A simple condition for L_2 stability and stabilization of networked control systems

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Abstract: The stability analysis and stabilization of networked control systems subject to data loss and time-varying transmission delays are explored. The stability result is based on quadratic separation and operator theory, which allows to capture the above phenomena into the single formalism of aperiodic sampling. The obtained stability conditions are expressed in terms of LMIs. The stabilization problem is a bit more involved due to the inherent structure of the obtained LMI for stability. A dilated result is then proposed to obtain a more tractable LMIs for stabilization. Several examples illustrate the effectiveness of the proposed approach.

Keywords: Networked Control Systems; Quadratic Separation; LMIs

1. INTRODUCTION

Networked control systems (NCS) [Hespanha et al., 2007, Heemels et al., 2009] is a wide class of physical systems controlled or interconnected through a network, wired or not. The idea behind NCS is the development of new paradigms and problems considering the interconnection of networks and systems; remote/decentralized/distributed control are important examples. The presence of networks has an important influence on the overall systems behavior by inducing delays, data loss and imposing restrictions, like the use of coded data and transmission channels with finite capacity, etc. It is well-known that the above phenomena deteriorate the systems stability/performance and must be considered in the analysis and the controller design.

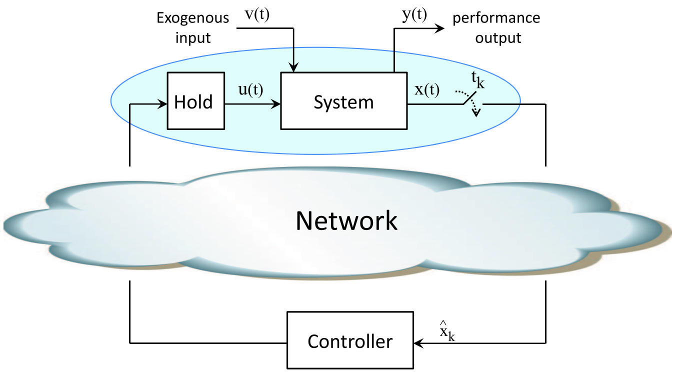

Fig. 1. Example of a networked control system (remote control).

Amongst all the problems which can be imagined in this framework, the remote control of processes through wireless networks (Fig. 1) is a very important one which faces to almost all the problems a wireless network can induce: data loss (due to collisions and interferences), time-varying propagation delays (due to interferences, computation delays and possibly a varying distance between the transmitter and the receiver). In such a set-up, the controller receives the data from the systems sensors through a network and sends back the control input (Fig. 1). Such a problem has already been studied in the literature, for instance in [Yu et al., 2004, Yue et al., 2004, Naghshtabrizi and Hespanha, 2006, Hespanha et al., 2007, Cloosterman et al., 2009, Heemels et al., 2009].

In this paper, the stability and L_2-gain analysis of linear systems remotely controlled by a state-feedback controller is considered. From the nominal expression of the sampled-data control law, the network effects are successively added to the control law in order to build an accurate model for the closed-loop system (i.e. system+network+controller). The overall system is then rewritten as an interconnection of uncertain/dynamical operators (describing the network effects and the performance criterion) with an implicit algebraic expression. In order to consider the operators accurately for the stability analysis of the NCS, they are implicitly characterized through the behavior of their input/output signals using IQCs [Rantzer and Megretski, 1997]. The stability conditions, expressed as LMIs, are then obtained using recent results on quadratic separation (or well-posedness) [Goh and Safonov, 1995, Iwasaki and Hara, 1998, Peaucelle et al., 2007]. Finally, a LMI-based stabilization result is obtained from a dilation of the stability conditions.

The paper is structured as follows, Section 2 introduces preliminary results on quadratic separation, IQCs and the closed-loop system expression. Section 3 is devoted to the stability analysis of the NCS with corresponding examples. Finally, in Section 4, a stabilization result is derived.

