Modelling, identification and control of a denitrifying biofilter

S. Bourrel a, D. Dochain a,*, J.P. Babary b, I. Queinnec b

aCESAME, Université Catholique de Louvain Bât. Euler, av. G. Lemaître 4-6, 1348 Louvain-La-Neuve, Belgium
bLAAS/CNRS 7 av. du Colonel Roche, 31077 Toulouse Cedex 4, France

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Abstract

Biological denitrification in a fixed bed reactor is a process gaining popularity in the area of water treatment. The advent of more stringent criteria on nitrate and nitrite concentrations of treated water (along with the need to control and optimize the addition of a carbon source needed to denitrify) make the use of dynamical models able to represent the behaviour of the process not only of interest; it is also compulsory for optimizing control. The paper is concerned with the process dynamic modelling, the parameter identification and the control of the fixed bed biofilter with real-life experiments.

Keywords: Bioprocess; Fixed-bed reactor; Adaptive linearizing control

1. Introduction

Denitrification plays a key role in the treatment of drinkable water, and is largely motivated by the possibly high concentrations of nitrogen in drinkable water which may be particularly detrimental to human health. Indeed the quality of the presently available drinkable water is largely determined by the damaging effects of human activities, e.g. the intensive use of nitrogen-based fertilizers in agriculture and for domestic use. The regulations about the nitrogen content of drinkable water and their control tend to be more and more stringent.

Fixed bed biofilters are destined to play an increasing role in the denitrification of drinkable water. The main advantages of fixed bed biofilters are the compactness of the process, their efficiency for biological water treatment, and their low energy consumption. On the other hand, there is a progressive clogging of the filter during the treatment due to biomass growth and retention of suspended particles. Therefore the filter has to be washed frequently (when the pressure drop reaches an upper limit): this induces a cyclic behaviour, which is the main drawback of biofilters.

A submerged granular fixed bed biofilter can be considered as a continuous flow bioreactor, in which the water to be treated flows through a cylindrical reactor. The microorganisms fixed in the reactor absorb the nutrients in such a way that the substrate concentrations (here, the nitrate and nitrite) decrease in the outflowing water.

The need for efficient controllers in order to optimize the operation of such processes is obvious. Strictly speaking, the dynamics of such processes are described by partial differential equations [derived from material balance considerations (first principle models)], i.e. the dynamical model is a distributed parameter one. Moreover, as any biological reactor, the dynamics are intrinsically nonlinear. If the distributed parameter nature of the process dynamics appears to be non-negligible (due to spatial concentration gradients or transport delays), it is important to take it into account in the design of controllers. In addition to the process nonlinearities, the distributed parameter dimension of the process makes the control problem even more difficult and requires a careful design.

This paper is concerned with the design and application of an adaptive linearizing controller that intends to compensate for the lack of modelling accuracy (basically about the process kinetics) and of reliable on-line sensors for the key process variables (here, the biomass and the ethanol concentrations). The paper is organised as follows.

A description of the biofilter is given in Section 2. Section 3 is devoted to the dynamical modelling via a general mathematical model and a simplified model of the process behaviour. The identification of the process parameters will be the topic of Section 4. And finally
Section 5 is concerned with the control design and the illustration of its performance in numerical simulation and real life experiments.

2. Description of the biofilter

The denitrification process under study is a submerged granular fixed bed biofilter (see Fig. 1); it is packed with pouzzolane, a very porous volcanic material which retains a large amount of microorganisms during the washing. Thus the initial biomass concentration is often very close to the maximum active biomass concentration. The substrates to be removed can freely move along the reactor. The inside temperature is kept almost constant via a double insulation jacket. An ethanol solution is mixed with the feeding nitrate in a tank at the inlet of the reactor. Eight measurement points are distributed at every 30 cm along the column for the measurement of nitrate and nitrite concentrations.

The control variable can be the feeding flowrate $F(t)$ (or equivalently the superficial velocity $u(t) = \frac{F(t)}{A}$, $A$ being the cross-section of the biofilter) or the ethanol concentration $S_{C,\text{eth}}(t)$ at the inlet of the reactor ($z = 0$).

Preliminary hydraulic experiments showed that the axial dispersion is negligible, and that the biofilter can be described by a series of 20–30 stirred tank reactors and can be considered to be running in plug flow conditions.

3. Modelling of the biofilter

3.1. The dynamical model

The denitrification processing is defined as being the reduction of nitrates (electron acceptor) $\text{NO}_3^-$ into gaseous nitrogen $\text{N}_2$ by using organic carbon (electron donor) with production of intermediate compounds, namely nitrites $\text{NO}_2^-$. In order to dissociate nitrates and nitrites, two sequential biological reactions are considered in the model, denitratation (1) and denitritation (2):

$$\text{NO}_3^- \xrightarrow{(1)} \text{NO}_2^- \xrightarrow{(2)} \text{N}_2$$

- The denitratation [(1) $\iff$ (nitrate reductase)], in terms of reduction degree is written as follows:

$$\frac{1}{Y_{h1}} \text{carbon} + \frac{1}{\alpha_1} \frac{1 - Y_{h1}}{Y_{h1}} \text{nitrates} \rightarrow \text{biomass} + \frac{1}{\alpha_2} \frac{1 - Y_{h2}}{Y_{h2}} \text{nitrites}$$

For $\frac{1}{Y_{h1}}$ moles [Chemical oxygen demand (COD)] provided by the organic substrate, one mole (COD) of biomass is produced and the rest is used by the nitrates (COD conversion coefficient: $\alpha_1 = 1.14$) in order to be oxidized in nitrites (COD conversion coefficient: $\alpha_2 = 1.71$).

Fig. 1. Experimental denitrification process.
The denitrification [(2) \(\leftrightarrow\) (nitrite reductase)], in terms of reduction degree, is written as follows:

\[
\frac{1}{Y_{h2}} \text{carbon} + \frac{1}{\alpha_2} \frac{1 - Y_{h2}}{Y_{h2}} \text{nitrites} \rightarrow \text{biomass} + \text{nitrogen}
\]

For \(\frac{1}{Y_{h2}}\) moles (COD) provided by the organic substrate, one mole (COD) of biomass is produced. The above reaction equations are based on the IAWQ model [1]. The denitrification is performed under anaerobic conditions.

