Combinatorial optimization with energy constraints

Sandra Ulrich NGUEVEU

Université de Toulouse / INP-Toulouse / LAAS-CNRS
ngueveu@laas.fr

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Plan

1. Introduction
2. Literature review
3. Resolution scheme
4. Focus on a scheduling problem with an energy source
5. Conclusion
Hybrid-electric vehicles

Electric propulsion motor powered by:
- onboard generator:
  - internal combustion engine or
  - hydrogen fuel cell (FC)
- reversible source:
  - battery or
  - supercapacitor (SE)

Architectures (hybrid-series, hybrid-parallel, ...)

Problem description

- Given the **power request** of a driver on a predefined road section ...

- ... and the characteristics of the energy sources: **power limitations (kW)**, **efficiency (%)**, **storage capacity (kWs)** ...

... Find at each instant the **optimal power split** between the energy sources to **minimize** the total fuel consumption.
Findings

- Example of solution

- Better modeling hypothesis and efficient reformulations
- 20% improvement over the previous state-of-the-art
Water pumping and desalination process

Figure : Source : (Sareni et al., 2012)
Mechanic-hydraulic-electric models

**Electrical model**
- $V_m, I_m$: electrical tension, courant
- $T_m$: motor electromag. torque
- $\Omega$: rotation speed
- $k_\Phi$: torque equivalent coefficient
- $r$: stator resistance

Electric motor equations (inertia neglected):
\[
V_m = rI_m + k_\Phi \Omega \quad (1)
\]
\[
T_m = \Phi_m I_m \quad (2)
\]
Electrical power needed: $P_e = V_m I_m$.

**Mechanical-Hydraulic conv.**
- $P_p$: output pressure
- $q$: debit of water
- $a, b$: non linear girator coefs
- $c$: hydraulic friction
- $p_0$: suction pressure
- $f_p + f_m$: mechanical losses

Static equations of the motor-pump (mechanical inertia neglected):
\[
P_p = (a\Omega + bq)\Omega - (cq^2 + p_0) \quad (3)
\]
\[
T_m = (a\Omega + bq)q + (f_m + f_p)\Omega \quad (4)
\]

**Pressure drop in the pipe**
- $\Delta\text{Pipe}$: pressure drop
- $h$: height of water pumping
- $\rho$: water density

Static+Dynamic pressure
\[
\Delta\text{Pipe} = kq^2 + \rho gh \quad (5)
\]
Efficiency function of pump 2 + RO

The subsystem resulting from the combination of pump 2 and the Reverse Osmosis module is modeled with equation:

\[
\text{power required} = r \cdot \mathcal{K}(q_c, h) + ((f_m + f_p) \cdot \Omega(q_c, h) + (q_c + \mathcal{F}(q_c)/R_{Me}) \cdot \mathcal{M}(q_c, h)) \cdot \Omega(q_c, h)
\]

where

\[
\begin{align*}
\mathcal{F}(q_c) &= (R_{Mod} + R_{Valve}) \cdot q_c^2 \\
G(q_c) &= (b \cdot (q_c + \mathcal{F}(q_c)/R_{Me})) \\
\mathcal{M}(q_c, h) &= a \cdot \Omega(q_c, h) + G(q_c) \\
\Omega(q_c, h) &= \frac{-G(q_c) + \sqrt{G(q_c)^2 - 4a*(-(p_0 + \rho g*(h-l_{out})+(k+c)*(q_c+\mathcal{F}(q_c)/R_{Me})^2+\mathcal{F}(q_c)))}}{2a} \\
\mathcal{K}(q_c, h) &= (((f_m + f_p) \cdot \Omega(q_c, h) + (q_c + \mathcal{F}(q_c)/R_{Me}) \cdot (a \cdot \Omega(q_c, h) + G(q_c))))/k_{\phi})^2
\end{align*}
\]
Literature review

Mathematical programming-based resolution methods on similar problems

Generic MINLP resolution methods

Hybrid algorithms and frameworks
PGMO project OREM

- Previous studies involve multiple energy sources and general non-linear efficiency functions, but no scheduling.
- All our previous work on scheduling under energy constraint considered linear (and even identical) energy efficiency functions, which oversimplifies the problem.
- We want to solve explicitly and in an integrated fashion energy resource allocation problems and energy-consuming activity scheduling problems with non-linear energy efficiency functions.

