Detection of time shift failures in (max, +)-linear systems with time intervals applied to the supervision of assembly lines

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***Abstract*—In this paper, we address the problem of failure detection in assembly lines modeled as Timed Event Graphs (TEG). The proposed method represents TEGs as (max,+)-linear systems with time intervals and aims at detecting time shift failures in the underlying assembly lines. To do so, we propose the definition of a set of indicators relying on the residuation theory on (max,+) linear systems that handle certain and uncertain observable outputs.**

***Index Terms*— (max,+)-linear system, assembly line, fault detection, discrete event system, timed event graph**

I. Introduction

In industrial systems such as assembly lines, fault diagnosis is usually automatized by using Discrete Event System (DES). Industry also requires that failures are rapidly identified, to avoid systems unavailability for too long. The various system failures on which we proposed to work can be loss of event information, loss of time information. Among those failures, timing issues can be a problem for instance, an assembly line that slows down will put out fewer pieces. STMicroelectronics is a company that develops, produces and commercializes microchips for electronic system. The semiconductor manufacturing process is extremely complex and constantly innovating. One of the challenges is the monitoring of time drifts for the supervision of an instrumented production chain. This means detecting as soon as possible the time differences between the production plan and actual production performed in order to be able to apply the corrections quickly to prevent too much delay in the delivery of the product.

This problem is a subclass of the problems called failure diagnosis in timed discrete event systems. One of the first diagnostic methods used to solve such problems is the method extracted from [SSL$^+$95] which is applied to timed automata [Tri02]. This diagnostic method refines decisions on the diagnosis by taking into account dated observations. In [GTY09] the diagnostic method uses time Petri net that models competition and/or system parallelism. Among the classes of Petri Nets, Timed Event Graph is a good candidate to represent assembly lines. TEGs are one of the subclasses of Petri nets where places are associated with a punctual duration; they can be modeled by (max,+) algebra as introduced in [BCOQ92], [Max91]. More recently the survey [KLBvdB18] summarizes the history of (max,+) algebra within the field of discrete event systems. For example, [KL15] uses (max,+) algebra to represent TEGs to control cluster tools in semiconductor manufacturing.

Recently, in the article [SLCP17] the method uses (max,+) algebra to model the normal behavior of the system in a linear state representation and proposes a method to perform offline failure diagnosis. This method performs diagnosis on a fixed time (max,+)-linear systems. In this article, we extend the diagnostic method of the article [SLCP17] by dealing with time intervals in TEGs and by still using a representation with (max,+) algebra. We first propose a method for detecting time lags in system with certain outputs (i.e. we know exactly what are the output events of the system) that is secondly extended to deal with a method for detecting time lags in system with uncertain outputs (i.e. the occurrence dates of the output events are within a given time interval).

The paper is organized as follows. Section II presents a motivation example inspired from the microchip industry. Section III summarizes the necessary mathematical background about (max,+)-linear systems. Section IV introduces the acceptable outputs in (max,+)-linear system with time intervals. Section V then defines detection in (max,+)-linear systems with time intervals with indicators for certain outputs and indicators for uncertain outputs.

II. Motivation example

In STMicroelectronics plants, several products are produced at the same time. For the manufacturing of products there are several different production plans for the same product and variations in production time. Production performed may change during manufacture depending on equipment availability. These changes may cause delays depending on the delivery date of the product, the production performed have to be corrected. The purpose of the proposed method is to detect *a posteriori* when the delay becomes significant and that it is absolutely necessary to make a correction of the production performed in order not to have a delay on the delivery.

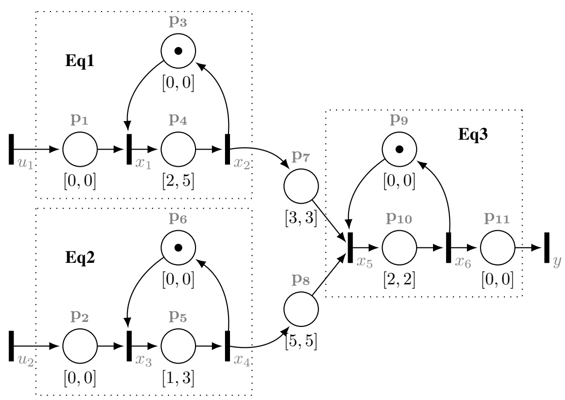

Fig. 1. Representation of the normal behavior of an assembly line as a Timed Event Graph with time intervals

Example 1. *Figure 1 shows a part of such a plant. It is an assembly line composed of equipments represented as a Timed Event Graph where places hold time intervals. Equipments 1 and 2 (Eq1 and Eq2) do the same treatment but not with the same time. The treatment on Equipment 3 (Eq3) needs a sufficient number of wafers that requires synchronization. Input u_1 (resp. u_2) is a flow of timed events corresponding to the arrivals of the wafers on Eq1 (resp. Eq2). If Eq1 (resp. Eq2) is ready, this corresponds to a token in p_3 (resp. p_6), the processing of the wafer by Eq1 (resp. Eq2) is carried out on p_4 (resp. p_5) and the process duration is between 2 and 5 hours (resp. 1 and 3 hours). The processing by Eq3 corresponds to p_{10} and lasts exactly 2 hours. This operation can only be processed if there is a sufficient number of wafers coming from Eq1 and Eq2. Wafers take 3 hours from Eq1 (p_8) to arrive on Eq3 and 5 hours from Eq2 (p_7).*

Now suppose that on u_1 we have 4 wafers: one at t=1, one at t=2 and two at t=3; the same on u_2. Final products are available on the output y respectively at time 12, 18, 24 and 30. Then, the question is: is the schedule respected according to the TEG of Figure 1 or is there a time drift?