Notations: Throughout the paper, the following notations are used. The set of L_2^n consists of all measurable functions $f : \mathbb{R}^+ \to \mathbb{R}^n$ such that the L_2 norm $\|f\|_{L_2} = \left(\int_0^\infty f^*(t)f(t)dt\right)^{1/2}$ is finite. When no ambiguity may occur, the superscript n will be omitted. The truncation operator $\mathbb{P}_T$ is defined as $\mathbb{P}_T(f) = f_T$ with $f_T(t) = f(t)$ when $t \le T$ and 0 otherwise. The set L_{2e}^n denotes the extended set of L_2^n which consists of the functions whose time truncation lies in L_2^n. For two symmetric matrices, A and B, $A \succ (\succeq)\ B$ means that $A - B$ is positive (semi)definite. $\mathsf{1_n}$ and $\mathsf{0_{m\times n}}$ denote the identity matrix of size n and zero matrix of size $m\times n$ respectively. $A_\perp$ is a full rank matrix spanning the null-space of A, i.e. $AA_\perp = \mathsf{0}$. For a square matrix A, $A^{\mathcal{S}}$ stands for the sum $A + A^T$.

2. PRELIMINARY RESULTS

2.1 Quadratic Separation

As mentioned in the introduction, the original NCS (Fig. 1) will be rewritten as the interconnection of Fig. 2. This section provides then a fundamental result on quadratic separation [Iwasaki and Hara, 1998, Peaucelle et al., 2007, Ariba and Gouaisbaut, 2009], which will be used as a basis for the stability analysis.

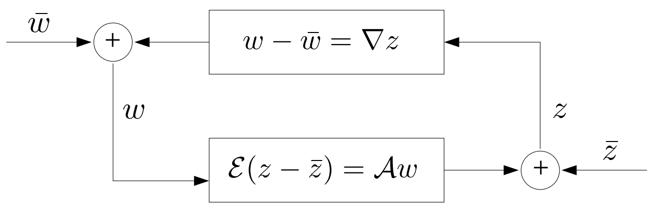

Fig. 2. Feedback system.

Theorem 1. The interconnected system of Figure 2 is well-posed if there exists a symmetric matrix Θ satisfying both conditions

$$[\,\mathcal{E}\ -\mathcal{A}\,]_\perp^T\,\Theta\,[\,\mathcal{E}\ -\mathcal{A}\,]_\perp \succ 0 \tag{1}$$

and

$$\langle \begin{bmatrix} 1 \\ \mathbb{P}_T\nabla \end{bmatrix} u_T, \Theta \begin{bmatrix} 1 \\ \mathbb{P}_T\nabla \end{bmatrix} u_T \rangle \le 0 \tag{2}$$

for all $u \in L_{2e}$ and all $T > 0$.

Proof : The proof can be found in [Ariba et al., 2008, Peaucelle et al., 2009]. $\diamondsuit$

The above result considers the well-posedness of the interconnection, that is, that the loop signals w and z are uniquely defined by the input signals $\bar{w}$ and $\bar{z}$. When the operator ∇ consists of a dynamic operator, for instance an integral operator, then the interconnection becomes a dynamical system and in such case, well-posedness can be made equivalent to stability provided that the structure of the separator Θ is chosen accordingly. For more details, see e.g. [Iwasaki and Hara, 1998].

2.2 Closed-loop system expression

Let us consider the following LTI continuous-time process

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ev(t) \\ y(t) &= Cx(t) + Du(t) + Fv(t) \\ x(0) &= x_0 \end{aligned} \tag{3}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $v \in \mathbb{R}^p$, $y \in \mathbb{R}^q$ and $x_0 \in \mathbb{R}^n$ are the system state, the control input, the exogenous input, the controlled output and the initial condition. Let us consider a bounded time-varying propagation delay $\tau(t) \in [0, \tau_{max}]$ and assume also that the maximal (time-varying) sampling period is T_{max}. In such a case, the control law is given in the following proposition:

Proposition 2. The control law incorporating the network effects is given by

$$\begin{aligned} u(t) &= Kx(t_k) \\ t &\in [t_k + \tau_k, t_{k+1} + \tau_{k+1}) \\ t_{k+1} - t_k &\le (1+m)T_{max} \\ \tau(t_k) &\in [0, \tau_{max}] \end{aligned} \tag{4}$$

where $\tau_k = \tau(t_k)$, $k \in \mathbb{N}$ and m is the number of consecutive dropouts.

