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Information constraints in multiple agent problems with i.i.d. states

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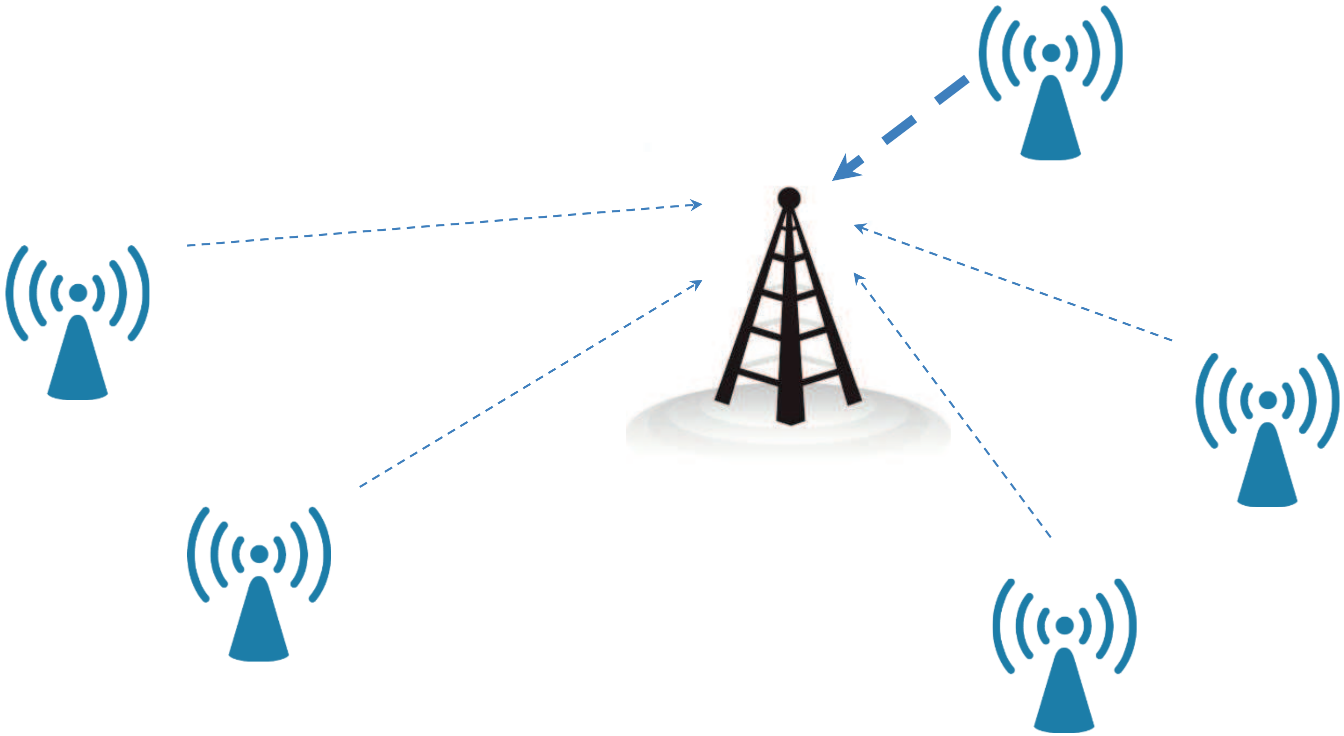
Motivation example: Power control

Power control in wireless networks



A typical distributed power control setup

- Set of transmitters: $\mathcal{K} = \{1, ..., K\}$
- What Transmitter k does:

$$\begin{array}{cccc} g_k(1) & g_k(2) & g_k(3) & \cdots \\ \hline p_k(1) & p_k(2) & p_k(3) & \cdots \end{array}$$

- Performance criteria: data rate, energy, etc.

$$u_k(\underbrace{g_1, ..., g_K}_{a_0}, p_1, ..., p_K)$$

- **Local decision, local observation $\rightarrow$ Ultimate average performance? (Cf Opt. Ctrl.)**

General problem statement

- Set of agents: $\mathcal{K} = \{1, ..., K\}$

- Stage/instantaneous perf. criteria (utilities):

$$u_k(a_0, a_1, ..., a_K)$$

- Action sets: $\mathcal{A}_k$ with $|\mathcal{A}_k| < \infty$

- T iterations/samples/stages: $t \in \{1, ..., T\}$

- Observation: $(a_0(t), a_1(t), ..., a_K(t)) \to o_k(t)$

Ultimate Goal: characterize the feasible long-term utility region

- Long-term utilities:

$$v_k^\infty = \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} u_k(a_0(t), a_1(t), ..., a_K(t))$$

Ultimate goal: characterize the feasible long-term utility region

- Long-term utilities:

$$v_k^{\infty} = \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[u_k(a_0(t), a_1(t), ..., a_K(t))\right]$$

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$$v_k^{\infty}(f_1, ..., f_K) = \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[u_k(a_0(t), a_1(t), ..., a_K(t))\right]$$

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- $f_1, ..., f_K =$???

