Observer-based detection and localization of time shift failures in (max,+)-linear systems*

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***Abstract*— This paper addresses a problem of failure detection and localization in production lines modeled, through Timed Event Graphs (TEG), as (max,+) linear systems with disturbances, over which observers can be developed. The state of the observed system is estimated and an indicator that returns true if a time shift failure is detected is defined. The localization step is proposed for elementary structures of TEG through the results of the detection.**

I. INTRODUCTION

In the industry, Discrete Event Systems (DES) can be used to model and diagnose faults, malfunctions on production lines. The objective is to detect, locate and identify failures as soon as possible to avoid further equipment unavailability. At STMicroelectronics Crolles 300 plant, we investigate this problem in order to detect production drifts, especially failures that generate time shifts in the production lines. STMicroelectronics is among the world's largest semiconductor companies, serving all electronics segments. Semiconductor manufacturing is a complex industries and one of its most important challenges is to succeed in detecting production drifts before they have real impact on production plans.

Fault diagnosis involving time in DES has been introduced in [Tri02] using timed automata to refine diagnosis decisions based on timed observations. In [GTY09], time Petri nets allow to easily model concurrency. This paper represents production lines as Timed Event Graphs (TEG) which are a subclass of Petri nets where places are associated with a duration. A TEG can be modeled by (max,+) algebra as introduced in [BCOQ92], [Max91], [KLBvdB18]. For example [KL15] uses (max,+) algebra techniques to control wafer delays in cluster tools for semiconductor production.

In this paper, we propose an observer-based (max,+) algebra method that detects and then localizes the potential sources of time shifts in a TEG. Our proposal extends the following results. [LCPSP21] presents a detection method of time shift failures by simply comparing simulated and real outputs without estimating the internal state of the system. Once the detection made, the localization proposes a - possibly large - set of admissible sources of the failure using signature matrices and characteristic signatures. To improve the precision of the detection, [PLCPV20] uses a (max,+) observer [HMCL10] to actually compare the estimated state with a fault-free simulated state. Our proposal goes one step further by providing suspected sources of the time shifts in a more precise way, by analysing elementary structures of TEG (tandem, synchronization and parallelism). Using (max,+) algebra makes this work of polynomial complexity.

The paper is organized as follows. Section II recalls the necessary mathematical background. Section III refines the detection method detailed in [PLCPV20] that will be used to setup the rules for localizing the source of time shift failure in TEG defined in Section IV.

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II. MATHEMATICAL BACKGROUND

This section recalls the mathematical background used for describing (max,+)-linear systems [BCOQ92], [Max91].

A. Dioid theory

Dioid theory is the mathematical framework for modeling Timed Event Graphs (TEG) as (max,+) algebra. A dioid $\mathcal{D}$ is a set composed of two internal operations $\oplus$ and $\otimes$. The addition $\oplus$ is associative, commutative, idempotent (i.e. $\forall a \in \mathcal{D}, a \oplus a = a$) and has a neutral element ε. The multiplication $\otimes$ is associative, distributive on the right and the left over the addition $\oplus$ and has a neutral element e. Element ε is absorbing by $\otimes$. When there is no ambiguity, the symbol $\otimes$ is omitted. A dioid is complete if it is closed for infinite sums and if $\otimes$ is distributive over infinite sums.

For instance, the set $\mathbb{Z}_{max} = (\mathbb{Z} \cup -\infty)$, endowed with the max operation as addition $\oplus$ and the addition as multiplication $\otimes$ with neutral element $\varepsilon = -\infty$ and $e = 0$ is a dioid. By adding $+\infty$ to the dioid $\mathbb{Z}_{max}$, we get the complete dioid $\overline{\mathbb{Z}}_{max}$ where $(-\infty) + (+\infty) = (-\infty)$.

Definition 1: For a dioid $\mathcal{D}$, $\preceq$ denotes the order relation such that $\forall a, b \in \mathcal{D}, a \preceq b \Leftrightarrow a \oplus b = b$.

Theorem 1 ([BCOQ92]): Let $\mathcal{D}$ be a complete dioid, $x = a^*b$ is the solution of $x = ax \oplus b$ where $a^* = \bigoplus_{i \geq 0} a^i$ is the Kleene star operator with $a^0 = e$ and $a^{i+1} = a \otimes a^i$.

To model TEG as (max,+)-linear systems, a specific dioid has to be defined: the dioid $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$. It is a particular dioid of formal series with two commutative variables γ and δ that will represent event and time shifts of a TEG as explained in the next subsection. This dioid comes from the following $\mathbb{B}[\![\gamma, \delta]\!]$ dioid.

The complete dioid $\mathbb{B}[\![\gamma, \delta]\!]$ is the set of formal series with two commutative variables γ and δ with Boolean coefficients in $\{\varepsilon, e\}$ and exponents in $\mathbb{Z}$. A series $s \in \mathbb{B}[\![\gamma, \delta]\!]$ is written $s = \bigoplus_{(n,t) \in \mathbb{Z}^2} s(n,t)\gamma^n\delta^t$ where $s(n,t) = e$ or ε (respectively representing the presence or the absence of the monomial). The neutral elements are $\varepsilon(\gamma, \delta) = \bigoplus_{(n,t) \in \mathbb{Z}^2} \varepsilon\gamma^n\delta^t$ and $e(\gamma, \delta) = \gamma^0\delta^0$.