Proof : Ignoring first the network presence, the following sampling-based control law is considered

$$u(t) = Kx(t_k),\ t \in [t_k, t_{k+1}) \tag{5}$$

where $\{t_k\}_{k\in\mathbb{N}}$ is an increasing sequence of time-instants, with not necessarily constant but bounded increments (varying sampling period).

Now, the idea is to add the network effects to the above nominal control law, namely the time-varying propagation delays and the data dropouts. Assuming that the propagation delay $\tau(t)$, $t \in \mathbb{R}_+$ is time-varying, and since the control input is transmitted only at the pointwise time instants t_k, then only the delay value at these instant has an influence on the transmission of the control law, which becomes

$$u(t) = Kx(t_k),\ t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1}). \tag{6}$$

where $\tau_k = \tau(t_k)$. Finally, assuming that the receiver is driven by an event-based system [1], then the data losses can be easily incorporated in the above control law simply by noting that a dropout will be reflected in an extension of the holding duration of the actuator input. In other words, if m consecutive dropouts occur while a constant sampling period T_s is used, then the actuator input will remain the same for mT_s seconds. When the sampling period is time varying, the holding extension is bounded from above by mT_{max} where T_{max} is the maximal value of the varying sampling period. Gathering all the above facts yields (4). $\diamondsuit$

Remark 1. To derive the above result, we have exploited the fact that the controller is static and time-invariant. In such a case, the network effects on both forward and backward paths can be merged together as in a *one-channel feedback NCS* [Hespanha et al., 2007]. Hence, only the network effects on one path needs to be considered (e.g. the forward path) and thus the control law (4) can be considered without loss of generality. Note that this is

[1] that is the control input on the process side is updated only when a new data come, otherwise the previous data is maintained

however not the case when a time-varying or a dynamic output feedback controller is considered.

The quantity referred to as the *maximum allowable transfer interval* (MATI, [Walsh et al., 1999]) denoted by μ and defined as

$$\mu := (1+m)T_{max} + \tau_{max} \tag{7}$$

is very often used to compare the different methods. The term *communication outage* [Henriksson et al., 2009] is also employed to refer to the time interval during which sensors data and controller signals do not reach the controller and the actuator respectively. The set of admissible communication outages is parameterized by the MATI as

$$\mathscr{S}_\mu := \{(m,\tau,T) \in \mathbb{N} \times \mathbb{R}_+ \times \mathbb{R}_{++} : \ (1+m)T + \tau \leq \mu\}. \tag{8}$$

At the light of the above discussion, it turns out that the control law behaves as if the 'sampling' was asynchronous and bounded from above by the MATI. Problems related to asynchronous sampling have been studied in several papers with many different approaches: time-delay systems [Yu et al., 2004, Fridman et al., 2004], impulsive systems [Naghshtabrizi et al., 2008, Seuret, 2009], sampled-data techniques [Mirkin, 2007], robust techniques [Fujioka, 2009]. In the following, robust techniques involving IQC coupled with well-posedness techniques will be used.

Remark 2. A classical approach [Fridman et al., 2004, Hespanha et al., 2007] consists in rewriting the sampled-data system as an input-delay system with a time-varying delay $h(t) = t - t_k$, $\forall t \in [t_k+\tau_k,\, t_{k+1}+\tau_{k+1})$ (Fig. 3) yielding then the comparison system

$$\dot{x} = Ax(t) + BKx(t-h(t)),\ \forall t \in [t_k+\tau_k,\, t_{k+1}+\tau_{k+1}).$$

Then, time-delay system analysis tools like Lyapunov-Krasovskii functionals [Tang et al., 2008, Naghshtabrizi and Hespanha, 2006] and Razumikhin functions [Yu et al., 2004] can be used to study stability.

