Model-based diagnosis of time shift failures in discrete event systems: a (max,+) observer-based approach

Claire Paya[123], Euriell Le Corronc[23], Yannick Pencolé[2], and Philippe Vialletelle[1]

[1] STMicroelectronics, Crolles, France,
claire.paya@st.com,philippe.vialletelle@st.com
[2] LAAS-CNRS, Univ. Toulouse, CNRS, Toulouse, France
euriell.le.corronc@laas.fr,yannick.pencole@laas.fr
[3] Université Toulouse 3, Paul Sabatier, Toulouse, France

Abstract. This paper addresses the problem of diagnosing the occurrence of time shift failures in systems like automated production lines. The model of the system is represented as a Timed Event Graph (TEG) that is characterized as a (max,+)-linear system. The proposed method aims at detecting and localizing the source of time shift failures by the design of a set of indicators. These indicators rely on the residuation theory on (max,+)-linear systems and a (max,+) observer that estimates the internal state of the observed system.

Keywords: model-based diagnosis, (max,+)-linear system

1 Introduction

Discrete Event Systems (DES) can be used to model and solve fault diagnosis problems in automated production lines. In systems like production lines, failures can be not only caused by complete equipment breakdowns but also by the occurrence of time shifts so that the production line can dramatically slow down and not be able to comply with the specified production objectives. This paper addresses the problem of how to automatically detect and localize the source of such time shifts based on a subclass of time Petri Nets, called Timed Event Graph (TEG). In TEGs, places are associated with a punctual duration and they can be modeled by (max,+) algebra as introduced in [1, 6]. The history of DES with the use of (max,+) algebra is presented in [5]. For example, [4] uses (max,+) algebra to control wafer delays in cluster tools for semiconductor production. The problem of failure diagnosis by the use of (max,+) algebra has been introduced in [7] where the proposed detection method relies on the residuation theory and compares observable outputs with expected ones to detect output time shifts (Section 3). Failure localisation is then performed by an ad hoc structural analysis of the underlying TEG that does not use (max,+) algebra. The objective of this paper is to design a new set of time shift failure indicators

(Section 5) that are not based on the observable outputs of the system only but on the estimation of the internal state of the system (Section 4) so that the failure localisation problem is also solved in an algebraic way. To do so, the proposed failure indicator will rely on an observer that is proposed in [2] and aims at rebuilding system states based on the observations.

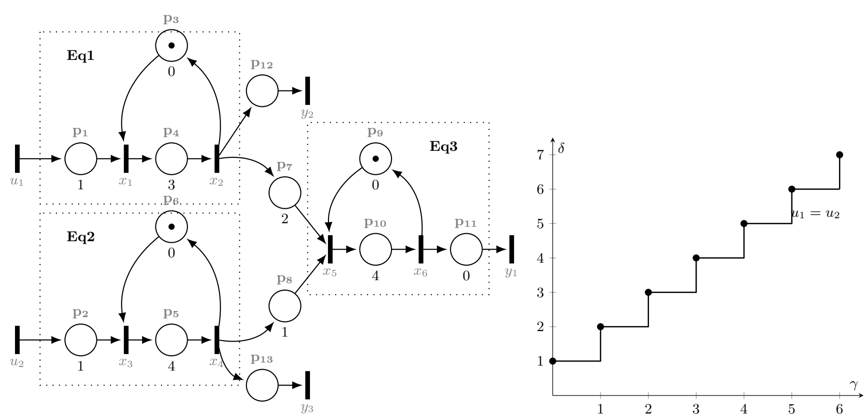

Fig. 1. Fault free model (left) and a graphical representation of the inputs u_1 ($=u_2$) in the proposed scenario (right).