Graphically, a series of $\mathbb{B}[\![\gamma,\delta]\!]$ is described by a collection of point of coordinates (n,t) in $\mathbb{Z}^2$ with γ as horizontal axis and δ as vertical axis. For instance, Figure 1 shows series $u_1 = u_2 = \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^5 \oplus \gamma^5\delta^6 \oplus \gamma^6\delta^7 \oplus \gamma^7\delta^{+\infty}$ (monomials with e as Boolean coefficient).

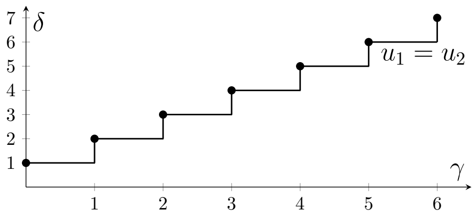

Fig. 1. Representation of series $u_1 = u_2$

The complete dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ is the quotient of $\mathbb{B}[\![\gamma,\delta]\!]$ modulo $\gamma^*(\delta^{-1})^*$ where $\forall a,b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$: $a = b \Leftrightarrow a\gamma^*(\delta^{-1})^* = b\gamma^*(\delta^{-1})^*$. Internal operations and neutral elements are identical to those of $\mathbb{B}[\![\gamma,\delta]\!]$. All series can be expressed by the following canonical form.

Definition 2: The canonical form of $s \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ is

$$s = \bigoplus_{k=0}^{K} \gamma^{n_k}\delta^{t_k} \text{ with } K \in \mathbb{N}\cup\{+\infty\} \text{ and } \begin{cases} n_0 < n_1 < \dots \\ t_0 < t_1 < \dots \end{cases}$$

B. Models of (max,+)-linear systems

Figure 2 presents a TEG with two inputs u_1 and u_2 and one output y_1. Durations associated to places are expressed here by letters a,b,c,d,f and transition x_3 is a synchronization between paths from u_1 and u_2. The entire structure of such a TEG can be modeled by equations in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ through a set of matrices A, B and C. Then, relations between input u, state x and output y transitions are expressed by its state representation. Let $u \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times 1}$, $x \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times 1}$ and $y \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times 1}$, the state representation of a TEG is:

$$\begin{cases} x = Ax \ \oplus \ Bu, \\ y = Cx, \end{cases}$$

where $A \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times n}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times p}$ and $C \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times n}$. Equality $x = Ax \oplus Bu$ can be transformed to $x = A^*Bu$ thanks to Theorem 1 so we have

$$y = CA^*Bu.$$

Matrix $H = CA^*B$ represents the transfer function that is the dynamic of the system between inputs and outputs.

Example 1: In the TEG of Figure 2, suppose that the durations are $a = b = d = 1$ for places p_1, p_3, p_4, $c = 2$ for p_2 and $f = 0$ for p_5. The state representation's matrices are:

$$A = \begin{pmatrix} . & . & . \\ . & . & . \\ \gamma^0\delta^1 & \gamma^0\delta^1 & . \end{pmatrix}, B = \begin{pmatrix} \gamma^0\delta^1 & . \\ . & \gamma^0\delta^2 \\ . & . \end{pmatrix},$$
$$C = \begin{pmatrix} . & . & \gamma^0\delta^0 \end{pmatrix}.$$

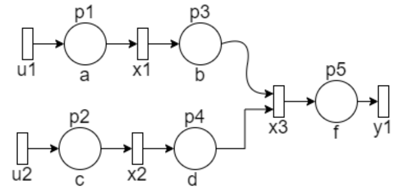

Fig. 2. A Timed Event Graph (TEG)

The indices of γ represent the backward event shift between transitions and the indices of δ represent the backward time shift. For instance in $B(1,1) = \gamma^0\delta^1$, the $n+1^{th}$ firing of x_1 depends on the n^{th} firing of u_1 (no event shift); the firing date of x_1 is exactly shifted of duration $a = 1$ from the firing date of u_1. A trigger of an input transition u_i represents the occurrence of an event. Then, input flow events of TEG are represented by series of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ as series $u1 = u2$ illustrated in Figure 1. In u_1 and u_2, index 0 of γ is the first event occurrence of this input (index 1 is the second, etc.); index 1 of δ is the date of this first occurrence. The absence of an 8^{th} event is indicated by $+\infty$ in monomial $\gamma^7\delta^{+\infty}$.

C. Time comparison of series

In the rest of the paper, time comparison of series are made thanks to a particular element called residual.