Strategies

- Strategies informal definition: history $\mapsto$ action

- Observation structure: $o_k = (s_k, y_k)$ where s_k is given by $\daleth_k(s_k|a_0)$ and y_k by $\Gamma_k(y_k|a_0, a_1, ..., a_K)$

- Example: $s_k = \widehat{g}_k$ and $y_k = \widehat{u}_k$

Strategies. Continued

- Definition (causal scenario):

$$a_k(t) = f_{k,t}(s_k(1), ..., s_k(t), y_k(1), ..., y_k(t-1))$$

Strategies. Continued

- Definition (causal scenario):

$$a_k(t) = f_{k,t}(s_k(1), ..., s_k(t), y_k(1), ..., y_k(t-1))$$

- Definition (noncausal scenario):

$$a_k(t) = f_{k,t}(s_k(1), ..., s_k(T), y_k(1), ..., y_k(t-1))$$

Problem statement: recap

Find the feasible region $(v_1^{\infty}, ..., v_K^{\infty})$

- for the causal/**noncausal** scenario

- when $a_0(t)$ is i.i.d. and $\sim \rho_0$

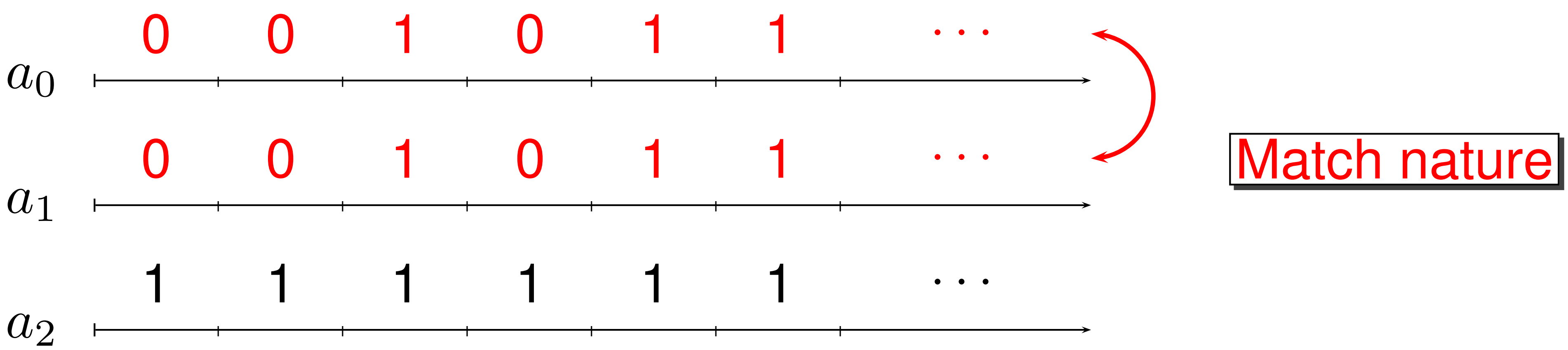
- with the memoryless observation structure given by $\daleth_k$ and Γ_k

Example (noncausal+asymmetrical scenario)

- Agents: $\{1, 2\}$; $0 \equiv$ nature.
- Action sets: $\mathcal{A}_0 = \mathcal{A}_1 = \mathcal{A}_2 = \{0, 1\}$ (think of PC).
- Stage utility function:

$$u(a_0, a_1, a_2) = \left| \begin{array}{ll} 1 & \text{if } a_0 = a_1 = a_2 \\ 0 & \text{otherwise} \end{array} \right. .$$

- Scheme 1:

