = \emptyset
\]

When diagnosability according to definition 1 holds, the observations emitted by the system always allow one to decide which faults have happened, and which faults have not. But when the signatures of two fault modes intersect, this means that there exists at least one observation that can be emitted by the system under two different fault modes. There are two possible explanations for this observation, and a diagnosis process would output two diagnostic candidates.

3.5 Macrofault diagnosability

Another definition of diagnosability is given in Cordier et al. [2007] as an extension of definition 1. It is based on the notion of macrofault, which is a set of fault modes. It is based on the idea that not all pairs of fault modes need to be discriminable: fault modes that do not need to be discriminated one from another are gathered into a macrofault. The set of states in which the macrofault \(F_i\) is present is noted \(SD_{F_i} = \bigcup_{f \in F_i} SD_f\).

This raises a significant difference compared to the previous approach. Whereas fault modes are disjoint, macrofaults may overlap (e.g. macrofaults \(\{f_1, f_2\}\) and \(\{f_2, f_3\}\) share the states of \(f_2\)). In the macrofault approach, it is considered that when the system state belongs to several macrofaults, it belongs to an overlapping fault mode, and identifying only one of the macrofaults with certainty is enough for the system to be diagnosable.

In this approach, only covering sets of macrofaults are considered, i.e. sets of macrofaults such that every fault mode belongs to a macrofault. Consequently, there is always at least one present macrofault, whatever the system state is.

Definition 2. (Macrofault, Characteristic signature). A macrofault \(F_i\) is a set of fault modes. \(F_i\) is present if and only if the system is in one of the fault modes \(f_j \in F_i\).

A characteristic signature \(cSig(F_i)\) is a set of observations that allow one to assess with certainty that the macrofault \(F_i\) is present.

\[
cSig(F_i) \subseteq \left( \bigcup_{f_j \in F_i} (\text{Sig}(f_j)) \setminus \bigcup_{f_k \in F_i \setminus F_j} (\text{Sig}(f_k)) \right)
\]

Note that there are several possible characteristic signatures for each macrofault. If \(O\) is a characteristic signature for a macrofault \(F_i\), then any \(O' \subseteq O\) is also a characteristic signature for \(F_i\).

Definition 3. (Macrofault Diagnosability). A covering set of macrofaults \(\{F_i\}\), i.e. a set of macrofaults that cover all
the fault modes, is diagnosable if and only if there exists a set of characteristic signatures for these macrofaults that form a partition of \( OB \).

When such a partition is established as illustrated in Fig. 1, it is always possible to find out at least one present macrofault. As a state may belong to several macrofaults, an observation can also correspond to several macrofaults. However, it is only needed, for each observation, to assess with certainty that one macrofault is present.

This definition is a generalization of definition 1, since fault modes are particular macrofaults. Because macrofaults may overlap when they contain the same fault mode, this definition applies to a greater range of sets of states than the fault mode definition.

This definition is also less constrained than fault mode diagnosability (definition 1), in the sense that in a system verifying fault mode diagnosability, any set of macrofaults is diagnosable.

4. DIAGNOSABILITY REVISITED

This section presents a new definition of diagnosability, which applies to any state dependent property. It is based upon the analysis of the set of states in which a property holds. It is a generalization of existing diagnosability definitions which only apply to sets of states characterized by the presence or absence of some faults. Comparisons show that this new definition is consistent with the existing ones.

4.1 Diagnosability of a property

Definition 4. (Diagnosable block). Let \( =_{OB} \) be the equivalence relation defined on \( SD \) by:

\[
\forall s_1, s_2 \in SD, s_1 =_{OB} s_2 \iff \mathcal{P}_{OB}(s_1) = \mathcal{P}_{OB}(s_2)
\]

Each equivalence class of \( =_{OB} \) is called a diagnosable block of the system. The set of diagnosable blocks of the system is the quotient set of \( SD \) by \( =_{OB} \).

Definition 5. (Diagnosability). A property or its corresponding set of states \( S \subseteq SD \) is diagnosable if and only if \( S \) is exactly a union of diagnosable blocks.

Fig. 2 depicts a system with 7 states and 4 possible observations. The diagnosable blocks are represented by white sets with dashed lines. Observation \( o_2 \) is received in two different states, one inside \( S_1 \) and one outside. Thus, when observing \( o_2 \), a supervisor is unable to decide whether the system is in \( S_1 \) or not. On the other hand, it is always possible to decide from the observations whether the system state belongs to \( S_2 \) or not.

