Distributed hybrid control synthesis for multi-agent systems from high level specifications

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Background

- Multi-agent control: motivated by a large variety of engineering applications: transportation systems, robotics, smart grids
- Multi-agent control objectives: simple/control type (consensus, formation control, ...)
- Formal methods based planning: higher level objectives for single agent
- Based on discrete represenations (aka abstractions) of control systems

State of the Art

Single Agent-Single Task

$$\dot{X} = f(X, u)$$
$$\varphi = (\Diamond(r_1 \wedge \Diamond r_2)) \wedge (\Diamond\Box r_3)$$

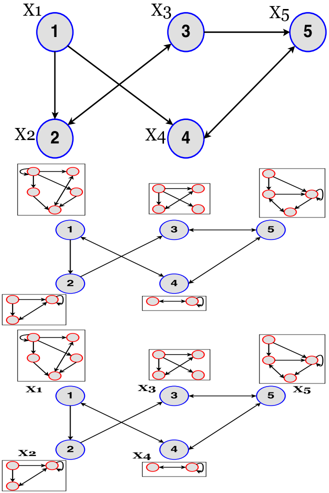
- High-level task specs using formal languages
- Planning on discrete abstraction of agent dynamics
- Implemented by continuous control sequence

Multiple Agents-Multiple Tasks

- Need for distributed, bottom-up solutions to deal with:
    - Distributed tasks and abstractions
    - Couplings, limited communication

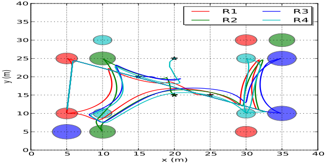
Proposed approach

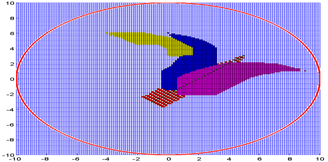
- Multi-agent control layer: distributed control through continuous state information
- Formal methods based planning: distributed task planning based on discrete information exchange
- Hybrid control: blending continuous and discrete information, need for abstractions of multi-agent control systems



Today's talk

- Task planning and control through specification-based abstraction
- Abstractions of dynamically coupled multi-agent systems
- Distributed task planning for task-level dependencies





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Problem Formulation

- A team of N mobile agents, $x_i(t), u_i(t) \in \mathbb{R}^2$:

$$\dot{x}_i(t) = u_i(t), \qquad i \in \mathcal{N} = \{1, \cdots, N\}.$$

- Agents i can observe agent j's position $x_j(t)$ only if:

$$\|x_i(t) - x_j(t)\| \leq r.$$

  Initial network G_0.

- Sphere regions of interest: $\mathcal{R}_i = \{R_{i\ell},\, \ell \in \{1, \cdots, M_i\}\}$.
  $R_{i\ell} = (c_{i\ell},\, r_{i\ell})$.

- Assumptions on the workspace.

- Services Σ_i available at each region in $\mathcal{R}_i$.

Problem Formulation, cont'd

- Local LTL task specification φ_i, over Σ_i.
- Note that φ_i can be co-safe or general LTL formulas.
- φ_i specifies the sequences at which the services should be done at certain regions.

Problem

How to synthesize the control input $u_i(t)$ and the discrete plan $\mathbb{S}_i$ such that

$$\varphi_i \text{ is satisfied}, \quad \forall i \in \mathcal{N}$$

and $\|x_i(t) - x_j(t)\| \leq r$, $\forall (i, j) \in E_0$, $\forall t \in [0, \infty)$.

Challenges

- Discrete task planning
- Continuous motion constraints
- Sensing limitations

Solution: three main steps.

- High-level discrete plan synthesis.
- Distributed potential-field-based motion control.
- Hybrid control strategy.

Step1. Discrete Plan Synthesis

Aim

Each agent synthesizes a local discrete plan that satisfies φ_i and minimizes a cost function.

- Automata-based model-checking algorithm[1]
- Discrete plan synthesized locally by each agent $i \in \mathcal{N}$:

$$\mathbb{S}_i = \sigma_{i1} \cdots \sigma_{is_i}(\sigma_{i(s_i+1)} \cdots \sigma_{iN_i})^{\omega}, \qquad \sigma_{is_i} = (R_{is_i}, \Sigma_{is_i}).$$

- Our algorithm minimizes the maximal distance between two consecutive regions along the plan[2].

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[1] C. Baier, J.-P Katoen. *Principles of model checking*, 2008.

[2] S. L. Smith, J. Tumova, C. Belta, D. Rus. Optimal Path Planning for Surveillance with Temporal Logic Constraints. The International Journal of Robotics Research, 2011.

Step2. Distributed Motion Control

- Setup for motion control:
  - Each agent has its goal region $\sigma_{ig} = (R_{ig}, \Sigma_{ig})$, but only known locally.
  - Relative-distance constraints.

Goal

Design a distributed control law $u_i(t)$ such that one agent arrives at its goal region in finite time, given the relative-distance constraints.

- *Time-varying* connectivity graph $G(t) = (\mathcal{N}, E(t))$, where $E(t) \subseteq \mathcal{N} \times \mathcal{N}$.
  - initially $G(0) = G_0$; dynamically add new edges.