For the wafer that arrives at t=1 on Eq1 (Eq2), in the best case it is processed in 2 hours (1 hour) and then takes 3 hours (5 hours) to arrive on Eq3. Synchronization between wafers is done at t=6 and they are processed in 2 hours on Eq3. So, the wafers come out at t=8. In the worst case, the wafers come out at t=11. However, the first observable output comes out one hour later the worst case hence the time drift.

Timed Event Graphs, as the one presented in Figure 1, can be formally defined as (max,+)-linear systems that are introduced in the next section. This formalization will be used to the definition of the indicators of time shift failures as the one of Example 1.

III. Scientific background

This section recalls the mathematical background used in this paper for describing (max,+)-linear systems with time intervals [BCOQ92], [Max91], [BHMR12] and [LHCJ04].

A. Dioid theory

The dioid theory is used to describe the inputs, the outputs and the behavior of the system studied. In particular, series of a specific dioid are defined to obtain the trajectories of inputs and outputs flows of timed events.

Definition 1. *A dioid $\mathcal{D}$ is a set composed of two internal operations $\oplus$ and $\otimes$. The addition $\oplus$ is associative, commutative, idempotent (i.e. $\forall a \in \mathcal{D}, a \oplus a = a$) and has a neutral element ε. The multiplication $\otimes$ is associative, distributive on the right and the left over the addition $\oplus$ and has a neutral element e. When there is no ambiguity, the symbol $\otimes$ is omitted.*

Definition 2. *A dioid is complete if it is closed for infinite sums and if $\otimes$ is distributive over infinite sums.*

Example 2. *The dioid $\mathbb{Z}_{max} = (\mathbb{Z} \cup -\infty)$ endowed with the max operation as addition $\oplus$ and the addition as multiplication $\otimes$ with neutral element denoted $\varepsilon = -\infty$ and $e = 0$. The dioid $\mathbb{Z}_{max}$ is not complete because $+\infty$ does not belong to the set $\mathbb{Z}_{max}$ so the infinite sum is not set to $+\infty$. By adding $+\infty$ to the dioid $\mathbb{Z}_{max}$, we get the complete dioid $\overline{\mathbb{Z}}_{max}$.*

Theorem 1 ([BCOQ92])**.** *Let $\mathcal{D}$ be a complete dioid, $x = a^*b$ is the solution of $x = ax \oplus b$, where $x = a^*b$, and $a^* = \bigoplus_{i \geq 0} a^i$ is the Kleene star operator with $a^0 = e$ and $a^{i+1} = a \otimes a^i$.*

Definition 3. *For a dioid $\mathcal{D}$, $\preceq$ denotes the order relation such that $\forall a, b \in \mathcal{D}, a \preceq b \Leftrightarrow a \oplus b = b$.*

Example 3. *The complete dioid $\mathbb{B}[\![\gamma, \delta]\!]$ is the set of formal series with two commutative variables γ and δ with boolean coefficients in $\{\varepsilon, e\}$ and exponents in $\mathbb{Z}$. A series $s \in \mathbb{B}[\![\gamma, \delta]\!]$ is written $s = \bigoplus_{n,t \in \mathbb{Z}} s(n,t)\gamma^n\delta^t$ where $s(n,t) = e$ or ε. The neutral elements are $\varepsilon = \bigoplus_{n,t \in \mathbb{Z}} \gamma^n\delta^t$ and $e = \gamma^0\delta^0$.*

Graphically, a series of $\mathbb{B}[\![\gamma, \delta]\!]$ is described by a collection of point of coordinates (n, t) in $\mathbb{Z}^2$ with γ as horizontal axis and δ as vertical axis. For instance, Figure 2 shows a couple of series $\underline{p} = \gamma^3\delta^0 \oplus \gamma^4\delta^1 \oplus \gamma^5\delta^2$ and $\overline{p} = \gamma^0\delta^1 \oplus \gamma^2\delta^3 \oplus \gamma^4\delta^5$.

In the following, we will consider the dioid $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$. It is the quotient of the dioid $\mathbb{B}[\![\gamma, \delta]\!]$ by the modulo $\gamma^*(\delta^{-1})^*$. The dioid $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ is a complete dioid with $\forall a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$: $a = b \Leftrightarrow a\gamma^*(\delta^{-1})^* = b\gamma^*(\delta^{-1})^*$. The internal operations are the same as in $\mathbb{B}[\![\gamma, \delta]\!]$ and neutral elements ε and e are identical to those of $\mathbb{B}[\![\gamma, \delta]\!]$.

Definition 4. *Let $s \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ be a series, the dater function of s is the non-decreasing function $D_s(n)$ from $\mathbb{Z} \mapsto \overline{\mathbb{Z}}$ such that $s = \bigoplus_{n \in \mathbb{Z}} \gamma^n\delta^{D_s(n)}$.*

Example 4. *Considering Example 1, a first wafer arrives on u_1 at time t=1, a second at t=2 and finally a third and a fourth at t=3 represented by series $u_1 = \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^4\delta^{+\infty}$. The absence of a fifth wafer is indicated by $+\infty$ in* *monomial $\gamma^4\delta^{+\infty}$. Series u_1 has for dater function $D_{u_1}(0) = 1$, $D_{u_1}(1) = 2$, $D_{u_1}(2) = 3$ and $D_{u_1}(3) = 3$.*

In the problem of detection of time differences between a plan and the reality, the residuation theory is used and provides time comparison between series.

