Optimizing energy allocation using lot sizing models (OPAL)

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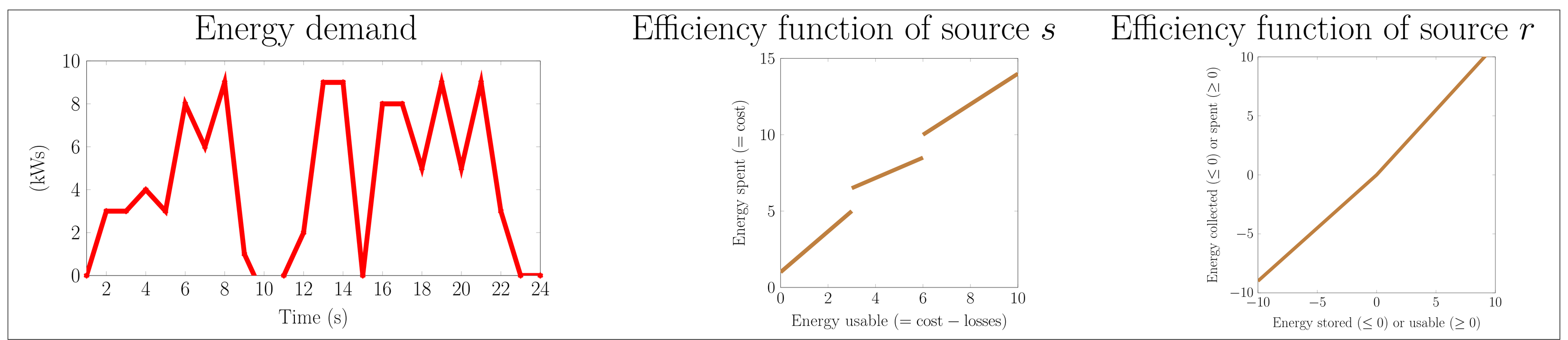
Introduction

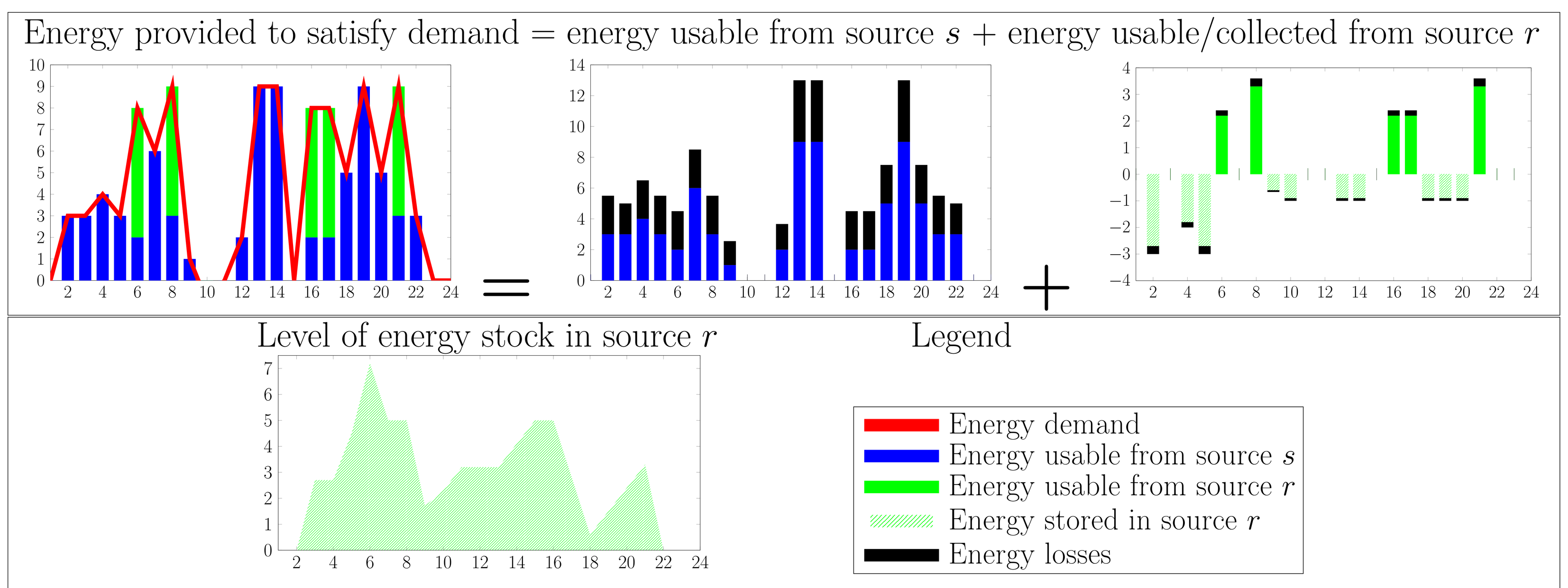
- The aim of the project is to study the **allocation problem of renewable and non-renewable energy sources** as a **new variant of single-item lot sizing problem**.
- State-of-the-art methods from the literature no longer apply because of the **piecewise linear inventory conversion functions**. Other characteristics to take into account are (i) continuous production/inventory quantities, (ii) no inventory cost, (iii) inventory bounds and (iv) piecewise linear production costs.
- Solving the problem efficiently would contribute to the solution of more complex scheduling problems involving energy constraints/objectives via **decomposition methods**.