2 Motivation

The problem that we address is motivated by a real production line that is at STMicroelectronics Crolles300 plant. Semiconductor manufacturing is complex and one of its most important challenges is to succeed in detecting production drifts before they have real impact on production plan. STMicroelectronics has complex production lines of wafer batches with many pieces of equipment running in parallel. One of the objectives is to detect as soon as possible that an equipment is late to ensure that products (wafer batches) are delivered on time or at least with minimal delays. Figure 1 presents a fault-free behavioural model of such a production line defined by a TEG. This production line corresponds to three pieces of equipment (namely Eq1, Eq2, Eq3). Eq1 is modeled with a couple of places p_3, p_4: it is available (i.e. no current processing) if a token is in place p_3 while it is processing its input if a token is in place p_4. The process of Eq1 is carried out in 3 hours. Similarly Eq2 and Eq3 are respectively modeled by the couple of places p_6, p_5 (processing time: 4 hours) and p_9, p_{10} (processing time 4 hours). Places p_7, p_8 model wafer batch transportation between Eq1, Eq2 and Eq3. For $i = \{1, 2\}$, a trigger of an input transition u_i represents the occurrence of an event from sensors on the production line that indicates the arrival of unprocessed wafer batches in front of Eqi. The output to the production line is a stream of fully processed wafer batches modeled by firing transition y_1. Outputs y_2 and y_3 provide observable information about the end of the process of Eq1 and Eq2. Suppose a scenario where a stream of 7 wafer batches arrive at Eq1 (input u_1) respectively at time $t \in \{1, 2, 3, 4, 5, 6, 7\}$ and suppose it is similar for u_2. As detailed later, this sequence of events can graphically be represented as a set of points (event of index $\gamma = 0$ arrives at time $\delta = 1$...), see Figure 1. Then, suppose that processed wafer batches are successively available at time $\{12, 17, 22, 27, 32, 37, 42\}$ (output y_1), and process information is available at time $\{5, 8, 11, 14, 17, 20, 23\}$ for output y_2 and at time $\{7, 12, 17, 22, 27, 32, 37\}$ for output y_3. Then, the question is: based on the fault-free model of Figure 1, can we detect and localize a time shift failure in the underlying production line? This paper aims at designing a model-based (max,+)-algebraic decision method for the detection of such time shifts relying on the use of the dioid $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$.

3 Mathematical background

About dioid theory The dioid theory is used to describe the inputs and the behavior of a system.

Definition 1. *A dioid $\mathcal{D}$ is a set composed of two internal operations $\oplus$ and $\otimes$.*[4] *The addition $\oplus$ is associative, commutative, idempotent (i.e. $\forall a \in \mathcal{D}, a \oplus a = a$) and has a neutral element ε. The multiplication $\otimes$ is associative, distributive on the right and the left over the addition $\oplus$ and has a neutral element e. Element ε is absorbing by $\otimes$ (i.e. $\forall a \in \mathcal{D}, a \otimes \varepsilon = \varepsilon = \varepsilon \otimes a$).*

For instance, let $\mathbb{Z}_{max}$ be the set $(\mathbb{Z} \cup \{-\infty\})$ associated with the max operation denoted $\oplus$ ($2 \oplus 3 = 3$...) with neutral element $\varepsilon = -\infty$ and the integer addition denoted $\otimes$ ($2 \otimes 3 = 5$...) with neutral element $e = 0$, then $\mathbb{Z}_{max}$ is a dioid.

Definition 2. *A dioid is complete if it is closed for infinite sums and if $\otimes$ is distributive over infinite sums.*

$\mathbb{Z}_{max}$ is not complete as $+\infty$ does not belong to $\mathbb{Z}_{max}$. Let $\overline{\mathbb{Z}}_{max} = \mathbb{Z}_{max} \cup \{+\infty\}$, $\overline{\mathbb{Z}}_{max}$ defines a complete dioid where $(-\infty) \otimes (+\infty) = (-\infty)$. By construction, a dioid $\mathcal{D}$ is partially ordered with respect to $\preceq$ with: $\forall a, b \in \mathcal{D}, a \preceq b \Leftrightarrow a \oplus b = b$. The exponential term $a^i, a \in \mathcal{D}, i \in \mathbb{N}$ is defined as follows: $a^0 = e$ and $\forall i > 0, a^{i+1} = a \otimes a^i$. Finally, in a complete dioid, the Kleene star operator is $a^* = \bigoplus_{i \geq 0} a^i$. From this, it follows a fundamental result (Theorem 1):

Theorem 1. *Let $\mathcal{D}$ be a complete dioid, $x = a^* b$ is the solution of $x = ax \oplus b$.*

[4] When there is no ambiguity, the symbol $\otimes$ is omitted.

About dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ The complete dioid $\mathbb{B}[\![\gamma,\delta]\!]$ is the set of formal series $s_J = \bigoplus_{(n,t)\in J} \gamma^n\delta^t$ with $J \subseteq \mathbb{Z}^2$ where $\gamma^n\delta^t$ is a monomial composed of two commutative variables γ and δ. Neutral elements are $\varepsilon = \bigoplus_{(n,t)\in\varnothing} \gamma^n\delta^t$ and $e = \gamma^0\delta^0$. Graphically, the series s_J of $\mathbb{B}[\![\gamma,\delta]\!]$ represents any collection J of point of coordinates (n,t) in $\mathbb{Z}^2$ with γ as horizontal axis and δ as vertical axis (see Figure 1). The dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ is defined as the quotient of the dioid $\mathbb{B}[\![\gamma,\delta]\!]$ by the modulo $\gamma^*(\delta^{-1})^*$. The internal operations are the same as in $\mathbb{B}[\![\gamma,\delta]\!]$ and neutral elements ε and e are identical to those of $\mathbb{B}[\![\gamma,\delta]\!]$ and $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ is also complete. By construction, any series of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ represents a *non-decreasing function* over γ and its canonical form is $s = \bigoplus_{k=0}^{K} \gamma^{n_k}\delta^{t_k}$ with $K \in \mathbb{N}\cup\{+\infty\}$ with $n_0 < n_1 < \ldots$, $t_0 < t_1 < \ldots$. Throughout this paper, we will use series of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ to represent the occurrence of an event type over time. For instance, Figure 1 shows such a series $u_1 = u_2 = \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^5 \oplus \gamma^5\delta^6 \oplus \gamma^6\delta^7 \oplus \gamma^7\delta^{+\infty}$ as described in the scenario of Section 2.

Definition 3. *Let $s \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ be a series, the dater function of s is the non-decreasing function $D_s(n)$ from $\mathbb{Z} \mapsto \mathbb{Z}\cup\{+\infty\}$ such that $s = \bigoplus_{n\in\mathbb{Z}} \gamma^n\delta^{D_s(n)}$.*

Series u_1 (similarly for u_2) has for dater function $D_{u_1}(0) = 1$, $D_{u_1}(1) = 2$, $D_{u_1}(2) = 3$, $D_{u_1}(3) = 4$, $D_{u_1}(4) = 5$, $D_{u_1}(5) = 6$, $D_{u_1}(6) = 7$ and $D_{u_1}(7) = +\infty$. This dater function lists all the dates of the event occurrences.

About time comparison in series: residuation theory Let $\mathcal{D}$ and $\mathcal{C}$ denote two complete dioids. $I_{d\mathcal{C}}$ and $I_{d\mathcal{D}}$ are the identity mappings on $\mathcal{C}$ and $\mathcal{D}$.

Definition 4. *Let $\Pi : \mathcal{D} \mapsto \mathcal{C}$ be an isotone mapping*[5]*, Π is residuated if for all $c \in \mathcal{C}$ there exists a greatest solution of $\Pi(x) = c$. Moreover this solution is $\Pi^\sharp(c)$ where $\Pi^\sharp : \mathcal{C} \to \mathcal{D}$ is the unique isotone mapping such that $\Pi \circ \Pi^\sharp \preceq I_{d\mathcal{C}}$ and $\Pi^\sharp \circ \Pi \succeq I_{d\mathcal{D}}$. $\Pi^\sharp$ is called the residual of Π.*

Consider the isotone mapping $L_y : \mathcal{D} \to \mathcal{D} : x \mapsto y \otimes x$, it is residuated and its residual is $L_y^\sharp(z)$ also denoted $y \backslash\!\!\circ z$. That is $y \backslash\!\!\circ z$ is the greatest solution on x of $y \otimes x = z, \forall z \in \mathcal{D}$. Similarly, for the residuated isotone mapping $R_y : \mathcal{D} \to \mathcal{D} : x \mapsto x \otimes y$, $R_y^\sharp(z) = z \,/\!\!\!\circ\, y$. Intuitively speaking, $y \backslash\!\!\circ z$ and $z \,/\!\!\!\circ\, y$ provide a way to formally compare z and y in a complete dioid.

Theorem 2. *Let $A \in \mathcal{D}^{n\times m}$ be a matrix of series. Then, $A \backslash\!\!\circ A = (A \backslash\!\!\circ A)^*$.*

Time comparison between series of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ can be defined with residuals.

Definition 5. *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ and their respective dater functions $\mathcal{D}_a$ and $\mathcal{D}_b$. The time shift function representing the time shift between a and b for each $n \in \mathbb{Z}$ is defined by $\mathcal{T}_{a,b}(n) = \mathcal{D}_a - \mathcal{D}_b$.*

[5] Recall that dioids are ordered sets. Let $\Pi : \mathcal{S} \mapsto \mathcal{S}'$ be an application defined on ordered sets, Π is isotone if $\forall x, x' \in \mathcal{S}$, $x \preceq x' \Rightarrow \Pi(x) \preceq \Pi(x')$.