Using the complete model for the control input, the closed-loop system writes

$$\begin{array}{rcl} \dot{x}(t) &=& Ax(t) + BKx(t_k) + Ev(t) \\ y(t) &=& Cx(t) + DKx(t_k) + Fv(t) \\ x(0) &=& x_0 \\ t &\in& [t_k+\tau_k, t_{k+1}+\tau_{k+1}). \end{array} \tag{9}$$

with the conditions given in Proposition 2.

In order to tackle the problem in the well-posedness framework, the following transformed equivalent model is used instead:

$$\begin{array}{rcl} \dot{x}(t) &=& (A+BK)x(t) - BK\delta(t) + Ev(t) \\ y(t) &=& (C+DK)x(t) - DK\delta(t) + Fv(t) \\ \delta(t) &=& \Delta_{sh}[\dot{x}](t) \\ x(0) &=& x_0 \\ t &\in& [t_k+\tau_k, t_{k+1}+\tau_{k+1}). \end{array} \tag{10}$$

where the operator $\Delta_{sh}(\cdot)$ is defined as:

$$\Delta_{sh}[\eta](t) = \int_{t_k}^{t} \eta(s)ds,\ t \in [t_k+\tau_k, t_{k+1}+\tau_{k+1}) \tag{11}$$

with $t_{k+1}+\tau_{k+1}-t_k \leq \mu$ with $\mu > 0$. The signals involved in the above system can be related through the dynamical expression

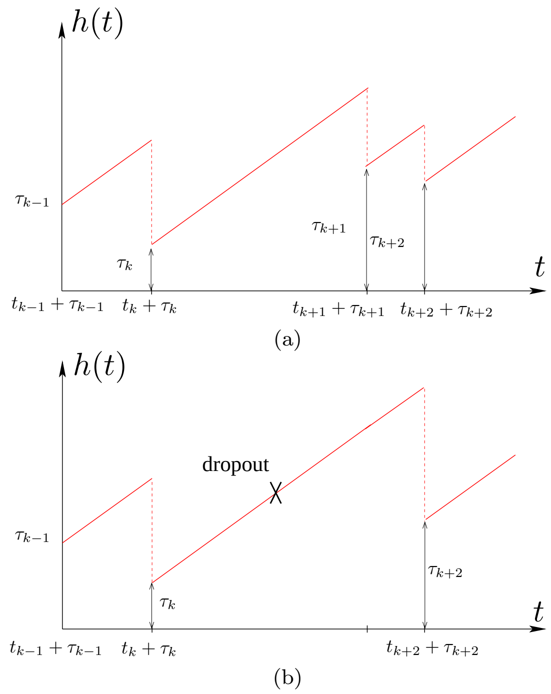

Fig. 3. Delay pattern in the continuous-time system with input delay approach.

$$\underbrace{\begin{bmatrix} x(t) \\ \delta(t) \\ v(t) \end{bmatrix}}_{w(t)} = \underbrace{\begin{bmatrix} \mathcal{I}\mathbb{1}_n & & \\ & \Delta_{sh}\mathbb{1}_n & \\ & & \Delta_\gamma \end{bmatrix}}_{\nabla} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t) \\ y(t) \end{bmatrix}}_{z(t)}, \tag{12}$$

and the implicit algebraic expression

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t) \\ y(t) \end{bmatrix}}_{z(t)} = \underbrace{\begin{bmatrix} A+BK & -BK & E \\ 0 & 0 & 0 \\ C+DK & -DK & F \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{bmatrix} x(t) \\ \delta(t) \\ v(t) \end{bmatrix}}_{w(t)}. \tag{13}$$

where $\mathcal{I}$ is the integral operator and Δ_γ is a virtual operator characterizing the L_2 gain of the transfer $v \to y$ (detailed further).

2.3 Characterization of the operators

In this section, the IQCs defining the operators involved in ∇ are derived.