4.2 Comparison with fault mode diagnosability

Since definition 5 applies to any set of states, it applies in particular to fault modes. It is shown now that when applied to fault modes, this definition is equivalent to definition 1.

Proposition 1. A system is diagnosable according to definition 1 if and only if for every fault mode \( f \), \( SD_f \) is diagnosable according to definition 5.

Proof The signatures of two fault modes \( f_i \) and \( f_j \) intersect if and only if there exists a state \( s_i \in SD_{f_i} \) and another state \( s_j \in SD_{f_j} \) leading to the same observation. These two states obviously belong to the same diagnosable block, say \( d \), and, since \( SD_{f_i} \) and \( SD_{f_j} \) are disjoint, none is a subset of \( d \). Since diagnosable blocks form a partition of \( SD \), \( s_i \) (resp. \( s_j \)) does not belong to any other diagnosable block than \( d \). Hence, \( SD_{f_i} \) (resp. \( SD_{f_j} \)) is not a union of diagnosable blocks.

If one fault mode \( f_i \) is not a union of diagnosable blocks, then since fault modes form a partition of \( SD \), there exists a diagnosable block \( d \) containing a state \( s_i \) of \( f_i \) and at least one state \( s_j \) belonging to another fault mode \( f_j \). These two states lead to the same observation \( o \), which necessarily belongs to both \( Sig(f_i) \) and \( Sig(f_j) \). Consequently the signatures of all fault modes are not disjoint.

4.3 Signature and preemptability

Definition 5 expresses the diagnosability of a single property. This definition is interesting when evaluating independent or disjoint sets of states. Quality of service is an example, the system states are partitioned, each part corresponding to one level of quality of service (good, fair, poor, etc.). In this case, quality of service levels are very similar to fault modes, and their diagnosability should be evaluated wrt. definition 5.

Some other properties, in particular repair preconditions, are related one to another. First, they may overlap, i.e. a state may belong to several repair preconditions, which means that there are several ways to repair the system in this state. Second, they may preempt, i.e. if two repairs can be applied and for some reason repair \( r_1 \) is preferable to repair \( r_2 \), then \( r_1 \) preempts \( r_2 \). Among the various reasons that may lead the supervisor to prefer one repair, we focus on the diagnosability of the repair preconditions: if among two repair candidates, one of the candidates' precondition is diagnosable (i.e. we know when the system state belong to the precondition or not) and the other is not (the actual system state may be inside or outside of the repair precondition), there is a point in choosing the repair with the diagnosable precondition. Macrofault diagnosability takes into account these overlapping and preemption aspects: a set of macrofaults is diagnosable when for every observation, the supervisor can assess the presence of at least one macrofault with certainty.

Our general definition is now extended to a set of properties. For this, the classical notion of signature is extended and the notion of preemptability is introduced. The new
definition of the signature applies to sets of states as opposed to definition 1 that applies to fault modes.

Definition 6. (Signature of a set of states). The signature of a set of states \( S \), or of the property \( p \) mapped to \( S \), is the set of observations that can be obtained when the system is under one of these states:

\[
\text{Sig}(S) = \{ p_{\text{obs}}(s), s \in S \}
\]

This definition applies equally to the complement set \( \overline{S} \). As sets of states generally overlap, comparing their signatures with one another does not bring much information. It is worthwhile to compare their signatures with the signatures of their respective complements. Indeed, if a set of states corresponds to a given property of the system, its complement corresponds to the negation of the property.

Definition 7. (Diagnosable space, Undiagnosable space). The diagnosable space \( D(S) \) (resp. undiagnosable space \( UD(S) \)) of a set of states \( S \) (resp. \( \overline{S} \)) is the subset of \( S \) in which it is possible (resp. impossible) to assert whether the property \( p \) holds.

\[
UD(S) = S \cap p_{\text{obs}}^{-1}(\text{Sig}(S) \cap \text{Sig}(\overline{S}))
\]

\[
D(S) = S \setminus UD(S)
\]

As illustrated in Fig. 3, \( D(S) \) can also be defined as the union of the diagnosable blocks included in \( S \). The diagnosable blocks that intersect but are not included in \( S \) form \( UD(S) \cup UD(\overline{S}) \). The intersection of this set with \( S \) gives \( UD(S) \). Hence, when a set of states is diagnosable, its undiagnosable space is empty.

