Observer-based detection of time shift failures in (max,+)-linear systems

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Abstract

In this paper, we address the problem of failure detection in production lines modeled as Timed Event Graphs (TEG). The proposed method represents TEGs as (max,+)-linear systems with disturbance and aims at detecting time shift failures in the underlying production lines. To do so, we will reconstruct the state of the observed system and define an indicator relying on the residuation theory on (max,+)-linear systems.

1 Introduction

In the industry, Discrete Event Systems (DES) are used to diagnose potential problems on production lines. The objective is then to detect, identify and locate failures as soon as possible to avoid further equipment unavailability. In those cases, the different failures of the system are basically the loss of information on a given event or the loss of time information on the considered event. Between those failures, timing issues can be a problem for a production line that slows down, putting out fewer pieces.

At STMicroelectronics Crolles300 plant, the approach was adapted in order to detect production drifts. STMicroelectronics is among the world's largest semiconductor companies, serving all electronics segments. Semiconductor manufacturing is complex. One of its most important challenges is to succeed in detecting production drifts before they have real impact on production plan.

One of the first diagnostic methods used to solve failure is taken from [SSL+95] on untimed automata. [Tri02] uses a method on on timed automata to refine diagnostic decisions using dated observations. In [GTY09], the diagnosis is based on Petri Nets which allows to model competition and parallelism to be done. The subclass of Petri Nets referred to Timed Event Graph (TEG) can also be used to specifically represent systems as a set of production lines. TEGs are one of the subclasses of Petri Nets where places are associated with a punctual duration; they can be modeled by (max,+) algebra as introduced in [BCOQ92, Max91]. [KLBvdB18] presents the history of DES with the use of (max,+) algebra. For example, [KL15] uses (max,+) algebra to control wafer delays in cluster tools for semiconductor production.

Back to the purpose of this article, [SLCP17] presents a detection method using (max,+) algebra to detect time shift failures from the system output whereas [HMCL10] presents a method for calculating an observer. In this paper, we will use the theory of (max,+) algebra to compute an observer and detect time shift failures on the rebuild states thanks to the observer. In particular, we propose a method for detecting time shift failures in system with disturbance.

The paper is organized as follows. Section 2 presents a motivation example inspired from the semiconductor industry. Section 3 summarizes the necessary mathematical background about (max,+)-linear systems. Section 4 describes the time shift failures in system with disturbance and gives the construction of an observer of such system. Section 5 then defines detection in (max,+)-linear systems. Finally, Section 6 provides conclusions.

2 Motivation example

STMicroelectronics has complex production lines with many pieces of equipment running in parallel. One of the objectives is to detect as soon as possible that an equipment is late to ensure that products are delivered on time or at least with minimal delays.

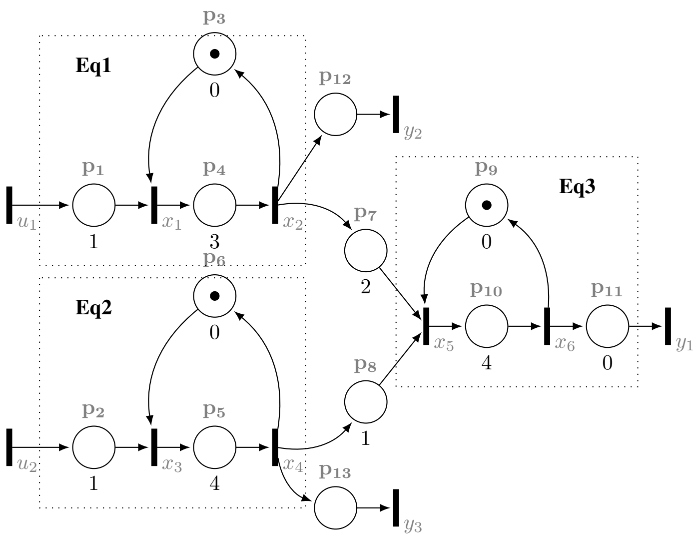

Figure 1: Fault free model

The fault-free behavioural model of the production lines is defined by the TEG shown in Figure 1. This production line corresponds to two pieces of equipment that process the same type of manufacturing step, but the processing time is not the same. The process of Equipment 1 corresponds to the place p_4 and is carried out in 3 hours. The availability of Equipment 1 corresponds to the token in the place p_3. Similarly the processes of Equipments 2 and 3 are respectively modeled by place p_5 (4 hours) and place p_{10} (4 hours) while their availabilities are modeled by p_6 and p_9 respectively. There is a synchronization between Equipment 1 and 2, that corresponds to the operation of Equipment 3 which can only start working with a sufficient number of wafers. Wafers take 2 hours from Equipment 1 to arrive on Equipment 3 and 1 hour from Equipment 2. The input to the production line is a wafer stream modeled by firing transitions u_1, u_2. A trigger of an input transition represents the occurrence of an event from sensors on the production line that indicates the arrival of the wafer. The inputs u_1 and u_2 correspond to the arrival of the wafers on Equipment 1 for entry u_1 and the arrival of the wafers on Equipment 2 for entry u_2. The output to the production lines is a wafer stream modeled by firing transition y_1. Outputs y_2 and y_3 provide information about the end of the manufacturing process of Equipments 1 and 2 respectively.