http://homepages.laas.fr/sungueve/PGMOOREM.html
Resolution scheme: (Ngueveu et al., 2014)

Step 1: Piecewise linear bounding of the nonlinear energy transfer/efficiency functions

(a) Linear approximation

(b) Piecewise bounding

Step 2: Reformulation of the problem into two mixed integer problems (MILP)

- the problem is originally a MINLP
- using the pair of bounding functions previously defined
Piecewise bounding (Ngueveu et al., 2014)

Mathematical formulations
Specificities of piecewise bounding with a tolerance $\epsilon$
Proof of optimality
How to perform the bounding
Resulting bounding algorithms proposed
Results on a practical problem
What is the impact of the (piecewise linear) energy function on the nature and the structure of a problem?

Can it render NP-hard an initially polynomial pb?

Can it render polynomial an initially NP-hard problem?

If polynomial what is the best algorithm to solve the resulting pb?

If NP-hard, what is the best formulation and best approach for the resulting pb?
1. Introduction

2. Literature review

3. Resolution scheme

4. Focus on a scheduling problem with an energy source
   - Problem definition
   - Dantzig-Wolfe decomposition
   - Branch-and-Price
   - Results

5. Conclusion
Back to basics: A pre-emptive scheduling problem

Data

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Initial solution: cost = 30

Absolute efficiency function
Back to basics: A pre-emptive scheduling problem

Optimal solution: cost = 26
Notations and definition

Data

- Set of time instants \( T \)
- Set of activities \( \mathcal{A} \)
  - \( r_i, d_i, p_i \) : release date, due date, duration of activity \( i \)
  - \( b_i \) : constant instantaneous energy demand of activity \( i \)
- Set of non-reversible energy sources \( \mathcal{S} \)
  - \( \rho^s \) : piecewise-linear efficiency function for source \( s \) (x-axis = cost, y-axis = demand and \( \rho^s(x) = 0, \forall x < 0, \forall s \in \mathcal{S} \)).

Useful constants

- \( a_{it} \) : constant term equal to 1 if \( t \in [r_i, d_i] \) and 0 otherwise

Decision variable

- \( x_{it} \) : binary, = 1 iff activity \( i \) is ongoing at instant \( t \)
Formulation

Minimize the total energy cost

\[(CF) \min \sum_{t \in T} \rho^{-1}(\sum_{i \in A} b_i x_{it}) \quad (8)\]

s.t.

Satisfaction of the demand for each activity

\[\sum_{t \in T} a_{it} x_{it} \geq p_i, \quad \forall i \in A \quad (9)\]

Validity domain

\[x_{it} \in \{0, 1\}, \quad \forall i \in A, t \in T \quad (10)\]
Equivalence to a single-source problem

**Theorem**

\[ \forall (P) \text{ with } |A'| > 1, \exists (P') \text{ with } |A'| = 1 \text{ such that } (P) \text{ and } (P') \text{ are equivalent}. \]

**Proof outline**

For all \( x \), \( \rho'(x) \) can be defined as the solution cost of the problem :

\[
\min \left( \sum_{i \in S} \rho^i(y_i) \right) \quad (11)
\]

s.t.

\[
\sum_{i \in S} y_i = x \quad (12)
\]

\[
y_i \in \mathbb{R}, \quad \forall i \in S \quad (13)
\]
Proof of Complexity

Theorem

\((P')\) is NP-hard.

Proof outline

• Any decisional instance of the discrete bin packing problem can be transformed into a particular decisional instance of \((P')\).
  
  • Decisional discrete BPP : \(n\) items of size \(b_i, \forall i \in 1..n\); bin capacity \(C\). Does a solution exists with at most \(B\) bins?
  
  • equivalent to the following \((\tilde{P}')\) problem : \(n\) activities, each with energy demand \(b_i\); an energy source of efficiency function

\[
\tilde{\rho}'(x) = 1 \text{ if } 0 \leq x \leq C \text{ and } B \text{ if } x \geq C
\]

Does it exists a solution of \((\tilde{P}')\) with a cost not exceeding \(B\)?
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Based on Activity sets

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**Demand**, **Release date**, **Due date**, **Cost**
Based on Activity sets

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Additional Notations and definitions