When a property \( p \) is undiagnosable, it can be preemptable if its undiagnosable space is included in the diagnosable space of other properties. In this case these other properties may preempt \( p \) in the sense that when the validity of \( p \) is uncertain, one of these other properties is valid, which makes \( p \) unnecessary, as illustrated in Fig. 4.

Definition 8. (Preemptability). A property or its corresponding set of states \( S \) is preemptable by a set of properties \( \{S'\} \) if and only if:

\[
UD(S) \subseteq \bigcup_{S'} D(S')
\]

4.4 Diagnosability of a set of properties

This section presents a definition of diagnosability for a set of properties that accounts for the mutual influence that properties may have with one another by the means of preemptability.

Definition 9. (Diagnosability of a set of properties). A set of properties is diagnosable if and only if each property is either diagnosable or preemptable.

Considering a diagnosable set of properties, the union of all the corresponding sets of states is diagnosable.

Let \( S_i \) be the set of states corresponding to the \( i \)-th property of a diagnosable set of properties. For each \( i \), \( UD(S_i) \) is either empty or included in the diagnosable sets of other sets of states. Hence, \( \bigcup S_i \) is diagnosable since each \( D(S_i) \) is a union of diagnosable blocks.

4.5 Comparison with macrofault diagnosability

Now it is shown that definition 9 is equivalent to definition 3 when applied to macrofaults.

Proposition 2. A covering set of macrofaults is diagnosable according to definition 3 if and only if it is diagnosable according to definition 9.

Proof. First, given a macrofault \( F_i \), let us consider \( \text{Sig}(D(F_i)) \). This set contains no observation from a state in which \( F_i \) is absent, and is hence a characteristic signature for \( F_i \). Let us map each macrofault \( F_i \) to the set \( \Sigma_i = \text{Sig}(D(F_i)) \setminus \bigcup_{j<i} \text{Sig}(D(F_j)) \). \( \Sigma_i \) is a characteristic signature for \( F_i \) since it is a subset of \( \text{Sig}(D(F_i)) \).

Second, let \( o \in \text{Sig}(D(F_i)) \). We have either \( o \not\in \Sigma_i \), or \( o \in \Sigma_i \) with \( j<i \), and if \( o \in \Sigma_i \) then necessarily \( o \not\in \Sigma_k \) with \( k \neq i \). Hence, the set of all \( \Sigma_i \) forms a partition of the set \( \bigcup \text{Sig}(D(F_i)) \).

The previous statements are now used to establish the equivalence. The covering set of macrofaults \( \{F_0 \ldots F_n\} \) is diagnosable according to definition 9 if and only if the set of all \( D(F_i) \) covers \( D(S) \) (see section 4.4), which is equivalent to the set \( \bigcup \text{Sig}(D(F_i)) \) covering \( OBS \). Consequently, from the statements above, it follows that the set \( \Pi = \{\Sigma_0 \ldots \Sigma_n\} \) partitions \( OBS \), and the set of macrofaults is diagnosable according to definition 3.

5. APPLICATION TO REPAIR PRECONDITIONS

When analysing diagnosability of some system property, the designer needs to choose whether to take preemptability into account. This article provides no formal or informal criterion for the relevance of preemptability. We have proved in last section that the definition of self-healability, and more precisely the diagnosability of macrofaults given in Cordier et al. [2007] relies implicitly on preemptability.
Two faults are considered. First, the pipe may be clogged, which reduces greatly the flow capacity of the pipe. Second, the tank may be always able to deliver water, however in exceptional conditions, the tank may occur to be empty. When this occurs, the input flow is directed in priority to the tank. If the input flow is sufficiently high, it can supply both the empty tank and the output.

**Sensors** A pressure sensor is placed in the pipe, in order to detect abnormally high pressures. This happens when the pipe is clogged, and there is input flow.

**Model** The model of this system contains five variables:

- **if** describes the input flow and has 3 values: *none*, *low*, and *high*.
- **of** is Boolean and equals 1 when there is an output flow.
- **hp** is Boolean and equals 1 when the pressure inside the pipe is abnormally high.
- **pc** is Boolean and equals 1 when the pipe is clogged.
- **te** is Boolean and equals 1 when the tank is empty.