Definition 5. *Let $\Pi : \mathcal{D} \mapsto \mathcal{C}$ be an isotone mapping, where $\mathcal{D}$ and $\mathcal{C}$ are complete dioids. The largest solution of $\Pi(x) = b$ is called the residual of Π and is noted $\Pi^\sharp$. When Π is residuated, $\Pi^\sharp$ is the unique isotone mapping such that $\Pi \circ \Pi^\sharp \preceq I_{d\mathcal{C}}$ and $\Pi^\sharp \circ \Pi \succeq I_{d\mathcal{D}}$ where $I_{d\mathcal{C}}$ and $I_{d\mathcal{D}}$ are respectively the identity mappings on $\mathcal{C}$ and $\mathcal{D}$.*

Example 5. *The mappings $L_a \mapsto a \otimes x$ and $R_a \mapsto x \otimes a$ defined over a complete dioid $\mathcal{D}$ are both residuated. Their residuals are denoted by $L_a^\sharp(x) = a \backslash\!\!\circ\, x$ and $R_a^\sharp(x) = x \phi a$.*

Thanks to the residuals defined above we will be able to define time comparison between series.

Definition 6. *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ and their respective dater functions $\mathcal{D}_a$ and $\mathcal{D}_b$. The time shift function representing the time shift between a and b for each $n \in \mathbb{Z}$ is defined by $\mathcal{T}_{a,b}(n) = \mathcal{D}_a - \mathcal{D}_b$.*

Theorem 2 ([Max91]). *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, the time shift function $\mathcal{T}_{a,b}(n)$ can be bounded by:*

$$\forall n \in \mathbb{Z}, \quad \mathcal{D}_{b\phi a}(0) \leq \mathcal{T}_{a,b}(n) \leq -\mathcal{D}_{a\phi b}(0),$$

where $\mathcal{D}_{b\phi a}(0)$ is obtained from monomial $\gamma^0\delta^{\mathcal{D}_{b\phi a}(0)}$ of series $b\phi a$ and $\mathcal{D}_{a\phi b}(0)$ is obtained from $\gamma^0\delta^{\mathcal{D}_{a\phi b}(0)}$ of series $a\phi b$.

Definition 7. *Let $a, b \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, the time shift between series a and b is*

$$\Delta(a, b) = [\mathcal{D}_{b\phi a}(0); -\mathcal{D}_{a\phi b}(0)], \tag{1}$$

where $\gamma^0\delta^{\mathcal{D}_{b\phi a}(0)} \in b\phi a$ and $\gamma^0\delta^{\mathcal{D}_{a\phi b}(0)} \in a\phi b$. In this interval, the series from which the time offset is measured is the series a. It is called the reference series *of the interval.*

From this definition, if the time shift interval needs to be defined with series b as the reference series, the interval will be $\Delta(b, a) = [\mathcal{D}_{a\phi b}(0); -\mathcal{D}_{b\phi a}(0)]$.

Example 6. *Let us consider two different observable outputs y from the system (Figure 1): $y_1 = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{18} \oplus \gamma^3\delta^{21} \oplus \gamma^4\delta^{+\infty}$ (i.e delivery of final products at times 12, 15, 18, 21), and $y_2 = \gamma^0\delta^{12} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{19} \oplus \gamma^3\delta^{23} \oplus \gamma^4\delta^{+\infty}$ (i.e delivery of final products at times 12, 15, 19, 23). The minimal time shift between y_1 and y_2 is $\mathcal{D}_{y_2\phi y_1}(0) = 0$ and is found in the monomial where the degree of γ is 0 of $y_2\phi y_1 = \gamma^0\delta^{\mathbf{0}} \oplus \gamma^1\delta^3 \oplus \gamma^2\delta^7 \oplus \gamma^3\delta^{11} \oplus \gamma^4\delta^{+\infty}$. The maximal time shift is $-\mathcal{D}_{y_1\phi y_2}(0) = 2$ and is found in $\gamma^0\delta^{-\mathbf{2}}$ of $y_1\phi y_2 = \gamma^0\delta^{-\mathbf{2}} \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^6 \oplus \gamma^3\delta^9 \oplus \gamma^4\delta^{+\infty}$.*

B. Models of (max,+)-linear systems with time intervals

This section presents a (max,+)-linear system with time intervals and the graphical representation (see TEG of Figure 1).

Definition 8. *A closed interval in a dioid $\mathcal{D}$ is a set of the form $\mathbf{x} = \{t \in \mathcal{D} | \underline{x} \preceq t \preceq \overline{x}\}$ denoted by $\mathbf{x} = [\underline{x}, \overline{x}]$.*

Definition 9. *The set of intervals in $\mathcal{D}$ denoted by $I(\mathcal{D})$, endowed with the operations*

$$\mathbf{x} \overline{\oplus} \mathbf{y} = [\underline{x} \oplus \underline{y}, \overline{x} \oplus \overline{y}], \quad \mathbf{x} \overline{\otimes} \mathbf{y} = [\underline{x} \otimes \underline{y}, \overline{x} \otimes \overline{y}],$$

where $\mathbf{x} = \{x \in \mathcal{D} | \underline{x} \preceq x \preceq \overline{x}\}$ and $\mathbf{y} = \{y \in \mathcal{D} | \underline{y} \preceq y \preceq \overline{y}\}$. The interval $\boldsymbol{\varepsilon} = [\varepsilon, \varepsilon]$ is neutral element of $\overline{\oplus}$ and the interval $\mathbf{e} = [e, e]$ is neutral element of $\overline{\otimes}$.