Now, consider a stream of 6 wafers on input u_2: respectively at time t=1,2,3,4,5,6. Consider a similar stream on u_1. Suppose now that products are available on the output y_1 respectively at time 12, 17, 22, 27, 32, 37 and 42; on the output y_2 respectively at time 5, 8, 11, 14, 17, 20 and 23; and finally on the output y_3 respectively at time 7, 12, 17, 22, 27, 32 and 37.

Then, the question is: if there is a time shift failure on the TEG of the figure, can we detect it?

Considering output y_1, the wafer that arrives at t=1 on Equipment 2 (Equipment 1), is processed in 4 hours (3 hours) and then takes 1 hour (2 hours) to arrive on Equipment 3. Synchronization between wafers is done at t=7 and they are processed in 4 hours on Equipment 3. So, the wafers come out at t=11. However, the first observable output comes out one hour later, hence a time drift happened.

In the same idea, considering output y_3, the wafer that arrives at t=1 on Equipment 2, is processed in 4 hours. So, the wafers come out at t=6. However, the first observable output comes out one hour later, hence another time drift happened.

In production lines, such time drifts are considered as time shift failures. This paper aims at providing an indicator that will detect them by using TEG as the one presented in Figure 1. TEG can be formally defined as (max,+)-linear systems that are introduced in the next section. Thanks to the dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ inputs and outputs are defined by series representing the events γ and the timings δ.

3 Mathematical background

This section recalls the mathematical background used in this paper for describing (max,+)-linear systems [BCOQ92, Max91].

3.1 Dioid theory

The dioid theory is used to describe the inputs and the behavior of the studied system. In particular, series of a specific dioid are defined to obtain the trajectories of inputs and states flows of timed events.

Definition 1. *A dioid $\mathcal{D}$ is a set composed of two internal operations $\oplus$ and $\otimes$. The addition $\oplus$ is associative, commutative, idempotent (i.e. $\forall a \in \mathcal{D}, a \oplus a = a$) and has a neutral element ε. The multiplication $\otimes$ is associative, distributive on the right and the left over the addition $\oplus$ and has a neutral element e. When there is no ambiguity, the symbol $\otimes$ is omitted.*

Definition 2. *A dioid is complete if it is closed for infinite sums and if $\otimes$ is distributive over infinite sums.*

Example 1. *The dioid $\mathbb{Z}_{max} = (\mathbb{Z} \cup -\infty)$ endowed with the max operation as addition $\oplus$ and the addition as multiplication $\otimes$ with neutral element denoted $\varepsilon = -\infty$ and $e = 0$. The dioid $\mathbb{Z}_{max}$ is not complete because $+\infty$ does not belong to the set $\mathbb{Z}_{max}$ so the infinite sum is not set to $+\infty$. By adding $+\infty$ to the dioid $\mathbb{Z}_{max}$, we get the complete dioid $\overline{\mathbb{Z}}_{max}$.*

Theorem 1 ([BCOQ92])**.** *Let $\mathcal{D}$ be a complete dioid, $x = a^*b$ is the solution of $x = ax \oplus b$, where $x = a^*b$, and $a^* = \bigoplus_{i\geq 0} a^i$ is the Kleene star operator with $a^0 = e$ and $a^{i+1} = a \otimes a^i$.*

Definition 3. *For a dioid $\mathcal{D}$, $\preceq$ denotes the order relation such that $\forall a, b \in \mathcal{D}, a \preceq b \Leftrightarrow a \oplus b = b$.*

Example 2. *The complete dioid $\mathbb{B}[\![\gamma,\delta]\!]$ is the set of formal series with two commutative variables γ and δ with boolean coefficients in $\{\varepsilon, e\}$ and exponents in $\mathbb{Z}$. A series $s \in \mathbb{B}[\![\gamma,\delta]\!]$ is written $s = \bigoplus_{n,t\in\mathbb{Z}} s(n,t)\gamma^n\delta^t$ where $s(n,t) = e$ or ε. The neutral elements are $\varepsilon = \bigoplus_{n,t\in\mathbb{Z}} \gamma^n\delta^t$ and $e = \gamma^0\delta^0$.*

Graphically, a series of $\mathbb{B}[\![\gamma,\delta]\!]$ is described by a collection of point of coordinates (n,t) in $\mathbb{Z}^2$ with γ as horizontal axis and δ as vertical axis. For instance, Figure 2 shows a series $u_1 = u_2 = \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^5 \oplus \gamma^5\delta^6 \oplus \gamma^6\delta^7 \oplus \gamma^7\delta^{+\infty}$.

In the following, we will consider the dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$. It

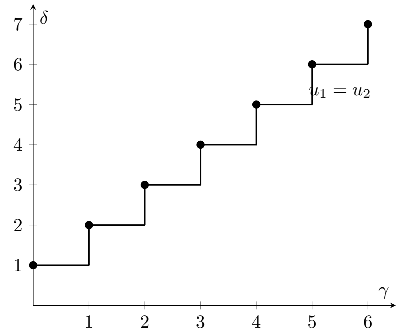

Figure 2: Representation of inputs $u_1 = u_2$

is the quotient of the dioid $\mathbb{B}[\![\gamma,\delta]\!]$ by the modulo $\gamma^*(\delta^{-1})^*$. The dioid $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ is a complete dioid with $\forall a, b \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$: $a = b \Leftrightarrow a\gamma^*(\delta^{-1})^* = b\gamma^*(\delta^{-1})^*$. The internal operations are the same as in $\mathbb{B}[\![\gamma,\delta]\!]$ and neutral elements ε and e are identical to those of $\mathbb{B}[\![\gamma,\delta]\!]$.