Theorem 3 ([6]). *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, the time shift function $\mathcal{T}_{a,b}(n)$ can be bounded by:*

$$\forall n \in \mathbb{Z}, \quad \mathcal{D}_{b \not\phi a}(0) \leq \mathcal{T}_{a,b}(n) \leq -\mathcal{D}_{a \not\phi b}(0),$$

where $\mathcal{D}_{b \not\phi a}(0)$ is obtained from monomial $\gamma^0\delta^{\mathcal{D}_{b \not\phi a}(0)}$ of series $b \not\phi a$ and $\mathcal{D}_{a \not\phi b}(0)$ is obtained from $\gamma^0\delta^{\mathcal{D}_{a \not\phi b}(0)}$ of series $a \not\phi b$.

Definition 6. *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, the time shift between series a and b is*

$$\Delta(a,b) = [\mathcal{D}_{b \not\phi a}(0); -\mathcal{D}_{a \not\phi b}(0)], \tag{1}$$

where $\gamma^0\delta^{\mathcal{D}_{b \not\phi a}(0)} \in b \not\phi a$ and $\gamma^0\delta^{\mathcal{D}_{a \not\phi b}(0)} \in a \not\phi b$. In this interval, the series from which the time offset is measured is the series a. It is called the reference series *of the interval.*

From this definition, if the time shift interval needs to be defined with series b as the reference series, the interval will be $\Delta(b,a) = [\mathcal{D}_{a \not\phi b}(0); -\mathcal{D}_{b \not\phi a}(0)]$.

Example 1. Let us consider series $a = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$ and $b = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{+\infty}$. The minimal time shift between a and b is $\mathcal{D}_{b \not\phi a}(0) = 0$ (found in $\gamma^0\delta^{\mathbf{0}}$ in $b \not\phi a = \gamma^0\delta^{\mathbf{0}} \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^7 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{+\infty}$). The maximal time shift is $-\mathcal{D}_{a \not\phi b}(0) = 2$ (found in $\gamma^0\delta^{-\mathbf{2}}$ from $a \not\phi b = \gamma^0\delta^{-\mathbf{2}} \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^9 \oplus \gamma^4\delta^{+\infty}$). Therefore $\Delta(a,b) = [0,2]$: series b is later than a with a minimum of 0 and a maximum of 2 hours.

Models of (max,+)-linear systems The elements of the TEG are represented by equations in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$. The equations can be grouped into a set of matrices A, B and C that contain information about the structure of the TEG. The state representation defines relations between any set of input event flows u and the state x, and the relations between the state x and the output event flows y. Let $u \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times 1}$ be the input vector of size p, $x \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times 1}$ be the state vector of size n and $y \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times 1}$ be the output vector of size q. The state representation is:

$$\begin{cases} x = Ax \ \oplus \ Bu, \\ y = Cx, \end{cases}$$

where $A \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times n}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times p}$ and $C \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times n}$. Equality $x = Ax \oplus Bu$ can be transformed to $x = A^*Bu$ thanks to Theorem 1 so we have

$$y = CA^*Bu.$$

Matrix $H = CA^*B$ represents the transfer function of the TEG, that is the dynamic of the system between the inputs and the outputs. For the system of Figure 1 the matrices $A \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{6\times 6}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{6\times 2}$ and $C \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{3\times 6}$ of the state representation are:

$$A : \begin{pmatrix} . & \gamma^1\delta^0 & . & . & . & . \\ \gamma^0\delta^3 & . & . & . & . & . \\ . & . & . & \gamma^1\delta^0 & . & . \\ . & . & \gamma^0\delta^4 & . & . & . \\ . & \gamma^0\delta^2 & . & \gamma^0\delta^1 & . & \gamma^1\delta^0 \\ . & . & . & . & \gamma^0\delta^4 & . \end{pmatrix} \; B : \begin{pmatrix} \gamma^0\delta^1 & . \\ . & . \\ . & \gamma^0\delta^1 \\ . & . \\ . & . \\ . & . \end{pmatrix} \; C : \begin{pmatrix} . & . & . & . & . & \gamma^0\delta^0 \\ . & \gamma^0\delta^0 & . & . & . & . \\ . & . & . & \gamma^0\delta^0 & . & . \end{pmatrix}.$$

The exponent n of γ represents the backward event shift between transitions (the $n+1^{th}$ firing of x_1 depends on the n^{th} firing of x_2) and the exponent of δ represents the backward time shift between transition (the firing date of x_2 depends on the firing date of x_1 and time between 2 and 5).