The order relation $\preceq$ in $I(\mathcal{D})$ induced by the additive law $\overline{\oplus}$ is such that

$$\mathbf{x} \overline{\oplus} \mathbf{y} = \mathbf{y} \Leftrightarrow \mathbf{x} \preceq \mathbf{y} \Leftrightarrow \begin{cases} \underline{x} \preceq \underline{y} \text{ in } \mathcal{D}, \\ \overline{x} \preceq \overline{y} \text{ in } \mathcal{D}. \end{cases}$$

Let $\mathrm{I}(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!])$ denote the set of intervals of the dioid $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$. An interval defined over $\mathrm{I}(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!])$ corresponds to all the series between the series of the minimum bound and the series of the maximum bound. For this purpose, the representation of the series in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ is used for the plotting of the series of the lower and upper bounds.

Example 7. *In Figure 2, the couple of polynomials represents the interval $\boldsymbol{p} = [\gamma^3\delta^0 \oplus \gamma^4\delta^1 \oplus \gamma^5\delta^2 \ , \ \gamma^0\delta^1 \oplus \gamma^2\delta^3 \oplus \gamma^4\delta^5]$.*

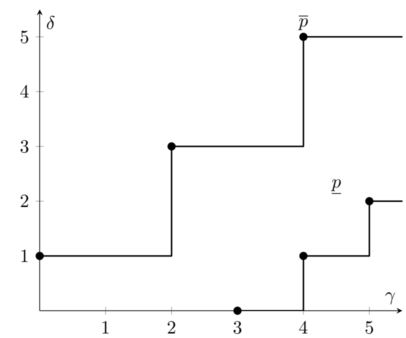

Fig. 2. Interval representation of $\mathbf{p} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!])$

The elements of the TEG will be represented by equations in $I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!])$. The equations can be grouped into a set of matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ called the state representation of the system that defines the relations between any set of input event flows $\mathbf{u}$ and the state $\mathbf{x}$, and the relations between the state $\mathbf{x}$ and the output event flows $\mathbf{y}$. Let $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p\times 1})$ be the input vector of size p, $\mathbf{x} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times 1})$ be the state vector of size n and $\mathbf{y} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q\times 1})$ be the output vector of size q. The state representation is:

$$\begin{cases} \mathbf{x} = \mathbf{A}\mathbf{x} \ \overline{\oplus} \ \mathbf{B}\mathbf{u}, \\ \mathbf{y} = \mathbf{C}\mathbf{x}, \end{cases}$$

where $\mathbf{A} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times n})$, $\mathbf{B} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{n\times p})$ and $\mathbf{C} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q\times n})$. Equality $\mathbf{x} = \mathbf{A}\mathbf{x} \overline{\oplus} \mathbf{B}\mathbf{u}$ can be transformed to $\mathbf{x} = \mathbf{A}^*\mathbf{B}\mathbf{u}$ thanks to Theorem 1 so we have

$$\mathbf{y} = \mathbf{C}\mathbf{A}^*\mathbf{B}\mathbf{u}.$$

Matrix $\mathbf{H} = \mathbf{C}\mathbf{A}^*\mathbf{B}$ represents the transfer function of the TEG.

Example 8. *For the system of Figure 1 the matrices* $\mathbf{A} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{6\times 6})$, $\mathbf{B} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{6\times 2})$ *and* $\mathbf{C} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{1\times 6})$ *of the state representation are:*

$$\begin{array}{ll} \mathbf{A(1;2)} = [\gamma^1\delta^0, \gamma^1\delta^0]; & \mathbf{A(2;1)} = [\gamma^0\delta^2, \gamma^0\delta^5]; \\ \mathbf{A(3;4)} = [\gamma^1\delta^0, \gamma^1\delta^0]; & \mathbf{A(4;3)} = [\gamma^0\delta^1, \gamma^0\delta^3]; \\ \mathbf{A(5;2)} = [\gamma^0\delta^3, \gamma^0\delta^3]; & \mathbf{A(5;4)} = [\gamma^0\delta^5, \gamma^0\delta^5]; \\ \mathbf{A(5;6)} = [\gamma^1\delta^0, \gamma^1\delta^0]; & \mathbf{A(6;5)} = [\gamma^0\delta^2, \gamma^0\delta^2]; \\ \mathbf{B(1;1)} = [\gamma^0\delta^0, \gamma^0\delta^0]; & \mathbf{B(3;2)} = [\gamma^0\delta^0, \gamma^0\delta^0]; \\ \mathbf{C(1;6)} = [\gamma^0\delta^0, \gamma^0\delta^0]. & \end{array}$$

Other entries in matrices are equal to ε. The exponent n of γ represents the backward event shift between transitions (the $n+1^{th}$ firing of x_1 depends on the n^{th} firing of x_2) and the exponent of δ represents the backward time shift between transition (the firing date of x_2 depends on the firing date of x_1 and time between 2 and 5). The transfer function is:

$$\begin{array}{ll} \mathbf{H(1,1)} = & [(\gamma^0\delta^7)[\gamma^1\delta^2]^*, (\gamma^0\delta^10)[\gamma^1\delta^5]^*]; \\ \mathbf{H(1,2)} = & [(\gamma^0\delta^8)[\gamma^1\delta^2]^*, (\gamma^0\delta^10)[\gamma^1\delta^3]^*]. \end{array}$$

IV. Acceptable outputs in (max,+)-linear systems with time intervals

This section introduces the acceptable outputs of a (max,+)-linear systems with time intervals. Acceptable outputs correspond to the outputs that are included in the interval of the predicted output. The hypothesis for the different definitions are known inputs and known transfer function of the (max,+)-linear systems with time intervals.

