

INTRODUCTION TO SIGNAL PROCESSING WITH MATLAB

1 Introduction

This section recalls some basic MATLAB commands, useful for the lab session :

- `t = first:step:last` generates a vector with components going from value `first` to `last` with a step of `step`. `length(t)` returns the number of elements of the vector. `t(k)` selects the k^{th} element of t .
- `A = [1 2 3 ; 4 5 6]` defines a matrix A with its components (here, 2 rows and 3 columns). The size of the matrix is given by `size(A)`.
- `plot(x,y)` draw y with respect to x on a 2D graph. x and y are two vectors of the same size. A third argument may be specified to set the color, the line type, or the plot symbol. The instruction `hold on` allows to draw several consecutive plots on the same figure. `figure(x)` selects figure number x to plot a graph on. Title and labels can be added on a figure with commands `title`, `xlabel`, `ylabel`, `legend`.
- Table 1 shows some elementary mathematical functions.
- `fft(x)` is the discrete Fourier transform of vector x .
- `tf(B,A)` defines a transfer function with B and A being vectors of the coefficients of the numerator and denominator, respectively.

<code>sin</code>	<code>cos</code>	<code>tan</code>	<code>atan2</code>
<code>asin</code>	<code>acos</code>	<code>atan</code>	<code>sec</code>
<code>sinh</code>	<code>cosh</code>	<code>tanh</code>	<code>csc</code>
<code>abs</code>	<code>real</code>	<code>imag</code>	<code>conj</code>
<code>exp</code>	<code>log</code>	<code>log10</code>	<code>log2</code>
<code>sign</code>	<code>mod</code>	<code>sqrt</code>	<code>lcm</code>
<code>round</code>	<code>floor</code>	<code>ceil</code>	<code>gcd</code>

TABLE 1

Remember that you can type `help acommand` in the “command window” to have more information and examples on the command `acommand`.

2 Sampling

Let consider the following continuous-time function :

$$s(t) = 3 + \cos(2\pi f_1 t) + 2 \cos(2\pi f_2 t)$$

with $f_1 = 2 \text{ Hz}$ and $f_2 = 7 \text{ Hz}$.

1. Draw the frequency spectrum of $s(t)$.
2. What is the minimum sampling frequency F_e to capture all the information from the continuous-time signal?
3. Let choose $F_e = 10 \text{ Hz}$. Draw the corresponding spectrum of the sampled signal $s_e(t)$.
4. In that case, what is expression of the resulting signal?

In MATLAB, compute signal $s(t)$ and plot the time representation for $t \in [0, 1]$. On the same graph, plot the sampled version with a sampling rate of $F_e = 10 \text{ Hz}$. Then, plot the spectral representation and check that it is consistent with your previous answer. Back on the first graph, add the plot of the resulting signal and check that the curve matches all the sampling points.

3 Digital filtering

The Butterworth low pass filter is defined by a frequency response of the form

$$G_n(\omega) = \frac{1}{\sqrt{1 + (\omega/\omega_c)^{2n}}}.$$

G_n is the gain of the filter, ω_c is the cut-off frequency in rad/s and n is the order of the filter. It will be used for noise reduction of an audio signal. MATLAB comes with a speech data set of someone saying “matlab”. This data set can be loaded with the command `load mt1b` and provides a vector named `mt1b`. The speech data set was obtained with a sampling frequency of $F_s = 7418 \text{ Hz}$. The command `soundsc(mt1b)` allows to play the signal on the computer.

1. Plot the signal `mt1b`. Then, compute and plot the magnitude of the spectrum. What are the predominant frequencies in the speech signal? The width of spectrum?
2. This speech is now corrupted with a high-frequency noise. The corresponding data set is stored in the file¹ “noisyspeech.txt”. Plot the signal in time and its spectrum. What is the difference?

In order to filter the noise, we define a low-pass filter of the form

$$H(s) = \frac{1}{\frac{1}{\omega_c^2} s^2 + \frac{\sqrt{2}}{\omega_c} s + 1},$$

where ω_c is the cut-off frequency.

3. Show that $H(s)$ is a Butterworth filter.
4. Calculate the corresponding digital IIR filter $H(z)$ based on the bilinear transform.
5. Then, apply the digital filter to the noisy speech with an appropriate cut-off frequency. Plot the filtered signal in time and its spectrum.

1. The file must be downloaded at <https://dl.dropboxusercontent.com/u/7582995/noisyspeech.txt>.

4 Amplitude modulation

Let consider the following message to be sent :

$$m(t) = \cos(2\pi f_1 t) + 0.5 \cos(2\pi f_2 t),$$

with $f_1 = 10 \text{ Hz}$ and $f_2 = 15 \text{ Hz}$. A sampling frequency $f_e = 1 \text{ kHz}$ will be used to represent all signals. In order to transmit this message, we define a carrier wave with a frequency f_0 :

$$c(t) = \cos(2\pi f_0 t).$$

We set f_0 to 200 Hz .

1. Plot the signal $m(t)$ over a duration of $4s$. What does the spectrum look like ?
2. Express the modulated signals $x_{am}(t)$ and $x_{sc}(t)$ based on the above carrier wave, respectively, with and without the carrier component. In the case of amplitude modulation with the carrier, use a modulation index $k = 0.6$. What are the spectral representations ?

For each question, compute and plot graphs with MATLAB. Verify your results.

Then, on the receiver side, we want to demodulate $x_{am}(t)$ and $x_{sc}(t)$ with the synchronous demodulation principle (see Figure 1).

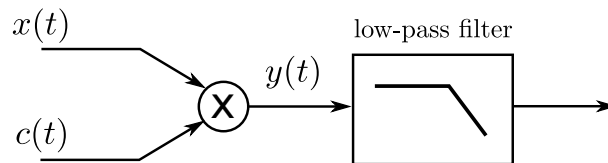


FIGURE 1: *Synchronous demodulation.*

3. Calculate the expression of signal $y(t)$ in the case of an amplitude modulated signal without carrier $x_{sc}(t)$ and draw the resulting spectrum.
4. Is the sampling frequency still valid ?
5. What should be the cut-off frequency of the low-pass filter ?
6. Define an appropriate filter and apply it to $y(t)$. Plot the resulting final demodulated signal.