Jamming-resilient self-triggered coordination

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Outline

1. Coordination, consensus, constraints
2. Hybrid coordination system: self-triggered interactions
   - System description
   - Interpretation
   - Jamming signals (DoS)
   - DoS-resilient consensus
3. Conclusion

Coordination problem: consensus

The simplest and best known example of coordination:

- Consider n systems

$$\dot{x}_i = u_i \qquad i \in I := \{1, \ldots, n\}$$

  linked by an undirected connected graph $G = (I, E)$.
  $\mathcal{N}_i$ is the set of neighbors of system i

- **Control problem:** Design inputs u_i, $i \in I$,
  - which depend on x_i and $\{x_j : j \in \mathcal{N}_i\}$ (local information),
  - such that

$$x_i - x_j \to 0 \quad \forall i, j$$

Why (still) studying consensus?

- It is a prototypical problem:
  solutions can be extended to more complex scenarios
- It is useful in many application fields:
  - power networks,
  - flow networks,
  - opinion dynamics,
  - load balancing
  - robotic networks,
  - sensors networks,
- It is well studied:

Proposition (Standard consensus)

If the graph G is connected, the control law $u_i = \sum_{j \in \mathcal{N}_i} (x_j - x_i)$ *guarantees that* $\lim_{t \to \infty} x_i(t) = c$ *for all* i*, where* $c = \sum_{j=1}^{n} \frac{x_j(0)}{n}$

A constrained coordination problem

Standard requires continuous acquisition of information from neighbors.... yes this is not strictly required...

Average max-min consensus

$$\dot{x} = \mathrm{sign}\left(\sum_{j \in \mathcal{N}_i} (x_j - x_i)\right)$$

Convergence to

$$\frac{\max_i(x_i(0)) + \min_i(x_i(0))}{2}$$

Cortès. "Finite-time convergence gradient flows with applications to network consensus." Automatica 42, 1993-2000, 2006.

Binary control protocols

$$\dot{x} = \sum_{j \in \mathcal{N}_i} \mathrm{sign}(x_j - x_i) + \mathrm{sign}(x_0 - x_i)$$

Chen, Lewis, Xie. "Finite-time distributed consensus via binary control protocols". Automatica 47, 1962-1968, 2011.

A constrained coordination problem

Standard requires continuous acquisition of information from neighbors

This is too demanding!

We instead want a scenario in which

- sensors collect information only upon need
- the continuous-time systems "naturally" interacts with the discrete-time information acquisition
- the whole system is robust against network uncertainties (delays, poor synchronization of local clocks, limited data rate communication, noise)

Nowzari, Cortés. "Self-triggered coordination of robotic networks for optimal deployment". Automatica, 48(6), 1077–1087, 2012.
Seyboth, Dimarogonas, Johansson. "Event-based broadcasting for multi-agent average consensus". Automatica 49(1), 245–252, 2013.

Self-triggered coordination

A hybrid coordination system I

State variables ($i \in I$)

- consensus variables: $x_i \in \mathbb{R}$
- control variables: $u_i^j \in \{-1, 0, +1\}$ (ternary controls)
- local clock variables: $\theta_i^j \in \mathbb{R}$

Continuous evolution when no information exchange occurs

$$\begin{cases} \dot{x}_i = \sum_{j \in \mathcal{N}_i} u_i^j \\ \dot{u}_i^j = 0 \\ \dot{\theta}_i^j = -1 \end{cases}$$

Jumps occur at every t such that the set

$$\mathcal{J}(\theta, t) = \{\{i, j\} \in I \times I \ : \ j \in \mathcal{N}_i \text{ and } \theta_i^j(t) = 0\} \neq \emptyset$$

Note: the law $u_i = \sum_{j \in \mathcal{N}_i} \text{sign}(x_j - x_i)$ implies finite-time convergence

A hybrid coordination system II

Discrete evolution: how the exchange of information affects the systems

$$\begin{cases} x_i(t^+) = x_i(t) \quad \forall i \in I \\ u_i^j(t^+) = \begin{cases} \mathrm{sign}_\varepsilon\big(x_j(t) - x_i(t)\big) & \text{if } \{i,j\} \in \mathcal{J}(\theta, t) \\ u_i^j(t) & \text{otherwise} \end{cases} \\ \theta_i^j(t^+) = \begin{cases} f_i^j(x(t)) & \text{if } \{i,j\} \in \mathcal{J}(\theta, t) \\ \theta_i^j(t) & \text{otherwise} \end{cases} \end{cases}$$