- Solution: the two-mode control law
  (1) the *active* mode:

$$\mathbf{C}_{act}: \quad u_i(t) \triangleq -d_i\, p_i - \sum_{j \in \mathcal{N}_i(t)} h_{ij}\, x_{ij},$$

  (2) the *passive* mode:

$$\mathbf{C}_{pas}: \quad u_i(t) \triangleq -\sum_{j \in \mathcal{N}_i(t)} h_{ij}\, x_{ij},$$

  where $x_{ij} \triangleq x_i - x_j$; $p_i \triangleq x_i - c_{i\mathrm{g}}$; $R_{ig} = (c_{i\mathrm{g}},\, r_{i\mathrm{g}})$.

$$d_i \triangleq \frac{\varepsilon^3}{(\|p_i\|^2 + \varepsilon)^2} + \frac{\varepsilon^2}{2\,(\|p_i\|^2 + \varepsilon)}; \quad h_{ij} \triangleq \frac{r^2}{(r^2 - \|x_{ij}\|^2)^2}$$

- $\varepsilon > 0$ is a key design parameter.
- u_i is local w.r.t. $\mathcal{N}_i(t)$.

Convergence results

Considering a potential-field like Lyapunov function it can be shown that:

- $G(t)$ remains connected.
- There exists a finite time T_f and one active agent $i^\star \in \mathcal{N}_a$, such that $x_j(T_f) \in R_{i^\star \mathrm{g}}$, $\forall j \in \mathcal{N}$.
- All agents will enter $R_{i^\star \mathrm{g}}$, i.e., $x_j \in R_{i^\star \mathrm{g}}$, $\forall j \in \mathcal{N}$.
- The above holds for any number of active agents that $1 \leq N_a \leq N$.

Potential-field-based Design

Consider the following potential-field function:

$$V(x(t)) \triangleq \frac{1}{2}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}_i(t)}\phi_c(x_{ij}) + b_i\sum_{i\in\mathcal{N}}\phi_g(x_i)$$

- $\phi_c(x_{ij})$ is an attractive potential to agent i's neighbors.
- $\phi_g(\cdot)$ is an attractive force to agent i's goal:
- $b_i = 1$, $\forall i \in \mathcal{N}_a$ and $b_i = 0$, $\forall i \in \mathcal{N}_p$. $\mathcal{N} = \mathcal{N}_a \cup \mathcal{N}_p$.

Connectivity Results

$G(t)$ remains connected.
No existing edges within $E(T_s)$ will be lost.

- Proof shows that $V(t)$ remains bounded for $t \in [T_s, \infty)$.
  New edges might be added but no existing edges will be lost.

Convergence (non-switching case)

Constant sets of passive and active agents. Analysis of the critical points of V:

- Regions around the critical points:

  $$\mathcal{S}_i \triangleq \{\mathbf{x} \in \mathbb{R}^{2N} \mid \|\mathbf{x} - \mathbf{1}_N \otimes c_{i\mathrm{g}}\| \leq r_S(\varepsilon)\}, \qquad \forall i \in \mathcal{N}_a.$$

  Let $\mathcal{S} \triangleq \cup_{i \in \mathcal{N}_a} \mathcal{S}_i$ and $\mathcal{S}^{\neg} \triangleq \mathbb{R}^{2N} \setminus \mathcal{S}$.

- Lemma 1: There exists $\varepsilon_1 > 0$ such that if $\varepsilon < \varepsilon_1$, all critical points of V in $\mathcal{S}^{\neg}$ are non-degenerate saddle points.

- Lemma 2,3: There exists $\varepsilon < \min\{\varepsilon_2, \varepsilon_6\}$ such that regions $\{\mathcal{S}_i\}$ are sufficiently far. Critical points are close to the region center.

Lemma 4: There exists $\varepsilon_{\min} > 0$ such that if $\varepsilon < \varepsilon_{\min}$, all critical points of V within $\mathcal{S}$ are local minima.

Convergence Results

There exists a finite time $T_f \in [T_s, \infty)$ and one active agent $i^\star \in \mathcal{N}_a$, such that $x_j(T_f) \in R_{i^\star \mathrm{g}}$, $\forall j \in \mathcal{N}$, while $\|x_i(t) - x_j(t)\| < r$, $\forall (i, j) \in E(T_s)$ and $\forall t \in [T_s,\ T_f]$.

- The system converge to the set of local minima within $\mathcal{S}_{i^\star}$ for one active agent $i^\star \in \mathcal{N}_a$.
- All agents would enter $R_{i^\star \mathrm{g}}$, i.e., $x_j \in R_{i^\star \mathrm{g}}$, $\forall j \in \mathcal{N}$.
- All edges within $E(T_s)$ will be preserved for all $t > T_s$

The above theorem holds for any number of active agents that $1 \leq N_a \leq N$.

Step3. Hybrid Control: sc-safe LTL task case

Case one

All tasks $\{\varphi_i\}$ are given as sc-safe LTL formulas.

- If φ_i is sc-safe, every agent has a finite plan

$$\tau_i = (R_{i1}, \Sigma_{i1})(R_{i2}, \Sigma_{i2}) \cdots (R_{iN_i}, \Sigma_{iN_i}).$$

Local switching policy

- When R_{ik} is reached, provide the services Σ_{ik} and then set goal to $R_{i(k+1)}$.
- After $(R_{iN_i}, \Sigma_{iN_i})$, set $b_i = 0$ and be passive.

- Guaranteed that $\forall i \in \mathcal{N}$, φ_i is eventually satisfied, and $\|x_i(t) - x_j(t)\| < r$, $\forall (i, j) \in E(0)$ and $\forall t \geq 0$.