A. The system time shift

For a (max,+)-linear system with time intervals where $\mathbf{H} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times p})$ is the transfer function and $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times 1})$ is a given input, the predicted outputs are :

$$\mathbf{y_p} = \mathbf{Hu} = [\underline{y}_p, \overline{y}_p].$$

It is important to notice that the predicted outputs of the system are intervals of series because of the interval description of the system through the interval transfer function. In this interval of predicted outputs, the lower bound corresponds to the fastest execution of the system. That is, the tokens travel through the system with the smallest bound of the time intervals associated with the places. The upper bound corresponds to the slowest execution of the system. The tokens consume the largest bound of the time intervals associated with the places. The time shift between $\overline{y}_p$ and $\underline{y}_p$ can be computed thanks to the time shift bounds given in Definition 7.

Definition 10. *Let* $\mathbf{y_p} = \mathbf{Hu} = [\underline{y}_p, \overline{y}_p]$, *with* $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times 1})$, $\mathbf{H} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times p})$. *The system time shift with $\overline{y}_p$ as reference series is:*

$$\Delta_{\overline{y}_p} = [\mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0); -\mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0)] = [\underline{\Delta}_{\overline{y}_p}, \overline{\Delta}_{\overline{y}_p}].$$

The system time shift with $\underline{y}_p$ as reference series is:

$$\Delta_{\underline{y}_p} = [\mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0); -\mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0)] = [\underline{\Delta}_{\underline{y}_p}, \overline{\Delta}_{\underline{y}_p}].$$

The system time shift with $\overline{y}_p$ as reference series corresponds to the time shift from $\underline{y}_p$ to $\overline{y}_p$ where $\underline{y}_p \preceq \overline{y}_p$ so

$$\mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0) \leq 0 \text{ and } -\mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0) \leq 0.$$

The system time shift with $\underline{y}_p$ as reference series corresponds to the time shift from $\overline{y}_p$ to $\underline{y}_p$ where $\overline{y}_p \succeq \underline{y}_p$ so

$$\mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0) \geq 0 \text{ and } -\mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0) \geq 0.$$

Example 9. *Given the TEG of Figure 1, with the following inputs* $\mathbf{u_1} = \mathbf{u_2} = [\gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^4\delta^{+\infty}\ ,\ \gamma^0\delta^1 \oplus \gamma^1\delta^2 \oplus \gamma^2\delta^3 \oplus \gamma^4\delta^{+\infty}]$ *where* $\underline{u}_1 = \overline{u}_1 = \underline{u}_2 = \overline{u}_2$, *we get the predicted output* $\mathbf{y_p} = [\underline{y}_p, \overline{y}_p] = [\gamma^0\delta^9 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{13} \oplus \gamma^3\delta^{15} \oplus \gamma^4\delta^{+\infty}\ ,\ \gamma^0\delta^{11} \oplus \gamma^1\delta^{16} \oplus \gamma^2\delta^{21} \oplus \gamma^3\delta^{26} \oplus \gamma^4\delta^{+\infty}]$ *where* $\underline{y}_p \preceq \overline{y}_p$. *The system time shift with $\overline{y}_p$ as reference is* $\Delta_{\overline{y}_p} = [\mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0); -\mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0)] = [\underline{\Delta}_{\overline{y}_p}, \overline{\Delta}_{\overline{y}_p}] = [-11, -2]$. *The system time shift with $\underline{y}_p$ as reference is* $\Delta_{\underline{y}_p} = [\mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0); -\mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0)] = [\underline{\Delta}_{\underline{y}_p}, \overline{\Delta}_{\underline{y}_p}] = [2, 11]$.

B. Acceptable outputs

Definition 11. *Let* $\boldsymbol{H} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times p})$ *a known transfer function,* $\boldsymbol{u} \in I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{p\times 1})$ *a known input and* $\boldsymbol{y_p} = \boldsymbol{Hu} = [\underline{y}_p, \overline{y}_p]$ *the predicted output. A given output y is said acceptable if $\underline{y}_p \preceq y \preceq \overline{y}_p$. Such an output is denoted y_{acc} and belongs to the set denoted $\mathcal{Y}_{acc}$.*

Proposition 1. *An output $y \in \mathcal{Y}_{acc}$ is an acceptable output if and only if the following four conditions hold:*

$$\mathcal{D}_{y \nleftarrow \overline{y}_p}(0) \in [\underline{\Delta}_{\overline{y}_p}\ ,\ 0], \quad (2) \qquad -\mathcal{D}_{\overline{y}_p \nleftarrow y}(0) \in [\overline{\Delta}_{\overline{y}_p}\ ,\ 0], \quad (3)$$

$$\mathcal{D}_{y \nleftarrow \underline{y}_p}(0) \in [0\ ,\ \underline{\Delta}_{\underline{y}_p}], \quad (4) \qquad -\mathcal{D}_{\underline{y}_p \nleftarrow y}(0) \in [0\ ,\ \overline{\Delta}_{\underline{y}_p}]. \quad (5)$$