- $\mathrm{sign}_\varepsilon(z) = \begin{cases} \mathrm{sign}(z) & \text{if } |z| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases}$
- $\varepsilon > 0$ is a *sensitivity* parameter

Note: the law $u_i = \sum_{j \in \mathcal{N}_i} \mathrm{sign}(x_j - x_i)$ implies finite-time convergence

A hybrid coordination system III

Next sampling time is chosen by $\theta_i^j(t^+) = \begin{cases} f_i^j(x(t)) & \text{if } \{i,j\} \in \mathcal{J}(\theta, t) \\ \theta_i^j(t) & \text{otherwise} \end{cases}$

$$f_i^j(x(t)) = \begin{cases} \dfrac{|x_j - x_i|}{2(\deg_i + \deg_j)} & \text{if } |x_j - x_i| \geq \varepsilon \\ \dfrac{\varepsilon}{2(\deg_i + \deg_j)} & \text{if } |x_j - x_i| < \varepsilon \end{cases}$$

so that

- $\mathrm{sign}(x_j - x_i)$ is constant during inter-sampling interval $[t_k^{ij}, t_{k+1}^{ij}]$
- **"dwell time"** property holds: $t_{k+1}^{ij} - t_k^{ij} \geq \dfrac{\varepsilon}{2(\deg_i + \deg_j)}$

Main result

Theorem (Practical consensus)

For every initial condition $\bar{x}$, let $x(t)$ be the solution to the self-triggered control algorithm such that $x(0) = \bar{x}$. Then $x(t)$ converges in finite time to a point x^* belonging to the set

$$\mathcal{E} = \{x \in \mathbb{R}^n \,:\, |x_j - x_i| < \varepsilon \; \forall \{i,j\} \in E\}$$

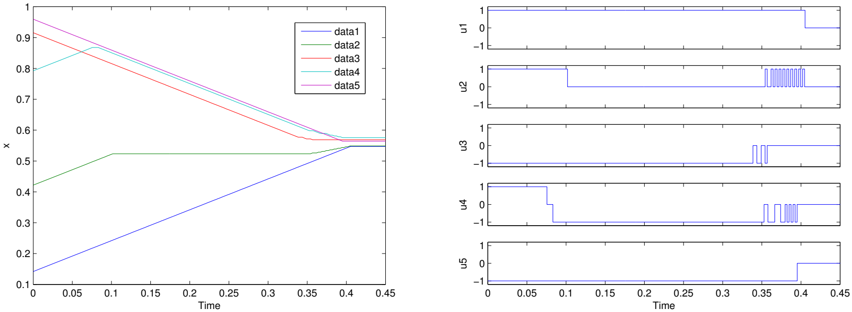
Time cost (time to converge)

$$T := \inf\{t \geq 0 \,:\, x(t) \in \mathcal{E}\} \leq \frac{\deg_{\max} + 1}{\varepsilon} \sum_{i \in I} \bar{x}_i^2$$

Communication cost (# updates to converge)

De Persis, Frasca. "Robust self-triggered coordination with ternary controllers". IEEE Transactions on Automatic Control, 58(12), 3024–3038, 2013.

Simulations



Sample evolutions of states x and corresponding controls u on a ring with $n = 5$ nodes, $\varepsilon = 0.02$

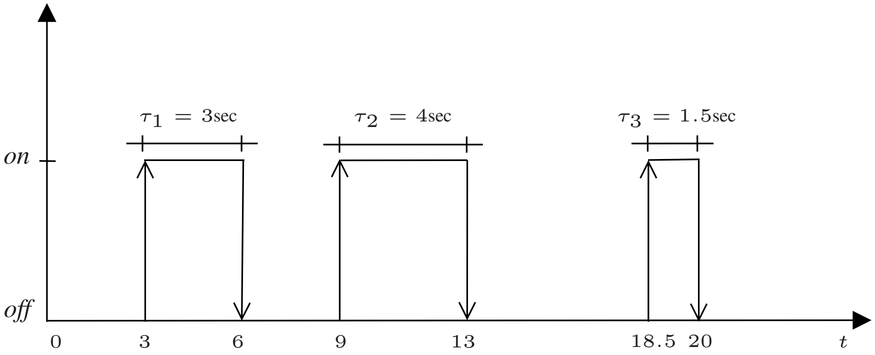
Extra features

- Asymptotic consensus ($\varepsilon(t) \to 0$, $\dot{x}_i = \gamma(t) \sum_{j \in \mathcal{N}_i} u_i^j$, $\varepsilon(t)/\gamma(t)) \geq c$)
- Robustness to bounded delays, quantization, clock skews ($\dot{\theta}_i^j = -R_i^j$)
- Node-based polling algorithms (mimics $\dot{x} = \mathrm{sign}(\sum_{j \in \mathcal{N}_i}(x_j - x_i))$)