Lemma 3. The operator $\mathcal{I}$ is characterized by the IQC:

$$\Pi_1 := \langle \begin{bmatrix} \mathbb{1}_n \\ \mathcal{I}\mathbb{1}_n \end{bmatrix} x_T, \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} \begin{bmatrix} \mathbb{1}_n \\ \mathcal{I}\mathbb{1}_n \end{bmatrix} x_T \rangle \leq 0.$$

for al $x \in L_{2e}^n$ and for any matrix $P \in \mathbb{S}_{++}^n$.

Proof : Expanding the expression, we get $\forall T > 0, \forall x \in L_{2e}^n$, $(x_T = \mathbb{P}_T(x))$

$$\begin{array}{rcl} \Pi_1 &=& -2\displaystyle\int_0^{+\infty} x_T(t)^T P \int_0^t x_T(s)dsdt \\ &=& -2\displaystyle\int_0^{+\infty} \frac{d}{dt}(\mathcal{I}x_T)^T P(\mathcal{I}x_T)dt \\ &=& -\displaystyle\int_0^T x_T^T(s)dsP\int_0^T x_T(s)ds \leq 0 \end{array}$$

$\diamondsuit$

Lemma 4. The operator Δ_{sh} can be characterized by the IQC:

$$\langle \begin{bmatrix} 1_n \\ \Delta_{sh} 1_n \end{bmatrix} x_T, \begin{bmatrix} -\frac{4}{\pi^2}\mu^2 S_1 & -S_2 \\ -S_2 & S_1 \end{bmatrix} \begin{bmatrix} 1_n \\ \Delta_{sh} 1_n \end{bmatrix} x_T \rangle \leq 0.$$

for all $x \in L_{2e}^n$ and for any matrices $S_1, S_2 \in \mathbb{S}_{++}^n$.

Proof : Using the same arguments as in [Chen and Francis, 1995, Mirkin, 2007], the L_2-induced norm of the operator Δ_{sh} is equal to $\frac{2}{\pi}\mu$, where μ is the largest interval of integration for the operator Δ_{sh} given in (7). Therefore, for all $r \in L_2^n$, the inequality

$$\|\Delta_{sh} r\|_{L_2}^2 \leq \frac{4}{\pi^2}\mu^2 \|r\|_{L_2}^2$$

holds or equivalently there exists $S_1 \in \mathbb{S}_{++}^n$ such that

$$\int_0^{+\infty} \varphi_{S_1}(\Delta_{sh}[r](t))dt \leq \frac{4}{\pi^2}\mu^2 \int_0^{+\infty} \varphi_{S_1}(r(t))dt$$

where $\phi_X(\alpha) = \alpha^T X \alpha$ for any matrices $X = X^T$ and vectors α of appropriate dimensions. Considering now $x(t) \in L_{2e}^n$, $x_T(t) \in L_2^n$, we have

$$\int_0^{+\infty} \left\{ -\varphi_{S_1}(\Delta_{sh}[x_T](t)) + \frac{4}{\pi^2}\mu^2 \varphi_{S_1}(x_T(t)) \right\} dt \geq 0. \tag{14}$$

Moreover, the passivity of the operator Δ_{sh} has been proved in [Fujioka, 2009]. So, for any $S_2 \in \mathbb{S}_{++}^n$ we have

$$\int_0^{\infty} r^T(t) S_2 \Delta_{sh}[r](t) dt \geq 0.$$

for all $r \in L_2$. As previously, for all $x(t) \in L_{2e}^n$,

$$\int_0^{\infty} x_T^T(t) S_2 \Delta_{sh}[x_T](t) dt \geq 0. \tag{15}$$

Finally, the sum of the inequalities (14) and (15) can be easily arranged as the result of the lemma. ♢

Lemma 5. The operator Δ_γ, which has a L_2-induced norm equal to γ^{-1}, is characterized by the IQC given by the following inequality:

$$\langle \begin{bmatrix} 1_r \\ \Delta_\gamma \end{bmatrix} x_T, \begin{bmatrix} -\eta 1_q & 0 \\ 0 & 1_r \end{bmatrix} \begin{bmatrix} 1_q \\ \Delta_\gamma \end{bmatrix} x_T \rangle \leq 0,$$

for all $x \in L_{2e}^q$ where $\eta = \gamma^{-2}$.