Proof. ($\Rightarrow$) Consider an acceptable output y. When $y = \overline{y}_p$, the time shifts between $\mathbf{y_p}$ and y are $\mathcal{D}_{y \nleftarrow \overline{y}_p}(0) = 0$, $\mathcal{D}_{\overline{y}_p \nleftarrow y}(0) = 0$, $\mathcal{D}_{y \nleftarrow \underline{y}_p}(0) = \mathcal{D}_{\overline{y}_p \nleftarrow \underline{y}_p}(0) = \underline{\Delta}_{\underline{y}_p}$ and $\mathcal{D}_{\underline{y}_p \nleftarrow y}(0) = \mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0) = \overline{\Delta}_{\underline{y}_p}$ thanks to Definitions 7 and 10. So Equations (2)-(5) hold in this limit case. When $y = \underline{y}_p$, the time shifts between $\mathbf{y_p}$ and y are $\mathcal{D}_{y \nleftarrow \underline{y}_p}(0) = 0$, $\mathcal{D}_{\underline{y}_p \nleftarrow y}(0) = 0$, $\mathcal{D}_{y \nleftarrow \overline{y}_p}(0) = \mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0) = \underline{\Delta}_{\overline{y}_p}$ and $\mathcal{D}_{\underline{y}_p \nleftarrow y}(0) = \mathcal{D}_{\underline{y}_p \nleftarrow \overline{y}_p}(0) = \overline{\Delta}_{\overline{y}_p}$ thanks to Definitions 7 and 10. So Equations (2)-(5) hold in this limit case. Then, for any acceptable output y such as $\underline{y}_p \preceq y \preceq \overline{y}_p$, since y evolves between $\underline{y}_p$ and $\overline{y}_p$, the time shift distances $\mathcal{D}(0)$ can only evolve between the bounds of the intervals of Equations (2)-(5).

($\Leftarrow$) If $\mathcal{D}_{y \nleftarrow \overline{y}_p}(0) = 0$ and $-\mathcal{D}_{\overline{y}_p \nleftarrow y}(0) = 0$, then there is no time shift between y and $\underline{y}_p$ because of Definition 10 of the system time shift with $\overline{y}_p$ as reference. So $y = \overline{y}_p$. It implies that $\mathcal{D}_{y \nleftarrow \underline{y}_p}(0) = \underline{\Delta}_{\underline{y}_p}$ and $-\mathcal{D}_{\underline{y}_p \nleftarrow y}(0) = \overline{\Delta}_{\underline{y}_p}$ thanks to Definition 10 of the system time shift with $\underline{y}_p$ as reference. Now if $\mathcal{D}_{y \nleftarrow \underline{y}_p}(0) = 0$ and $-\mathcal{D}_{\underline{y}_p \nleftarrow y}(0) = 0$, then there is no time shift between y and $\overline{y}_p$ because of Definition 10 of the system time shift with $\underline{y}_p$ as reference. So $y = \underline{y}_p$. It implies that $\mathcal{D}_{y \nleftarrow \overline{y}_p}(0) = \underline{\Delta}_{\overline{y}_p}$ and $-\mathcal{D}_{\overline{y}_p \nleftarrow y}(0) = \overline{\Delta}_{\overline{y}_p}$ thanks to Definition 10 of the system time shift with $\overline{y}_p$ as reference. Thus, if the time shift distances $\mathcal{D}(0)$ evolve between these limit bounds, then the output y evolves between $\underline{y}_p$ and $\overline{y}_p$. So for any set of time shift distances that respect Equations (2)-(5), the output y is an acceptable output. ☐

Proposition 2. *If $-\mathcal{D}_{\underline{y}_p \nmid y}(0) \notin [0\ ,\ \overline{\Delta}_{\underline{y}_p}]$ then $-\mathcal{D}_{\overline{y}_p \nmid y}(0) \notin [\overline{\Delta}_{\overline{y}_p}\ ,\ 0]$. If $\mathcal{D}_{y \nmid \overline{y}_p}(0) \notin [\underline{\Delta}_{\overline{y}_p}\ ,\ 0]$ then $\mathcal{D}_{y \nmid \underline{y}_p}(0) \notin [0\ ,\ \underline{\Delta}_{\underline{y}_p}]$. So, an output is said acceptable if Equations (3) and (4) are correct.*

Proof. In the case $-\mathcal{D}_{\underline{y}_p \nmid y}(0) \notin [0\ ,\ \overline{\Delta}_{\underline{y}_p}]$, $-\mathcal{D}_{\underline{y}_p \nmid y}(0) \succeq \overline{\Delta}_{\underline{y}_p}$ implies $-\mathcal{D}_{\overline{y}_p \nmid y}(0) \preceq \overline{\Delta}_{\overline{y}_p}$ because $\overline{\Delta}_{\underline{y}_p}$ and $\overline{\Delta}_{\overline{y}_p}$ are the same time shift to the nearest sign caused by the reference taken into account in the calculation according to Definition 7.

In the case $\mathcal{D}_{y \nmid \overline{y}_p}(0) \notin [\underline{\Delta}_{\overline{y}_p}\ ,\ 0]$, $\mathcal{D}_{y \nmid \overline{y}_p}(0) \succeq \underline{\Delta}_{\overline{y}_p}$ implies $\mathcal{D}_{y \nmid \underline{y}_p}(0) \preceq \underline{\Delta}_{\underline{y}_p}$ because $\underline{\Delta}_{\overline{y}_p}$ and $\underline{\Delta}_{\underline{y}_p}$ are the same time shift to the nearest sign caused by the reference taken into account in the calculation according to Definition 7. ☐

Example 10. *The predicted output $\mathbf{y_p} = [\gamma^0\delta^7 \oplus \gamma^1\delta^9 \oplus \gamma^2\delta^{11} \oplus \gamma^3\delta^{13} \oplus \gamma^4\delta^{+\infty}\ ,\ \gamma^0\delta^{10} \oplus \gamma^1\delta^{15} \oplus \gamma^2\delta^{20} \oplus \gamma^3\delta^{25} \oplus \gamma^4\delta^{+\infty}]$ is represented by the series with plain lines in Figure 3 with the system time shift $\Delta_{\overline{y}_p} = [-12, -3]$ and $\Delta_{\underline{y}_p} = [3, 12]$. The output $y_1 = \gamma^0\delta^8 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{14} \oplus \gamma^3\delta^{17} \oplus \gamma^4\delta^{+\infty}$ is represented with dotted line and the output $y_2 = \gamma^0\delta^7 \oplus \gamma^2\delta^9 \oplus \gamma^3\delta^{13} \oplus \gamma^4\delta^{+\infty}$ is represented with dashed line.*