Definition 3: Let $\Pi : \ \mathcal{D} \mapsto \mathcal{C}$ be an isotone[1] mapping, where $\mathcal{D}$ and $\mathcal{C}$ are complete dioids. The largest solution of $\Pi(x) = b$, if it exists, is called the residual of Π and is denoted $\Pi^\sharp$. When Π is residuated, $\Pi^\sharp$ is the unique isotone mapping such that $\Pi \circ \Pi^\sharp \preceq I_{d\mathcal{C}}$ and $\Pi^\sharp \circ \Pi \succeq I_{d\mathcal{D}}$ where $I_{d\mathcal{C}}$ and $I_{d\mathcal{D}}$ are resp. the identity mappings on $\mathcal{C}$ and $\mathcal{D}$.

The right product $R_a : x \mapsto x \otimes a$ defined over a complete dioid $\mathcal{D}$ is residuated. Its residual is $R_a^\sharp(x) = x \phi a$ and corresponds to the pseudo-inverse of the product.

Now, time comparison between series can then be defined based on residuals. It requires the use of dater functions to get times of $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ series.

Definition 4: Let $s \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ be a series, its dater function is the non-decreasing function $D_s(n)$ from $\mathbb{Z} \mapsto \overline{\mathbb{Z}}$ such that $s = \bigoplus_{n\in\mathbb{Z}} \gamma^n\delta^{D_s(n)}$.

Example 2: Series u_1 has for dater function $D_{u_1}(0) = 1$, $D_{u_1}(1) = 2$, $D_{u_1}(2) = 3$, $D_{u_1}(3) = 4$, $D_{u_1}(4) = 5$, $D_{u_1}(5) = 6$ and $D_{u_1}(6) = 7$ that lists all the dates of the event occurrences. As $u_1 = u_2$, u_2 has obviously the same dater function.

Definition 5: Let $a,b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ and their respective dater functions $\mathcal{D}_a$ and $\mathcal{D}_b$. The time shift function representing the time shift between a and b for each $n \in \mathbb{Z}$ is defined by $\mathcal{T}_{a,b}(n) = \mathcal{D}_a - \mathcal{D}_b$.

Theorem 2 ([Max91]): Let $a,b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, the time shift function $\mathcal{T}_{a,b}(n)$ can be bounded by:

$$\forall n \in \mathbb{Z}, \quad \mathcal{D}_{b\phi a}(0) \leq \mathcal{T}_{a,b}(n) \leq -\mathcal{D}_{a\phi b}(0),$$

where $\mathcal{D}_{b\phi a}(0)$ is obtained from monomial $\gamma^0\delta^{\mathcal{D}_{b\phi a}(0)}$ of series $b\phi a$ and $\mathcal{D}_{a\phi b}(0)$ is obtained from $\gamma^0\delta^{\mathcal{D}_{a\phi b}(0)}$ of $a\phi b$.

[1] Π isotone $\equiv \forall s,s' \in \mathcal{D}\ s \preceq s' \Rightarrow \Pi(s) \preceq \Pi(s')$.

Definition 6: Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, the time shift interval between series a and b is

$$\Delta(a, b) = [\mathcal{D}_{b \phi a}(0); -\mathcal{D}_{a \phi b}(0)], \tag{1}$$

where $\gamma^0\delta^{\mathcal{D}_{b\phi a}(0)} \in b \phi a$ and $\gamma^0\delta^{\mathcal{D}_{a\phi b}(0)} \in a \phi b$.

Example 3: Generally speaking, let us consider two different series $a = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$ and $b = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{+\infty}$. The minimal time shift between a and b is $\mathcal{D}_{b\phi a}(0) = 0$ from $b \phi a = \gamma^0\delta^{\mathbf{0}} \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^7 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{+\infty}$. The maximal time shift is $-\mathcal{D}_{a\phi b}(0) = 2$ from $a \phi b = \gamma^0\delta^{-\mathbf{2}} \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^9 \oplus \gamma^4\delta^{+\infty}$. The time shift interval is $\Delta(a, b) = [0; 2]$ meaning that a is equal or faster than b by a maximum of 2 time units.

III. Detection of time shift failures of (max,+)-linear systems with disturbances

This section presents an observer-based indicator that will be used for detecting and then localizing time shift failures. Like the indicator proposed in [PLCPV20], it uses observers of (max,+)-linear systems as defined in [HMCL10] (see Section III-A) and relies on the fact that time shift failures can be characterized by input disturbances (Section III-B). This new indicator is also based on a refinement of the estimated states (Section III-C) that is used to improve the accuracy of the localization information.

A. Observer of a disturbed (max,+) linear system

An observer of a Timed Event Graph, as defined by [HMCL10] and shown in Figure 3, aims at estimating the internal states of a system based on the measurement of its inputs u and outputs y_o in the presence of unobservable disturbances characterized by specific inputs w. Then, the state representation of the system is:

$$\begin{cases} x = Ax \oplus Bu \oplus Rw, \\ y = Cx. \end{cases} \tag{2}$$

where $w \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{l\times 1}$ with l the number of disturbed internal transitions and matrix $R \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times l}$ is filled with $\gamma^0\delta^0$ monomials where internal transitions are disturbed, with ε otherwise. The observer's equations are:

$$\begin{cases} x_e = Ax_e \oplus Bu \oplus L(y_e \oplus y_o) \\ \quad = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw, \\ y_e = Cx_e, \end{cases} \tag{3}$$

in which x_e and y_e are the estimated state and the estimated output of the system and $L \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times q}$ is a new matrix called the observer matrix.

