

Relationships between Petri nets and constraint graphs: application to manufacturing

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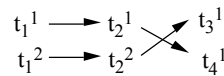
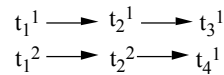
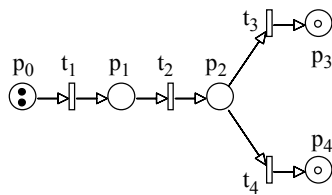
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Objective

- **(Time(d)) Petri nets seen as (T)Constraint Satisfaction Problems**
- **TCSP algorithms and Constraint Propagation used in the context of Petri nets**
- **Clear separation between discrete (logic) and continuous (time)**
- **Reverse the classical approach (Linear prog. and then Tree based search) : Discrete constraints are satisfied first**
- **Simultaneously solve Assignment and Scheduling**

Ordinary Petri net and CSP

- **Petri net defines precedence relations in a procedural way**
 - They are not explicitly enumerated

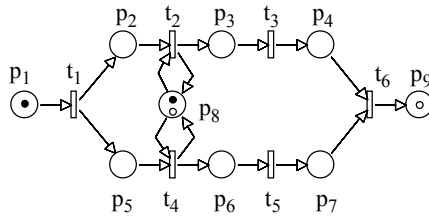


- **Two consistent sets of precedence relations**
- **Avoid interleaving (firing sequence)**

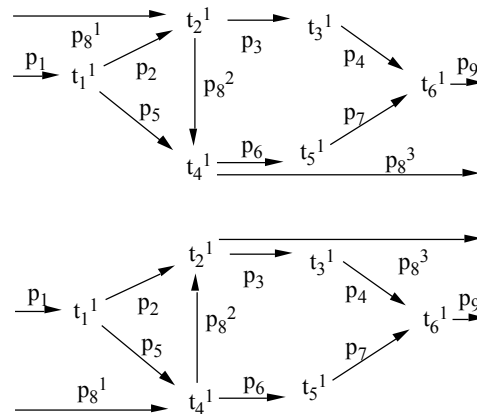
Linear logic and Petri nets

- **Equivalence between reachability in Petri nets and the provability of specific sequents in Linear logic (Girard's)**
- **Tokens have to be produced before being consumed**
- **Checked during the proof => precedence relations derived**
- **One proof => one consistent set of precedence relations (A fragment of one process in the Petri net unfolding)**
- **One proof = a sequence of decisions i.e. conflict (tokens and transitions) resolutions**

An example



Conflict t2 t4 : decision
Two proof trees
Two partial orders

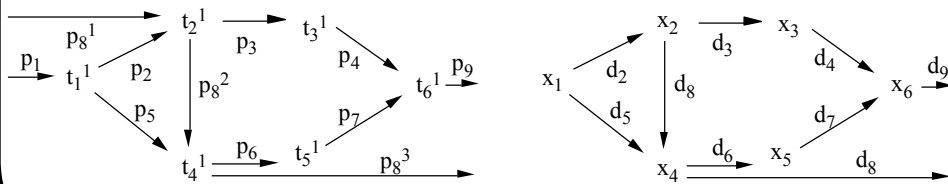


Introducing time

- **If one consistent set of precedence relations has been selected time constraints are introduced in a linear way (AOA-graphs)**
- **The following steps are executed:**
 - Petri net model (static definition of manufacturing system and specification of how resource assignments can be done)
 - From the current marking to another marking => defines an assignment/scheduling problem
 - Derive one partial order (heuristic based?) => a set of consistent precedence relations
 - Take continuous time constraint into account => Activity on arcs graphs (AOA)
 - Use Linear Programming or Constraint Propagation (arc consistency) on AOA

p-timed Petri nets => AOA

- d_i minimal sojourn time in place p_i
- Transition firing nodes => firing dates (variables)
- Arc labels (place names) => constraints (duration: $x_3 \geq x_2 + d_3$)