Step3. Hybrid Control: general LTL task case

- LTL and mixed sc-safe LTL/LTL tasks can be also tackled under different switching policies
- Account for infiniteness of satisfying plans
- Further ongoing extension to double-integrator dynamics with collision avoidance and quantified specs (MITL and STL formulas)

Four agents with co-safe or general LTL tasks:

Workspace

- $\Pi_1 = \{\pi_{1\mathtt{tl}}, \pi_{1\mathtt{tr}}, \pi_{1\mathtt{br}}, \pi_{1\mathtt{bl}}\}$. $\Sigma_1 = \{\sigma_{11}, \sigma_{12}\}$.
- $\Pi_2 = \{\pi_{2\mathtt{tl}}, \pi_{2\mathtt{tr}}, \pi_{2\mathtt{bl}}\}$. $\Sigma_2 = \{\sigma_{21}, \sigma_{22}, \sigma_{23}\}$.
- $\Pi_3 = \{\pi_{3\mathtt{tr}}, \pi_{3\mathtt{br}}, \pi_{3\mathtt{bl}}\}$. $\Sigma_3 = \{\sigma_{31}, \sigma_{32}, \sigma_{33}\}$.
- $\Pi_4 = \{\pi_{4\mathtt{tl}}, \pi_{4\mathtt{tr}}, \pi_{4\mathtt{br}}, \pi_{4\mathtt{bl}}\}$. $\Sigma_4 = \{\sigma_{41}, \sigma_{42}, \sigma_{43}\}$.

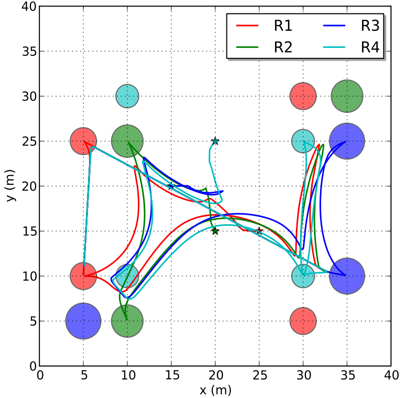
Sc-safe LTL task

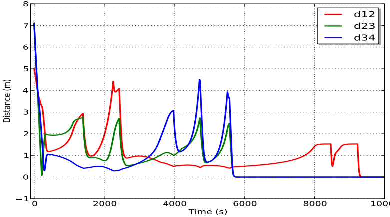
- $\varphi_1 = \Diamond(\sigma_{12} \wedge \Diamond(\sigma_{11} \wedge \Diamond\sigma_{12}))$.
- $\varphi_2 = \Diamond(\sigma_{21} \vee \sigma_{22}) \wedge \Diamond\sigma_{23}$.
- $\varphi_3 = \Diamond(\sigma_{31} \vee \sigma_{32}) \wedge \Diamond\sigma_{33}$.
- $\varphi_4 = \Diamond(\sigma_{42} \wedge \Diamond(\sigma_{41} \wedge \Diamond\sigma_{42}))$.

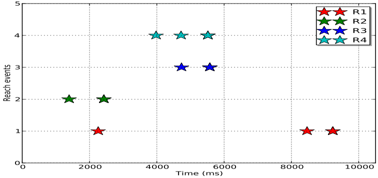
General LTL task

- $\varphi_1 = \Box\Diamond\sigma_{11} \wedge \Box\Diamond\sigma_{12}$.
- $\varphi_2 = \Box\Diamond(\sigma_{21} \vee \sigma_{22} \vee \sigma_{23})$
- $\varphi_3 = \Box\Diamond(\sigma_{31} \vee \sigma_{32} \vee \sigma_{33})$
- $\varphi_4 = \Box\Diamond\sigma_{41} \wedge \Box\Diamond\sigma_{42}$

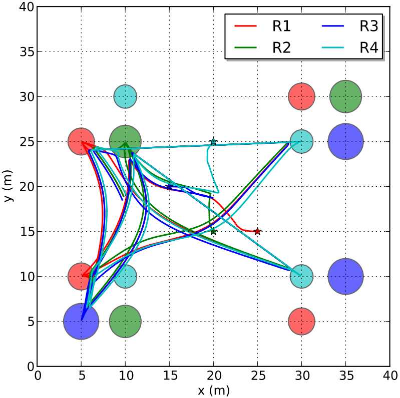
Scenario one

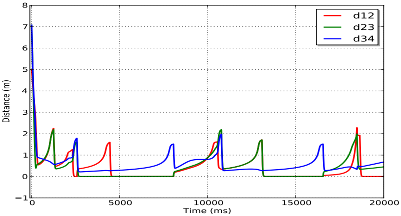


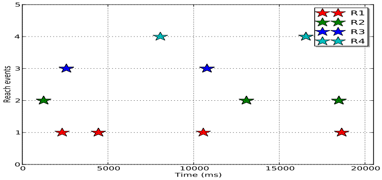




Scenario two







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Motivation

- Coupled multi-agent control systems
- Define discrete representations irrespective of given high-level specs
- May lead to trade-offs or fundamental limits to what can be requested from the system

Systems Description and objective

- Consider the multi-agent system

$$\dot{x}_i = u_i = f_i(x_i, \mathbf{x}_j) + v_i, \mathbf{x}_j = (x_{j_1}, \ldots, x_{j_{N_i}}), i = 1, \ldots, N$$

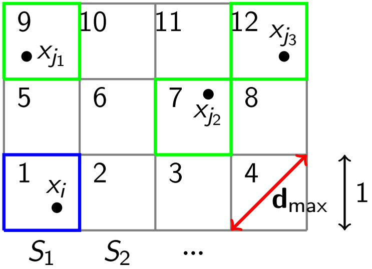
- Closed loop system with coupled constraints $f_i(x_i, \mathbf{x}_j)$ and free inputs v_i
- Goal: abstract continuous space-time system properties in a discrete Transition System
- Goal: find finite abstractions for the multi-agent system in a distributed way that makes sense