The dynamics of the biofilter can be deduced from mass balance considerations for the different components: nitrate \(S_{\text{NO}_3}\), nitrite \(S_{\text{NO}_2}\), carbon \(S_c\) and active biomass \(X_a\). In addition to the above two reactions of denitrification and denitrification, we have considered, in accordance with the experimental reality, a death/detachment reaction for the active biomass. The mass balances of the above components through an infinitesimal volume of Adz result in the following dynamical partial differential equations (PDE):

\[
\begin{align*}
\frac{\partial S_{\text{NO}_3}(z, t)}{\partial t} &= F \frac{\partial S_{\text{NO}_3}(z, t)}{\partial z} - \frac{1 - Y_{h1}}{1.14 Y_{h1}} \mu_{\text{NO}_3}(S_{\text{NO}_3}, S_c) X_a(z, t) \\
\frac{\partial S_{\text{NO}_2}(z, t)}{\partial t} &= -F \frac{\partial S_{\text{NO}_2}(z, t)}{\partial z} + \frac{1 - Y_{h1}}{1.14 Y_{h1}} \mu_{\text{NO}_3}(S_{\text{NO}_3}, S_c) X_a(z, t) \\
\frac{\partial S_c(z, t)}{\partial t} &= -F \frac{\partial S_c(z, t)}{\partial z} - \frac{1}{Y_{h1}} \mu_{\text{NO}_3}(S_{\text{NO}_3}, S_c) X_a(z, t) \\
\frac{\partial X_a(z, t)}{\partial z} &= (\mu_{\text{NO}_3}(S_{\text{NO}_3}, S_c) + \mu_{\text{NO}_2}, S_c)) \\
\left(1 - \frac{X_a(z, t)}{X_{a_{\text{max}}}}\right) X_a(z, t)
\end{align*}
\]

for \(0 < z \leq L\).

The boundary conditions, given at the input of the system, are written as follows:

\[
S_{\text{NO}_3}(z = 0, t) = S_{\text{NO}_3_{\text{in}}}(t), S_{\text{NO}_2}(z = 0, t) = S_{\text{NO}_2_{\text{in}}}(t), S_c(z = 0, t) = S_{c_{\text{in}}}(t).
\]

In the above equations, \(z\) represents the space variable, \(\varepsilon, Y_{h1}, Y_{h2}, \mu_{\text{NO}_3}\), and \(\mu_{\text{NO}_2}\) represent the porosity, the yield coefficients (g/g) and the specific growth rates (g/\(M^3/\text{h})\) of the denitrification and denitrifications reactions, respectively. Concentrations of substrates and biomass are expressed in g/m\(^3\) and the influent flow rate in m\(^3\)/h (or equivalently, the superficial velocity \(u\) in m/h). \(X_{a_{\text{max}}}\) is the maximal (g/M\(^3\)) active biomass concentration. The term \((\mu_{\text{NO}_3} + \mu_{\text{NO}_2}) \frac{X_a(z, t)}{X_{a_{\text{max}}}}\) represent the death/detachment of the biomass; it will be slightly modified in the control design section.

The specific growth rates are typically described with a double Monod-type model [2]:

\[
\mu_{\text{NO}_3}(S_{\text{NO}_3}, S_c) = \mu_{\text{NO}_3_{\text{max}}} \frac{S_{\text{NO}_3}}{S_{\text{NO}_3} + K_{\text{NO}_3}} \frac{S_c}{S_c + K_c} (2)
\]

\[
\mu_{\text{NO}_2}(S_{\text{NO}_2}, S_c) = \mu_{\text{NO}_2_{\text{max}}} \frac{S_{\text{NO}_2}}{S_{\text{NO}_2} + K_{\text{NO}_2}} \frac{S_c}{S_c + K_c} (3)
\]

where \(\mu_{\text{NO}_3_{\text{max}}}\) and \(\mu_{\text{NO}_2_{\text{max}}}\) are the maximum values of the specific growth rates \(\mu_{\text{NO}_3}\) and \(\mu_{\text{NO}_2}\), \(K_{\text{NO}_3}\) and \(K_{\text{NO}_2}\) are the affinity constants associated to nitrate, nitrite and carbon, respectively. The above kinetic expressions will be considered in the simulation model (used to test the performance of the control algorithm), and in the identification procedure (performed for the calibration of the model parameters), but not in the control design, due to the large uncertainty associated to the process kinetics (e.g. [2]). A more detailed model taking biological reaction schemes and deep filtration into account, that includes six state variables (nitrate, nitrite, carbon, porosity, active biomass and total biomass) has been developed and published [3].

3.2. The lumped parameter model

For simulation needs, the nonlinear distributed parameter model Eq. (1) can be transformed into an ordinary differential equation system by applying an orthogonal collocation method, which is one of the weighted residual methods [4,5]. The choice of this method has been dictated by two main reasons. First of all, this method is widely used and accepted in chemical engineering [6], especially for the reduction of dynamical models of tubular reactors; secondly, orthogonal collocation is known to be an efficient and powerful method, provided that the base functions and other parameters are carefully chosen [7,8]. Furthermore, it offers the advantage that its implementation is fairly easy: the minimization of the residual function is carried out without any integral or averaging computation (the weights are delta functions applied at given collocation points). Another advantage is that the nature and the dimension of state variables remain unchanged after the reduction procedure. Finally, it has been shown that...
mass and heat balance equations are preserved, at least in steady-state [7].

The collocation method consists of expanding each state variable as a finite sum of products of time functions and space functions, as shown in the following expression $S(z, t) = \sum_{i=0}^{N} l_i(z)S_i$. The implementation of the collocation method needs the choice of three parameters: the base functions $l_i(z)$, the position $z$ of collocation points and the number $N$ of internal collocation points. A certain number of quantitative and qualitative rules must be applied [9]. Here, we have considered the Lagrange interpolation polynomials as base functions. The best choice of $z$ and $N$ has been found by comparing the model solution with the solution obtained by using a finite difference method (see [3]). So, the lumped parameter model has been obtained by setting:

$$S_{NO_i}(z, t) = \sum_{i=0}^{N} l_i(z)S_{NO_i}, \quad S_C(z, t) = \sum_{i=0}^{N} l_i(z)S_C$$  (4)

$$S_{NO_2}(z, t) = \sum_{i=0}^{N} l_i(z)S_{NO_2}, \quad X_a(z, t) = \sum_{i=0}^{N} l_i(z)X_a$$  (5)

with $S_{NO_i} = S_{NO}(z = z_i, t)$, $S_{NO_i} = S_{NO}(z = z_i, t)$, $S_C(z = z_i, t)$, $X_a = X_a(z = z_i, t)$.