Definition 4. *Let $s \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ be a series, the dater function of s is the non-decreasing function $D_s(n)$ from $\mathbb{Z} \mapsto \overline{\mathbb{Z}}$ such that $s = \bigoplus_{n\in\mathbb{Z}} \gamma^n\delta^{D_s(n)}$.*

Considering the TEG of Figure 1, a first wafer arrives on u_1 and u_2 at time t=1, a second at t=2, a third at t=3, a fourth at t=4, a fifth at t=5 and finally a sixth at t=6 represented by series $u_1 = u_2 = \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^5 \oplus \gamma^5\delta^6 \oplus \gamma^6\delta^7 \oplus \gamma^7\delta^{+\infty}$ modeled in the $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$ dioid. The absence of a seventh wafer is indicated by $+\infty$ in monomial $\gamma^7\delta^{+\infty}$. The u_1 and u_2 inputs are shown in Figure 2. The series is composed of monomials $\gamma\delta$, where γ represents the events of the series, and δ represents the dates of the series. Series u_1 has for dater function $D_{u_1}(0) = 1$, $D_{u_1}(1) = 2$, $D_{u_1}(2) = 3$, $D_{u_1}(3) = 4$, $D_{u_1}(4) = 5$, $D_{u_1}(5) = 6$ and $D_{u_1}(6) = 7$. This dater function list all the dates of the event occurrences. u_2 has the same dater function as the u_1 series because $u_1 = u_2$.

Definition 5. *Let $\Pi : \mathcal{D} \mapsto \mathcal{C}$ be an isotone mapping, where $\mathcal{D}$ and $\mathcal{C}$ are complete dioids. The largest solution of $\Pi(x) = b$ is called the residual of Π and is noted $\Pi^\sharp$. When Π is residuated, $\Pi^\sharp$ is the unique isotone mapping such that $\Pi \circ \Pi^\sharp \preceq I_{d\mathcal{C}}$ and $\Pi^\sharp \circ \Pi \succeq I_{d\mathcal{D}}$ where $I_{d\mathcal{C}}$ and $I_{d\mathcal{D}}$ are respectively the identity mappings on $\mathcal{C}$ and $\mathcal{D}$.*

Theorem 2 ([Max91]). *Let $\mathcal{D}$ be a complete dioid and $A \in \mathcal{D}^{n\times m}$ be a matrix. Then,*

$$A \backslash A = (A \backslash A)^* \tag{1}$$

Example 3. *The mappings $L_a \mapsto a \otimes x$ and $R_a \mapsto x \otimes a$ defined over a complete dioid $\mathcal{D}$ are both residuated. Their residuals are denoted by $L_a^\sharp(x) = a \backslash x$ and $R_a^\sharp(x) = x / a$.*

Thanks to the residuals defined above we will be able to define time comparison between series.

Definition 6. *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ and their respective dater functions $\mathcal{D}_a$ and $\mathcal{D}_b$. The time shift function representing the time shift between a and b for each $n \in \mathbb{Z}$ is defined by $\mathcal{T}_{a,b}(n) = \mathcal{D}_a - \mathcal{D}_b$.*

Theorem 3 ([Max91]). *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, the time shift function $\mathcal{T}_{a,b}(n)$ can be bounded by:*

$$\forall n \in \mathbb{Z},\ \ \mathcal{D}_{b/a}(0) \leq \mathcal{T}_{a,b}(n) \leq -\mathcal{D}_{a/b}(0),$$

where $\mathcal{D}_{b/a}(0)$ is obtained from monomial $\gamma^0\delta^{\mathcal{D}_{b/a}(0)}$ of series b/a and $\mathcal{D}_{a/b}(0)$ is obtained from $\gamma^0\delta^{\mathcal{D}_{a/b}(0)}$ of series a/b.

Definition 7. *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, the time shift between series a and b is*

$$\Delta(a, b) = [\mathcal{D}_{b/a}(0); -\mathcal{D}_{a/b}(0)], \tag{2}$$

where $\gamma^0\delta^{\mathcal{D}_{b/a}(0)} \in b/a$ and $\gamma^0\delta^{\mathcal{D}_{a/b}(0)} \in a/b$. In this interval, the series from which the time offset is measured is the series a. It is called the reference series *of the interval.*

From this definition, if the time shift interval needs to be defined with series b as the reference series, the interval will be $\Delta(b, a) = [\mathcal{D}_{a/b}(0); -\mathcal{D}_{b/a}(0)]$.

From the system of Figure 1, let us consider two different outputs y. Output y_1 delivers one wafer at time t=12, one wafer at t=15, one wafer at t=18 and one wafer at t=21 which gives us the following series $y_1 = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$. Output y_2 delivers one wafer at time t=12, one wafer at t=15, one wafer at t=19 and one wafer at t=23 which gives us the following series $y_2 = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{+\infty}$. The minimal time shift between y_1 and y_2 is $\mathcal{D}_{y_2/y_1}(0) = 0$ and is found in the monomial where the degree of γ is 0 of $y_2/y_1 = \gamma^0\delta^{\mathbf{0}} \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^7 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{+\infty}$. The maximal time shift is $-\mathcal{D}_{y_1/y_2}(0) = 2$ and is found in $\gamma^0\delta^{-\mathbf{2}}$ of $y_1/y_2 = \gamma^0\delta^{-\mathbf{2}} \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^9 \oplus \gamma^4\delta^{+\infty}$. The time shift interval is $\Delta(y_1, y_2) = [0; 2]$. The distance between y_1 and y_2 is a minimum of 0 and a maximum of 2 hours. The y_1 series is faster than the y_2 series, meaning that y_1 produces faster than y_2.