De Persis, Frasca. "Robust self-triggered coordination with ternary controllers". IEEE Transactions on Automatic Control, 58(12), 3024–3038, 2013.

Jamming (Denial-of-Service)

DoS signals

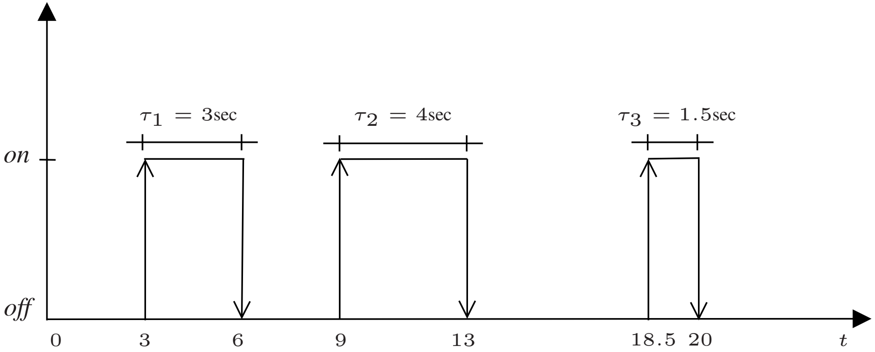


For each link $\{i, j\}$

on/off DoS instants $\{h_n^{ij}\}_{n \in \mathbb{Z}_{\geq 0}}$

***n*-th DoS interval** $H_n^{ij} := \{h_n^{ij}\} \cup \left[h_n^{ij}, h_n^{ij} + \tau_n^{ij}\right[$ (duration $\tau_n^{ij} \geq 0$)

DoS signals



For each link $\{i,j\}$, for any $t \geq \tau \geq 0$

DoS frequency $\quad n^{ij}(\tau,t) \leq \eta^{ij} + \dfrac{t-\tau}{\tau_f^{ij}} \qquad n^{ij}(\tau,t)$ # on/off transitions

DoS duration $\quad |\Xi^{ij}(\tau,t)| \leq \kappa^{ij} + \dfrac{t-\tau}{\tau_d^{ij}} \qquad \Xi^{ij}(\tau,t) = \bigcup_{n\in\mathbb{Z}_{\geq 0}} H_n^{ij} \bigcap [\tau,t]$

Problem

Jamming-resilient self-triggered coordination

Given a class of DoS signals characterized by

frequency τ_f^{ij}
duration τ_d^{ij}

- redesign the self-triggered control algorithm to preserve consensus
- quantify performance deterioration

De Persis, Tesi. "Resilient control under Denial-of-Service". Proc. 19th IFAC World Congress, pp. 134–139, 2014.
De Persis, Tesi. "Input-to-state stabilizing control under denial-of-service". IEEE Transactions on Automatic Control, 60, pp. 2930–2944, 2015.

De Persis, Tesi. "Networked control of nonlinear systems under Denial-of-Service". Systems & Control Letters, 96, pp. 124–131, 2016.

DoS-resilient consensus

DoS-resilient hybrid coordination I

Continuous dynamics as before

$$\begin{cases} \dot{x}_i = \sum_{j \in \mathcal{N}_i} u_i^j \\ \dot{u}_i^j = 0 \\ \dot{\theta}_i^j = -1 \end{cases}$$

Jumps occur at every t such that the set

$$\mathcal{J}(\theta, t) = \{\{i, j\} \in I \times I \ : \ j \in \mathcal{N}_i \text{ and } \theta_i^j(t) = 0\} \neq \emptyset$$