3. STABILITY WITH L_2 PERFORMANCE

In this section, the stability result based on quadratic separation is provided and illustrated through several examples.

3.1 Main Result

Theorem 6. The system (9) is asymptotically stable for all $(m, \tau, T) \in \mathscr{S}_\mu$ if there exist matrices $P, S_1, S_2 \in \mathbb{S}_{++}^n$ and a scalar $\eta > 0$ such that the LMI

$$[\,\mathcal{E}\ -\mathcal{A}\,]_\perp^T\, \Theta\, [\,\mathcal{E}\ -\mathcal{A}\,]_\perp \prec 0 \tag{16}$$

holds where $\mathcal{E}, \mathcal{A}$ are defined in (13) and

$$\Theta = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & -P & 0 & 0 \\ 0 & -\frac{4}{\pi^2}\mu^2 S_1 & 0 & 0 & -S_2 & 0 \\ 0 & 0 & -\eta 1_q & 0 & 0 & 0 \\ \hline & & & 0 & 0 & 0 \\ & * & & 0 & S_1 & 0 \\ & & & 0 & 0 & 1_r \end{array}\right]. \tag{17}$$

Moreover, the closed-loop system satisfies $||y||_{L_2} \leq \sqrt{1/\eta}||v||_{L_2}$.

Proof : The result is based on Theorem 1. The matrices $\mathcal{E}$, $\mathcal{A}$ and ∇ describing the interconnection are defined in (12) and (13). The inequality (2) is difficult to check since ∇ is an operator, thus the idea is to set a specific structure for Θ in order to have (2) automatically verified. This specific structure is given by (17) for which it can be shown, using Lemmas 3, 4 and 5, that (2) is indeed satisfied. Hence, the remaining condition to check is (1). The proof is complete. ♢

Remark 3. Thanks to the framework in use, it is easy to consider uncertainties on the system. So, extensions to robust analysis with respect to parametric uncertainties is possible [Peaucelle et al., 2007].

3.2 Numerical examples

Example 1: Let us consider the system

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.8 & -0.01 \\ 1 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} F x(t_k) \\ F &= \begin{bmatrix} -2.0348 & -1.8108 \end{bmatrix} \end{aligned} \tag{18}$$

The results obtained using Theorem 6 are compared to [Tang et al., 2008] and those reported in [Hespanha et al., 2007], namely [Yu et al., 2004, Yue et al., 2004]. They are represented in the Figure 4 where we can see that the proposed approach outperforms the previous ones. Figure 4 shows the maximal allowable sampling period T_{max} for a given maximal value of the delay τ_{max} (when no dropout occurs). The determined stability region for (18) assessed by the different approaches is the surface below the corresponding line.

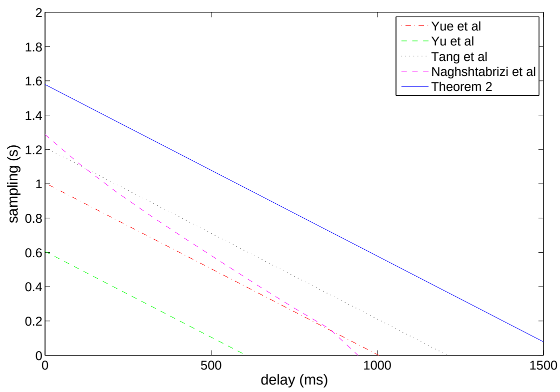

Fig. 4. Stability regions of (18) using different approaches (when no dropout occurs).

Assuming now the delay $\tau(t) \in [0, 0.43]$ and dropouts may occur. The maximal number of consecutive dropouts is depicted in Fig. 5 and, as expected, the smaller the maximal sampling period is, the larger is the number of admissible consecutive dropouts.