Problem with a single energy source of each type

Example of data

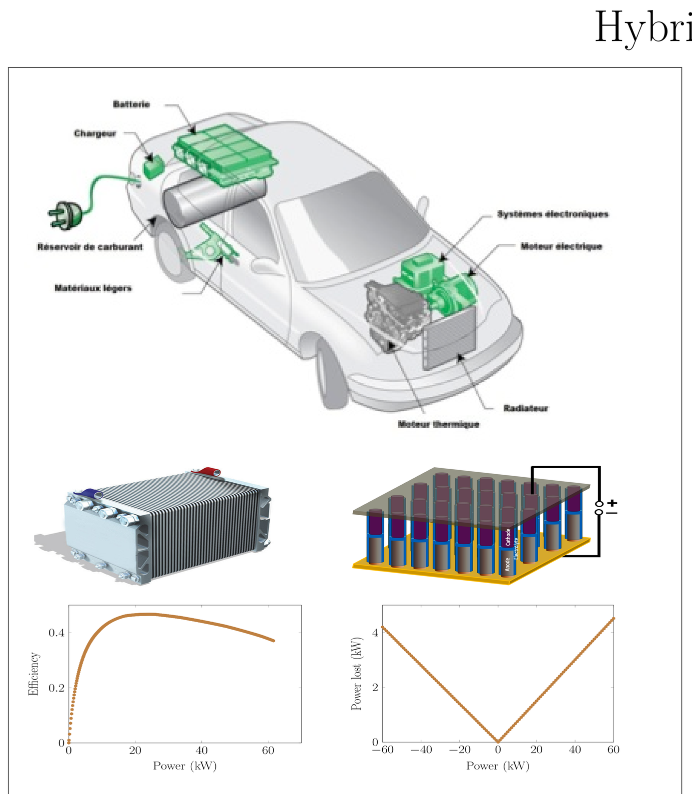


Example of solution

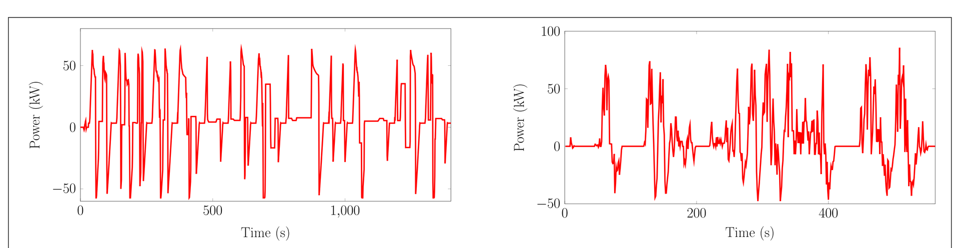


Some applications

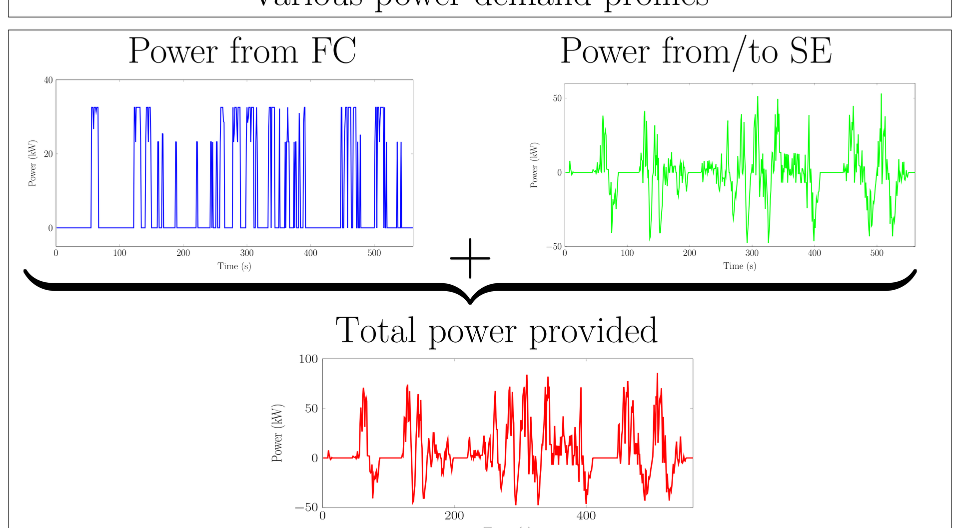
Hybrid-Electric Vehicles



Multiple energy sources available for the vehicle propulsion

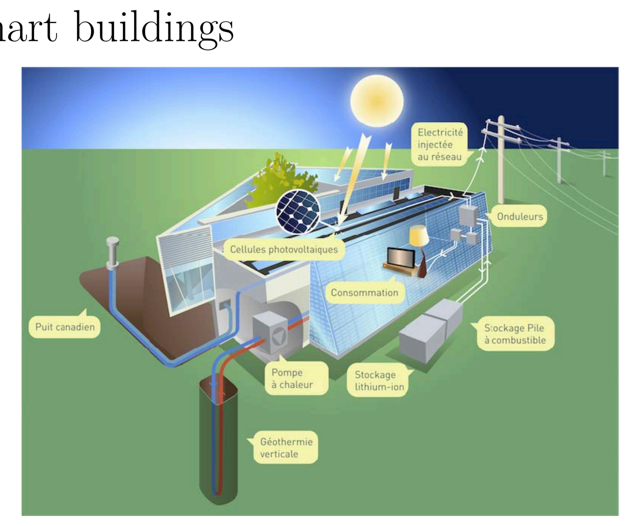


Various power demand profiles



Objective: find best power split among sources

Smart grids in smart buildings



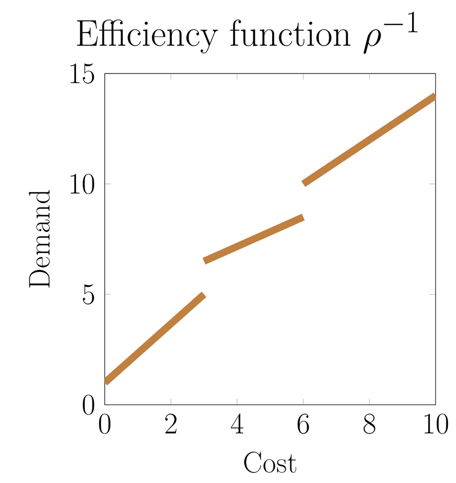
The smart building ADREAM of the LAAS

Objective: find the best energy management strategy

Generic simplified problem: ideal reversible energy source

Data

- Set of time periods $\mathcal{T}$
- d_t: fixed energy demand at time period t
- One non-reversible energy source s
  - ρ: piecewise-linear efficiency function for source s (x-axis = cost, y-axis = demand and $\rho(x) = 0, \forall x < 0$).
- One ideal reversible energy source r
  - C: storage capacity for r
  - initial assumptions:
    - negligible losses (identity efficiency function) for r
    - zero initial stock in source r



Decision variables

- x_t: energy produced by non-reversible energy source s at time period t
- s_t: energy inventory carried in reversible source r from period t to period $t+1$.

Our Problem Formulation (P)

$$(\mathrm{P}) \min \sum_{t=1}^{|\mathcal{T}|} \rho^{-1}(x_t) \quad (1)$$

$$\text{s.t. } x_t - s_t + s_{t-1} = d_t, \quad \forall t \in \mathcal{T} \quad (2)$$

$$s_t \leq C, \quad \forall t \in \mathcal{T} \quad (3)$$

$$x_t, s_t \in \mathbb{R}^+, \quad \forall t \in \mathcal{T} \quad (4)$$