DoS-resilient hybrid coordination II

Discrete dynamics

$$
\begin{cases}
x^i(t^+) = x^i(t) \quad \forall i \in I \\
u_i^j(t^+) = \begin{cases}
\operatorname{sign}_\varepsilon(x_j(t) - x_i(t)) & \text{if } (i,j) \in \mathcal{J}(\theta, t) \wedge t \notin \Xi^{ij}(0,t) \\
0 & \text{if } (i,j) \in \mathcal{J}(\theta, t) \wedge t \in \Xi^{ij}(0,t) \\
u_i^j(t) & \text{otherwise}
\end{cases} \\
\theta_i^j(t^+) = \begin{cases}
f_i^j(x(t)) & \text{if } (i,j) \in \mathcal{J}(\theta, t) \wedge t \notin \Xi^{ij}(0,t) \\
\dfrac{\varepsilon}{2(\deg^i + \deg^j)} & \text{if } (i,j) \in \mathcal{J}(\theta, t) \wedge t \in \Xi^{ij}(0,t) \\
\theta_i^j(t) & \text{otherwise}
\end{cases}
\end{cases}
$$

Convergence of the solutions

Proposition (Point convergence)

Let $x(t)$ be the solution to the DoS-resilient self-triggered control algorithm. Then there exists a finite time $T_\star$ such that, for any $i \in I$, it holds that

$$u_i(t) := \sum_{j \in \mathcal{N}_i} u_i^j(t) = 0, \quad \text{for all} \quad t \geq T_\star$$

Consider the Lyapunov function

$$V(x) = \frac{1}{2} x^\top x$$

Then, for $t_k^{ij} := \max\{t_\ell^{ij} : t_\ell^{ij} \leq t, \ell \in \mathbb{Z}_{\geq 0}\}$

$$\dot{V}(x(t)) \leq - \sum_{\substack{\{i,j\} \in \mathcal{E}: \\ |x_j(t_k^{ij}) - x_i(t_k^{ij})| \geq \varepsilon \,\wedge\, t_k^{ij} \notin \Xi^{ij}(0,t)}} \frac{|x_j(t_k^{ij}) - x_i(t_k^{ij})|}{2}$$

Persistence of communication (PoC) I

To prevent persistent lack of communication after an unsuccessful transmission

$$\boxed{\alpha^{ij} := \frac{1}{\tau_d^{ij}} + \frac{\Delta_*^{ij}}{\tau_f^{ij}} < 1}$$

where

$$\Delta_*^{ij} := \frac{\varepsilon}{2(\deg^i + \deg^j)}$$

is the length of the sampling interval after an unsuccessful transmission

$$\theta_i^j(t^+) = \begin{cases} f_i^j(x(t)) & \text{if } (i,j) \in \mathcal{J}(\theta, t) \wedge t \notin \Xi^{ij}(0,t) \\ \dfrac{\varepsilon}{2(\deg^i + \deg^j)} & \text{if } (i,j) \in \mathcal{J}(\theta, t) \wedge t \in \Xi^{ij}(0,t) \\ \theta_i^j(t) & \text{otherwise} \end{cases}$$

Persistence of communication (PoC) II

Proposition (Link PoC)

For any link $\{i,j\} \in E$*, if the DoS signal parameters* τ_d^{ij}, τ_f^{ij} *satisfy*

$$\alpha^{ij} < 1$$

then for any given unsuccessful transmission attempt t_k^{ij}*, at least one successful transmission occurs over the link* $\{i,j\}$ *within the interval* $[t_k^{ij}, t_k^{ij} + \Phi^{ij}]$*, where*

$$\Phi^{ij} := \frac{\kappa^{ij} + (\eta^{ij} + 1)\Delta_*^{ij}}{1 - \alpha^{ij}}$$

Main result

Theorem (DoS-resilient practical consensus)

For every initial condition $\bar{x}$, let $x(t)$ be the solution to the DoS-resilient self-triggered control algorithm such that $x(0) = \bar{x}$. For each $\{i,j\} \in E$, consider any DoS sequence with η^{ij} and κ^{ij} arbitrary, and τ_d^{ij} and τ_f^{ij} such that $\alpha^{ij} < 1$. Then x converges in finite time to a point x^* belonging to the set

$$\mathcal{E} = \{x \in \mathbb{R}^n \, : \, |x_j - x_i| < \varepsilon \, \forall \, \{i,j\} \in E\}$$