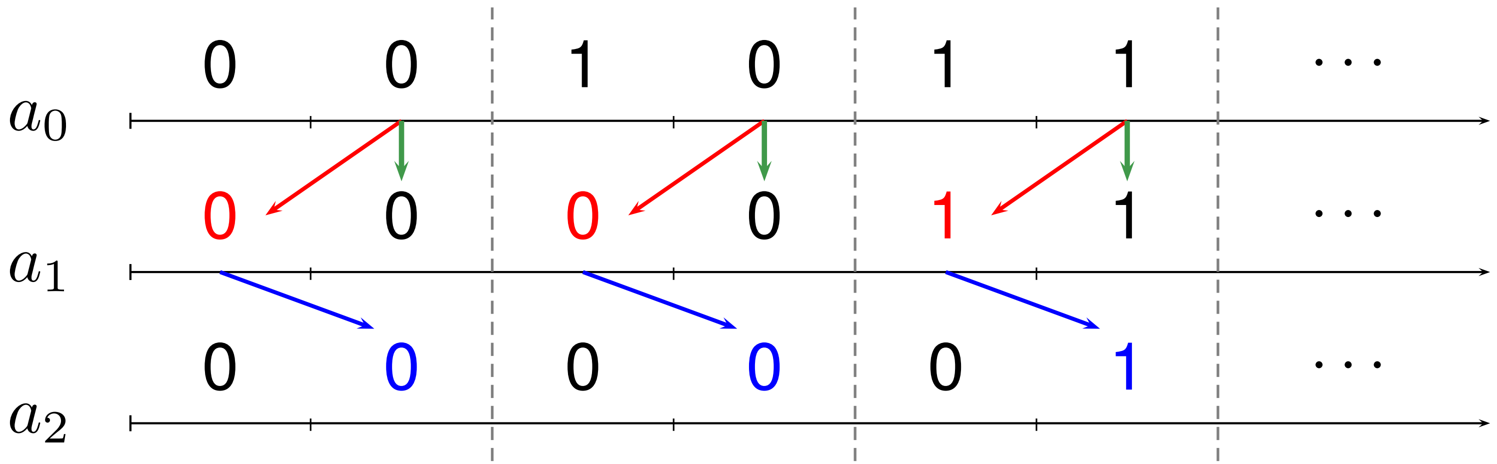


- Long-term utility

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} u(a(t))\right] \to \frac{1}{2} = 0.5. \qquad \text{for } A_0 \sim \mathcal{B}\left(\frac{1}{2}\right)$$

Long-term utility

- Scheme 2:



- Long term utility:

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} u(a(t))\right] \to \frac{5}{8} = 0.625.$$

Maximal long-term utility

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} u(a(t))\right] \to x^\star \simeq 0.81$$

where

$$x^\star \quad \text{is the solution of} \quad \frac{h(x)-1}{x-1} = \log_2 3$$

and $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$.

Performance characterization of stochastic games with i.i.d. states

Noncausal scenario

Key observation. Say $\mathcal{A} = \{0, 1\}$, $K = 1$

$$\frac{1}{T}\sum_{t=1}^{T} u(a(t))$$

$$= \frac{1}{T}\left[u(0) + u(1) + u(1) + u(0) + \ldots + u(1)\right]$$

$$= \frac{N_0}{T} u(0) + \frac{N_1}{T} u(1)$$

$$= q_0 u(0) + q_1 u(1) \qquad (q_0 \geq 0, q_1 \geq 0, q_0 + q_1 = 1)$$

$$= \sum_{a \in \mathcal{A}} q_a u(a) = \mathbb{E}[u(a)]$$

More formally

$$
\begin{aligned}
& v_i^{\infty}(f_1, ..., f_K) \\
&= \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[u_k(a_0(t), a_1(t), ..., a_K(t))\right] \\
&= \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{a_0, ..., a_K} P_t(a_0, ..., a_K) u_k(a_0, ... a_K) \\
&= \sum_{a_0, ..., a_K} u_k(a_0, ..., a_K) \lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} P_t(a_0, ..., a_K)
\end{aligned}
$$

Implementable coordination

Definition $Q(a_0, a_1, ..., a_K)$ is implementable if $\exists$ $(f_1, ..., f_K)$ s.t.

$$\frac{1}{T}\sum_{t=1}^{T} P_t(a_0, ..., a_K) \to Q(a_0, a_1, ..., a_K)$$

Reminder:

$$a_k(t) = f_{k,t}(s_k(1), ..., s_k(T), y_k(1), ..., y_k(t-1))$$

Extreme cases of distribution

- Team control $u_k = u$

- Zero correlation (lower bound):

$$Q(a_0, a_1, ..., a_K) = \rho_0(a_0) \prod_{k=1}^{K} P_{A_k}(a_k)$$

- Full correlation (upper bound):

$$(a_1(t), ..., a_K(t)) \in \text{arg} \max_{a_1,...,a_K} u(a_0(t), a_1, ..., a_K)$$

$$\rightarrow Q(a_0, \underbrace{a_1, ..., a_K}_{a}) = \rho_0(a_0)\delta(a - m(a_0))$$

Implementable distribution set characterization

Proposition 1 Solving the problem is at least as hard as solving the two-way channel.