Example 2: For the second example, the following system is considered:

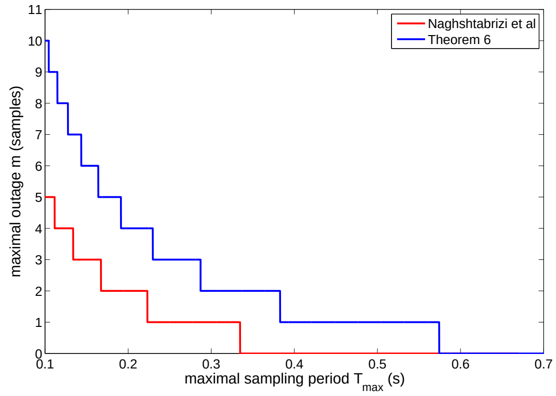

Fig. 5. Stability regions in term of number of consecutive dropouts for system (18) with a maximal delay $\tau_{max} = 430ms$.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [\,-1.006 \;\; -1.006\,]\, x(t_k). \quad (19)$$

The results are compared to [Yu et al., 2004, Yue et al., 2004, Tang et al., 2008, Naghshtabrizi and Hespanha, 2006]. In Table 1, the corresponding different MATI are given and we can see that the proposed technique leads to better results both in terms of efficiency and computational complexity.

Example 3: In [Cloosterman et al., 2009], the set of feedback gain K_2 which stabilizes the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 10.018 \end{bmatrix} [\,-50 \;\; -K_2\,]\, x(t_k), \quad (20)$$

is sought. In the considered scenario, we have set $T_s = 1ms$ and $\tau_k \in [0, 2T_s]$, $k \in \mathbb{N}$. The corresponding results are depicted in Figure 6 where we can see that the proposed approach improves the stability region.

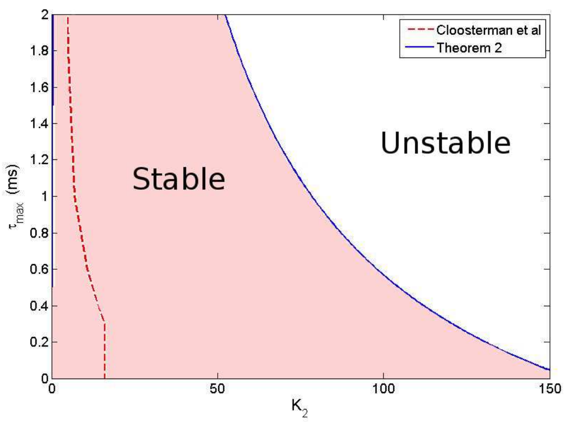

Fig. 6. Stability regions of (20) on the delay vs feedback gain K_2.

4. STABILIZATION

This section is devoted to the stabilization of NCS via state-feedback. A dilated LMI is devised from (16)-(17) in order to make the stabilization problem tractable. This is stated in the following theorem:

Theorem 7. There exists a matrix $K \in \mathbb{R}^{m\times n}$ such that the closed-loop system (9) is asymptotically stable for all $(m, \tau, T) \in \mathscr{S}_\mu$ if there exist matrices $P, S_1 \in \mathbb{S}^n_{++}$, $X \in \mathbb{R}^{n\times n}$, $U \in \mathbb{R}^{m\times n}$ and a scalar $\gamma > 0$ such that the LMI

$$\begin{bmatrix} -X^{\mathcal{S}} & P + A'_{cl} & -BU & E & 0 & X & \alpha S_1 \\ \star & -P & 0 & 0 & {C'_{cl}}^T & 0 & 0 \\ \star & \star & -S_1 & 0 & -(DU)^T & 0 & 0 \\ \star & \star & \star & -\gamma I & F^T & 0 & 0 \\ \star & \star & \star & \star & -\gamma I & 0 & 0 \\ \star & \star & \star & \star & \star & -P & -\alpha S_1 \\ \star & \star & \star & \star & \star & \star & -S_1 \end{bmatrix} \prec 0 \quad (21)$$

holds with $A'_{cl} = AX + BU$, $C'_{cl} = CX + DU$ and $\alpha = \mu\frac{\pi}{2}$. Furthermore, the closed-loop system satisfies $||y||_{L_2} \leq \gamma ||v||_{L_2}$.