3.2 Models of (max,+)-linear systems

The elements of the TEG will be represented by equations in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$. The equations can be grouped into a set of matrices A, B and C that contains information on the structure of TEG. The state representation defines relations between any set of input event flows u and the state x, and the relations between the state x and the output event flows y. Let $u \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p\times 1}$ be the input vector of size p, $x \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times 1}$ be the state vector of size n and $y \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q\times 1}$ be the output vector of size q. The state representation is:

$$\begin{cases} x = Ax \oplus Bu, \\ y = Cx, \end{cases}$$

where $A \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times n}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times p}$ and $C \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q\times n}$. Equality $x = Ax \oplus Bu$ can be transformed to $x = A^*Bu$ thanks to Theorem 1 so we have

$$y = CA^*Bu.$$

Matrix $H = CA^*B$ represents the transfer function of the TEG, that is the dynamic of the system between the inputs and the outputs.

For the system of Figure 1 the matrices $A \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{6\times 6}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{6\times 2}$ and $C \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{3\times 6}$ of the state representation are:

$$A = \begin{pmatrix} . & \gamma^1\delta^0 & . & . & . & . \\ \gamma^0\delta^3 & . & . & . & . & . \\ . & . & . & \gamma^1\delta^0 & . & . \\ . & . & \gamma^0\delta^4 & . & . & . \\ . & \gamma^0\delta^2 & . & \gamma^0\delta^1 & . & \gamma^1\delta^0 \\ . & . & . & . & \gamma^0\delta^4 & . \end{pmatrix},$$

$$B = \begin{pmatrix} \gamma^0\delta^1 & . \\ . & . \\ . & \gamma^0\delta^1 \\ . & . \\ . & . \\ . & . \end{pmatrix},$$

$$C = \begin{pmatrix} . & . & . & . & . & \gamma^0\delta^0 \\ . & \gamma^0\delta^0 & . & . & . & . \\ . & . & . & \gamma^0\delta^0 & . & . \end{pmatrix}.$$

The exponent n of γ represents the backward event shift between transitions (the $n + 1^{th}$ firing of x_1 depends on the n^{th} firing of x_2) and the exponent of δ represents the backward time shift between transition (the firing date of x_2 depends on the firing date of x_1 and time between 2 and 5).

4 How can a (max,+) observer be sensitive to time shift failures?

The objective of the paper is to propose a method that detects time shift failures as proposed in Section 2 and that uses an observer as introduced in [HMCL10] and [HMCSM10]. As later detailed in Section 4.2, this observer aims at computing a reconstructed state from the observation of the inputs and outputs of the system that is sensitive to a specific type of disturbance. These disturbances are characterized as new inputs w that slow down the system. Section 4.1 describes how time shift failures can be characterized as such disturbances. Section 4.2 then introduces the observer that will be used in the proposed detection method.

4.1 Characterisation of time shift failures as input disturbances

The time shift on a place corresponds to the injection of an offset to the duration of this place. When a time offset failure is injected, the duration of this place is modified. As shown on Figure 3, this place is characterized by a transition upstream x_{i-1}, a duration t and a transition downstream x_i.

Let $x_{i-1} = \bigoplus_{n=0}^{k} \gamma^{s_n}\delta^{h_n}$, where $s_j, j \in \{0, ..k\}$ is the transition firing number j, $h_j, j \in \{0, ..k\}$ is the firing date and k the number of firing events. The events s_n are numbered from 0 ($s_0 = 0$) so event number k never happens. The corresponding monomial is then $\gamma^k\delta^{+\infty}$. The downstream transition is $x_i = \bigoplus_{n=0}^{k-1} \gamma^{s_n}\delta^{h_n+t}$.

When a time shift on a place is characterized by a time offset $d > 0$ injected in this place, we can characterize this offset as follows: $x_i = \bigoplus_{n=0}^{k-1} \gamma^{s_n}\delta^{h_n+t+d}$.

To characterise the same time shift failure over a place p by a disturbance, we will first modify the TEG. To translate a time shift failure on a place such the one of Figure 3, we add to the downstream transition x_i after the place an input w_i which slows down this transition as shown in Figure 4. this new input w_i is not observed because it is related to a failure in an equipment.

Now, the input w_i have to be modified to slow down the transition x_i. To define this w_i, we take the trajectory of transition x_i while the fault occurs on place p. In other words, to obtain the offset d on the place p, the input w_i have to be defined as $w_i = \bigoplus_{n=0}^{k-1} \gamma^{s_n}\delta^{h_n+t+d}$. In case the offset is $d = 0$ we give it the value $w_i = \varepsilon$ so that the transition x_i has no disturbance.