Theorem 2

$- K = 2$

$- a_1(t) = f_{1,t}(s_1(1), ..., s_1(T))$

$- a_2(t) = f_{2,t}(s_2(1), ..., s_2(T), y_2(1), ..., y_2(t-1))$

► Then $Q(a_0, a_1, a_2)$ is implementable iff ... see [Larrousse et al ITW 2015].

Review of the entropy function

For a vector in the unit simplex $x = (x_1, ..., x_N)$**,** $x_n \geq 0$**:** $\sum_n x_n = 1$**.**

$$H(x) = -\sum_n x_n \log x_n.$$

For a random variable $A \sim P$**:**

$$H(A) = -\sum_{a \in \mathcal{A}} P(a) \log P(a).$$

Implementable distribution set characterization

Corollary 3

– $K = 2$

– $a_1(t) = f_{1,t}(a_0(1), ..., a_0(T))$

– $a_2(t) = f_{2,t}(a_1(1), ..., a_1(t-1))$

► Then $Q(a_0, a_1, a_2)$ is implementable iff its marginal w.r.t (a_1, a_2) is ρ_0 and

$$H_Q(A_0, A_1, A_2) \geq H_Q(A_0) + H_Q(A_2).$$

Utility region characterization

Pareto frontier: use $w_\alpha = \alpha u_1 + (1-\alpha) u_2$

$$
\begin{array}{ll}
\text{minimize} & -\sum_{a_0,a_1,a_2} Q(a_0,a_1,a_2) w_\alpha(a_0,a_1,a_2) \\
\text{subject to} & \begin{aligned} H_Q(A_0) + H_Q(A_2) - H_Q(A_0,A_1,A_2) &\le 0 \\ -Q(a_0,a_1,a_2) &\le 0 \\ -1 + \sum_{a_0,a_1,a_2} Q(a_0,a_1,a_2) &= 0 \\ -\rho_0(a_0) + \sum_{a_1,a_2} Q(a_0,a_1,a_2) &= 0 \end{aligned}
\end{array}
$$

Let us try to interpret by specializing further

- $H_Q(a_0) =$ constant $= -\sum_{a_0} \rho_0(a_0) \log \rho_0(a_0)$

- $H_Q(a_2) \sim$ constant

- Boltzmann-Gibbs is optimal:

$$Q^\star(a_0, a_1, a_2) = \frac{e^{\lambda u(a_0,a_1,a_2)}}{\sum_{a_0,a_1,a_2} e^{\lambda u(a_0,a_1,a_2)}}$$