(P) vs classic lot sizing

- (P) = uncapacitated lot sizing problem with inventory bounds, no holding costs, and stationary production costs
- if ρ is linear then (P) is a Linear Program that can be solved in $O(T)$.
- if ρ has a fixed cost then (P) can be solved in $O(T^2)$ ([2,4]).
- if ρ is piecewise concave then (P) can be solved in $O(T^2)$ ([1]).
- if ρ is a constant or a monotonic general piecewise function, then its complexity is **unknown**. This complexity depends on the structure of ρ, some special cases are addressed in [5].

Methodology and expected contributions

Analysis

- Complexity analysis, identification of properties, notable special cases, dominance rules/relations
- Identify the best suited computational models and design efficient solution methods

Possible extensions

- **Piecewise linear losses (γ) in energy source** r: replace (2) with (5)-(6).

$$x_t + y_t = d_t \quad (5)$$

$$s_t = s_{t-1} - \gamma(y_t) \quad (6)$$

- Time dependent costs, different time horizons introducing new constraints such as battery aging

Benchmarks with real-world energy sources

- Introduction of more realistic consumption/production curves
- Tests on data sets of realistic power demand profiles for hybrid electric vehicles from researchers in Electrical Engineering

Expected Contributions

- Solution of the resulting new variants of lot sizing problems with energy sources
- Lot-sizing-based decomposition method(s) for solving complex scheduling problems with energy sources

References

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