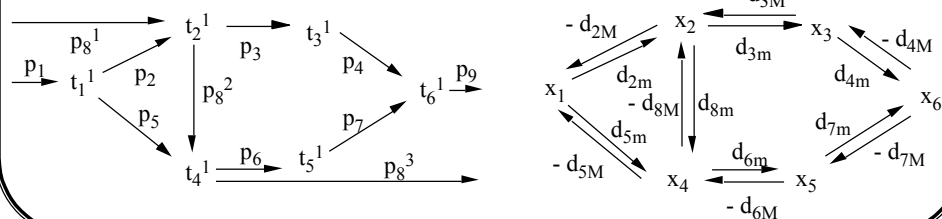
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p-time Petri net => AOA

- d_{im} minimal d_{iM} maximal sojourn time in place p_i
- Transition firing nodes => firing dates (variables)
- Arc labels (places) => constraints (duration) (2 arcs each place

$$x_3 \geq x_2 + d_{3m} \text{ and } x_3 \leq x_2 + d_{3M}$$

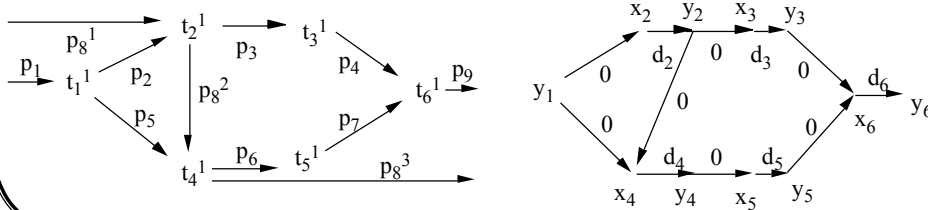


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t-timed Petri nets => AOA

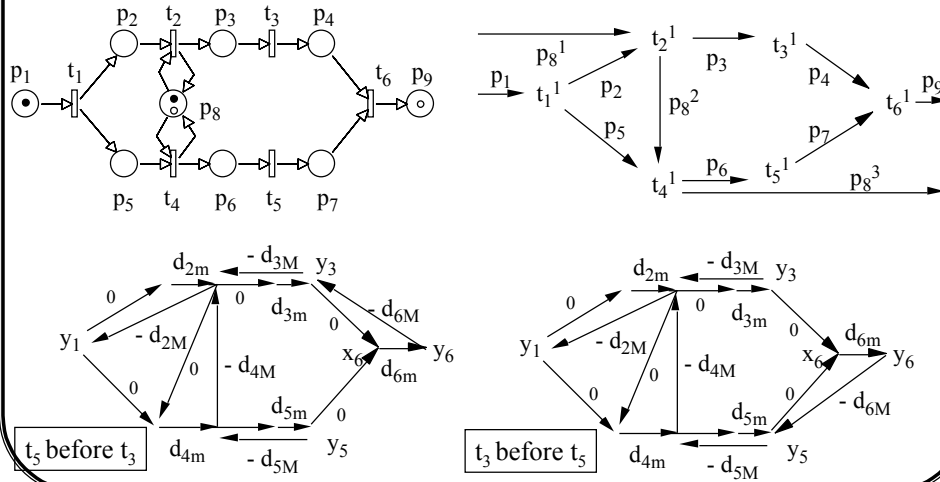
- d_i firing duration attached to each transition
- Each node broken down into 2: x: "begin firing" y: "end firing"
- Transition arc ($y_2 \geq x_2 + d_2$), token arc ($x_3 \geq y_2$)



t-time Petri nets => AOA (1)

- d_{im} minimal, d_{iM} maximal enabling duration transition t_i
- Each node broken down into 2:
x: "begin enabling" y: "end firing"
- Transition arc ($y_2 \geq x_2 + d_2$), token arc ($x_3 \geq y_2$)
- "Backward arcs" from "end firing" of t_i to the end of firing producing the last token (t_i becomes enabled when it arrives)