Back to Figure 1 where the place p_5 has a duration of $t = 4$. To characterize an offset, meaning a time shift failure, of $d = 1$, we add a disturbance w_4 to the transition x_4 after the place p_5 in the same configuration as Figure 4. Suppose that $x_3 = \gamma^0\delta^2 \oplus \gamma^1\delta^6 \oplus \gamma^2\delta^{10} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{18} \oplus \gamma^5\delta^{22} \oplus \gamma^6\delta^{26} \oplus \gamma^7\delta^{+\infty}$. Since an offset of 1 time unit is present on p_5, $x_4 = \gamma^0\delta^{2+4+1} \oplus \gamma^1\delta^{6+4+1} \oplus \gamma^2\delta^{10+4+1} \oplus \gamma^3\delta^{14+4+1} \oplus \gamma^4\delta^{18+4+1} \oplus \gamma^5\delta^{22+4+1} \oplus \gamma^6\delta^{26+4+1} \oplus \gamma^7\delta^{+\infty}$. By setting the disturbance $w_4 = x_4 = \gamma^0\delta^7 \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{17} \oplus \gamma^3\delta^{22} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{32} \oplus \gamma^6\delta^{37} \oplus \gamma^7\delta^{+\infty}$, the firing of transition x_4 is slowed down.

Based on this characterisation, the system will then be assumed to behave with respect to the following state representation where inputs w are unknown but generate disturbances as defined above.

$$\begin{cases} x = Ax \oplus Bu \oplus Rw, \\ y = Cx. \end{cases}$$

The equations have new matrix R for the state representation of the system and new relations with the set of input event flows w. Let $w \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{l\times 1}$ be the input vector of disturbances of size l. The input w corresponds to the transition that will be disturbed. Matrix $R \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times l}$ is filled with $\gamma^0\delta^0$ monomials that represent the connections between disturbances and internal transitions we want to disturb. All the other entries are set to ε. Equality $x = Ax \oplus Bu \oplus Rw$ can be transformed to $x = A^*Bu \oplus A^*Rw$ thanks to Theorem 1 so we have

$$y = CA^*Bu \oplus CA^*Rw.$$

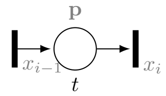

Figure 3: Representation of a place

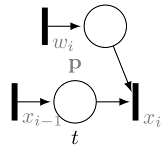

Figure 4: Representation of a place with disturbance

In the example of Section 2, all the internal transitions in Figure 1 will be disturbed so R is the matrix $R \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{6\times 6}$:

$$R = \begin{pmatrix} \gamma^0\delta^0 & . & . & . & . & . \\ . & \gamma^0\delta^0 & . & . & . & . \\ . & . & \gamma^0\delta^0 & . & . & . \\ . & . & . & \gamma^0\delta^0 & . & . \\ . & . & . & . & \gamma^0\delta^0 & . \\ . & . & . & . & . & \gamma^0\delta^0 \end{pmatrix}$$

4.2 Observer synthesis

In this paper we use the definition of an observer from the articles [HMCSM10], [HMCL10]. Figure 5 shows the system with disturbances w and from which we can observe the outputs y_o. The observer is a new model obtained from the fault-free model and that will estimate the states of the system x_r in the presence of such disturbances.

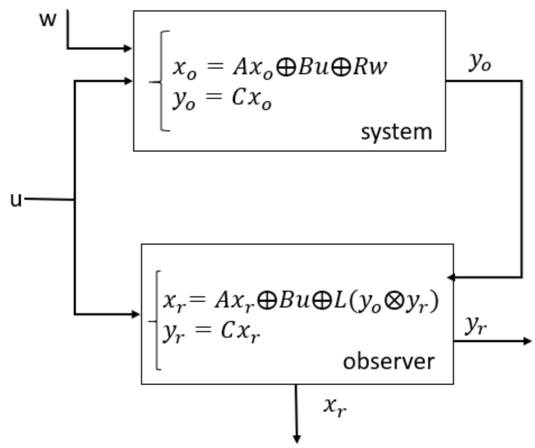

Figure 5: Observer structure with disturbance

From articles [HMCSM10], [HMCL10] we get the following observer's equations :

$$\begin{cases} x_r = Ax_r \oplus Bu \oplus L(y_r \oplus y_o) \\ = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw, \\ y_r = Cx_r. \end{cases} \quad (3)$$

To obtain the estimated vector x_r as close as possible to real state x, the largest observation matrix $L \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times q}$ is computed:

$$\begin{array}{ccc} x_r & \preceq & x_o \\ (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw & \preceq & A^*Bu \oplus A^*Rw \end{array}$$

The largest matrix L that satisfies the condition $x_r \preceq x_o$ is given by :

$$L_{opt} = (A^*B \phi CA^*B) \wedge (A^*R \phi CA^*R).$$

Considering the TEG of Figure 1 the matrix L is

$$
\begin{aligned}
L_{opt} &= (A^*B\phi CA^*B) \wedge (A^*R\phi CA^*R) \\
&= \begin{pmatrix}
. & \gamma^1\delta^0(\gamma^1\delta^3)^* & . \\
. & \gamma^0\delta^0(\gamma^1\delta^3)^* & . \\
. & . & \gamma^1\delta^0(\gamma^1\delta^4)^* \\
. & . & \gamma^0\delta^0(\gamma^1\delta^4)^* \\
\gamma^1\delta^0(\gamma^1\delta^4)^* & \gamma^0\delta^2(\gamma^1\delta^4)^* & \gamma^0\delta^1(\gamma^1\delta^4)^* \\
\gamma^0\delta^0(\gamma^1\delta^4)^* & \gamma^0\delta^6(\gamma^1\delta^4)^* & \gamma^0\delta^5(\gamma^1\delta^4)^*
\end{pmatrix}
\end{aligned}
$$