4. Parameter identification study

Different sets of steady-state experimental data of the concentrations of nitrate, nitrite and in some instances, of carbon, are available. These have been used for parameter identification. Note from Eq. (1) that in steady-state $X_a = X_{a,max}$ (and can then be considered as a parameter) and the porosity $\varepsilon$ is known. In steady-state, the model Eq. (1) is written as follows:

$$-\frac{F}{A}\frac{dS_{NO_i}(z)}{dz} - \frac{1 - Y_{h_i}\mu_{NO_i}(S_{NO_i}, S_C)X_{a,max}}{1.14Y_{h_i}} = 0$$
$$-\frac{F}{A}\frac{dS_{NO_2}(z)}{dz} - \frac{1 - Y_{h_i}\mu_{NO_2}(S_{NO_2}, S_C)X_{a,max}}{1.14Y_{h_i}} = 0$$
$$-\frac{1 - Y_{h_i}\mu_{NO_3}(S_{NO_3}, S_C)X_{a,max}}{1.14Y_{h_i}} = 0$$

4.1. Structural identifiability

4.1.1. Introduction

The notion of structural identifiability is related to the possibility of giving a unique value to each parameter of a mathematical model. In simple words, the structural identifiability of a model can be formulated as follows (a rigorous definition can be found e.g. in [10]): given a model structure and perfect data (i.e. data that fit the model perfectly) for model variables, are all the parameters of the model identifiable? From the structural identifiability analysis one may conclude that only combinations of the model parameters are identifiable. If the number of resulting combinations is lower than the original model parameters, or if there is not a one-to-one relationship between both parameter sets, then a-priori knowledge about some parameters may be required to achieve identifiability.

Different approaches for testing the structural identifiability of the model have been considered:

- the Taylor series expansion which consists of expanding the output function $y(z)$ around $z = 0$ [10,11], and then of calculating successive derivatives of this function, to check if they contain information about the parameters (or combinations of parameters) to be identified.
- Another approach for analysing the structural identifiability is to transform the nonlinear model into a model linear in the parameters, and then look at the identifiability of the linear model (e.g. [12]).
- The third considered approach is based on the theory of the observability of nonlinear systems: the study of the identifiability properties of a nonlinear system can be reduced to the study of the observability properties of the nonlinear system augmented by an auxiliary vector containing the model parameters to be identified [13].

The identifiability analysis has been performed in four cases (see Table 1): when the values of the concentrations of the three components are available and when only the nitrate and nitrite concentrations are available, and when the values of the concentrations at the reactor input are available additionally and when they are not. This is in accordance with the typically encountered experimental situations: indeed, for some experiments,
the ethanol has not been measured; it was also found out in some cases that there was a high uncertainty on the determined value of the concentrations at the reactor input.

The detailed calculations of the identifiability analysis are presented in [14]. For each approach, a symbolic calculation software (e.g. MAPLE V) must be used. In the following subsection, as an illustration, we present the second approach when only the nitrate and nitrite concentrations are available at a certain number of measurement points (including the reactor input).

4.1.2. Application of the nonlinear model transformation

Let us first consider in the following the more compact notation:

\[ S_{NO_3} \equiv S_1, \quad S_{NO_2} \equiv S_2, \quad S_C \equiv S_3, \quad \mu_{NO_{\text{max}}} \equiv \mu_1, \quad \mu_{NO_{\text{max}}} \equiv \mu_2, \quad X_{\text{max}} \equiv X, \quad 1.14 = \alpha_1 \]

and 1.71 = \alpha_2.

System Eq. (6) may then be rewritten under the following equivalent form:

\[ \frac{dS_1(z)}{dz} = -\frac{A}{\alpha_1 F} \frac{1 - Y_{h_1}}{Y_{h_1}} \mu_1 X \frac{S_1}{S_1 + K_{NO_3}} \frac{S_3}{S_3 + K_C} \quad (7) \]

\[ \frac{dS_2(z)}{dz} + \frac{dS_3(z)}{dz} = -\frac{A}{\alpha_2 F} \frac{1 - Y_{h_2}}{Y_{h_2}} \mu_2 X \frac{S_1}{S_2 + K_{NO_2}} \frac{S_3}{S_3 + K_C} \quad (8) \]

\[ \frac{dS_3(z)}{dz} = -\frac{A}{\alpha_1 F} \frac{1}{Y_{h_1}} \mu_1 X \frac{S_1}{S_1 + K_{NO_3}} \frac{S_3}{S_3 + K_C} \]

\[ -\frac{A}{\alpha_2 F} \frac{1}{Y_{h_2}} \mu_2 X \frac{S_1}{S_2 + K_{NO_2}} \frac{S_3}{S_3 + K_C} \quad (9) \]

The advantage of the above formulation for the identifiability analysis is that each equation contains a ratio of polynomial functions of \( S_1, S_2 \) and \( S_3 \) which are different for each equation. Moreover it avoids some redundancy [which appears with the original formulation Eq. (6)] in the parameter sets of the first two equations.

By multiplying each equation by its denominator and expanding the different terms, the system can be rewritten as follows:

\[ S_1 S_2 S_3 \frac{dS_1}{dz} + \beta_1 S_1 \frac{dS_1}{dz} + \beta_2 S_1 \frac{dS_3}{dz} + \beta_1 \beta_2 \frac{dS_1}{dz} = -\frac{1}{\alpha_1 u} \beta_3 S_1 S_3 \quad (10) \]

\[ S_2 S_3 \left( \frac{dS_1}{dz} + \frac{dS_3}{dz} \right) + \beta_1 S_3 \frac{dS_1}{dz} + \beta_2 S_3 \frac{dS_3}{dz} + \beta_2 S_3 \frac{dS_1}{dz} + \beta_1 S_3 \frac{dS_3}{dz} = -\frac{1}{\alpha_2 u} \beta_5 S_3 S_2 S_3 \quad (11) \]

\[ S_1 S_2 S_3 \frac{dS_3}{dz} + \beta_1 \beta_4 S_1 \frac{dS_3}{dz} + \beta_1 \beta_2 S_2 \frac{dS_3}{dz} + \beta_2 \beta_3 S_3 \frac{dS_1}{dz} + \beta_1 S_2 S_3 \frac{dS_3}{dz} + \beta_4 S_3 S_3 \frac{dS_1}{dz} + \beta_2 S_2 S_3 \frac{dS_3}{dz} = -\frac{1}{u} (\beta_6 + \beta_7) S_1 S_3 - \frac{1}{u} \beta_4 \beta_6 S_1 S_3 \]

\[ -\frac{1}{u} \beta_2 \beta_7 S_2 S_3 \]

\[ \beta_1 = K_C \]

\[ \beta_2 = K_{NO_1} \]

\[ \beta_3 = \frac{1 - Y_{h_1}}{Y_{h_1}} \mu_1 X \]

\[ \beta_6 = \frac{1}{Y_{h_1}} \mu_1 X \]

\[ \beta_4 = K_{NO_2} \]

\[ \beta_7 = \frac{1}{Y_{h_2}} \mu_2 X \]