The behaviour of the system is described by the following constraints:

- \( of = 1 \Leftrightarrow (te = 0 \lor (if = \text{high} \land pc = 0)) \)
- \( hp = 1 \Leftrightarrow (if \neq \text{none} \land pc = 1) \)

**Observables** The variables *if*, *of*, and *hp* are observable.

**Repairs** The following repairs are available:

1. It is possible to unclog the pipe thanks to a chemical action \((rp_1)\). This repair can be applied when the pipe is clogged. For safety reasons, it must not be applied when the pipe is not clogged. If the tank is empty, this repair is not sufficient to bring back the system in a normal state.

   \[ rp_1 : (pc = 1 \land te = 0) \]

2. It is also possible to unclog the pipe mechanically \((rp_2)\). The action consists in sending someone on site and clean the pipe. This repair can only be applied when the pipe is empty (no input flow), but is not sufficient if the tank is empty, since it will not bring the system to a normal state. This repair can if necessary be applied in the normal mode: once the cleaner is on site, if the pipe is not clogged, then the cleaner will do nothing.

   \[ rp_2 : (te = 0 \land if = \text{none}) \]

3. Finally, if the tank is empty, it is possible to redirect the whole flow through the tank \((rp_3)\). This permits to mechanically unclog the pipe if needed. However, the manipulations involved in this repair require that there is no flow in the pipe.

   \[ rp_3 : (te = 1 \land (if = \text{none} \lor pc = 1)) \]

The system has 12 different states, represented in Fig. 6 as well as the diagnosable blocks and their corresponding observations. The application of definitions 5 and 9 to the fault modes, macrofaults and repair preconditions are now illustrated on this system.

6.1 Fault mode diagnosability analysis

The system has 4 fault modes, named *normal*, *pipe clogged*, *tank empty* and *pipe clogged & tank empty* noted *pipe & tank*. They define 4 sets of states whose diagnosability is analyzed. The analysis details are described in Fig. 7.

According to definition 5, none of these fault modes is diagnosable. Moreover, fault modes are by definition disjoint sets, and the notion of preemptability is not relevant when dealing with disjoint sets of states, since disjoint sets cannot preempt one another. Consequently, no fault mode is diagnosable according to definition 9 either.
6.2 Macrofault diagnosability analysis

Macrofaults have been introduced in Cordier et al. [2007] under the hypothesis that a repair repairs a set of fault modes, i.e. a repair precondition is a union of fault modes. This is obviously not the case here, which makes macrofault diagnosability analysis not relevant.

For the sake of illustration, let us consider the following set of macrofaults \( \{F_1, F_2, F_3\} \) with \( F_1 = \{\text{normal, tank empty}\} \) and \( F_2 = \{\text{pipe clogged}\} \) and \( F_3 = \{\text{tank empty, pipe & tank}\} \). Fig. 7 shows that none of these macrofaults is diagnosable with respect to definition 5. Only \( F_3 \) is preemptable by \( F_1 \), hence \( \{F_1, F_2, F_3\} \) is not diagnosable according to definition 9 either.

6.3 Repair precondition diagnosability analysis

The sets of states corresponding to the repair preconditions are represented in figure 6. Diagnosability analysis provides the results indicated in Table 7.

The undiagnosable spaces of repair preconditions \( RP_2 \) and \( RP_3 \) are empty, which means these sets of states are unions of diagnosable blocks. They are diagnosable, according to definition 5. Moreover, \( RP_1 \) is not diagnosable, but \( UD(RP_1) \subseteq D(RP_2) \), it is preemptable. The set of repair preconditions \( \{RP_1, RP_2; RP_3\} \) is diagnosable with respect to definition 9.

7. CONCLUSION

This paper provides a new, general definition of diagnosability, that applies to any set of states and by extension to any state dependent property. Given a property that depends on the system state, it is possible to assess whether the observations are sufficient to deduce that the property holds or not. Such properties can be the presence or absence of faults, which falls back into existing diagnosability approaches. They can also be repair preconditions, which we expect to fall back into existing self-healability approaches Cordier et al. [2007]. Analysing repair preconditions diagnosability is shown to bring different information from analysing fault modes or macrofaults diagnosability.

This work covers other kinds of properties, for example the ability of the system to provide its function, or a subfunction, or to fulfill a given quality of service. Any property that can be mapped to a set of states can be checked with this approach.

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