Based on the previous observer, suppose that the system behaves with respect to the inputs u_1 and u_2 defined in Section 3 but disturbed with $w_4 = \gamma^0\delta^7 \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{17} \oplus \gamma^3\delta^{22} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{32} \oplus \gamma^6\delta^{37} \oplus \gamma^7\delta^{+\infty}$ then the reconstructed state is:
$x_r = (A \oplus LC)^*Bu \oplus (A \oplus LC)^*LCA^*Rw$ with
$x_{r1} = \gamma^0\delta^2 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^8 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{14} \oplus \gamma^5\delta^{17} \oplus \gamma^6\delta^{20} \oplus \gamma^7\delta^{+\infty}$,
$x_{r2} = \gamma^0\delta^5 \oplus \gamma^1\delta^8 \oplus \gamma^2\delta^{11} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{17} \oplus \gamma^5\delta^{20} \oplus \gamma^6\delta^{23} \oplus \gamma^7\delta^{+\infty}$,
$x_{r3} = \gamma^0\delta^2 \oplus \gamma^1\delta^7 \oplus \gamma^2\delta^{12} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{22} \oplus \gamma^5\delta^{27} \oplus \gamma^6\delta^{32} \oplus \gamma^7\delta^{+\infty}$,
$x_{r4} = \gamma^0\delta^7 \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{17} \oplus \gamma^3\delta^{22} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{32} \oplus \gamma^6\delta^{37} \oplus \gamma^7\delta^{+\infty}$,
$x_{r5} = \gamma^0\delta^8 \oplus \gamma^1\delta^{13} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{28} \oplus \gamma^5\delta^{33} \oplus \gamma^6\delta^{38} \oplus \gamma^7\delta^{+\infty}$,
$x_{r6} = \gamma^0\delta^{12} \oplus \gamma^1\delta^{17} \oplus \gamma^2\delta^{22} \oplus \gamma^3\delta^{27} \oplus \gamma^4\delta^{32} \oplus \gamma^5\delta^{37} \oplus \gamma^6\delta^{42} \oplus \gamma^7\delta^{+\infty}$.

The state x_r estimated by the observer takes into account the disturbance w_4. If the disturbance w_4 was not present, the estimated state would be different, for instance, without disturbance, $x_{r4} = \gamma^0\delta^6 \oplus \gamma^1\delta^{10} \oplus \gamma^2\delta^{14} \oplus \gamma^3\delta^{18} \oplus \gamma^4\delta^{22} \oplus \gamma^5\delta^{26} \oplus \gamma^6\delta^{30} \oplus \gamma^7\delta^{+\infty}$ (no time shift: monomial $\gamma^0\delta^6$ instead of $\gamma^0\delta^7$ for instance).

5 Time shift failure detection in (max,+)-linear systems with observer

5.1 Design of a time shift failure indicator

In Figure 6 we have three blocks: the system which is ruled by the observable inputs u, the unobservable disturbances w and produces the observable outputs y_o ; the observer, whose output y_r and state x_r are computed thanks to the input u and output y_o and the fault free model of the system as the one in Figure 1.

Thanks to the estimation of the state obtained by the observer, we compare the estimated state noted x_r with the fault-free model state denoted x_s, the comparison is denoted $\Delta(x_{ri}, x_{si})$. This gives us the following indicator.

Definition 8 (Indicator of time shift failure in state)**.** *Let $A \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times n}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times p}$, $C \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times n}$ and $R \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times l}$ be the matrix of a $(\max,+)$-linear system, let $u \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times 1}$ and $y_o \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times 1}$ be the observable input and output trajectories of the system. The indicator $I_x(u, y_o)$ is the function:*

$$
I_x(u, y_o) = \begin{cases} \textit{false if for } \Delta(x_{ri}, x_{si}) = [0;0], \\ \textit{true otherwise}, \end{cases}
$$

with $x_s = A^*Bu$, $x_r = Ax_r \oplus Bu \oplus LCx_r \oplus Ly_o$ and

$$
\Delta(x_{ri}, x_{si}) = [\mathcal{D}_{x_{ri}\phi x_{si}}(0); -\mathcal{D}_{x_{si}\phi x_{ri}}(0)].
$$

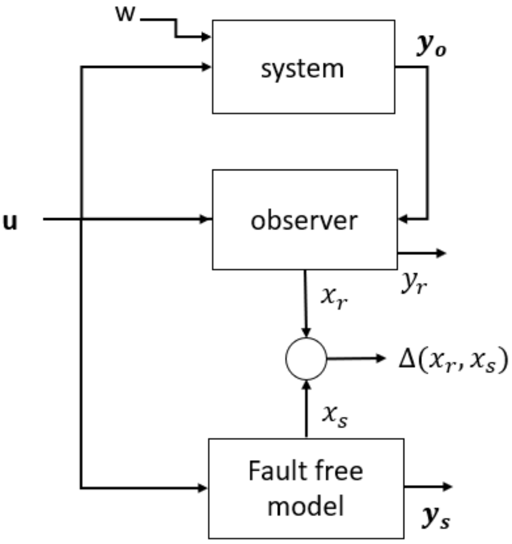

Figure 6: Detection method structure

The indicator uses the residuals (Definition 5), which allows calculating the interval (Definition 7) between two series.