This system is linear with respect to parameters that are sets of parameters \( \beta_i \) and depends on substrate concentrations. Since the regressors (i.e. the functions multiplying the unknown parameters \( \beta_i \)) are formally independent, the structural identifiability of parameters \( \beta_i \) follows. Let us write the expressions of the initial model parameters in terms of the parameters \( \beta_{i=1,2,...} \):

\[ K_{NO_1} = \beta_2 \]

\[ \mu_1 X = \beta_6 - \beta_3 \]

\[ Y_{h_1} = \beta_1 \]

\[ K_{NO_2} = \beta_4 \]

\[ K_C = \beta_1 \]

\[ \mu_2 X = \beta_7 - \beta_5 \]

\[ Y_{h_2} = \beta_6 - \beta_7 \]

We can then conclude that the above seven parameters \( (K_{NO_1}, K_{NO_2}, K_C, \mu_1 X, \mu_2 X, Y_{h_1}, Y_{h_2}) \) are identifiable. Note in particular that only the products \( \mu_1 X \) and \( \mu_2 X \) are identifiable.

4.2. Identification

4.2.1. Methodology and experimental data

The identification method used is based on the minimization of a quadratic performance index by using the Marquardt method [15] modified for the ordinary differential equation case (see [16]); the knowledge of the sensitivity functions with respect to the estimated parameters is needed. The performance index is a quadratic function of the error between calculated and measured concentrations of nitrate and nitrite at some internal points of the bioreactor and at the outlet.

A preliminary sensitivity analysis showed that the output of the model is relatively insensitive to changes in the saturation constants \( K_{NO_1}, K_{NO_2} \) and \( K_C \) of the Monod model. Consequently, we decided to fix the values of these parameters to values given in literature. So only four parameters have to be identified: \( \mu_{\text{max}}_{i=1,2} X_{\text{u}_{\text{max}}} \) and \( k_{h_{i=1,2}} \)
Component concentration data were obtained from five sets of planned experiments performed on the pilot-scale denitrification process, each set corresponding to various different operating conditions. In a first step, the identification study was made for each operating condition, i.e. identification with one set of data and validation with another one corresponding to the same operating condition. This preliminary study leads to four sets of estimated parameters. However, the aim of parameter identification is to validate a unique model under different operating conditions. So, from these four sets of parameters, a unique set of parameters has been determined by calculating their statistical means.

In the following we present the validation obtained from two sets of experiments, where the superficial velocity of the flowrate, $u(t)$, is equal, respectively, to 4 and 6 m/h and in which the value of the inlet ethanol concentration has been chosen so as to have a ratio of carbon over nitrogen (C/N) equal to 1.5. The nitrogen concentration at the inlet was equal to 16.93 g [N]m$^{-3}$ ($S_{1,\text{in}}=16.93$ g[N]m$^{-3}$, $S_{2,\text{in}}=0$ g[N]m$^{-3}$). The parameter identification has been performed with the following numerical values for the affinity constants, the influent flow rate and the biofilter section:

\[ K_{NO_2} = 2 \text{ g} \text{ m}^{-3}, \quad K_{NO_3} = 3 \text{ g} \text{ m}^{-3}, \quad K_C = 40 \text{ g} \text{ m}^{-3} \]
\[ F = 0.014 \text{ m}^3\text{h}^{-1}, \quad A = 0.0036 \text{ m}^2 \]

Finally the identification algorithm has been implemented so as to allow the identification of the initial conditions $S_1(z=0)$, $S_2(z=0)$ and $S_3(z=0)$ (by considering them as unknown parameters).

### 4.2.2. Numerical results

Table 2 summarizes the values obtained by identification with and without the estimated initial conditions. The values of the estimated parameters are in line with those generally considered and admitted in the scientific literature.

The estimated parameters have been validated by considering the remaining sets of experimental data. Figs. 2 and 3 represent the steady-state profiles of substrate concentrations compared with the experimental data for two experiments (at eight points of the bioreactor) after parameter identification.

The mean values of parameters have been considered for each set of data: if the quality of the validation does not prove to be perfect for some operating conditions (see Fig. 3), on the whole, the results are qualitatively satisfactory enough to be used for model simulation (see [14]).

Let us finally make some comments about the steady-state aspect of the above identification study in the context of an automatic control study (that basically relies on the knowledge of the process dynamics). Here the use of the steady-state model cannot be considered as a major drawback. First of all, recall that the model has a strong physical basis since it is derived from mass balances, and that as mentioned e.g. in [3], each dynamical equation is indeed composed of two terms: one for the hydrodynamics (via the convection term $-F \frac{\partial S}{\partial z}$) and one for the conversion (via the kinetics terms, e.g. $-1 \left[1 - Y_1 \right] \mu_{NO_3}(S_{NO_3}, S_C)X_a(z, t)$ for $S_{NO_3}$).

Part of the dynamics (the hydrodynamics) are known, and the identification concentrates only on the conversion term. This situation is very different to the classical identification problem when the whole process dynamics is unknown.

Finally, the identified kinetics will be used only for dynamical simulation and not for control design, because of the large uncertainty associated with the process kinetics in bioreactors [2]. This inherent uncertainty is reinforced in our application by the following two features: the unidentifiability of the parameters $\mu_{NO_3 \text{ max}}, \mu_{NO_2 \text{ max}}$ and $X_a \text{ max}$ (as shown above), and the difficulty to fit the model to the experimental data for the considered operating conditions with a unique set of model parameters (see [14]).

### 5. Control problem

#### 5.1. Statement of the control problem

The objective is to control the total concentration of nitrates and nitrites $y_L$ at the reactor output at some desired value $y_d$ (defined in accordance with the regulations on drinkable water)

\[ y_L = S_{NO_3} + S_{NO_2} \]  

(14)
Fig. 2. Identification results from one experiment $u = 4$ m/h and C/N = 1.5.

Fig. 3. Identification results from one experiment $u = 6$ m/h and C/N = 1.5.
each concentration being given in g[N]/m³. Two possible control inputs can be a priori considered: the influent flow rate \( F(t) \) and the ethanol concentration \( S_{c,\text{in}}(t) \) at the reactor input.

The design of the control algorithm takes the following assumptions into account:

1. concentrations of nitrate and nitrite are measured at the input and at the output of the reactor, and possibly at some internal points along the reactor;
2. the biomass concentration is not available for online measurement;
3. the ethanol concentration is measured only at the reactor input;
4. the specific growth rates \( \mu_{NO} \) and \( \mu_{NO} \) are unknown;
5. the yield coefficients and the death/detachment coefficient are known.

In the following, we shall consider two adaptive linearizing control algorithms corresponding to each control input. In both cases, the late lumping approach (see [17]) will be used, i.e. the design of the control law is based on the distributed parameter model and the space–time discretization is used only for numerical implementation.