Preliminaries - Notation

- Abstraction Requirements: find
  - cell decomposition $\rightarrow$ finite or countable "partition" $\mathcal{S} = \{S_l\}_{l \in \mathcal{I}}$ of the workspace by uniformly bounded sets
  - time step δt
  - which ensure that the discretized model of closed loop system is well posed - meaningful
- Notation
  - Cell Configuration CC of i and its neighbors $j_1, \ldots, j_{N_i}$
  - $N_i + 1$-tuple of cell indices $\mathbf{l}_i = (l_i, l_{j_1}, \ldots, l_{j_{N_i}}) \in \mathcal{I}^{N_i+1}$
- Cell decomposition diameter d_{max}:
  - "maximum" diameter of a cell $S_l \in \mathcal{S}$

$$d_{\max} := \sup\{|x - y| : x, y \in S_l, l \in \mathcal{I}\}$$

Cell Decomposition - Cell Configuration Example



- Cell decomposition: $\mathcal{S} = \{S_l\}_{l \in \{1,\ldots,12\}}$
- Cell configuration CC of i and its neighbors j_1, j_2, j_3:

$$\mathbf{l} = (l, l_1, l_2, l_3) = (1, 9, 7, 12) \in \{1, \ldots, 12\}^4$$

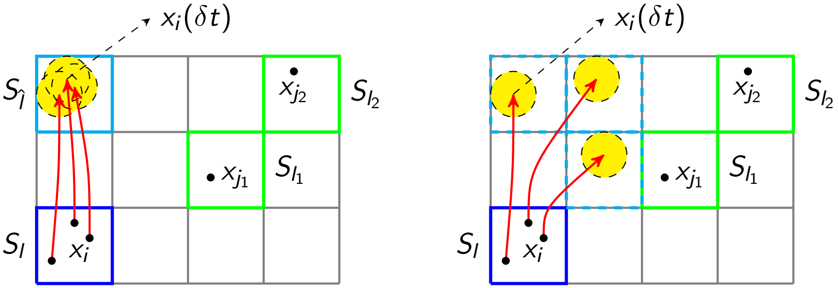
- Cell decomposition diameter: $d_{\text{max}} = \sqrt{2}$

Well Posed Discretizations

Given the cell decomposition $\mathcal{S} = \{S_l\}_{l \in \{\mathcal{I}\}}$ and the time step δt, we say that the space-time discretization $\mathcal{S}$-δt is well posed if for each $i = 1, \dots, N$ and CC $\mathbf{l}_i = (l_i, l_{j_1}, \dots, l_{j_{N_i}})$ of i

- there exists (at least one) cell $S_{l'_i}$
- and a control law assigned to the input v_i, such that for each $x_i(0) \in S_l$ and irrespectively of v_k, $k \neq i$ and the exact initial positions of the neighbors $x_{j_k}(0)$ in $S_{l_{j_k}}$
- agent i is driven to cell $S_{l'_i}$ exactly in time δt

Well Posed Discretizations



Sys. (A): $\dot{x}_i = f_{iA}(x_i, x_{j_1}, x_{j_2}) + v_{iA}$

Sys. (B): $\dot{x}_i = f_{iB}(x_i, x_{j_1}, x_{j_2}) + v_{iB}$

- The discretization is well posed for System (A)
- The discretization is not well posed for System (B)

Dynamics Properties[3]

- Lipschitz constants L_1, L_2

$$|f_i(x_i, \mathbf{x}_j) - f_i(x_i, \mathbf{y}_j)| \leq L_1 |\mathbf{x}_j - \mathbf{y}_j|$$

$$|f_i(x_i, \mathbf{x}_j) - f_i(y_i, \mathbf{x}_j)| \leq L_2 |x_i - y_i|$$

- Dynamics bounds

$$|f_i(x_i, \mathbf{x}_j)| \leq M$$

$$|v_i(t)| \leq v_{\max} \quad (< M)$$

$$\mathbf{x}_j := (x_{j_1}, \ldots, x_{j_{|\mathcal{N}_i|}})$$

[3] D. Boskos and D. V. Dimarogonas, Robust Connectivity Analysis for Multi-Agent Systems, CDC 2015

Analytical Results on Well Posed d_{max} - δt

QUESTION

- How do we quantify acceptable $d_{\max}$ - δt?

RESULT: Assuming that $v_{max} < M$, a sufficient condition which guarantees that the space-time discretization $d_{\max}$-δt is well posed, is that $d_{\max}$ and δt satisfy the following restrictions

$$
\begin{aligned}
d_{\max} &\in \left(0, \frac{v_{\max}^2}{4ML}\right] \\
\delta t &\in \left[\frac{v_{\max} - \sqrt{v_{\max}^2 - 4MLd_{\max}}}{2ML}, \frac{v_{\max} + \sqrt{v_{\max}^2 - 4MLd_{\max}}}{2ML}\right]
\end{aligned}
$$

with the dynamics dependent parameter L defined as

$$L := \max\{2L_2 + 4L_1\sqrt{N_i}, i = 1, \ldots, N\}$$

Analytical Results on Well Posed d_{max} - δt

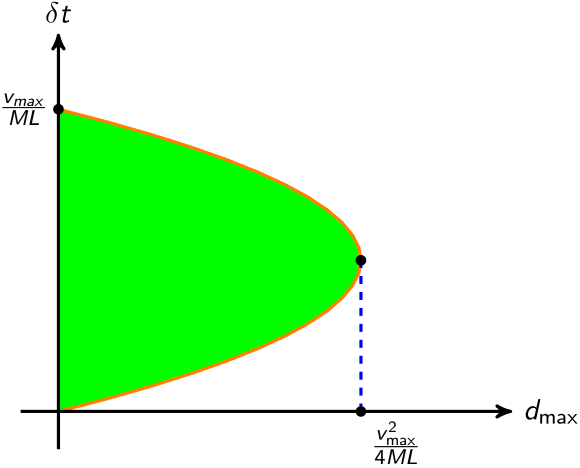


Figure: *Feasible $d_{\max} - \delta t$ region*

Selection of d_{max} - δt for Motion Planning

Transition possibilities can be quantified by employing additional d.o.f.!