Now, to obtain x_e as close as possible to real state x, the observer relies on the largest matrix $L \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times q}$ such that $x_e \preceq x$ meaning:

$$(A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw \preceq A^*Bu \oplus A^*Rw$$

which is given by $L = (A^*B \phi CA^*B) \wedge (A^*R \phi CA^*R)$. Matrix L actually represents the connections between the observed output y_o of the system and the internal transitions x_e of the observer.

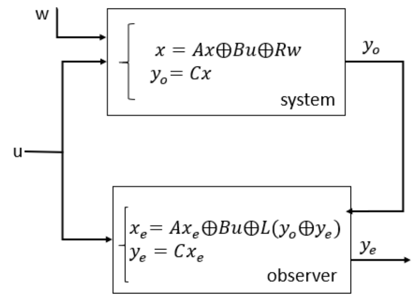

Fig. 3. Observer structure with disturbance

Example 4: In the TEG of Figure 2 with data of Example 1, the observer matrix is

$$L = \begin{pmatrix} . & . & \gamma^0\delta^0 \\ . & \gamma^0\delta^0 & . \\ \gamma^0\delta^0 & \gamma^0\delta^1 & \gamma^0\delta^1 \end{pmatrix}.$$

With inputs u_1 and u_2 as given in Figure 1, the estimated state $x_e = [x_{e1}, \ldots, x_{e3}]^T$ is

$$\begin{bmatrix} \gamma^0\delta^2 \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^4 \oplus \gamma^3\delta^5 \oplus \gamma^4\delta^6 \oplus \gamma^5\delta^7 \oplus \gamma^6\delta^8 \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^4 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^7 \oplus \gamma^4\delta^8 \oplus \gamma^5\delta^9 \oplus \gamma^6\delta^{10} \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^5 \oplus \gamma^1\delta^6 \oplus \gamma^2\delta^7 \oplus \gamma^3\delta^8 \oplus \gamma^4\delta^9 \oplus \gamma^5\delta^{10} \oplus \gamma^6\delta^{11} \oplus \gamma^7\delta^{+\infty} \end{bmatrix}$$

B. Time shift failure as input disturbance

To take advantage of the state estimation of an observer for the detection and the localization of time shift failures, we need to characterize a time shift failure as an input disturbance. We recall here the characterization of [PLCPV20]. A time shift failure is considered as a permanent phenomena and is formally defined by an unknown delay θ in a place p of a TEG. The place p, with an upstream transition x_{i-1} and a downstream transition x_i, is characterized by a number of o tokens and a duration t (see Figure 4). Upstream transition is described by $x_{i-1} = \bigoplus_{n=0}^{K} \gamma^{s_n}\delta^{h_n}$ (see Definition 2), where s_n is the transition firing number, h_n is the firing date and K the number of firing events. The downstream transition is $x_i = \bigoplus_{n=0}^{K} \gamma^{s_n+o}\delta^{h_n+t}$. If a time shift failure $\theta > 0$ holds in a place, the downstream transition then becomes: $x_i = \bigoplus_{n=0}^{K} \gamma^{s_n+o}\delta^{h_n+t+\theta}$. The firing dates of x_i will be slowed down by θ comparing to the firing dates of x_i during a nominal behavior. The same time shift failure over a place p can then be characterized by an input disturbance w_i as shown in Figure 5. If this disturbance is $w_i = \bigoplus_{n=0}^{K} \gamma^{s_n+o}\delta^{h_n+t+\theta}$, with s_n and h_n defined in series x_{i-1}, it has the same effect on transition x_i. In other words, having an unknown time shift failure $\theta > 0$ in place p is equivalent to say that such an unknown disturbance w_i exists.

Example 5: By taking back Example 4, to characterize a time shift failure in p_2 by an offset $\theta = 1$, an input disturbance w_2 is added to the transition x_2 as illustrated in Figure 5. The transition that takes into account this offset is $x_2 = \gamma^0\delta^{1+2+1} \oplus \gamma^1\delta^{2+2+1} \oplus \gamma^2\delta^{3+2+1} \oplus \gamma^3\delta^{4+2+1} \oplus$

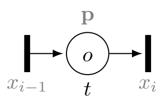

Fig. 4. Representation of a place

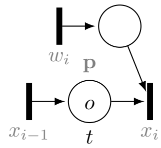

Fig. 5. Representation of a place with disturbance

$\gamma^4\delta^{5+1+1} \oplus \gamma^5\delta^{6+2+1} \oplus \gamma^6\delta^{7+2+1} \oplus \gamma^7\delta^{+\infty}$. Then, to copy this time shift failure effect, the disturbance has to be $w_2 = \gamma^0\delta^4 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^7 \oplus \gamma^4\delta^8 \oplus \gamma^5\delta^9 \oplus \gamma^6\delta^{10} \oplus \gamma^7\delta^{+\infty}$.