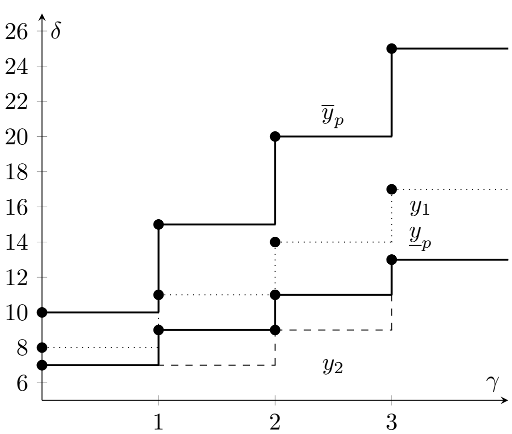

Fig. 3. Graphical representation of the acceptable output y_1 and the non-acceptable output y_2 with respect to interval $\mathbf{y_p} = [\underline{y}_p, \overline{y}_p]$

For y_1, the condition indicates $-\mathcal{D}_{\overline{y}_p \nmid y_1}(0) = -8 \in [\overline{\Delta}_{\overline{y}_p}\ ,\ 0] = [-12, 0]$ and $\mathcal{D}_{y_1 \nmid \underline{y}_p}(0) = 1 \in [0\ ,\ \underline{\Delta}_{\underline{y}_p}] = [0, 3]$ so the output y_1 belong to the set of acceptable outputs.

For y_2, we have $-\mathcal{D}_{\overline{y}_p \nmid y}(0) = -12 \in [\overline{\Delta}_{\overline{y}_p}\ ,\ 0] = [-12, 0]$ and $\mathcal{D}_{y \nmid \underline{y}_p}(0) = -3 \notin [0\ ,\ \underline{\Delta}_{\underline{y}_p}] = [0, 3]$ so the output y_2 does not belong to the set of acceptable outputs.

V. Time shift failure detection in (max,+)-linear systems with time intervals

This section finally presents the definition of indicators for the offline detection of time shift failures in systems modeled as TEGs (see Figure 1). Without loss of generality, we consider here that the system has one output only. In this framework, a *time shift failure* is characterized by the fact that for a given flow of input events the system does not generate an acceptable flow of output events (there are time shifts). Formally, a time shift failure has occurred in the system, if, for a given input vector $\mathbf{u} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p \times 1})$ where any element of $\mathbf{u}$ is a singleton interval like $[u, u]$, the real output $y \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q \times 1}$ of the system is not acceptable with regards to $\mathbf{y_p} = \mathbf{Hu}$.

As presented in Example 1, we suppose that the supervisor has a set of sensors to fully observe the inputs and the output of the system. These observations are denoted $\mathbf{u_o} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{p \times 1})$ and $\mathbf{y_o} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q \times 1})$. In the following, we consider that the sensors that observe the inputs are certain (no time uncertainty in the flow of input events) which means that $\mathbf{u_o} = \mathbf{u}$ with $\mathbf{u}$ the real input vector as defined hereabove.

The principle of the detection is to firstly predict the outputs $\mathbf{y_p}$ based on the model and the observed inputs ($\mathbf{y_p} = \mathbf{Hu_o}$ with $\mathbf{H} \in I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q \times p})$) and to secondly check whether the observed output $\mathbf{y_o}$ is acceptable or not. We consider two cases. The first one is when the observation of the output is *certain*: $\mathbf{y_o} = [y_o, y_o]$, with $y_o = y \in \mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]^{q \times 1}$ the real output. We then propose to extend the solution to deal with *uncertain observed outputs* : any output event of the system is observed at a time t by a sensor with an uncertainty d, which means that the output event has occurred within time interval $[t-d, t+d]$. In this case, $\mathbf{y_o} = [\underline{y}_o, \overline{y}_o]$ is part of $I(\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!])$.

A. Indicator in case of a certain observable output y_o

The indicator is a boolean function that aims at asserting whether there is a failure or not. It actually checks whether y_o is acceptable based on Equations (3) and (4).

Definition 12. *The indicator $Ind(\boldsymbol{u_o}, y_o)$ is:*

$$Ind(\boldsymbol{u_o}, y_o) = \begin{cases} \textit{false} & \textit{if for } \mathbf{y_p} = \boldsymbol{H}.\boldsymbol{u_o}, \\ & \quad -\mathcal{D}_{\overline{y}_p \nmid y_o}(0) \in [\overline{\Delta}_{\overline{y}_p}, 0] \\ & \quad \textit{and } \mathcal{D}_{y_o \nmid \underline{y}_p}(0) \in [0, \underline{\Delta}_{\underline{y}_p}] \\ \textit{true} & \textit{otherwise.} \end{cases}$$

The following proposition ensures that the indicator is correct, it does not generate any false positive/negative results.