The advantage of this type of method lies in the fact that the design of the control law takes into account the properties of the original dynamical model.

By combining the first two equations of model Eq. (1) with expression Eq. (14), the dynamical equation of the variable to be controlled is as follows:

\[
\frac{d\hat{y}_L}{dt} = -\frac{F}{\varepsilon A} \frac{\partial y(z,t)}{\partial z} \bigg|_{z=z_L} + \frac{k_2}{\alpha_2 \varepsilon} \mu_{NO_{\text{L}}} X_{aL} \tag{15}
\]

with \( k_2 = \frac{1}{\gamma} \).

5.2. Flow rate adaptive linearizing control

5.2.1. Controller design

Let us consider the following linear stable first-order closed loop dynamics:

\[
\frac{d\hat{y}_L}{dt} = \lambda (y_d - y_L) \text{ with } \lambda > 0 \tag{16}
\]

Combining Eq. (15) with Eq. (16) and assumptions 2 and 4, the following adaptive linearizing control law is derived:

\[
F(t) = -\lambda (y_d - y_L) + \frac{k_2}{\alpha_2 \varepsilon} \mu_{NO_{\text{L}}} \hat{X}_{aL} \tag{17}
\]

with:

\[
\lambda = \frac{1}{\varepsilon A} \frac{\partial y(z,t)}{\partial z} \bigg|_{z=z_L} \tag{18}
\]

This law does not depend on the ethanol concentration at the outlet of the reactor. Only one specific growthrate and the biomass concentration have to be estimated online.

5.2.2. Estimation of the biomass concentration

Let \( \hat{\xi}_1 \) be the vector of measured state variables:

\[
\hat{\xi}_1 = \hat{\xi}_1(z,t) = \begin{bmatrix} S_{NO_2}(z,t) \\ S_{NO_2}(z,t) \end{bmatrix} \tag{19}
\]

and \( \hat{\xi}_2 \) be the state variable (biomass concentration) to be estimated:

\[
\hat{\xi}_2 = \hat{\xi}_2(z,t) = X_a(z,t) \tag{20}
\]

Remark 1. Remind the active biomass concentration equation:

\[
\frac{dX_a(z,t)}{dt} = \mu_{NO_2} X_a(z,t) + \mu_{NO_2} X_a(z,t) - k_d(S_{NO_1}, S_{NO_2}, S_{C}, X_o) X_a(z,t)
\]

where \( k_d \) is equal to \( f \) from (2):

\[
k_d(S_{NO_1}, S_{NO_2}, S_{C}, X_o) = (\mu_{NO_1} + \mu_{NO_2}) \frac{X_a(z,t)}{X_{a_{\text{max}}}}
\]

The variations of the coefficient \( k_d \) can be assumed to be negligible \((1.12 < k_d < 1.17)\). Therefore, for the design of an asymptotic observer, \( k_d \) will be considered as a constant parameter. This assumption allows to eliminate the nonlinear term \( X_a(z,t)^2 \) in the equation of the biomass concentration and thus makes the design of the asymptotic observer possible.

The first two equations and the last equation of the dynamical model Eq. (1) can then be equivalently represented in the following matrix form:

\[
\frac{d\hat{\xi}_1}{dt} = K_{\hat{\xi}_1} \varphi - \frac{F}{\varepsilon A} \frac{d\hat{\xi}_1}{dz} \tag{21}
\]

\[
\frac{d\hat{\xi}_2}{dt} = K_{\hat{\xi}_2} \varphi - k_d \hat{\xi}_2 \tag{22}
\]

where \( K_{\hat{\xi}_1}, K_{\hat{\xi}_2} \) and \( \varphi \) are equal to:

\[
K_{\hat{\xi}_1} = -\frac{1}{\epsilon} \left[ \begin{array}{ccc} \frac{k_2 - 1}{\alpha_2 \varepsilon} & 0 \\ -\frac{k_2 - 1}{\alpha_1 \varepsilon} & \frac{k_2 - 1}{\alpha_2 \varepsilon} \end{array} \right] \quad K_{\hat{\xi}_2} = [1 \ 1] \cdot \varphi = \begin{bmatrix} \mu_{NO_1} X_o \\ \mu_{NO_2} X_o \end{bmatrix} \tag{23}
\]

The estimation of the biomass concentration \( X_a(= \hat{\xi}_2) \) can be performed by using an asymptotic observer [2] [18]. It is based on a state partition and the definition of a vector \( \xi = \xi(z,t) \) as a linear combination of the state variables, whose dynamics are independent of the reaction rate vector \( \varphi \):
\[ \xi = \xi_2 + A_0 \xi_1 \]  
with \( A_0 \) solution of:

\[ A_0 K_2 + K_2 = 0 \text{ and } K_2 \text{ full rank} \]  

This gives here:

\[ A_0 = [a_1 \ a_2] = \left[ \begin{array}{c} \alpha_1 (k_2 - 1) + \alpha_2 (k_1 - 1) \\ (k_1 - 1)(k_2 - 1) \end{array} \right] \]  

This transformation leads to the asymptotic observer of the biomass concentration:

\[ \frac{\partial z}{\partial t} = -k_0 z + a_1 S_{NO_3}(z, t) + a_2 S_{NO_2}(z, t) \]  

\[ \hat{x}_{a1} = \xi - a_1 S_{NO_3} - a_2 S_{NO_2} \]  

### 5.2.3. Estimation of specific growth rates

The specific growth rate \( \mu_{NO_2} \) can be written under the following form:

\[ \mu_{NO_2} = \beta_{NO_2} S_{NO_2} \]  

which simply expresses that there is no denitrification in absence of nitrite, say \( \mu_{NO_2} = 0 \) if \( S_{NO_2} = 0 \). \( \beta_{NO_2} \) is a parameter (possibly state dependent) which is assumed to be unknown. It will be estimated on the basis of the output Eq. (15) by using a recursive least square algorithm with a forgetting factor \( \gamma \), which is written as follows in discrete time:

\[ \hat{\beta}_{NO_2_{z+1}} = \hat{\beta}_{NO_2_z} - \gamma \frac{k_2 - 1}{\alpha_2 \epsilon} S_{NO_2_z} \hat{x}_{a1} \Delta t (y_{L_{z+1}} - y_{L_z}) \]  

\[ \gamma = \frac{\gamma_{t-1}}{\sigma + \gamma_{t-1}^2 \Delta t^2 \left( \frac{k_2 - 1}{\alpha_2 \epsilon} \right)^2 S_{NO_2_z}^2 \hat{x}_{a1}^2} \quad 0 < \sigma \leq 1 \]  

### 5.2.4. Simulation results

The numerical implementation of the control law requires the computation of space derivatives. This can be done by using:

- either the orthogonal collocation method:

\[ \frac{\partial S_{NO_3}(z, t)}{\partial z} \bigg|_{z_L} \approx \sum_{j=1}^{N+1} \frac{d l_j(z)}{d z} S_{NO_3}(z_j, t) \]  

\[ \Delta S_{NO_3}(z_L, t) \]  

\[ \frac{d l_j(z)}{d z} \bigg|_{z_L} \]  

\[ S_{NO_3,n} \]  

- or a finite difference method.