C. Refinement of the estimated state

The indicator proposed in [PLCPV20] is based on the computation of the time shift $\Delta(x_e, x_s)$ (see Definition 6) where x_e is the state estimated by the observer and x_s results from the simulation of a fault-free model of the system based on the real observable input u. As the effect of a time shift failure might not be immediate, the first monomials of x_e representing the first event occurrences might not be affected by the failure and then are considered as normal occurrences in the indicator of [PLCPV20] while the failure is present. The consequence is that the indicator in [PLCPV20] may generate ambiguous information and we propose here to filter out of x_e these unaffected monomials and only keep as the estimated state the *sensitive-to-disturbance* (STD for short) part of x_e. The way to extract this STD state is through a deeper analysis of the structure of the observer. It depends on values of matrix L that establishes connections between observed output y_o of the system and the internal transitions x_e of the observer. In the following, for a given matrix M, M_{ij} denotes the element of M at line i, column j; $M_{i\bullet}$ the i^{th} line of M and $M_{\bullet i}$ its i^{th} column. A row $L_{i\bullet}$ of matrix L represents an estimated state x_{ei} of the system.

Proposition 1: Suppose that $L_{i\bullet} = \begin{pmatrix}\varepsilon & \dots & \varepsilon\end{pmatrix}$, then $x_{ei} = A^*_{i\bullet} B_{\bullet i} u_i$, i.e the estimated state x_{ei} does not depend on disturbances.

Proof sketch. From (3), estimated state is $x_{ei} = (A_{i\bullet} \oplus L_{i\bullet} C_{\bullet i})^* (B_{\bullet i} u \oplus L_{i\bullet} C_{\bullet i} A^*_{i\bullet} R_{\bullet i} w_{i\bullet})$. Using algebraic operations of $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ and recalling that ε is absorbing by $\otimes$, it follows that $x_{ei} = A^*_{i\bullet} B_{\bullet i} u_i$ does not depend on w. $\square$

Let the canonical form $L_{ij} = \bigoplus_{k=0}^{K} \gamma^{n_k}\delta^{t_k}$ be an entry of L, in the following $(n_0)_{ij}$ denotes the smallest γ index n_0 in series L_{ij}. It is possible that $(n_0)_{ij}$ differs from 0 if tokens are already in the place represented by $\gamma^{n_0}\delta^{t_0}$ in L_{ij}.

Let $N_i = \min_{1 \leqslant j \leqslant q}\{(n_0)_{ij}\}$ where q be the number of outputs of the system. The following proposition gives the event occurrence of the estimated state trajectory from which the effect of the disturbance can be observed.

Proposition 2: Let $x_{ei} = \bigoplus_{\kappa=0}^{K} \gamma^{\nu_\kappa}\delta^{\tau_\kappa}$ be the estimated trajectory of x_i in its canonical form. The first event occurrence of x_{ei} that takes into account the unknown disturbances from $w_{i\bullet}$ is the event occurrence of index ν_d such that $\nu_d = N_i + \nu_0^b + \nu_0^{w_{i\bullet}}$ where ν_0^b and $\nu_0^{w_{i\bullet}}$ are the smallest γ indices of series $b = C_{\bullet i} A^*_{i\bullet} R_{\bullet i}$ and $w_{i\bullet}$.

Proof sketch. From (3), estimated state is $x_{ei} = (A_{i\bullet} \oplus L_{i\bullet} C_{\bullet i})^* (B_{\bullet i} u \oplus L_{i\bullet} b w_{i\bullet})$ where $b = C_{\bullet i} A^*_{i\bullet} R_{\bullet i}$ is the only part depending on disturbances w and ν_d is defined as the smallest γ index of series $L_{i\bullet} b w_{i\bullet}$. $\square$

Now, the STD state x_{edi} is based only on γ indices from the state x_{ei} that takes into account disturbances, which are the ones from ν_d.

Definition 7: The sensitive-to-disturbance (STD) estimated state x_{edi} is defined from x_{ei} by

$$x_{edi} = \bigoplus_{k=d}^{K} \gamma^{\nu_k}\delta^{\tau_k}$$

where ν_d is defined by Proposition 2.

Example 6: Generally speaking, if the complete estimate of a state x_1 is $x_{e1} = \gamma^0\delta^4 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^7 \oplus \dots$ and if $\nu_d = 1$, the event occurrence of index ν_d from which the disturbance is taken into account is the event numbered by 1. The STD estimated state is then $x_{ed1} = \gamma^1\delta^5 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^7 \oplus \dots$

D. Indicator of time shift failure

Based on dater and time shift functions introduced in Section 2, we propose a new time shift indicator that compares the fault-free model with the observed system through u and y_o, by only using information of the STD estimated state x_e and the fault-free model state x_{si}. To perform a consistent comparison between x_e and x_{si}, event occurrences from x_{si} before ν_d must also be filtered out.