Additional data

- Set of activity sets executable in parallel at any given instant $\mathcal{L}$
- Set of activities belonging to set $l \mathcal{A}_l$:
  - $\forall l \in \mathcal{L}$:
    - $b_l$ : energy demand ($= \sum_{i \in \mathcal{A}_l} b_i$)
    - $c_l$ : energy cost ($= \rho^{-1}(b_l)$)
- Set of activity sets executable at instant $t \mathcal{L}_t$

Useful constants

- $a_{il}$ : constant term equal to 1 if $i \in \mathcal{A}_l$ and 0 otherwise

Additional decision variables

- $y_{lt}$ : binary, $= 1$ iff activity set $l$ is being executed at time $t$
Extended formulation

Minimize the total energy cost

\[(EF) \min \sum_{t \in T} \sum_{l \in L_t} c_l y_{lt} \tag{14}\]

s.t.
Satisfaction of the demand for each activity

\[\sum_{l \in L} D_l a_{il} y_{lt} \geq p_i, \quad \forall i \in A \tag{15}\]

Only one set active at each instant-time

\[-\sum_{l \in L_t} y_{lt} \geq -1, \quad \forall t \in T \tag{16}\]

Validity domain

\[y_{lt} \in \{0, 1\} \quad \forall t \in T, l \in L_t \tag{17}\]
Extended formulation

Minimize the total energy cost

\[
(\text{EF}) \min \sum_{t \in T} \sum_{l \in L_t} c_l y_{lt} \tag{18}
\]

s.t.

Satisfaction of the demand for each activity

\[
\sum_{l \in L} \sum_{t = R_l} a_{il} y_{lt} \geq p_i, \quad \forall i \in A \tag{19}
\]

Only one set active at each instant-time

\[
- \sum_{l \in L_t} y_{lt} \geq -1, \quad \forall t \in T \tag{20}
\]

Link variables \(x\) and \(y\)

\[
x_{it} - \sum_{l \in L_t} a_{il} y_{lt} = 0, \quad \forall i \in A, t \in T \tag{21}
\]

Validity domain

\[
x_{it} \in \{0, 1\} \quad \forall t \in T, i \in A \tag{22}
\]

\[
y_{lt} \in \mathbb{R} \quad \forall t \in T, l \in L_t \tag{23}
\]
**Decomposition**

The linear relaxation of the master problem

\[(LRMP) \min \sum_{t \in T} \sum_{l \in L_t} c_l y_{lt} \quad (25)\]

s.t.

\[x_{it} - \sum_{l \in L_t} a_{il} y_{lt} = 0, \quad \forall i \in A, \, t \in T \quad (26)\]

\[D_l - 1 \sum_{l \in L} \sum_{t = R_l} a_{il} y_{lt} \geq p_i, \quad \forall i \in A \quad (27)\]

\[-\sum_{l \in L_t} y_{lt} \geq -1, \quad \forall t \in T \quad (28)\]

\[x_{it} \leq 1 \quad \forall i \in A, \, t \in T \quad (29)\]

\[y_{lt} \geq 0 \quad \forall t \in T, \, l \in L_t \quad (30)\]

\[x_{it} \geq 0 \quad \forall i \in A, \, t \in T \quad (31)\]

The resulting dual (DLMRP) is:

\[\max \sum_{i \in A} p_i u_i - \sum_{t \in T} v_t + \sum_{i \in A} \sum_{t \in T} z_{it} \quad (32)\]

s.t.

\[\sum_{i \in A} a_{il} (u_i - w_{it}) - v_t \leq c_l, \quad \forall t \in T, \, l \in L_t \quad (33)\]

\[w_{it} + z_{it} \leq 0, \quad \forall i \in A, \, t \in T \quad (34)\]

\[w_{it} \in \mathbb{R}, \quad \forall i \in A, \, t \in T \quad (35)\]

\[u_i \geq 0, \quad \forall i \in A \quad (36)\]

\[v_t \geq 0, \quad \forall t \in T \quad (37)\]

Therefore, the reduced cost of a column \(y_{lt}\) is:

\[\bar{c} = c_l - \sum_{i \in A} a_{il} (u_i) - v_t\]
Column Generation

Initialisation \( (\mathcal{L}_0, \text{LB}=0, \ldots) \)

Solve the Master Problem

\[ \text{dual variables } (u, v, w) \]

Generate/Add columns with \( \overline{c} < 0 \)

\[ \text{activity sets } \overline{L}_t + \text{insertion time } t \]

New? \( \text{YES} \rightarrow \text{LB} = f(u,v,w) \)

\( \text{NO} \rightarrow \text{Subproblem solution} \rightarrow \text{Any LP solver} \)
Subproblem

all dual values $u_i, v_t, w_{it}$

problem data $a_{it}, b_i, \rho^{-1}$

$\rightarrow$ \[ \text{best time } t' \]

$\rightarrow$ \{ \text{best task set } l' \}

Decision variables needed: $\alpha_i, \beta_t, \gamma_{it}, \in \{0, 1\}$

max $\sum_{i \in A} \alpha_i u_i - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \rho^{-1}(\sum_{i \in A} \alpha_i b_i)$ \ (38)

s.t.