In both cases, several measurement points are needed; from a practical point of view, this is a drawback for the implementation. It has been shown (through simulation runs and experiments on the biofilter) that it is possible to use the following (global finite difference) approximation:

\[ \frac{\partial S_{NO_3}(z, t)}{\partial z} \bigg|_{z_L} \approx \frac{S_{NO_3} - S_{NO_3,n}}{L} \]  

\[ = \frac{S_{NO_3}}{L_{\Delta S_{NO_3}(z_L, t)}} + \frac{1}{L_{b_{NO_3}}} S_{NO_3,n} \]  

\[ \frac{d l_j(z)}{d z} \bigg|_{z_L} \]  

\[ S_{NO_3,n} \]  

Simulation runs have been carried out with the following initial conditions:

\[ \forall i = 1, \ldots, N+1 \left\{ S_{NO_3}(z_i, t = 0) = 16.93 [N/m^3] \right\} \]  

\[ S_{NO_3}(z_i, t = 0) = 0 [N/m^3] \]  

\[ S_{C}(z_i, t = 0) = 101.5 [COD/m^3] \]  

\[ X_{a}(z_i, t = 0) = 625 [COD/m^3] \]  

The superficial velocity is \( u = 9 \text{ m/h} \) and the ratio \( C/N = 1.5 \). The values of parameters are:
Let us now consider the control of the nitrite and nitrate concentrations by acting on the influent ethanol concentration. First note that the control variable $S_{\text{C,in}}(t)$ does not appear in the output Eq. (15). One possible solution is to consider the ethanol mass balance equation of the dynamical model Eq. (1).

5.3.1. Controller design

The ethanol mass balance equation can be rewritten as follows:

$$
\frac{dS_C(z,t)}{dt} = \frac{F}{\varepsilon A} \frac{\partial S_C(z,t)}{\partial z} \bigg|_{z=z_L} + C_1 \frac{dS_L}{dr}
$$

Then the output dynamical equation can be rewritten as follows:

$$
\frac{dy_L}{dt} = \frac{F}{\varepsilon A} \frac{\partial y_L}{\partial z} \bigg|_{z=z_L} + C_1 \frac{dS_L}{dr}
$$

with $b_{0n+1}$ defined in Eqs. (32)–(35). The control input then explicitly appears in the output equation. Let us define:

$$
C_1 = \frac{k_1 k_2}{k_2 \alpha_2 \varepsilon}, \quad C_2 = \frac{1}{k_2} \frac{k_2 - 1}{\alpha_2}
$$

Then the output dynamical equation can be rewritten as follows:

$$
\frac{dy_L}{dt} = \frac{F}{\varepsilon A} \frac{\partial y_L}{\partial z} \bigg|_{z=z_L} + C_1 \frac{dS_L}{dr}
$$

By combining Eqs. (15) and (36), the dynamical output equation becomes:

$$
\frac{dy_L}{dt} = \frac{F}{\varepsilon A} \frac{\partial y_L}{\partial z} \bigg|_{z=z_L} + C_1 \frac{dS_L}{dr} + \frac{1}{k_2} \frac{k_2 - 1}{\alpha_2} \frac{dS_L}{dr} + \frac{1}{k_2} \frac{k_2 - 1}{\alpha_2} \frac{F}{\varepsilon A} \frac{\partial S_C(z,t)}{\partial z} \bigg|_{z=z_L}
$$

Whatever the chosen discretization method (orthogonal colocation or finite differences), $\frac{\partial S_C(z,t)}{\partial z} \bigg|_{z=z_L}$ can be approximated as follows:

$$
\frac{\partial S_C(z,t)}{\partial z} \bigg|_{z=z_L} \approx \Delta S_C + b_{0n+1} S_{\text{C,in}}(t)
$$

with $b_{0n+1}$ defined in Eqs. (32)–(35). The control input then explicitly appears in the output equation. Let us define:

$$
C_1 = \frac{k_1 k_2}{k_2 \alpha_2 \varepsilon}, \quad C_2 = \frac{1}{k_2} \frac{k_2 - 1}{\alpha_2}
$$

Then the output dynamical equation can be rewritten as follows:

$$
\frac{dy_L}{dt} = \frac{F}{\varepsilon A} \frac{\partial y_L}{\partial z} \bigg|_{z=z_L} + C_1 \frac{dS_L}{dr} + \frac{1}{k_2} \frac{k_2 - 1}{\alpha_2} \frac{dS_L}{dr} + \frac{1}{k_2} \frac{k_2 - 1}{\alpha_2} \frac{F}{\varepsilon A} \frac{\partial S_C(z,t)}{\partial z} \bigg|_{z=z_L}
$$
The control is readily derived by combining Eqs. (42) and (16):

\[
\frac{d y_L}{dt} = - \frac{F}{\varepsilon A} \left( \frac{\partial y(z, t)}{\partial z} \bigg|_{z=0} - C_2 \Delta S_{C_L} \right) + C_1 \mu_{NO_2} X_{al} \\
+ C_2 \frac{d S_{C_L}}{dt} + C_2 \frac{F}{\varepsilon A} b_{in} S_{C_{in}}(t)
\]

The control is readily derived by combining Eqs. (42) and (16):

\[
S_{C_{in}}(t) = \frac{\varepsilon A}{C_2 F b_{in}} \left[ \dot{y}(y_d - y_L) + \frac{F}{\varepsilon A} \left( \frac{\partial y(z, t)}{\partial z} \bigg|_{z=0} - C_2 \Delta S_{C_L} \right) - C_2 \frac{d S_{C_L}}{dt} - C_1 \mu_{NO_2} \dot{X}_{al} \right]
\]

This control law requires now estimated values of ethanol and biomass concentrations, and of the specific growth rate $\mu_{NO_2}$ at the reactor output.