Definition 8: Let $x_{si} = A^*Bu = \bigoplus_{k=0}^{K} \gamma^{\nu_k}\delta^{\tau_k}$ be the complete fault-free model state. Series x'_{si} denotes the fault-free model state comparable with x_{edi}:

$$x'_{si} = \bigoplus_{k=d}^{K} \gamma^{\nu_k}\delta^{\tau_k}.$$

Example 7: From Example 6, if $x_{s1} = \gamma^0\delta^3 \oplus \gamma^1\delta^4 \oplus \gamma^2\delta^5 \oplus \gamma^3\delta^6 \oplus \dots$, then $x'_{s1} = \gamma^1\delta^4 \oplus \gamma^2\delta^5 \oplus \gamma^3\delta^6 \oplus \dots$.

Definition 9: Let x_{edi} be the STD estimated state of x_i and x'_{si} its fault-free model state. Indicator $I_{x_i}(u, y_o)$ of state x_i is defined as the Boolean function:

$$I_{x_i}(u, y_o) = \begin{cases} \textit{false} \text{ if } \Delta(x_{edi}, x'_{si}) = [0; 0], \\ \textit{true} \text{ otherwise}, \end{cases}$$

with $\Delta(x_{edi}, x'_{si}) = [\mathcal{D}_{x_{edi} \oslash x'_{si}}(0); -\mathcal{D}_{x'_{si} \oslash x_{edi}}(0)]$.

Proposition 3: The indicator returns true only if a time shift failure has occurred in the system.

Proof sketch. By construction this indicator has the same detection capabilities as the indicator of [PLCPV20], hence the result. $\square$

Example 8: Back to Examples 1-4-5 where a time shift failure of 1 is in place p_2. The state x'_s comparable with the state x_{ed} is the vector $[x'_{s1}, \dots, x'_{s3}]^T =$

$$\begin{bmatrix} \gamma^0\delta^2 \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^4 \oplus \gamma^3\delta^5 \oplus \gamma^4\delta^6 \oplus \gamma^5\delta^7 \oplus \gamma^6\delta^8 \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^3 \oplus \gamma^1\delta^4 \oplus \gamma^2\delta^5 \oplus \gamma^3\delta^6 \oplus \gamma^4\delta^7 \oplus \gamma^5\delta^8 \oplus \gamma^6\delta^9 \oplus \gamma^7\delta^{+\infty} \\ \gamma^0\delta^4 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^7 \oplus \gamma^4\delta^8 \oplus \gamma^5\delta^9 \oplus \gamma^6\delta^{10} \oplus \gamma^7\delta^{+\infty} \end{bmatrix}.$$

A failure is then detected by the indicators whose computed intervals are:

$$\begin{cases} \Delta(x_{ed1}, x'_{s1}) & = [\mathcal{D}_{x_{ed1} \not\to x'_{s1}(0)}, -\mathcal{D}_{x'_{s1} \not\to x_{ed1}}(0)] = [0,0], \\ \Delta(x_{ed2}, x'_{s2}) & = [\mathcal{D}_{x_{ed2} \not\to x'_{s2}(0)}, -\mathcal{D}_{x'_{s2} \not\to x_{ed2}}(0)] = [1,1], \\ \Delta(x_{ed3}, x'_{s3}) & = [\mathcal{D}_{x_{ed3} \not\to x'_{s3}(0)}, -\mathcal{D}_{x'_{s3} \not\to x_{ed3}}(0)] = [1,1]. \end{cases}$$

IV. Time shift failure localization in (max,+)-linear systems with observer

This section details how to extract information from the computed intervals of the indicators to determine the source of the failure in the TEG. All along this section, we assume that only one permanent time shift failure is present in the TEG, in a place p with a delay θ. We propose in this section to analyse how to retrieve the localization for three types of elementary structures that are contained in any TEG: tandem (see Figure 6), parallelism (see Figure 7) and synchronization (see Figure 2). Computing the localization of the failure in a specific TEG then consists in assembling these elementary analyses to get the global conclusion.

A. Tandem and parallel structures

Table I summarizes the localization results that can be expected when a tandem structure is involved given all the possible configurations. The first set of configurations (Rows 1-4) is when x_1 and x_2 can be associated with an indicator (i.e. the observer is able to provide a STD estimated state for both x_1 and x_2). The second set of configurations (Rows 5-7) is when only the state x_2 of the tandem has an indicator. Each row corresponds to a possible configuration of the indicator intervals and the conclusion about where the failure is in the analysed structure. For instance, in Row 3, $\Delta(x_{ed1}, x'_{s1}) = [\theta, \theta]$ and $\Delta(x_{ed2}, x'_{s2}) = [\theta, \theta]$ the time shift failure θ is located either in place p_1 or in a place in the upstream of the tandem.[2] Indeed, $x_1 = \bigoplus_{k=0}^{K} \gamma^{n_k} \delta^{t_k + a}$ and the next transition is $x_2 = \bigoplus_{k=0}^{K} \gamma^{n_k} \delta^{t_k + a + b}$. If the time shift failure θ is in place p_1 (or in a place in its upstream), $x_{ed1} = \bigoplus_{k=d_1}^{K} \gamma^{n_k} \delta^{t_k + a + \theta}$ and $x_{ed2} = \bigoplus_{k=d_2}^{K} \gamma^{n_k} \delta^{t_k + a + \theta + b}$, hence the results in the indicators.