Proposition 1. *The indicator returns true only if a time shift failure has occurred in the system.*

Proof. To prove the result, we show that if the system has no failure then the indicator necessary returns false. Suppose the system does not have any failure then it means by definition of the observer that the estimated state x_r is the same as the fault-free model state x_s. If $x_{si} = x_{ri}$, then we have $x_{si}\phi x_{ri} = x_{ri}\phi x_{si} = x_{ri}\phi x_{ri}$ but $x_{ri}\phi x_{ri} = (x_{ri}\phi x_{ri})^*$ according to Theorem 2 and with Definition 1 of the Kleene star: $(x_{ri}\phi x_{ri})^* = e \oplus \cdots = \gamma^0\delta^0 \oplus \ldots$. So if $x_{ri} = x_{si}$, one has $\mathcal{D}_{x_{ri}\phi x_{si}}(0) = -\mathcal{D}_{x_{si}\phi x_{ri}}(0) = 0$. ☐

In the example of Section 2, based on the previous observer, suppose that the system behaves with respect to the inputs u_1 and u_2 defined in Section 3. Suppose that in reality there was an incident on Equipment 2: the operation lasts longer with a processing time of 5 hours in p_5 which does not in 4 hours (see Figure 1) which is equivalent to injecting the disturbance w_4 that is defined in Section 4.1.

The estimated state is the same as given at the end of Section 4.2. x_{r3} is represented with plain line in Figure 7. The fault free state is:
$x_{s1} = \gamma^0\delta^2 \oplus \gamma^1\delta^5 \oplus \gamma^2\delta^8 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{14} \oplus \gamma^5\delta^{17} \oplus \gamma^6\delta^{20} \oplus \gamma^7\delta^{+\infty}$,
$x_{s2} = \gamma^0\delta^5 \oplus \gamma^1\delta^8 \oplus \gamma^2\delta^{11} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{17} \oplus \gamma^5\delta^{20} \oplus \gamma^6\delta^{23} \oplus \gamma^7\delta^{+\infty}$,
$x_{s3} = \gamma^0\delta^2 \oplus \gamma^1\delta^6 \oplus \gamma^2\delta^{10} \oplus \gamma^3\delta^{14} \oplus \gamma^4\delta^{18} \oplus \gamma^5\delta^{22} \oplus \gamma^6\delta^{26} \oplus \gamma^7\delta^{+\infty}$,
$x_{s4} = \gamma^0\delta^6 \oplus \gamma^1\delta^{10} \oplus \gamma^2\delta^{14} \oplus \gamma^3\delta^{18} \oplus \gamma^4\delta^{22} \oplus \gamma^5\delta^{26} \oplus \gamma^6\delta^{30} \oplus \gamma^7\delta^{+\infty}$,
$x_{s5} = \gamma^0\delta^7 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{15} \oplus \gamma^3\delta^{19} \oplus \gamma^4\delta^{23} \oplus \gamma^5\delta^{27} \oplus \gamma^6\delta^{31} \oplus \gamma^7\delta^{+\infty}$
$x_{s6} = \gamma^0\delta^{11} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{27} \oplus \gamma^5\delta^{31} \oplus \gamma^6\delta^{35} \oplus \gamma^7\delta^{+\infty}$.

x_{s3} is represented with dotted line in Figure 7. The intervals computed by the indicator are:
$\Delta(x_{r1}, x_{s1}) = [\mathcal{D}_{x_{r1}\phi x_{s1}(0)}, -\mathcal{D}_{x_{s1}\phi x_{r1}}(0)] = [0,0]$,
$\Delta(x_{r2}, x_{s2}) = [\mathcal{D}_{x_{r2}\phi x_{s2}(0)}, -\mathcal{D}_{x_{s2}\phi x_{r2}}(0)] = [0,0]$,

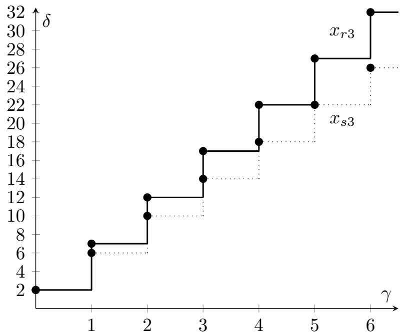

Figure 7: Graphical representation of x_{r3} and x_{s3}

$$
\begin{aligned}
\Delta(x_{r3}, x_{s3}) &= [\mathcal{D}_{x_{r3} \circ\!\!\!/ x_{s3}(0)}, -\mathcal{D}_{x_{s3} \circ\!\!\!/ x_{r3}}(0)] = [0, 6],\\
\Delta(x_{r4}, x_{s4}) &= [\mathcal{D}_{x_{r4} \circ\!\!\!/ x_{s4}(0)}, -\mathcal{D}_{x_{s4} \circ\!\!\!/ x_{r4}}(0)] = [1, 7],\\
\Delta(x_{r5}, x_{s5}) &= [\mathcal{D}_{x_{r5} \circ\!\!\!/ x_{s5}(0)}, -\mathcal{D}_{x_{s5} \circ\!\!\!/ x_{r5}}(0)] = [1, 7],\\
\Delta(x_{r6}, x_{s6}) &= [\mathcal{D}_{x_{r6} \circ\!\!\!/ x_{s6}(0)}, -\mathcal{D}_{x_{s6} \circ\!\!\!/ x_{r6}}(0)] = [1, 7].
\end{aligned}
$$

The indicator $I_x(u, y_o)$ returns true because $\Delta(x_{r3}, x_{s3}) = [0, 6] \neq [0, 0]$, $\Delta(x_{r4}, x_{s4}) = [1, 7] \neq [0, 0]$, $\Delta(x_{r5}, x_{s5}) = [1, 7] \neq [0, 0]$ and $\Delta(x_{r6}, x_{s6}) = [1, 7] \neq [0, 0]$. The time shift failure is detected.