PROPOSITION

Consider a cell decomposition $\mathcal{S}$ of D with diameter $d_{\max}$, a time step δt, the parameters $\lambda \in (0, 1)$, $\mu > 0$ and define

$$r := \lambda v_{\max} \delta t$$

We assume that r satisfies the design requirement

$$r \geq \frac{\mu}{2} d_{\max}$$

Then the space-time discretization is well posed for the multi-agent system, provided that λ, μ, $d_{\max}$ and δt satisfy certain algebraic sufficient conditions.

Selection of d_{max} - δt for Motion Planning

COROLLARY

Consider a cell decomposition $\mathcal{S}$ with diameter $d_{\max}$, a time step δt, and parameters $\lambda \in (0,1)$, $\mu > 0$ such that the hypotheses above are fulfilled. Then for each agent $i \in \{1, \ldots, N\}$ and each CC of i, there exist at least

$$\begin{aligned} \lfloor \mu^n \rfloor + 1, \text{ if } \mu^n \notin \mathbb{N}, \\ \lfloor \mu^n \rfloor, \text{ if } \mu^n \in \mathbb{N}, \end{aligned}$$

possible discrete transitions.

Corresponding Transition System

Agent's i individual transition system $TS_i := (Q, Act_i, \longrightarrow_i)$

- state set Q the indices $\mathcal{I}$ of the cell decomposition
- actions all possible cell indices of i and its neighbors

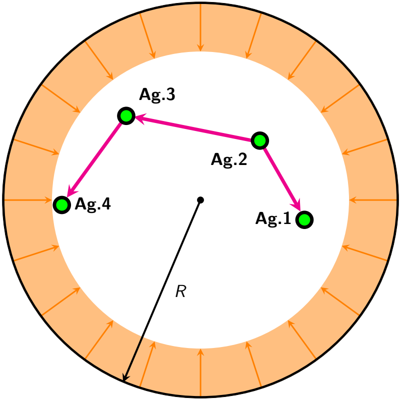
$$Act_i := \mathcal{I}^{N_i+1}$$

  (the set of all possible cell configurations of i)

- transition relation $\longrightarrow_i \subset Q \times Act_i \times Q$ as follows: For $l_i, l_i' \in Q$ and $\mathbf{l}_i = (l_i, l_{j_1}, \ldots, l_{j_{N_i}}) \in \mathcal{I}^{N_i+1}$,

$$l_i \xrightarrow{\mathbf{l}_i}_i l_i' \quad \text{iff} \quad l_i \xrightarrow{\mathbf{l}_i} l_i' \quad \text{is well posed.}$$

Example with Four Agents



- Network topology $\mathcal{N}_1 = \{2\}$, $\mathcal{N}_2 = \emptyset$, $\mathcal{N}_3 = \{2\}$, $\mathcal{N}_4 = \{3\}$
- Bounded circular domain of radius R
- Connectivity distance between neighboring agents ρ

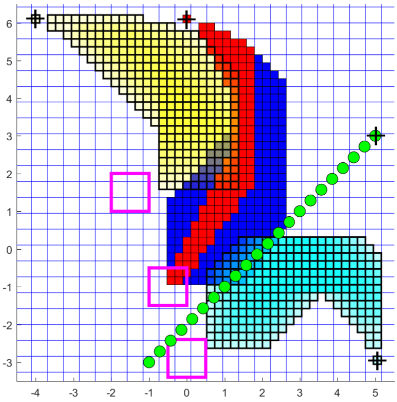
Dynamics and Selection of v_{max}

- Saturated dynamics

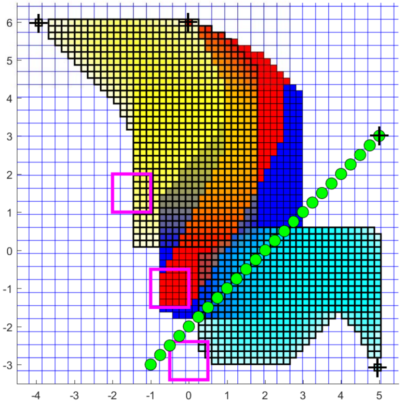
$$
\begin{aligned}
\dot{x}_1 &= \mathrm{sat}_\rho(x_2 - x_1) + g(x_1) + v_1 \\
\dot{x}_2 &= g(x_2) + v_2 \\
\dot{x}_3 &= \mathrm{sat}_\rho(x_2 - x_3) + g(x_3) + v_3 \\
\dot{x}_4 &= \mathrm{sat}_\rho(x_3 - x_4) + g(x_4) + v_4
\end{aligned}
$$

- $\mathrm{sat}_\rho(x) := x$ if $|x| \leq \rho$; $\mathrm{sat}_\rho(x) := \frac{\rho}{|x|}x$, if $|x| > \rho$
- Repulsion vector filed $g(x)$
- Selecting $v_{\mathrm{max}} = \frac{\rho}{2}$ ensures that initially connected configurations remain connected