	$\Delta(x_{ed1}, x'_{s1})$	$\Delta(x_{ed2}, x'_{s2})$	Localisation
2	[0,0]	[0,0]	∅
3	[θ,θ]	[θ,θ]	(p_1)
4	[0,0]	[θ,θ]	p_2
	$\Delta(x_{ed2}, x'_{s2})$		Localisation
6	[0,0]		∅
7	[θ,θ]		(p_1) or p_2

TABLE I

SUMMARY OF INTERVAL INTERPRETATIONS FOR TANDEM STRUCTURE

Similarly, Table III summarizes the results for the parallel structure (see Figure 7). Parallelism is an association of multiple tandems which shares a common transition. The results for parallelism are a direct consequence of the ones for the tandem structure.

[2]In every Table, notation p_i means that the localization is p_i while the notation (p_i) means the localization is p_i or a place in the upstream of p_i in the TEG.

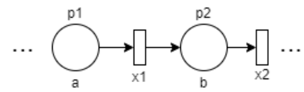

Fig. 6. Tandem

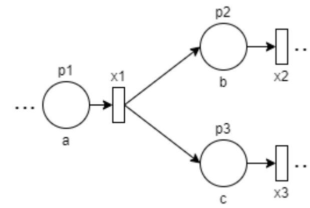

Fig. 7. Parallel

B. Synchronization structure

Figure 2 describes the synchronization structure. This one has many input places (p_1 and p_2)[3] and the localization results then depend on how tokens arrive in p_1 and p_2. To perform the exhaustive analysis, we suppose without loss of generality that the synchronization structure is governed by two inputs $u_1 = \bigoplus_{n \in \mathbb{Z}} \gamma^n \delta^{D_{u_1}(n)}$ and $u_2 = \bigoplus_{n \in \mathbb{Z}} \gamma^n \delta^{D_{u_2}(n)}$ where $D_{u_i}(n)$ is a dater function (see Definition 4). Table II details all the results into 3 blocks defining the three estimation configurations: in the first one, all the x_i's are estimated; in the second one, x_3 and only one among x_1 and x_2 are estimated; and finally in the last one, only x_3 is estimated. Consider for instance Row 5. First, as $\Delta(x_{ed3}, x'_{s3}) = [\theta, \theta]$, the effect of the time shift failure is always present at the synchronisation time. There are two possible situations. The first one is when $\forall n \in \mathbb{Z}$, $t_1 = D_{u_1}(n), t_2 = D_{u_2}(n)$ such that $a + \theta + b + t_1 > c + d + t_2$ (i.e. the time shift failure is along the path p_1, p_3 and its effect makes this path always slower than the path p_2, p_4). As $\Delta(x_{ed1}, x'_{s1}) = [0,0]$, the time shift failure θ is necessarily in place p_3. Indeed, $x_1 = \bigoplus_{k=0}^{K} \gamma^{n_k} \delta^{D_{u_1}(n_k) + a}$, $x_2 = \bigoplus_{k=0}^{K} \gamma^{n_k} \delta^{D_{u_2}(n_k) + c}$ and the next transition is $x_3 = \bigoplus_{k=0}^{K} \gamma^{n_k} \delta^{D_{u_1}(n_k) + a + b} \oplus \bigoplus_{k=0}^{K} \gamma^{n_k} \delta^{D_{u_2}(n_k) + c + d}$. If a time shift failure θ is in place p_3 the transition is $x_{ed1} = \bigoplus_{k=d_1}^{K} \gamma^{n_k} \delta^{D_{u_1}(n_k) + a}$, $x_{ed2} = \bigoplus_{k=d_2}^{K} \gamma^{n_k} \delta^{D_{u_2}(n_k) + c}$ and the next transition is $x_{ed3} = \bigoplus_{k=d_3}^{K} \gamma^{n_k} \delta^{D_{u_1}(n_k) + a + b + \theta} \oplus \bigoplus_{k=d_3}^{K} \gamma^{n_k} \delta^{D_{u_2}(n_k) + c + d}$. Therefore $x_{ed3} = \bigoplus_{k=d_3}^{K} \gamma^{n_k} \delta^{D_{u_1}(n_k) + a + b + \theta}$. The second situation is the symmetrical one that is if $a + b + t_1 < c + d + \theta + t_2$, and for the same reasons, the time shift failure is in p_4. Row 6 describes the case where the effect of the time shift failure is intermittent which leads to $\Delta(x_{ed3}, x'_{s3}) = [0, \theta]$ instead of $\Delta(x_{ed3}, x'_{s3}) = [\theta, \theta]$. In this case p_3 is suspected under the weaker condition that $\exists n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ such that $a + \theta + b + t_1 > c + d + t_2$. Note that here the indicator takes advantage of the underlying STD estimated state as the one of [PLCPV20] would generally return $[0, \theta]$ in any of these cases.