$\alpha_i + \beta_t \leq 1, \quad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i$ \ (39)

$\sum_{t \in T} \beta_t = 1$ \ (40)

$\gamma_{it} - 0.5\alpha_i - 0.5\beta_t \leq 0, \quad \forall i \in A, t \in T$ \ (41)

$\gamma_{it} - \alpha_i - \beta_t \geq -1, \quad \forall i \in A, t \in T$ \ (42)
Subproblem

all dual values $u_i, v_t, w_{it}$ problem data $a_{it}, b_i, \rho^{-1}$ } \rightarrow \rightarrow \{ \text{best time } t' \}

best task set \( l' \)

Decision variables needed: \( \alpha_i, \beta_t, \gamma_{it}, \in \{0, 1\} \)

$$\begin{align*}
\max & \sum_{i \in A} \alpha_i u_i - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \rho^{-1} \left( \sum_{i \in A} \alpha_i b_i \right) \\
\text{s.t.} & \quad \alpha_i + \beta_t \leq 1, \quad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i \\
& \quad \sum_{t \in T} \beta_t = 1 \\
& \quad \gamma_{it} - 0.5 \alpha_i - 0.5 \beta_t \leq 0, \quad \forall i \in A, t \in T \\
& \quad \gamma_{it} - \alpha_i - \beta_t \geq -1, \quad \forall i \in A, t \in T
\end{align*}$$
Subproblem SP2 with fixed $t$

A predefined time $t'$
related dual values $u_i, w_{it'}$
problem data $a_{it}, b_i, \rho^{-1}$

$\to$

Decision variables needed: $\alpha_i \in \{0, 1\}$

$$\max \sum_{i \in A} \alpha_i(u_i - w_{it'}) - v_{t'} - \rho^{-1}\sum_{i \in A} \alpha_i b_i$$

(43)

s.t.

$$\alpha_i \leq a_{it'}, \quad \forall i \in A$$

(44)

Note: $v_{t'}$ is constant
Subproblem SP2 with fixed $t$

A predefined time $t'$ related dual values $u_i, w_{it'}$ problem data $a_{it}, b_i, \rho^{-1}$

$\begin{align*}
\{ & \sum_{i \in A} \alpha_i (u_i - w_{it'}) - v_{t'} - \rho^{-1}(\sum_{i \in A} \alpha_i b_i) \\
\text{s.t.} & \alpha_i \leq a_{it'}, \quad \forall i \in A
\end{align*}$

Decision variables needed: $\alpha_i \in \{0, 1\}$

Note: $v_{t'}$ is constant
Subproblem SP3

Given a predefined time $t'$, a predefined sector $s$, related dual values $u_i, w_{it'}$, problem data $a_{it}, b_i, \rho^{-1}$, the decision variables needed are:

$$\alpha_i \in \{0, 1\}$$

The objective is to maximize:

$$(SP4_{\tilde{t}, \tilde{s}}) \max \left( \sum_{i \in A} \alpha_i (u_i - w_{it} - \tilde{a}_{\tilde{s}} b_i) \right) - v_{\tilde{t}} - \tilde{b}_{\tilde{s}}$$