Simulation Results



(i)



(ii)

- Reachable cells: (i) $\lambda = 0.2$ and (ii) $\lambda = 0.3$
- Agents: 1-cyan, 2-green, 3-blue, 4-yellow
- Agent 4 reaches its target box with the finer discretization, also due to the increased number of (red) paths of 3 that reach its target box

Ongoing and Future Work

- Abstractions of varying decentralization degree[4]
    - based on discrete positions up to a distance in the network graph
    - improved discretizations due to the reduction of the required control for the coupling terms
- Online abstractions
    - based on the discretization of each agent's reachable set over a time horizon
    - applicable to forward complete systems
    - improved discretizations and reachability properties for agents with weaker couplings over the horizon
- Future directions: higher order systems, special network structures ...

[4] D. Boskos and D. V. Dimarogonas, Abstractions of Varying Decentralization Degree for Coupled Multi-Agent Systems, CDC 2016

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Aim

- A team $\mathcal{N} = \{1, \ldots, N\}$ of agents
  - A finite discrete transition system $\mathcal{T}_i$
    - Abstraction of action capabilities
    - Example: transition system emerging from previous abstraction procedure
  - Synchronization capabilities
- High-level behavior specification
  - *Motion* LTL specification ϕ_i over the states
  - *Task* LTL specification ψ_i over the inputs/actions
- Efficiently synthesize controllers fulfilling the tasks
  - A *satisfying* trace of each $\mathcal{T}_i$
  - Necessary synchronizations
  - The catch: dependencies at the task (discrete) level

Problem Formulation

For each $i \in \mathcal{N}$, synthesize appropriate motion and action sequences so that

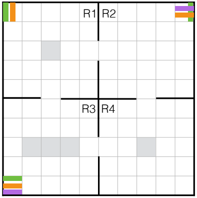
- the set of induced behaviors is nonempty
- the motion specification ϕ_i is satisfied
- the task specification ψ_i is locally satisfied

Example I

Agent 1 is a ground vehicle and has to avoid walls and obstacles.
Agent 2 and Agent 3 are UAVs and their environment is obstacle-free except for the walls.

Motion specifications

Agent 1: Keep avoiding R1, $\phi_1 = \mathcal{G}\neg \mathrm{R1}_1$.
Agent 2: Keep avoiding R2, $\phi_2 = \mathcal{G}\neg \mathrm{R2}_2$.
Agent 3: Periodically survey R1 and R2, $\phi_3 = \mathcal{G}\,\mathcal{F}\,\mathrm{R1}_3 \wedge \mathcal{G}\,\mathcal{F}\,\mathrm{R2}_3$.



Task specifications

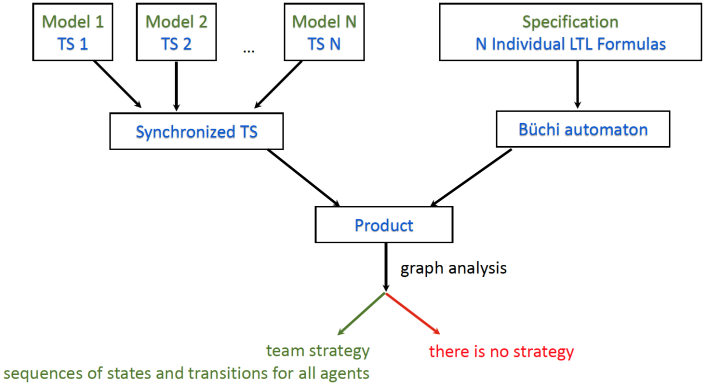
Agent 1: periodically *load*(–) with the *help* of agent 2 (–) and the *assistance* of agent 3 (–), then *unload* (|) with the *help* of agent 2 (–) or the *assistance* of agent 3 (–)

$$\psi_1 = \mathit{load} \wedge \mathit{help} \wedge \mathit{assist} \wedge \mathcal{G}\,(\mathit{load} \Rightarrow \mathcal{X}\,(\mathit{unload} \wedge (\mathit{help} \vee \mathit{assist}))) \wedge \mathcal{G}\,(\mathit{unload} \Rightarrow \mathcal{X}\,(\mathit{load} \wedge \mathit{help} \wedge \mathit{assist}))$$

Agent 2: Periodically provide *inform* service (|), $\psi_2 = \mathcal{G}\mathcal{F}\mathit{inform}$.

Agent 3: Nothing specific, $\psi_3 = \mathit{true}$.

Straightforward Approach



Computational infeasibility!

Our Hierarchical Approach I

- Each ϕ_i is translated to a Büchi automaton $\mathcal{B}_i^{\phi}$
- N motion products $\mathcal{P}_i = \mathcal{T}_i \otimes \mathcal{B}_i^{\phi}$ are built
- Each motion product is reduced to $\ddot{\mathcal{P}}_i$ by systematic removal of states, where no services of interest are available
- Each ψ_i is translated to a Büchi automaton $\mathcal{B}_i^{\psi}$
- N task and motion products $\bar{\mathcal{P}}_i = \ddot{\mathcal{P}}_i \otimes \mathcal{B}_i^{\psi}$
- Each motion and task product is reduced to $\widehat{\mathcal{P}}_i$ by systematic removal of states, where no dependent services are available
- A global product $\mathcal{P} = \widehat{\mathcal{P}}_1 \otimes \ldots \otimes \widehat{\mathcal{P}}_N$ containing only states relevant for planning of dependent tasks is constructed

Our Hierarchical Approach II

- An accepting run in the global product projected onto the original system gives
  - a motion plan
  - a task execution plan
  - a synchronization plan

  for each agent i, that is *correct-by-design with respect to ϕ_i and ψ_i.*

Example I Revisited

Agent 1 is a ground vehicle and has to avoid walls and obstacles.
Agent 2 and Agent 3 are UAVs and their environment is obstacle-free except for the walls.