V. Conclusions

This paper defines a method for detecting and localizing time shift failures in systems modeled as TEG using observer

[3]Note that transitions u_1 and u_2 are actually not part of the synchronization structure that we describe here.

	$\Delta(x_{ed1}, x'_{s1})$	$\Delta(x_{ed2}, x'_{s2})$	$\Delta(x_{ed3}, x'_{s3})$	Localisation
2	[0,0]	[0,0]	[0,0]	$\varnothing$ or $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_1 or p_3 if $a + \theta + b + t_1 < c + d + t_2$ p_2 or p_4 if $a + b + t_1 > c + d + \theta + t_2$
3	[θ,θ]	[0,0]	[θ,θ] or [0,0] or [0,θ]	p_1
4	[0,0]	[θ,θ]	[θ,θ] or [0,0] or [0,θ]	p_2
5	[0,0]	[0,0]	[θ,θ]	$\forall n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_3 if $a + \theta + b + t_1 > c + d + t_2$ p_4 if $a + b + t_1 < c + d + \theta + t_2$
6	[0,0]	[0,0]	[0,θ]	$\exists n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_3 if $a + \theta + b + t_1 > c + d + t_2$ p_4 if $a + b + t_1 < c + d + \theta + t_2$
	$\Delta(x_{ed1}, x'_{s1})$/$\Delta(x_{ed2}, x'_{s2})$		$\Delta(x_{ed3}, x'_{s3})$	Localisation
8	[0,0]		[0,0]	$\varnothing$ or $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_1 or p_3 if $a + \theta + b + t_1 < c + d + t_2$ p_2 or p_4 if $a + b + t_1 > c + d + \theta + t_2$
9	[θ,θ]		[0,0] or [θ,θ] or [0,θ]	p_1/p_2
10	[0,0]		[θ,θ]	$\forall n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_1 / (p_1 or p_3) if $a + \theta + b + t_1 > c + d + t_2$ (p_2 or p_4)/ p_4 if $a + b + t_1 < c + d + \theta + t_2$
11	[0,0]		[0,θ]	$\exists n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_3 / (p_1 or p_3) if $a + \theta + b + t_1 > c + d + t_2$ (p_2 or p_4)/ p_4 if $a + b + t_1 < c + d + \theta + t_2$
	$\Delta(x_{ed3}, x'_{s3})$			Localisation
12	[0,0]			$\varnothing$ or $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_1 or p_3 if $a + \theta + b + t_1 < c + d + t_2$ p_2 or p_4 if $a + b + t_1 > c + d + \theta + t_2$
13	[θ,θ]			$\forall n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_1 or p_3 if $a + \theta + b + t_1 > c + d + t_2$ p_2 or p_4 if $a + b + t_1 < c + d + \theta + t_2$
14	[0,θ]			$\exists n \in \mathbb{Z}$, $t_1 = D_{u_1}(n)$, $t_2 = D_{u_2}(n)$ p_1 or p_3 if $a + \theta + b + t_1 > c + d + t_2$ p_2 or p_4 if $a + b + t_1 < c + d + \theta + t_2$

TABLE II

SUMMARY OF INTERVAL INTERPRETATIONS FOR SYNCHRONIZATION

	$\Delta(x_{ed1}, x'_{s1})$	$\Delta(x_{ed2}, x'_{s2})$	$\Delta(x_{ed3}, x'_{s3})$	Localisation
2	[0,0]	[0,0]	[0,0]	$\varnothing$
3	[0,0]	[θ,θ]	[0,0]	p_2
4	[0,0]	[0,0]	[θ,θ]	p_3
5	[θ,θ]	[θ,θ]	[θ,θ]	(p_1)
	$\Delta(x_{ed2}, x'_{s2})$		$\Delta(x_{ed3}, x'_{s3})$	Localisation
7	[0,0]		[0,0]	$\varnothing$
8	[θ,θ]		[0,0]	p_2
9	[0,0]		[θ,θ]	p_3
10	[θ,θ]		[θ,θ]	(p_1)

TABLE III

SUMMARY OF INTERVAL INTERPRETATIONS FOR PARALLELISM

that estimates their sensitive-to-disturbance states. An indicator is proposed using these STD estimated states whose interval values help to localize the failures in three types of elementary TEG structures.

A first direct perspective is to improve the localization for an other TEG elementary structure called loop, meaning any elementary structure looped between two transitions. Then, all the conclusions made separately have to be unified in a general conclusion about where the failure is localized on the TEG. Finally, we will look for exploiting more deeply the interval values of the indicator to characterize the nature of the time shift failure.

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