Subject to:

$$x_{\tilde{s}}^{\min} \leq \left( \sum_{i \in S} b_i \alpha_i \right) \leq x_{\tilde{s}}^{\max}$$

$$\alpha_i \leq a_{it}, \quad \forall i \in A$$

$$\alpha_i \in \{0, 1\}, \quad \forall i \in A$$
Subproblem SP3

A predefined time $t'$
A predefined sector $s$
Related dual values $u_i, w_{it'}$
Problem data $a_{it}, b_i, \rho^{-1}$

\[
\left\{ \begin{array}{c}
\text{Decision variables needed: } \alpha_i \in \{0, 1\} \\
\end{array} \right. \\
\right. \rightarrow \text{best task set } l'
\]

\[
(SP_{4_{\tilde{t}, \tilde{s}}}) \max \left( \sum_{i \in A} \alpha_i (u_i - w_{i\tilde{t}} - \tilde{a}_s b_i) \right) - v_{\tilde{t}} - \tilde{b}_s
\] (45)

\[
\text{s.t.} \\
\]

\[
x_{\tilde{s}}^{\min} \leq \left( \sum_{i \in S} b_i \alpha_i \right) \leq x_{\tilde{s}}^{\max} \\
\]

(46)

\[
\alpha_i \leq a_{i\tilde{t}}, \quad \forall i \in A \\
\alpha_i \in \{0, 1\}, \quad \forall i \in A
\] (47) (48)
Subproblem SP4

A predefined sector $s$
related dual values $u_i, w_{it'}$
problem data $a_{it}, b_i, \rho^{-1}$

$\rightarrow \quad \rightarrow$

best time $t'$
best task set $l'$

Decision variables needed: $\alpha_i, \beta_t, \gamma_{i,t} \in \{0, 1\}$

$\begin{align*}
(SP3_s) \max & \sum_{i \in A} \alpha_i (u_i - \tilde{a}_s b_i) - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_{it} - \tilde{b}_s \\
\text{s.t.} & \quad x_s^{\min} \leq (\sum_{i \in S} b_i \alpha_i) \leq x_s^{\max} \\
& \quad \alpha_i + \beta_t \leq 1, \quad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i \\
& \quad \sum_{t \in T} \beta_t = 1
\end{align*}$
Subproblem SP4

A predefined sector $s$
related dual values $u_i, w_{it'}$
problem data $a_{it}, b_i, \rho^{-1}$

$\rightarrow$ \begin{align*}
  &\text{best time } t' \\
  &\text{best task set } l'
\end{align*}

Decision variables needed: $\alpha_i, \beta_t, \gamma_{i,t} \in \{0, 1\}$

\[
(\text{SP3}_{\tilde{s}}) \quad \max \sum_{i \in A} \alpha_i (u_i - \tilde{a}_s b_i) - \sum_{i \in A} \sum_{t \in T} \gamma_{it} w_{it} - \sum_{t \in T} \beta_t v_t - \tilde{b}_s
\] (49)

s.t.

\[
x_s^{\min} \leq (\sum_{i \in S} b_i \alpha_i) \leq x_s^{\max}
\] (50)

\[
\alpha_i + \beta_t \leq 1, \quad \forall i \in A, t \leq r_i - 1 \text{ or } t \geq d_i
\] (51)

\[
\sum_{t \in T} \beta_t = 1
\] (52)
**Subproblem Variants**

Notation a-b-c-d

a) type of subproblem
   1  for fixed t
   2  for variable t

b) column adding policy
   1  at instant t
   2  at all feasible instants
   3  only at feasible instants of negative reduced cost

c) multiple sets? (only available for a=1)
   0  stop the pricer as soon as one column is added
   1  try to generate multiple columns before exiting

d) time increment (only available for a=1)
   0  restart from $t = 0$
   1  restart from the instant where the last pricing stopped
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4. Focus on a scheduling problem with an energy source
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5. Conclusion
Branch-and-Price

- Column Generation

Initialisation ($\mathcal{L}_0$, LB=0, ...)

Solve the Master Problem

- dual variables $(u,v,w)$

Generate/Add columns with $\tilde{c} < 0$

- activity sets $\tilde{L}_t$ + insertion time $t$

YES

New ?

NO

$LB = f(u,v,w)$

- Branch-and-Price = Branch-and-Bound + Column Generation
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2 Literature review

3 Resolution scheme

4 Focus on a scheduling problem with an energy source
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5 Conclusion
Preliminary results and best parameters identification

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<td>1-2-0-0</td>
<td>53</td>
</tr>
<tr>
<td>1-3-0-0</td>
<td>52</td>
</tr>
<tr>
<td>1-1-0-0</td>
<td>41</td>
</tr>
</tbody>
</table>

- Basic settings: 2-1-0-0 (subproblem 1) and 1-1-0-0 (subproblem 2)
- All alternatives produced improvements over their respective basic settings (i.e., all 2-X-X-X are better than 2-1-0-0. All 1-X-X-X are better than 1-1-0-0)