4 How can a (max,+) observer be sensitive to time shift failures?

The objective of the paper is to propose a method that detects time shift failures as proposed in Section 2 and that uses an observer as introduced in [2] and [3]. As later detailed in Section 4.2, this observer aims at computing a reconstructed state from the observation of the inputs and outputs of the system that is sensitive to a specific type of disturbance. These disturbances are characterized as new inputs w that slow down the system. The system will then be assumed to behave with respect to the following state representation:

$$x = Ax \oplus Bu \oplus Rw, \; y = Cx. \tag{2}$$

4.1 Time shift failures as input disturbances

Throughout this paper, we consider that time shift failures are permanent phenomena that can occur at any step of the production. Formally speaking, a time shift failure is characterized by an unexpected and unknown delay $d > 0$ that is added to the normal duration time t of a place p. As shown in Figure 2, this place is characterized by a transition upstream x_{i-1}, a duration t, a number of tokens o and a transition downstream x_i. Let $x_{i-1} = \bigoplus_{n=0}^{K} \gamma^{s_n}\delta^{h_n}$, where s_n is the transition firing number, h_n is the firing date and K the number of firing events. The normal downstream transition is $x_i = \bigoplus_{n=0}^{K} \gamma^{s_n+o}\delta^{h_n+t}$. When a time shift failure $d > 0$ holds in a place, the downstream transition then becomes: $x_i = \bigoplus_{n=0}^{K} \gamma^{s_n+o}\delta^{h_n+t+d}$. To characterize the same time shift failure over the place p by a disturbance, we will first modify the TEG. We add to the downstream transition x_i an input w_i, as shown in Figure 3, which slows down this transition. This new input w_i is not observed because it is related to a failure in an equipment. To get the same effect of an offset $d > 0$ in the downstream transition, input w_i has to be defined as

$$w_i = \bigoplus_{n=0}^{k} \gamma^{s_n+o}\delta^{h_n+t+d}. \tag{3}$$

Back to Figure 1, to characterize a time shift failure of $d = 1$ on place p_5, a disturbance w_4 is added to transition x_4 as in Figure 3. Suppose that $x_3 = \gamma^0\delta^2 \oplus \gamma^1\delta^6 \oplus \gamma^2\delta^{10} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{18} \oplus \gamma^5\delta^{22} \oplus \gamma^6\delta^{26} \oplus \gamma^7\delta^{+\infty}$. Since an offset of 1 time unit is present on p_5, $x_4 = \gamma^0\delta^{2+4+1} \oplus \gamma^1\delta^{6+4+1} \oplus \gamma^2\delta^{10+4+1} \oplus \gamma^3\delta^{14+4+1} \oplus \gamma^4\delta^{18+4+1} \oplus \gamma^5\delta^{22+4+1} \oplus \gamma^6\delta^{26+4+1} \oplus \gamma^7\delta^{+\infty}$. By setting the disturbance $w_4 = x_4 = \gamma^0\delta^7 \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{17} \oplus \gamma^3\delta^{22} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{32} \oplus \gamma^6\delta^{37} \oplus \gamma^7\delta^{+\infty}$, the firing of transition x_4 is slowed down.

Based on this characterization, the faulty system that we consider will behave based on Equation (2) and input disturbances as defined by Equation (3). Let $w \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{l\times 1}$ be the input vector of disturbances of size l. The input w corresponds to the transition that will be disturbed. Matrix $R \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times l}$ is filled with $\gamma^0\delta^0$ monomials that represent the connections between disturbances and internal disturbed transitions. All the other entries are set to ε. Equality

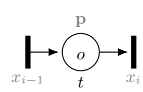

Fig. 2. Representation of a place

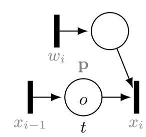

Fig. 3. Representation of a place with disturbance

$x = Ax \oplus Bu \oplus Rw$ can be transformed to $x = A^*Bu \oplus A^*Rw$ thanks to Theorem 1 so we have

$$y = CA^*Bu \oplus CA^*Rw.$$

In the example of Section 2, all the internal transitions in Figure 1 will be disturbed so R is the matrix $R \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{6\times 6}$ with $\forall i, j\{1, \ldots, 6\} i \neq j, R(i, j) = \varepsilon, R(i, i) = e = \gamma^0\delta^0$.

4.2 Observer synthesis

In this paper we use the definition of an observer from the articles [3],[2]. Figure 4 shows the system with disturbances w and from which we can observe the outputs y_o. The observer is a new model obtained from the fault-free model and that will estimate the states of the system x_r in the presence of such disturbances.