Proposition 3. *The indicator returns true iff a time shift failure has occurred in the system.*

Proof. Propositions 1-2 ensure y_o is acceptable with regards to $\mathbf{y_p} = \mathbf{Hu_o}$ iff Equations (3-4) hold. By Definition 12, Equations (3)-(4) hold iff $Ind(\mathbf{u_o}, y_o)$ is false. ☐

Example 11. *Consider the scenario that is defined by the observed inputs of Example 9. The TEG of Figure 1 has thus the following system time shift predictions: $\Delta_{\overline{y}_p} = [-11, -2]$ and $\Delta_{\underline{y}_p} = [2, 11]$. Suppose that in reality there was an incident on Equipment 1: the operation lasts longer with a processing time of 6 hours in p_4 which does not belong* *to interval [2,5] (see Figure 1). The following output is observed* $y_o = \gamma^0\delta^{12} \oplus \gamma^1\delta^{18} \oplus \gamma^2\delta^{24} \oplus \gamma^3\delta^{30} \oplus \gamma^4\delta^{+\infty}$ *(see Example 1).* $\mathcal{D}_{y_o \not\models \underline{y}_p(0)} = 3 \notin [0, \underline{\Delta}_{\underline{y}_p}] = [0,2]$ *and* $-\mathcal{D}_{\overline{y}_p \not\models y_o}(0) = 4 \notin [\overline{\Delta}_{\overline{y}_p}, 0] = [-11,0]$. *The indicator* $Ind(\boldsymbol{u_o}, y_o)$ *returns true because Equations (4) and (3) are false. The time shift failure is detected.*

B. Indicator in case of an uncertain observable output $\mathbf{y_o}$

As the observable output $\mathbf{y_o}$ in uncertain, it is characterized by an interval $\mathbf{y_o} = [\underline{y}_o, \overline{y}_o]$ from $I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!])$. Definition 13 extends Definition 12 to deal with uncertain observations.

Definition 13. *The indicator* $Ind(\boldsymbol{u_o},\boldsymbol{y_o})$ *is:*

$$Ind(\boldsymbol{u_o},\boldsymbol{y_o}) = \begin{cases} \textit{false if for } \mathbf{y_p} = \boldsymbol{Hu_o}, \\ \qquad\qquad -\mathcal{D}_{\overline{y}_p \not\models \overline{y}_o}(0) \in [\overline{\Delta}_{\overline{y}_p}, 0] \\ \qquad\qquad \textit{and } \mathcal{D}_{\underline{y}_o \not\models \underline{y}_p}(0) \in [0, \underline{\Delta}_{\underline{y}_p}]. \\ \textit{true} \quad \textit{otherwise}. \end{cases}$$

The following proposition ensures that the indicator is conservative and does not generate any false negative results.

Proposition 4. *If a time shift failure has occurred in the system then the indicator returns true.*

Proof. Suppose a time shift failure has occurred but the indicator returns false, then Equations (3-4) hold. By Propositions 1 and 2, any series of $\mathbf{y_o}$ is acceptable: no time shift failure could have happened, hence the contradiction. □

Example 12. *Looking back to the scenario described in Example 11 with the same observed inputs, but consider now that the observation of the outputs is uncertain:* $\boldsymbol{y_o}_1 = [\gamma^0\delta^9 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{13} \oplus \gamma^3\delta^{15} \oplus \gamma^4\delta^{+\infty} \ ; \ \gamma^0\delta^{12} \oplus \gamma^1\delta^{18} \oplus \gamma^2\delta^{24} \oplus \gamma^3\delta^{30} \oplus \gamma^4\delta^{+\infty}]$ *(i.e. the deliveries of the final products are estimated to be at time within* $[9,12]$, $[11,18]$, $[13,24]$, $[15,30]$*) and* $\boldsymbol{y_o}_2 = [\gamma^0\delta^9 \oplus \gamma^1\delta^{11} \oplus \gamma^2\delta^{13} \oplus \gamma^3\delta^{15} \oplus \gamma^4\delta^{+\infty} \ ; \ \gamma^0\delta^{10} \oplus \gamma^1\delta^{12} \oplus \gamma^2\delta^{14} \oplus \gamma^3\delta^{16} \oplus \gamma^4\delta^{+\infty}]$ *(i.e. the deliveries of the final products are estimated to be at time within* $[9,10]$, $[11,12]$, $[13,14]$, $[15,16]$*). For* $\boldsymbol{y_o}_1$, $\mathcal{D}_{\underline{y}_{o^1} \not\models \underline{y}_p(0)} = 2 \in [0, \underline{\Delta}_{\underline{y}_p}] = [0,2]$ *and* $-\mathcal{D}_{\overline{y}_p \not\models \overline{y}_{o^1}}(0) = 4 \notin [\overline{\Delta}_{\overline{y}_p}, 0] = [-11,0]$. *The indicator* $Ind(\boldsymbol{u_o},\boldsymbol{y_o}_1)$ *then returns true: the occurrence of a time shift failure is suspected. For* $\boldsymbol{y_o}_2$, $\mathcal{D}_{\underline{y}_{o^2} \not\models \underline{y}_p(0)} = 2 \in [0, \underline{\Delta}_{\underline{y}_p}] = [0,2]$ *and* $-\mathcal{D}_{\overline{y}_p \not\models \overline{y}_{o^2}}(0) = -1 \in [\overline{\Delta}_{\overline{y}_p}, 0] = [-11,0]$. *The indicator* $Ind(\boldsymbol{u_o},\boldsymbol{y_o}_2)$ *then returns false: the occurrence of no time shift failure.*

VI. Conclusion

In this article, we extend the problem of time failure diagnosis in TEG by using time intervals. Using (max,+) algebraic techniques we propose indicators that detect time shift with (un)certain observable outputs. The indicators are based on the dioid $I(\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!])$ but the indicator computations consist in comparing the min/max bounds of the observable outputs. This study is motivated by the development of algorithms for time shift failure detection in assembly lines. We have several perspectives for the use of indicators on a real production line such as that of STMicroelectronics. We will set up the C++ coding of this indicator in the *MaxPlusDiag* library of the article [LCSPP18] from the library MinMaxGD [CLHB00]. Another perspective is to deal also with event uncertainty (i.e. interval of tokens in a place of the TEG).

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