- 100 instances among 288
- time limit = 600 s
# Preliminary results and best parameters identification

<table>
<thead>
<tr>
<th></th>
<th>Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-1-1-0</td>
</tr>
<tr>
<td><strong>Ratio</strong></td>
<td>min 92.80 %</td>
</tr>
<tr>
<td></td>
<td>max 100 %</td>
</tr>
<tr>
<td></td>
<td>avg 99.79 %</td>
</tr>
<tr>
<td><strong>NbCols</strong></td>
<td>min 113</td>
</tr>
<tr>
<td></td>
<td>max 26507</td>
</tr>
<tr>
<td></td>
<td>avg 3236.3</td>
</tr>
<tr>
<td><strong>NbPrice</strong></td>
<td>min 6</td>
</tr>
<tr>
<td></td>
<td>max 761</td>
</tr>
<tr>
<td></td>
<td>avg 78.0</td>
</tr>
<tr>
<td><strong>Nbnodes</strong></td>
<td>min 1</td>
</tr>
<tr>
<td></td>
<td>max 488</td>
</tr>
<tr>
<td></td>
<td>avg 35.6</td>
</tr>
</tbody>
</table>

- 100 instances among the 288, time limit = 600 s
## Final results

<table>
<thead>
<tr>
<th></th>
<th>1-1-1-1</th>
<th>Compact</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Branch &amp; Price</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NbObs</td>
<td>288</td>
<td>288</td>
</tr>
<tr>
<td>NbOpt</td>
<td>261</td>
<td>0</td>
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<tr>
<td>Ratio</td>
<td>77.47 %</td>
<td>57.82 %</td>
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<tr>
<td></td>
<td>100%</td>
<td>83.54 %</td>
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<tr>
<td></td>
<td><strong>99.81%</strong></td>
<td>69.60 %</td>
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<tr>
<td></td>
<td>max</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>57.85 %</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>99.99 %</td>
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<td></td>
<td>min</td>
<td>69.60 %</td>
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<tr>
<td></td>
<td>max</td>
<td>84.21 %</td>
</tr>
<tr>
<td>Time</td>
<td>&lt; 1 s</td>
<td>&lt; 0.1 s</td>
</tr>
<tr>
<td></td>
<td>3603 s</td>
<td>2 s</td>
</tr>
<tr>
<td></td>
<td><strong>540 s</strong></td>
<td>1927 s</td>
</tr>
<tr>
<td></td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>avg</td>
</tr>
<tr>
<td></td>
<td>avg</td>
<td>avg</td>
</tr>
<tr>
<td>Nbnodes</td>
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<td>1</td>
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<tr>
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<tr>
<td></td>
<td>min</td>
<td>max</td>
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<tr>
<td></td>
<td>max</td>
<td>avg</td>
</tr>
<tr>
<td></td>
<td>avg</td>
<td>avg</td>
</tr>
</tbody>
</table>

- All 288 instances, time limit = 3600 s + node limit
<table>
<thead>
<tr>
<th></th>
<th>Introduction</th>
<th>Literature</th>
<th>Resolution scheme</th>
<th>Focus</th>
<th>Conclusion</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>Literature review</td>
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<tr>
<td>3</td>
<td>Resolution scheme</td>
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<tr>
<td>4</td>
<td>Focus on a scheduling problem with an energy source</td>
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</tr>
<tr>
<td></td>
<td>- Problem definition</td>
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</tr>
<tr>
<td></td>
<td>- Dantzig-Wolfe decomposition</td>
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<td></td>
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<tr>
<td></td>
<td>- Branch-and-Price</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Results</td>
<td></td>
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</tr>
</tbody>
</table>

**Conclusion**
Conclusion

Done

- Energy sources characteristics in (combinatorial) optimization pbs.
- Resolution scheme: piecewise bounding and integer programming
- Impact of non-reversible sources functions: aggregability, ...
- Complexity analysis, Extended formulation, Dantzig-Wolfe Decomposition, Branch-and-price

What next?

- Reversible energy sources
  - Previous theorems no longer valid and no direct adaptation
  - No clustering! Time horizon?
- Non-energy-related problems?
Integration of a real-world efficiency function
References and Acknowledgements

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S.U. Ngueveu, C. Artigues, P. Lopez,
Scheduling under multiple energy resources
*PGMO-COPI’14 accepted.*