Performance characterization of stochastic games with i.i.d. states

Causal scenario: omitted

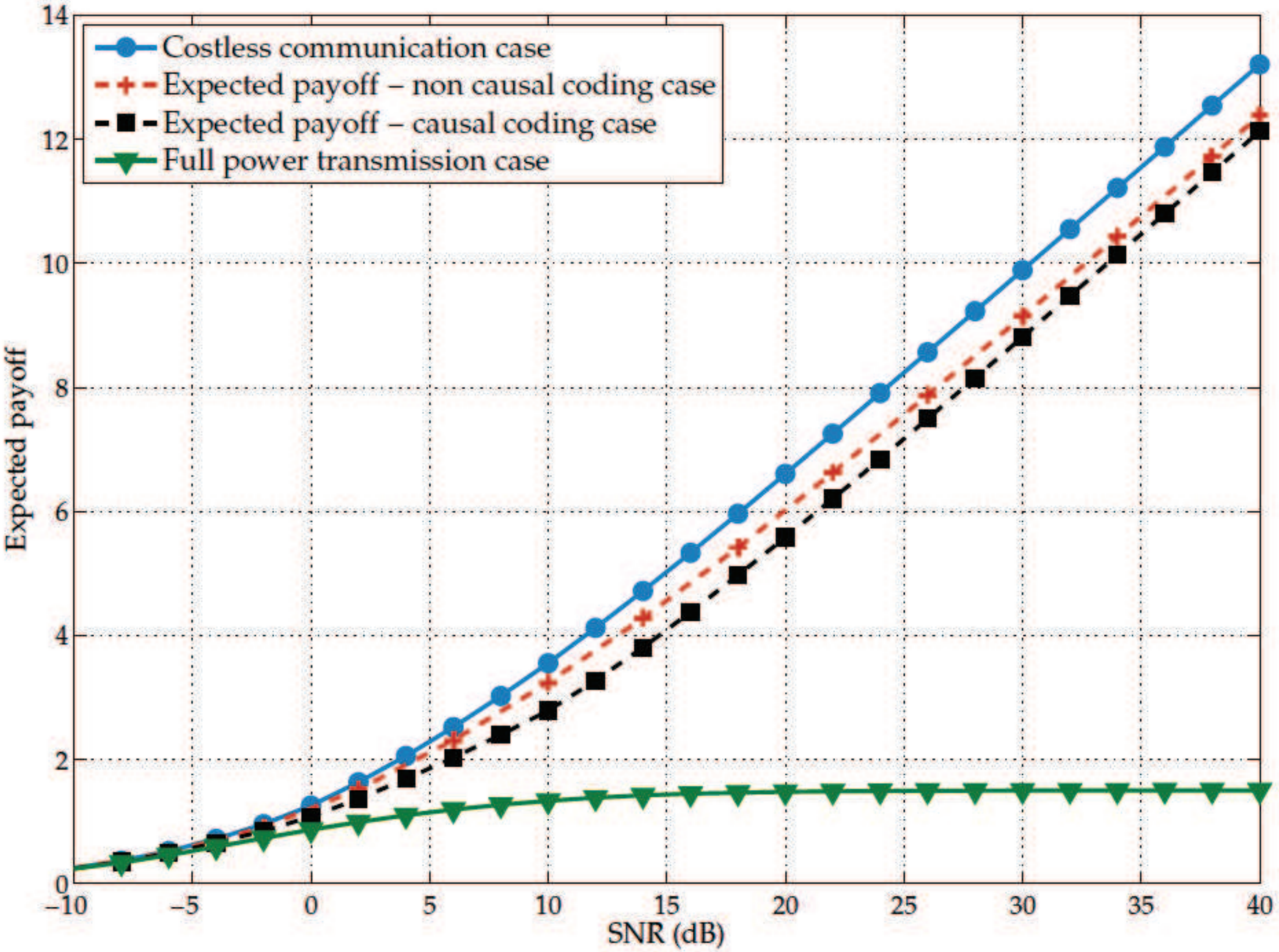
Application to power control over the interference channel

- $K = 2$ transmitter-receiver pairs

- Single-band case

- Utility $u = \log(1 + \mathrm{SINR}_1) + \log(1 + \mathrm{SINR}_2)$ with $\mathrm{SINR}_1 = \frac{g_{11}a_1}{\sigma^2 + g_{21}a_2}$, $\mathrm{SINR}_2 = \frac{g_{22}a_2}{\sigma^2 + g_{12}a_1}$.

Application to power control over the interference channel



Technical challenges

- ► Construct codes (see [Larrousse and Lasaulce ISIT 2013][Larrousse et al TIT 2016]). Joint control-communication problem.

- ► Controlled states.

- ► Nash equilibrium points.

Thank you for your attention!

Main references (1/2)

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Performance characterization of stochastic games with i.i.d. states

Causal scenario

Implementable distribution set characterization

Theorem 1 [Larrousse et al ITW 2015] Let $K \geq 2$. $Q(a_0, ..., a_K)$ is implementable if and only if it factorizes as

$$Q(a_0, ..., a_K) =$$

$$\sum_{s_1,...,s_k,w} \rho_0(a_0) \daleth(s_1, ..., s_k | a_0) P(w) \prod_{k=1}^{K} P_{A_k|S_k,W}(a_k | s_k, w)$$

Finding particular stationary strategies [Lasaulce and Visoz 2015]

- Set $u_k = u$

- Set $a_k(t) = \overline{f}_k(s_k(t), w(t))$

- Apply the sequential best response dynamics on

$$\mathbb{E}[u] = \sum_{a_0,a} Q(a_0, a) u(a_0, a) = U(P_{A_1|S_1,W}, ..., P_{A_K|S_K,W}, P(w))$$

$\rightarrow \overline{f}_1, ..., \overline{f}_K$