### 5.3.2. Estimation of ethanol and biomass concentrations

Let us consider again an asymptotic observer. The auxiliary variable is defined as a linear combination of the state variables:

\[
\zeta = \xi_2 + A_0 \xi_1
\]

and the matrix $A_0$ defined in Eq. (24) then becomes:

\[
A_0 = \begin{bmatrix}
-\frac{\alpha_1 k_1 + \alpha_2 k_2}{k_1 - 1} & -\frac{\alpha_1 k_1}{k_1 - 1} \\
\varepsilon \frac{\alpha_1}{k_1 - 1} + \frac{\alpha_2}{k_2 - 1} & \varepsilon \frac{\alpha_1}{k_2 - 1}
\end{bmatrix}
\]

The dynamics of the auxiliary variable are given by the following equations:

\[
\frac{\partial \xi_1}{\partial t} = - \frac{F}{\varepsilon A} \frac{\partial \xi_1}{\partial z}
\]

\[
\frac{\partial \xi_2}{\partial t} = -k_d \xi_2 + h_3 \left( k_d - \frac{F}{\varepsilon A} \frac{\partial}{\partial z} \right) S_{NO_3} + h_4 \left( k_d - \frac{F}{\varepsilon A} \frac{\partial}{\partial z} \right) S_{NO_2}
\]

where

\[
h_3 = \varepsilon \left( \frac{\alpha_1}{k_1 - 1} + \frac{\alpha_2}{k_2 - 1} \right)
\]

\[
h_4 = \varepsilon \frac{\alpha_2}{k_2 - 1}
\]

The estimated values of biomass and ethanol concentrations are given by the following expressions from the on-line measurements of $S_{NO_3}$ and $S_{NO_2}$ and the computed values of $\zeta = [\xi_1 \xi_2]$:

\[
\hat{S}_{C}(z, t) = \xi_1(z, t) - h_1 S_{NO_3}(z, t) - h_2 S_{NO_2}(z, t)
\]
\[ \dot{X}_a(z, t) = \xi_2(z, t) - h_1 S_{NO_2}(z, t) - h_4 S_{NO_3}(z, t) \] (47)

where

\[ h_1 = -\left( \frac{\alpha_1 k_1}{k_1 - 1} + \frac{\alpha_2 k_2}{k_2 - 1} \right) \]

\[ h_2 = -\frac{\alpha_2 k_2}{k_2 - 1} \]

5.3.3. Estimation of the specific growth rates

In the control law Eq. (41), only the specific growth rate for the denitrification is needed. We can apply the same method as previously (Section 5.2.3). The growth rate can be written under the following form:

\[ \mu_{NO_2} = \beta_{NO_2} S_{NO_2} \] (48)

\[ \beta_{NO_3} \] will be estimated on the basis of the nitrate mass balance equation by using a recursive least square algorithm with a forgetting factor \( \sigma \), which is written as follows in discrete time:

\[ \hat{\beta}_{NO_2} = \hat{\beta}_{NO_2} + \gamma_1 C_1 S_{NO_2} \hat{X}_{a,t} \Delta t (y_{L,t+1} - y_L) \]

\[ + \Delta t \left( \Delta S_{NO_2} + \Delta S_{NO_3} + b_{0b_{01}}(S_{NO_2,m} + S_{NO_3}) \right) \]

\[ - C_2 \Delta t ^{\hat{S}_{C,t}} + b_{0b_{11}} S_{C,m}(t) - C_2 \Delta t \frac{d \hat{S}_{C,t}}{dt} \]

\[ - C_1 \Delta t S_{NO_2} \hat{X}_{a,t} \]

\[ \gamma_1 = \frac{\gamma_1 \Delta t}{\sigma + \gamma_1 \Delta t (1 - \alpha \gamma_1)^2} S_{NO_2}^2 \hat{X}_{a,t}^2 \]

(49)

5.3.4. Simulation results

Simulation results have been carried out under the same conditions as in the preceding case. In Figs. 6–9, where the collocation method has been used, estimation and control algorithms have been initialized from Eqs. (41)–(47) as follows:

\[ \xi_1(z_{N+1}, 0) = 11.80, \]

\[ = S_{C}(z_{N+1}, 0) + h_1 S_{NO_2}(z_{N+1}, 0) \]

\[ + h_2 S_{NO_3}(z_{N+1}, 0), \]

\[ \xi_2(z_{N+1}, 0) = 666.43 \]

\[ = (\delta X_a(z_{N+1}, 0) + X_a(z_{N+1}, 0)) + h_3 S_{NO_2}(z_{N+1}, 0) \]

\[ + h_4 S_{NO_3}(z_{N+1}, 0), \]

\[ \hat{S}_{NO_2}(z_{N+1}, 0) = 17.000[N]/m^3, \]

\[ \hat{S}_{NO_3}(z_{N+1}, 0) = 0.000[N]/m^3, \]

\[ S_{C}(z_{N+1}, 0) = 101.5g(COD)/m^3, \]

\[ X_a(z_{N+1}, 0) = 625g(COD)/m^3, \]

\[ \hat{X}_{a}(z_{N+1}, 0) = 0.189. \]

\[ \sigma = 0.98, \gamma_0 = 20, \lambda = 4h^{-1}, \Delta t = 0.25h \]

These simulations have been performed under the same conditions as the on-line implementation of the ethanol concentration control algorithm, as it is shown in the next section. The initial values of the auxiliary variables \( \xi_1 \) and \( \xi_2 \) have been chosen so as to correspond to the initial state of the reactor, plus some initial error (about 3%: \( \delta X_a(z_{N+1}, 0) \) about 20g(COD)/m^3) on the value of the active biomass \( X_a \). The values of the design parameters \( \sigma, \lambda \) and \( \gamma_0 \) have been chosen via numerous numerical simulations so as to give a fair compromise between sufficiently fast closed loop dynamics and good disturbance rejection. In particular, the value of \( \lambda \) corresponds to a closed loop time constant of 15 min. Fig. 6 shows disturbances on the influent flow rate and on the inlet nitrate concentration. Figs. 7–9 represent the output variable and the input ethanol concentration, estimated biomass and ethanol concentrations, \( \beta_{NO_2} \) and \( \beta_{NO_3} \) respectively.

Figs. 10–12 illustrate the adaptive linearizing controller when a global finite difference approximation is used. Note that the performance of the adaptive linearizing controller is similar with both approximations, i.e. a proper choice can still give closed loop performance with a (simplest) global finite difference, however at the price of poor specific growth rate estimation (see Fig. 11). Note also that the model mismatch is compensated via the integral action of parameter (see Fig. 11) and state estimation (see Fig. 12). However, because \( \hat{\beta}_{NO_3}(\mu_{NO_2} = \beta_{NO_2} S_{NO_2}) \) is the estimate of a physical parameter whose value can be interesting for process monitoring and diagnosis, it is interesting to recover values closer to the true ones. This can be done by considering better evaluation of the space derivatives of \( S_{NO_2}, S_{NO_3} \) and \( S_C \) and by using the prediction model \( (\dot{y}_{L,t+1}, \ldots, \dot{X}_{a,t}) \) in Eq. (40) comparison between the values computed in the control scheme and the ones computed with the new space derivatives gives a new value for \( \hat{\beta}_{NO_3} \).