Time cost under DoS (time to converge)

$$T := \inf\{t \geq 0 \, : \, x(t) \in \mathcal{E}\} \leq \left[\frac{\deg_{\max} + \deg_{\min}}{\varepsilon} + \frac{4 \deg_{\max}}{\varepsilon^2} \Phi\right] \sum_{i \in I} \bar{x}_i^2$$

Time cost without DoS (time to converge)

$$T := \inf\{t \geq 0 \, : \, x(t) \in \mathcal{E}\} \leq \frac{\deg_{\max} + 1}{\varepsilon} \sum_{i \in I} \bar{x}_i^2$$

Main result

Theorem (DoS-resilient practical consensus)

For every initial condition $\bar{x}$, let $x(t)$ be the solution to the DoS-resilient self-triggered control algorithm such that $x(0) = \bar{x}$. For each $\{i, j\} \in E$, consider any DoS sequence with η^{ij} and κ^{ij} arbitrary, and τ_d^{ij} and τ_f^{ij} such that $\alpha^{ij} < 1$. Then x converges in finite time to a point x^* belonging to the set

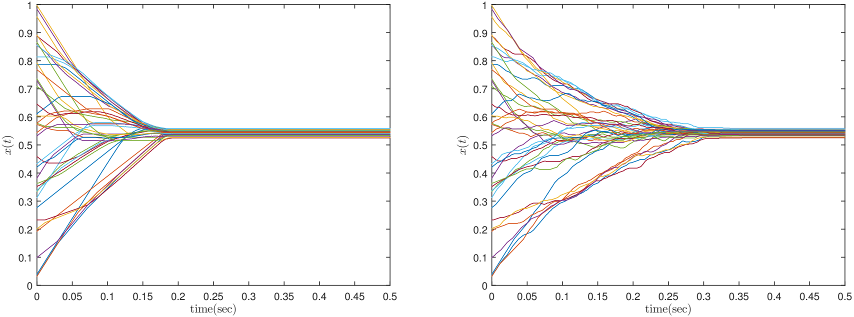
$$\mathcal{E} = \{x \in \mathbb{R}^n \,:\, |x_j - x_i| < \varepsilon \,\forall \{i, j\} \in E\}$$

Time cost under DoS (time to converge)

$$T := \inf\{t \geq 0 \,:\, x(t) \in \mathcal{E}\} \leq \left[\frac{\deg_{\max} + \deg_{\min}}{\varepsilon} + \frac{4\deg_{\max}}{\varepsilon^2}\Phi\right] \sum_{i \in I} \bar{x}_i^2$$

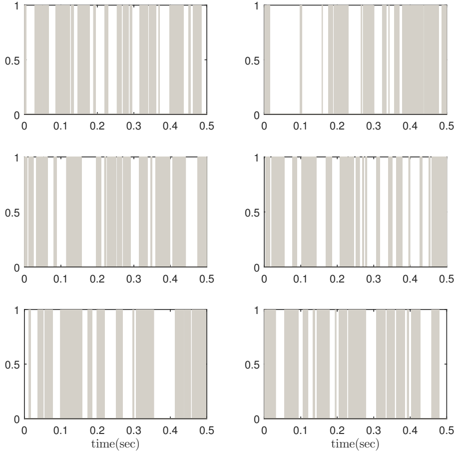
Senejohnny, Tesi, De Persis. "A jamming-resilient algorithm for self-triggered network coordination". IEEE Transaction on Control of Network Systems, under review, arXiv:1603.02563.

Simulations I



Sample evolutions of x on a random graph with $n = 40$, $\varepsilon = 0.005$, with (right) and without (left) DoS.

Simulations II



DoS pattern for the network links $\{29, 34\}$, $\{5, 33\}$, $\{9, 18\}$, $\{2, 8\}$, $\{22, 40\}$ and $\{1, 17\}$ generated as PWM signals with variable period (max 0.15 sec.) and maximum duty cycle equal to 100% . Vertical gray stripes = DoS time-intervals.

Conclusions

Conclusions

Results

- Coordination with self-triggered information collection (upon need)
- Link DoS signals
- Persistency of Communication
- Robustness to DoS attacks

Extensions

- Asymptotic consensus
- Scalability
- Estimates of the consensus value
- Counteracting DoS attacks
- Other attacks

De Persis, Postoyan. "A Lyapunov redesign of coordination algorithms for cyber-physical systems." IEEE Transactions on Automatic Control, in press, 2016.

Thank you for your attention