From articles [3], [2] we get the following observer's equations:

$$\begin{cases} x_r = Ax_r \oplus Bu \oplus L(y_r \oplus y_o) = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw, \\ y_r = Cx_r. \end{cases} \tag{4}$$

To obtain the estimated vector x_r as close as possible to real state x, the observer relies on the largest matrix $L \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times q}$ such that:

$$\begin{aligned} x_r &\preceq x_o \\ (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw &\preceq A^*Bu \oplus A^*Rw \end{aligned}$$

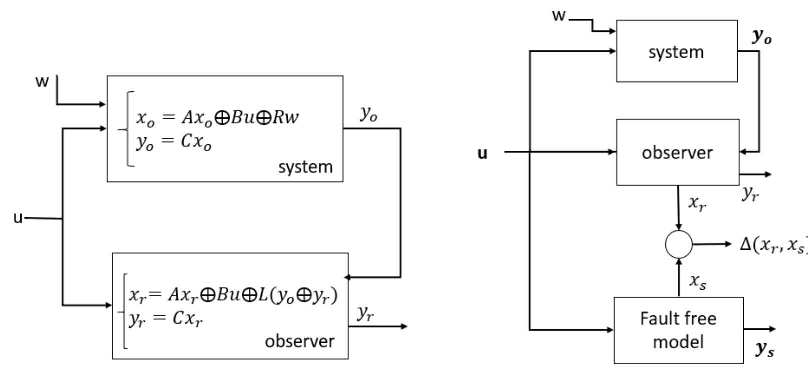

Fig. 4. Observer structure with disturbance (on the left) and the global architecture of the detection method (on the right).

where $L = (A^* B \phi C A^* B) \wedge (A^* R \phi C A^* R)$.

The observer matrix L of the TEG of Figure 1 is

$$L = \begin{pmatrix} . & \gamma^1\delta^0(\gamma^1\delta^3)^* & . \\ . & \gamma^0\delta^0(\gamma^1\delta^3)^* & . \\ . & . & \gamma^1\delta^0(\gamma^1\delta^4)^* \\ . & . & \gamma^0\delta^0(\gamma^1\delta^4)^* \\ \gamma^1\delta^0(\gamma^1\delta^4)^* & \gamma^0\delta^2(\gamma^1\delta^4)^* & \gamma^0\delta^1(\gamma^1\delta^4)^* \\ \gamma^0\delta^0(\gamma^1\delta^4)^* & \gamma^0\delta^6(\gamma^1\delta^4)^* & \gamma^0\delta^5(\gamma^1\delta^4)^* \end{pmatrix}$$

Suppose the system behaves with respect to the inputs u_1 and u_2 defined in Section 3 but transition x_4 is disturbed with $w_4 = \gamma^0\delta^7 \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{17} \oplus \gamma^3\delta^{22} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{32} \oplus \gamma^6\delta^{37} \oplus \gamma^7\delta^{+\infty}$ then the reconstructed state is $x_r = (A \oplus LC)^* Bu \oplus (A \oplus LC)^* LCA^* Rw$ which is the vector $x_r = [x_{r1}, \ldots, x_{r6}]^T =$

$$\begin{bmatrix} \gamma^0\delta^2 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^8 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{14} \oplus \gamma^5\delta^{17} \oplus \gamma^6\delta^{20} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^5 \oplus \gamma^1\delta^8 \oplus \gamma^2\delta^{11} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{17} \oplus \gamma^5\delta^{20} \oplus \gamma^6\delta^{23} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^2 \oplus \gamma^1\delta^7 \oplus \gamma^2\delta^{12} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{22} \oplus \gamma^5\delta^{27} \oplus \gamma^6\delta^{32} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^7 \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{17} \oplus \gamma^3\delta^{22} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{32} \oplus \gamma^6\delta^{37} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^8 \oplus \gamma^1\delta^{13} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{28} \oplus \gamma^5\delta^{33} \oplus \gamma^6\delta^{38} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^{12} \oplus \gamma^1\delta^{17} \oplus \gamma^2\delta^{22} \oplus \gamma^3\delta^{27} \oplus \gamma^4\delta^{32} \oplus \gamma^5\delta^{37} \oplus \gamma^6\delta^{42} \oplus \gamma^7\delta^{+\infty} \end{bmatrix}$$

The reconstructed state x_r takes into account the disturbance w_4. If w_4 were not present, the monomial $\gamma^0\delta^7$ in x_{r4} would be $\gamma^0\delta^6$ (no time shift).