Proof : The proof is based on the projection lemma [Gahinet and Apkarian, 1994] similarly as in [Tuan et al., 2003, Briat, 2008, Briat et al., 2009]. Since it relies on standard but tedious algebraic manipulations, the proof is only sketched. Denote the matrix (21) by Ω and a congruence transformation with respect to $\mathcal{C} := \text{diag}(I_3 \otimes Y, I, I, I_2 \otimes Y)$, $Y = X^{-1}$ yields

$$\begin{aligned} \Omega' &= \mathcal{C}^T \Omega \mathcal{C} \\ &= \mathcal{C}^T \left[\Omega|_{Y=0} + U^T Y V + (U^T Y V)^T \right] \mathcal{C} \end{aligned} \quad (22)$$

where

$$\begin{aligned} U &= [1\ 0\ 0\ 0\ 0\ 0\ 0], \\ V &= [-1\ A_{cl}\ -BK\ E\ 0\ 1\ 0], \end{aligned}$$

with $A_{cl} = A + BK$ and $K = UY^{-1}$. The projection lemma implies

$$U_\perp^T \left[\Omega'|_{Y=0} \right] U_\perp \prec 0, \quad (23)$$
$$V_\perp^T \left[\Omega'|_{Y=0} \right] V_\perp \prec 0. \quad (24)$$

So, assuming that (21) holds then so do (23) and (24). After some manipulations, it can be shown that (24) is equivalent to (16) with $S_2 = 0$ (using Schur complements) and thus, stability is proved. ♢

Example 1. Let us consider the open-loop system

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.8 & -0.01 \\ 1 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v(t) \\ y(t) &= x_1(t) \end{aligned} \quad (25)$$

In Yu et al. [2004], it is shown that the system is stabilizable provided that $\mu \leq 0.6011$. Using Theorem 7, it is shown that the system is still stabilizable for $\mu \leq 3.64826$ with a controller gain $K = [-0.3482\ \ -0.3097]$. When the closed-loop system stability is checked using Theorem 6, we find that the system remains stable if $\mu \leq 9.19286$ showing that, as usual, stabilization results are more conservative than stability ones. In Figure 7, the L_2 norm of the closed-loop system is plotted with respect to the MATI μ.

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Table 1.

	μ	no. of var.	for $n = 2$
[Yu et al., 2004]	unfeasible	$4\frac{n(n+1)}{2}$	12
[Yue et al., 2004]	0.970	$2\frac{n(n+1)}{2} + 6n^2$	30
[Tang et al., 2008]	0.995	$4\frac{n(n+1)}{2} + 16n^2$	76
[Naghshtabrizi and Hespanha, 2006] (without delay)	1.272	$7\frac{n(n+1)}{2} + 16n^2$	85
Theorem 6	1.561	$3\frac{n(n+1)}{2} + 1$	10

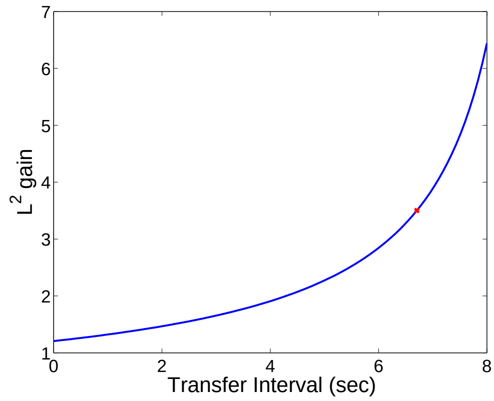

Fig. 7. L_2-gain of the controlled system (25) with respect to the MATI

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