Motion specifications

Agent 1: Keep avoiding R1, $\phi_1 = \mathcal{G}\neg \mathrm{R1}_1$.
Agent 2: Keep avoiding R2, $\phi_2 = \mathcal{G}\neg \mathrm{R2}_2$.
Agent 3: Periodically survey R1 and R2, $\phi_3 = \mathcal{G}\,\mathcal{F}\,\mathrm{R1}_3 \wedge \mathcal{G}\,\mathcal{F}\,\mathrm{R2}_3$.

Task specifications

Agent 1: periodically *load*(–) with the *help* of agent 2 (–) and the *assistance* of agent 3 (–), then *unload* (|) with the *help* of agent 2 (–) or the *assistance* of agent 3 (–)

$$\psi_1 = \mathit{load} \wedge \mathit{help} \wedge \mathit{assist} \wedge \mathcal{G}\,(\mathit{load} \Rightarrow \mathcal{X}\,(\mathit{unload} \wedge (\mathit{help} \vee \mathit{assist}))) \wedge \mathcal{G}\,(\mathit{unload} \Rightarrow \mathcal{X}\,(\mathit{load} \wedge \mathit{help} \wedge \mathit{assist}))$$

Agent 2: Periodically provide *inform* service (|), $\psi_2 = \mathcal{G}\mathcal{F}\mathit{inform}$.
Agent 3: Nothing specific, $\psi_3 = \mathit{true}$.

Example I Revisited

Centralized approach

- Each TS: 100 states
- Product TS: 100^3 states
- $\mathcal{B}_1^\phi, \mathcal{B}_2^\phi, \mathcal{B}_3^\phi, \mathcal{B}_1^\psi, \mathcal{B}_2^\psi, \mathcal{B}_3^\psi$: $2, 2, 3, 2, 2, 1$ states, respectively respectively
- Intersection BA: $2 \cdot 2 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 7 = 330$ states
- The overall product $\mathcal{P}$: ≈ 330 mil. states

Our approach:

- $\mathcal{P}_1$, $\mathcal{P}_2$, $\mathcal{P}_3$: $200, 200, 300$ states, respectively
- $\widehat{P}_1, \widehat{P}_2, \widehat{P}_3$: 27,17,8 states, respectively
- The largest structure handled has cca 15000 states.

Remarks

- Worst-case complexity meets the complexity of the centralized solution
- Suitable for sparsely distributed services of interest and occasional needs for collaboration
- The bottleneck is still the product $\mathcal{P}$ and (some) synchronization
- Extension to event-based receding horizon approach: uses local versions of product and synchronizations in an event-based fashion

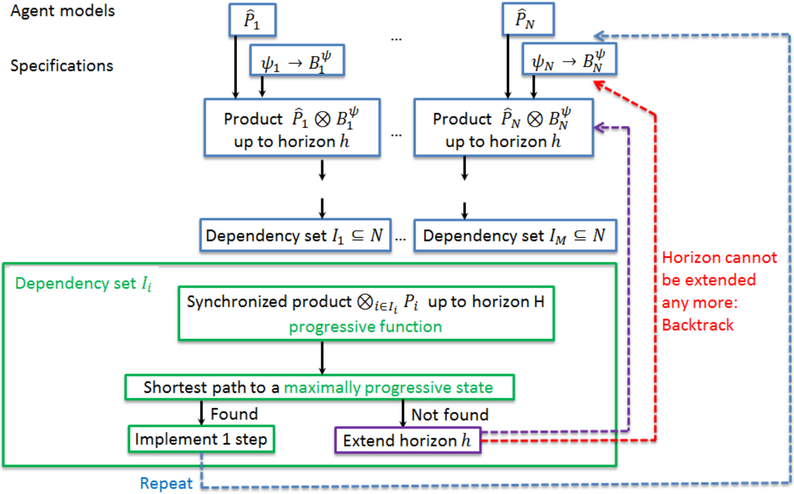
Event-triggered Receding Horizon Approach

- Each ϕ_i is translated to a Büchi automaton $\mathcal{B}_i^{\phi}$
- N motion products $\mathcal{P}_i = \mathcal{T}_i \otimes \mathcal{B}_i^{\phi}$ are built
- Each motion product is reduced to $\ddot{\mathcal{P}}_i$ by systematic removal of states, where no services of interest are available
- Each ψ_i is translated to a Büchi automaton $\mathcal{B}_i^{\psi}$
- N task and motion products $\bar{\mathcal{P}}_i = \ddot{\mathcal{P}}_i \otimes \mathcal{B}_i^{\psi}$
- Each motion and task product is reduced to $\widehat{\mathcal{P}}_i$ by systematic removal of states, where no dependent services are available
- A global product $\mathcal{P} = \widehat{\mathcal{P}}_1 \otimes \ldots \otimes \widehat{\mathcal{P}}_N$ containing only states relevant for planning of dependent tasks is constructed