5 Time shift failure detection in (max,+)-linear systems

Figure 4 on the right shows how the proposed set of indicators is designed: the system is ruled by the observable inputs u, the unobservable disturbances w and produces the observable outputs y_o; the observer estimates the states x_r based on the observation of u and y_o. States x_s result from the simulation of the fault-free model (as in Figure 1) based on u, the proposed indicator then relies on a series comparison denoted $\Delta(x_{ri}, x_{si})$ (see Definition 6) for every transition x_i.

Definition 7. *The indicator for state x_i is $I_{x_i}(u, y_o)$ defined as the Boolean function that returns true iff $\Delta(x_{ri}, x_{si}) = [0, 0]$ with $x_s = [x_{s1} \dots x_{sn}]^T = A^* Bu$, $x_r = [x_{r1} \dots x_{rn}]^T = Ax_r \oplus Bu \oplus LCx_r \oplus Ly_o$, and $\Delta(x_{ri}, x_{si}) = [\mathcal{D}_{x_{ri} \not\phi x_{si}}(0), -\mathcal{D}_{x_{si} \not\phi x_{ri}}(0)]$.*

Theorem 4. *The indicator $I_{x_i}(u, y_o)$ returns true only if a time shift failure involving x_i with $\Delta(x_{ri}, x_{si}) \neq [0, 0]$ has occurred in the system. A time shift failure involves a transition x_i if the time shift failure occurs in a place of the TEG that is in the upstream[6] of transition x_i.*

Proof: We show that, if the system has no failure in the upstream of x_i, $I_{x_i}(u, y_o)$ necessarily returns false. Suppose the system does not have such a time shift failure, by definition of the observer, the reconstructed state x_{ri} is the same as the fault-free model state x_{si} as no place in the upstream of x_i is disturbed. If $x_{si} = x_{ri}$, then we have $x_{si} \not\phi x_{ri} = x_{ri} \not\phi x_{si} = x_{ri} \not\phi x_{ri}$ but $x_{ri} \not\phi x_{ri} = (x_{ri} \not\phi x_{ri})^*$ according to Theorem 2 and with Definition 1 of the Kleene star: $(x_{ri} \not\phi x_{ri})^* = e \oplus \dots = \gamma^0 \delta^0 \oplus \dots$. So if $x_{ri} = x_{si}$, one has $\mathcal{D}_{x_{ri} \not\phi x_{si}}(0) = -\mathcal{D}_{x_{si} \not\phi x_{ri}}(0) = 0$. $\square$

In the example of Section 2, based on the previous observer, suppose that the system behaves with respect to the inputs u_1 and u_2 defined in Section 3. Suppose that in reality there was an incident on Equipment 2: the operation lasts longer with a processing time of 5 hours in p_5 instead of 4 hours (see Figure 1). The real system is then characterized by Equation (2) with the disturbance w_4 that is defined in Section 4.1. The estimated state is the same as given at the end of Section 4.2. In particular, x_{r3} is represented with plain line in Figure 5 (as well as series x_{s3} with dotted line). The expected state x_s is the vector:

$$\begin{bmatrix} x_{s1} \\ x_{s2} \\ x_{s3} \\ x_{s4} \\ x_{s5} \\ x_{s6} \end{bmatrix} = \begin{bmatrix} \gamma^0\delta^2 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^8 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{14} \oplus \gamma^5\delta^{17} \oplus \gamma^6\delta^{20} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^5 \oplus \gamma^1\delta^8 \oplus \gamma^2\delta^{11} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{17} \oplus \gamma^5\delta^{20} \oplus \gamma^6\delta^{23} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^2 \oplus \gamma^1\delta^6 \oplus \gamma^2\delta^{10} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{18} \oplus \gamma^5\delta^{22} \oplus \gamma^6\delta^{26} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^6 \oplus \gamma^1\delta^{10} \oplus \gamma^2\delta^{14} \oplus \gamma^3\delta^{18} \oplus \gamma^4\delta^{22} \oplus \gamma^5\delta^{26} \oplus \gamma^6\delta^{30} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^7 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{15} \oplus \gamma^3\delta^{19} \oplus \gamma^4\delta^{23} \oplus \gamma^5\delta^{27} \oplus \gamma^6\delta^{31} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^{11} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{31} \oplus \gamma^6\delta^{35} \oplus \gamma^7\delta^{+\infty} \end{bmatrix}$$

and the computed intervals are in Figure 5. Indicators that return true are associated with transitions x_3, x_4, x_5, x_6. Indicators for transitions x_1, x_2 return false. Now, if we assume that there is only one type of time shift failure in the system, Proposition 4 ensures that the time shift failure occurs in a place that is in the upstream of every transition x_3, x_4, x_5, x_6. The time shift failure occurs in Eq2, either in place p_2 (transportation delay before the arrival in front of Eq2), or in place p_6 (processing start of Eq2 is delayed), or in place p_5 (process of Eq2 longer than expected).