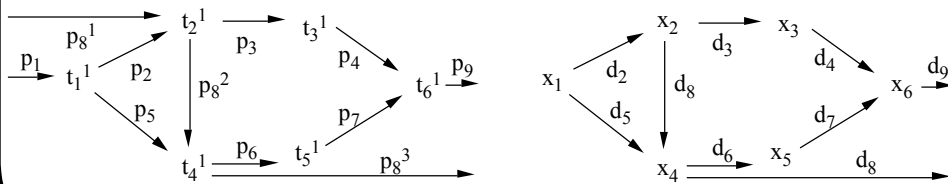
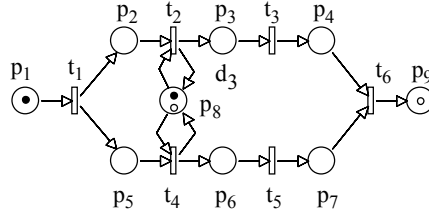
t-time Petri nets => AOA (2)



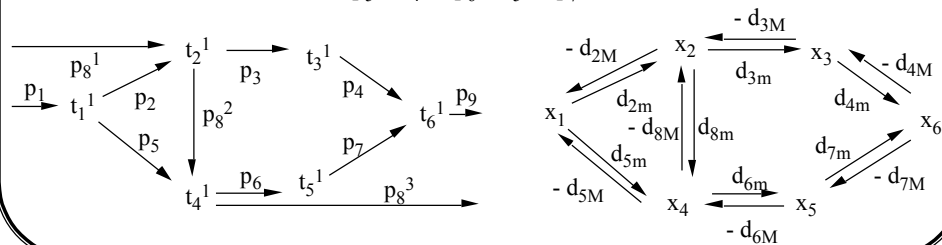
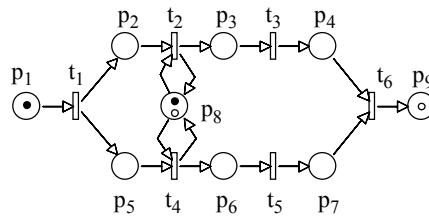
Conclusion

- **Comparing the time extensions of Petri nets / constraints:**
 - p-timed, t-timed and t-time Petri nets are such that derived AOA-graphs are without positive oriented circuits, "logical" reachability entails the existence of at least one sequence verifying all the temporal constraints
 - t-time Petri nets are not adequate because it is necessary to know the firing sequence (combinatorial explosion)
 - p-time Petri nets are the more general (possibility of temporal inconsistencies) and AOA graphs have the same structure as the precedence graphs
- **From a Petri net model: simultaneous assignment + scheduling**
- **A basis to elaborate hybrid approaches (Petri net + Constraints)**

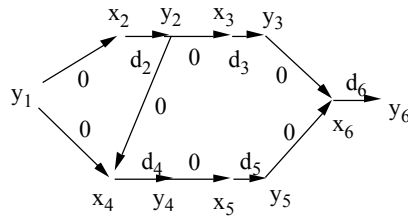
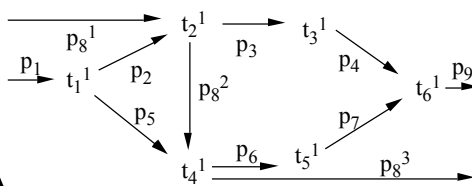
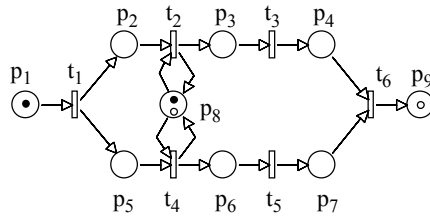
p-timed Petri nets => AOA



p-time Petri net => AOA



t-timed Petri nets => AOA



t-time Petri nets