5.2 Towards failure localization: a discussion

The previous section shows that the proposed indicator is able to detect the presence of time shift failures but does not discuss about their potential localizations. A further analysis about the bounds of the intervals actually provide more information about failure localizations. Before starting this interval analysis, let us go back to the definition of matrix L_{opt} which the state estimation relies on. Matrix L_{opt} actually represents the connections between the observed output y_o and the internal transitions x_r of the observer. In the matrix L_{opt} of Section 4.2, we can notice that all the rows are filled which means that L_{opt} is able to provide an estimate of any of the states x_i's (one row per x_i). Looking at the first monomial $\gamma^j \delta^l$ of a series $\gamma^j \delta^l (...)^*$ it is firstly possible to know when a state x_i starts to be reconstructed by L_{opt}. A monomial $\gamma^j \delta^l$ asserts that the estimation of the corresponding state only starts after the $j^{th} + 1$ event. For instance if $j = 0$, the estimation starts with the first event, meaning that we know exactly when the first trigger of the transition x_i happened. If $j = 1$, the estimation starts with the second event... In our example, by looking at L_{opt}, the estimation of x_2, x_4 and x_6 starts from the first event while the estimation of x_1 and x_3 starts from the second event. x_5 gets feedback from the 3 outputs (2 monomials with $j = 0$ and one with $j = 1$). Therefore, in the worst case, the estimation of x_5 starts from the second event. For the estimation of the states starting from the first event we can directly draw the following conclusions. Interval [0,0] on x_2 asserts that there is no time shift failure in place p_3. Intervals [1,7] on x_4 and x_6 assert that the failure has an effect on the transition firing time between the estimated state x_r and the expected state x_s. For the other estimations x_1, x_3 and x_5, intervals are not conclusive as the estimation does not start from the first event but the second. Further computations on the estimated state and so on the interval are necessary to prune any value that is not part of the estimation.

6 Conclusion

In this paper, we define a method for detecting time shift failures in systems modeled as Timed-Event Graphs using an observer that estimates the real states of the system based on the observations. Our work is motivated by the monitoring and the detection of time shift in production lines like in semiconductor manufacturing industry. The method defines a formal (max,+) algebraic indicator on the residuation theory. As a perspective, we aim at better exploiting the results returned by the indicator by a further analysis of the interval bounds to get better information about failure localization and identification as suggested in the previous discussion. Another perspective is to extend the method to deal with dynamical failures and fully exploit the capabilities of the observer.

References

[BCOQ92] F. Baccelli, G. Cohen, G.J Olsder, and J.-P Quadrat. *Synchronization and linearity: an algebra for discrete event systems*. Wiley and sons, 1992.

[GTY09] M. Ghazel, A. Toguyéni, and P. Yim. State observer for des under partial observation with time petri nets. *Discrete Event Dynamic Systems*, 19(2):137–165, 2009.

[HMCL10] L. Hardouin, C A. Maia, B. Cottenceau, and M. Lhommeau. Observer design for (max,+) linear systems. *IEEE Transactions on Automatic Control*, 55(2):538–543, 2010.

[HMCSM10] L. Hardouin, C.A. Maia, B. Cottenceau, and R. Santos-Mendes. Max-plus linear observer: application to manufacturing systems. In *10th International Workshop on Discrete Event Systems, WODES'10*, 2010.

[KL15] C. Kim and T.E. Lee. Feedback control of cluster tools for regulating wafer delays. *IEEE Transactions on Automation Science and Engineering*, 13(2):1189–1199, 2015.

[KLBvdB18] J. Komenda, S. Lahaye, J.-L. Boimond, and T. van den Boom. Max-plus algebra in the history of discrete event systems. *Annual Reviews in Control*, 45:240 – 249, 2018.

[Max91] MaxPlus. Second order theory of min-linear systems and its application to discrete event systems. In *Proceedings of the 30th IEEE Conference on Decision and Control. CDC'91*, 1991.

[SLCP17] A. Sahuguède, E. Le Corronc, and Y. Pencolé. Design of indicators for the detection of time shift failures in (max, +)-linear systems. In *20th World Congress of the International Federation of Automatic Control*, 2017.

[SSL+95] M. Sampath, R. Sengupta, S. Lafortune, K. Sinnamohideen, and D. Teneketzis. Diagnosability of discrete-event systems. *IEEE Transactions on automatic control*, 40(9):1555–1575, 1995.

[Tri02] S. Tripakis. *Fault Diagnosis for Timed Automata*, pages 205–221. Springer Berlin Heidelberg, Berlin, Heidelberg, 2002.