[6] A place p is in the upstream of a transition x in a TEG if there is a path of arcs from p to x.

$$
\begin{aligned}
\Delta(x_{r1}, x_{s1}) &= [\mathcal{D}_{x_{r1} \not\phi x_{s1}(0)}, -\mathcal{D}_{x_{s1} \not\phi x_{r1}}(0)] = [0, 0],\\
\Delta(x_{r2}, x_{s2}) &= [\mathcal{D}_{x_{r2} \not\phi x_{s2}(0)}, -\mathcal{D}_{x_{s2} \not\phi x_{r2}}(0)] = [0, 0],\\
\Delta(x_{r3}, x_{s3}) &= [\mathcal{D}_{x_{r3} \not\phi x_{s3}(0)}, -\mathcal{D}_{x_{s3} \not\phi x_{r3}}(0)] = [0, 6],\\
\Delta(x_{r4}, x_{s4}) &= [\mathcal{D}_{x_{r4} \not\phi x_{s4}(0)}, -\mathcal{D}_{x_{s4} \not\phi x_{r4}}(0)] = [1, 7],\\
\Delta(x_{r5}, x_{s5}) &= [\mathcal{D}_{x_{r5} \not\phi x_{s5}(0)}, -\mathcal{D}_{x_{s5} \not\phi x_{r5}}(0)] = [1, 7],\\
\Delta(x_{r6}, x_{s6}) &= [\mathcal{D}_{x_{r6} \not\phi x_{s6}(0)}, -\mathcal{D}_{x_{s6} \not\phi x_{r6}}(0)] = [1, 7].
\end{aligned}
$$

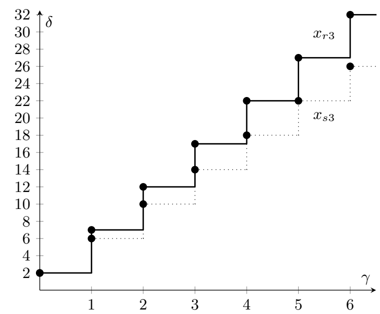

Fig. 5. Computed intervals for the scenario and representation of x_{r3} and x_{s3}.

6 Conclusion

This paper defines a method for detecting time shift failures in systems modeled as Timed-Event Graphs using an observer that estimates the real states of the system. The method defines a formal (max,+) algebraic indicator based on the residuation theory. The proposed indicator is able to detect the presence of time shift failures as soon as it returns true and provides first localisation results. As a perspective, we aim at improving the accuracy of this indicator to better exploit the quantitative information contained in the interval $\Delta(x_{ri}, x_{si})$. We expect that a further analysis about the bounds of the intervals may actually provide more information about failure localization and identification.

References

1. F. Baccelli, G. Cohen, G.J Olsder, and J.-P Quadrat. *Synchronization and linearity: an algebra for discrete event systems.* Wiley and Sons, 1992.
2. L. Hardouin, C A. Maia, B. Cottenceau, and M. Lhommeau. Observer design for (max,+) linear systems. *IEEE Transactions on Automatic Control*, 55(2):538–543, 2010.
3. L. Hardouin, C.A. Maia, B. Cottenceau, and R. Santos-Mendes. Max-plus linear observer: application to manufacturing systems. In *10th International Workshop on Discrete Event Systems, WODES'10*, pages 161–166, 2010.
4. C. Kim and T.E. Lee. Feedback control of cluster tools for regulating wafer delays. *IEEE Transactions on Automation Science and Engineering*, 13(2):1189–1199, 2015.
5. J. Komenda, S. Lahaye, J.-L. Boimond, and T. van den Boom. Max-plus algebra in the history of discrete event systems. *Annual Reviews in Control*, 45:240 – 249, 2018.
6. MaxPlus. Second order theory of min-linear systems and its application to discrete event systems. In *Proceedings of the 30th IEEE Conference on Decision and Control. CDC'91*, pages 1511–1516, 1991.
7. A. Sahuguède, E. Le Corronc, and Y. Pencolé. Design of indicators for the detection of time shift failures in (max, +)-linear systems. In *20th World Congress of the International Federation of Automatic Control*, pages 6813–6818, 2017.