One practical constraint is that the true values of the space derivatives are assumed to be unknown on-line: that is why we have decided to consider a polynomial approximation for each concentration \( S(z) \) on the basis of the steady-state data considered before:

\[ S(z) = az^2 + bz + c \] (50)

The second step consists of changing each term of spatial derivative of nitrate, nitrite and carbon concentrations in Eq. (13) by the following approximations:

\[ \Delta S_{NO_2} + b_{0b_{01}} S_{NO_2,m} \approx 2a_1 z_L + b_1 \]

\[ \Delta S_{NO_3} + b_{0b_{11}} S_{NO_3} \approx 2a_2 z_L + b_2 \]

\[ \Delta \hat{S}_{C,t} + b_{0b_{11}} S_{C,m}(t) \approx 2a_3 z_L + b_3 \] (51)

It has been found: \( a_1 = 8.45, b_1 = -25.00, a_2 = -7.90, b_2 = 13.16, a_3 = 21.32 \) and \( b_3 = -93.04 \).
Fig. 6. Adaptive linearizing control law; perturbations applied on the system.

Fig. 7. Adaptive linearizing control law (discretization by orthogonal collocation): —.— Desired value.
Fig. 8. Adaptive linearizing control law (discretization by orthogonal collocation): — simulated values; –– estimated values.

Fig. 9. Adaptive linearizing control law (discretization by orthogonal collocation): — real values; –– estimated values.
Fig. 10. Adaptive linearizing control law (discretization by global finite differences): — Desired value.

Fig. 11. Adaptive linearizing control law (discretization by global finite differences): — real values; – – estimated values.
As it is shown in Fig. 13, the second order polynomial approximation (derived from steady state concentration profiles) allows a rather fair estimation of the specific growth rate. The collocation approximation gives better results but several measurement points are needed along the biofilter. So, practically speaking, by using only two measurement points (input and output of the reactor) considered in the global finite difference for the control algorithm, the residual substrate concentrations have been successfully controlled on the pilot plant.

6. Experimental results

The experimental denitrification process is a fixed-bed biofilter of 2.1 m high (Fig. 1) as previously described. The polluting substrates (nitrite and nitrate) moved freely along the reactor, with a superficial velocity $\frac{U}{d}$ of 6 or 9 m/h. The ethanol solution supplied at the reactor inlet is considered hereafter as the control variable.

6.1. Nitrite and nitrate measurements

Nitrite and nitrate concentrations are measured by molecular absorption spectrometry (Technicon sensor); the automatic determination method uses a process of nitrate and nitrite reduction by segmented continuous flow. A set of electromagnetic valves is controlled by the microcomputer, allowing to handle measurements of concentrations. The duration of a cycle of measurements depends on the number of sampling points (from 10–20 min).

6.2. Computer system

A software package has been developed [19] to manage the different tasks relative to measurements, actions, parameter estimation, control action evaluation, graphic display, storage. This package has been implemented in C language on a PC compatible microcomputer (low equipment cost). An Advantech PC-LabCard PCL-812PG has been used for the communications between the microcomputer and the process.

6.3. Control objective

The control objective is expressed as a regulation problem of the sum of nitrate and nitrite concentrations at the outlet of the biofilter. The set point has been arbitrarily set to 2 g[N]/M$^3$ which is lower than the European standard (5.65 g[N]/m$^3$). In the current case, measurements have been performed at two points, the first one at 60 cm from the bottom of the column and the second one at 180 cm. It has been assumed that these two points are considered as the input and the output of the bioreactor. Indeed, measurements are not
Fig. 13. Parameter estimation (three types of approximation): — real values.

Fig. 14. Experimental results.
always reliable at the real input, and as the biofilter was overdimensioned, all the nitrate was already consumed at the outlet of the reactor.

6.4. Implementation form of space derivative

The choice for global finite differences approximation for discretizing spatial derivatives of concentrations was dictated by two main reasons:

- the closed-loop simulation study showed that this kind of approximation lead to less sensitive results for regulation performance,
- in industrial plants, measurements are generally available only at the input and the output of the biofilter.

The sampling period was equal to 15 min. The control variable, i.e. the influent ethanol concentration $S_{C,\text{in}}$ acted on the process by using a pump working at computer defined rates. It has been bounded from 10 to 189 g(COD)/m$^3$. Estimation and control algorithms have been initialized after simulation studies, as given in section 5.3.4.

6.5. Results

Several experiments have been carried out to examine the response of the control scheme to various practical stimuli. The first experiment presented here has been done with the superficial velocity $\frac{F}{A} = 9$ m/h. Fig. 14 (a) and (c) shows the nitrate and nitrite concentrations at the outlet of the biofilter, the setpoint and the controlled output and the control variable, respectively. The output has been converging towards the setpoint in 2 h. At this time, a perturbation occurred, due to the breakdown of a pressure pump. Some other disturbances took place between four and five hours which have not been identified. In spite of these disturbances, the setpoint has been reached and tracked. The control input increased towards its maximal value to force the output variable to reach the setpoint.

The second experiment (see Fig. 15) shows the closed-loop response after a load disturbance (the superficial

![Graph of nitrate and nitrite concentrations](image1)

![Graph of controlled variable](image2)

![Graph of control variable](image3)

Fig. 15. Experimental results.
velocity increases from 6 to 9 m/h after 1 h): the output converges towards the setpoint after a transient period of 2 h and Fig. 15 shows the evolution of the control variable and the measurements of nitrate and nitrite concentrations. The control action remained smooth and slowly decreased.

7. Conclusion

This paper was concerned with the mathematical modelling, the parameter identification and the control of a fixed bed denitrifying process.

The modelling lead to a nonlinear distributed parameter system of hyperbolic type. This system has been transformed into a lumped parameter model by applying a functional approximation method (orthogonal collocation).

In order to know whether this model is identifiable or not, an identifiability study was developed by using the nonlinear transformation of the model into a linear one with respect to sets of parameters to be estimated. The identification was based on the minimization of a quadratic performance index (the Marquardt method); this step was a very important one for dynamical simulation in view to performing control design.

The purpose of the control problem was to regulate the output total concentration of nitrates and nitrites by acting either on the superficial velocity or on the inlet ethanol concentration.

Adaptive linearizing controllers have been designed and tested in simulation before the on-line implementation on the process. In the experiments, the inlet ethanol concentration has been chosen to regulate the residual sum of nitrate and nitrite concentration at the outlet of the biofilter.

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