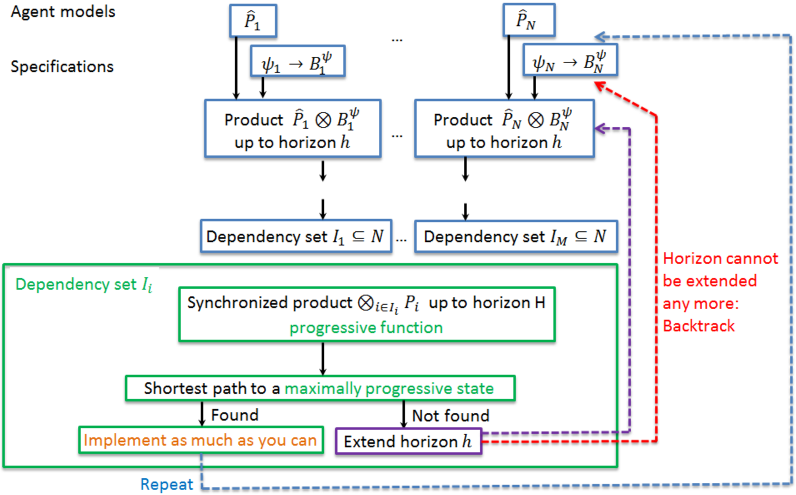
Event-triggered Receding Horizon Approach

- Translate the infinite-horizon problem into an infinite sequence of finite-horizon problems
- Dynamically partition the agents based on dependency
- Define progressive function to indicate closeness to goal satisfaction
- Introduce event-triggered synchronization

Stepwise Receding Horizon



Stepwise Receding Horizon



Example II

- Agent 1 can load (l_H, l_A, l_B), carry, and unload (u_H, u_A, u_B) a heavy object H or a light object A, B, in the green cells.

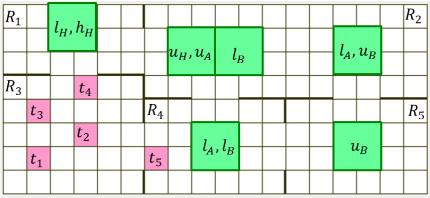
$$\psi_1 = \mathcal{F}(l_H \wedge h_H \wedge \mathcal{X}\, u_H \wedge \bigwedge_{i \in \{A,B\}} \mathcal{G}\mathcal{F}\,(l_i \wedge \mathcal{X} u_i)))$$

- Agent 2 is capable of helping the agent 1 to load object H (h_H), and to execute simple tasks in the purple regions $(t_1 - t_5)$.

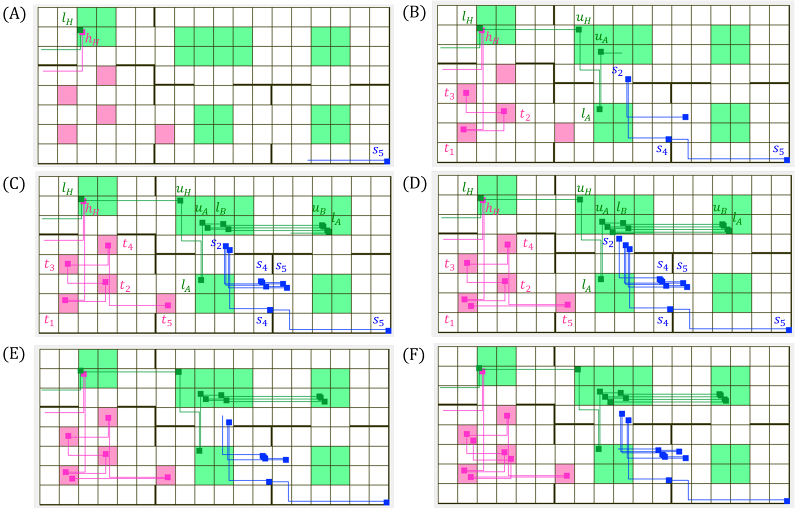
$$\psi_2 = \mathcal{G}\mathcal{F}\ (t_1 \wedge \mathcal{X}\ (t_2 \wedge \mathcal{X}\ (t_3 \wedge \mathcal{X}\ (t_4 \wedge \mathcal{X}\ \ t_5 \wedge s_4))))))$$

- Agent 3 is capable of taking a snapshot of the rooms $(s_1 - s_5)$ when being present in there.

$$\psi_3 = \bigwedge_{i \in \{2,4,5\}} \mathcal{G}\mathcal{F}\, s_i$$



Example II



cca 3 mil. vs. hundreds to thousands of states

Remarks

- The worst-case complexity still the same as for the centralized case
- Suitable for collaborations executed in small (dynamically changing) subgroups

Conclusion and Future Work

- Conclusion
  - Decentralized abstractions and planning for multi-agent systems
  - Consideration of dynamics and continuous-time constraints
  - Decomposition of formulas and event-based horizon framework for decentralized LTL based planning
- Future and current Work
  - Further reduction of complexity in distributed task planning
  - More general dynamics and combination with dependent tasks
  - Online version of abstraction framework
  - Quantifying space and time constraints at the task level (MITL and STL specs)

References and acks

- First part: discrete specs and coupled constraints: Guo et al., CDC14-15, IJRR15, TAC16
- Second part: locally defined abstractions for MAS: Boskos and Dimarogonas, CDC15,CDC 16, TAC16 under revision
- Third part: distributed task planning: Tumova and Dimarogonas ACC14, Automatica16, CDC15
- EU Projects: ERC StG BUCOPHSYS, FP7 RECONFIG, H2020 AEROWORKS and Co4Robots. National projects: VR, SSF, KAW IPSYS, KAW Fellowship.
- Contact: `http://people.kth.se/~dimos/`