Robust optimal control of bilinear DC–DC converters

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A B S T R A C T

This paper addresses the control problem of dc–dc converters. The control law synthesis considered here exploits the potential of LMI-based control approaches, which allow to cope with model uncertainty, disturbances and bilinearities to synthesize simple state-feedback controllers with a priori guarantee of stability in a large domain of initial and operating conditions. The aim of the paper is to contribute with a robust control framework to deal with the common requirements of regulated dc–dc converters. The correctness of the results has been verified both with numerical simulations and with experimental measurements from a laboratory prototype.

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1. Introduction

Switched mode dc–dc converters are power electronics devices employed to adapt the voltage and current levels between sources and loads, while maintaining a low power loss in the conversion process (Erickson & Maksimovic, 1999). Such devices are usually regulated with a control subsystem that maintains the desired levels of output current or voltage, despite the uncertainty of the system and the distortions or parametric changes that might appear. The objectives of the control subsystem are (1) to maintain a tight regulation of the output, (2) to maximize the bandwidth of the closed-loop response in order to reject disturbances, (3) to satisfy desirable transient characteristics, as for example, to minimize the system settling time or overshoot, as well as (4) to ensure the convergence of the closed-loop system to the desired equilibrium point(s) during startup or in case of recovery from a system fault.

Despite the existence of some recent works on robust control of dc–dc converters using the hybrid modeling approach (Iannelli, Henrik Johansson, Jönsson, & Vasca, 2008), the models used to derive such a control strategy usually neglect the switching ripple, considering that the switching period is small enough. In conventional control approaches, these models are linearized at an operating point, in order to derive linear output or state-feedback laws, which are simpler and of lower cost than nonlinear approaches. Nevertheless, a design that disregards the nonlinearities of the converter, issued from the switching action, feedback and saturation of the duty-cycle, may result in deteriorated output or undesired behavior in presence of disturbances, and such approaches may require auxiliary circuits to ensure the stability of the converter during startup. In addition, converter dynamics are also affected by component uncertainty, since some parameters such as the energy storage elements or the load are usually time-dependent or partially unknown.

The aforementioned characteristics of dc–dc converters and the requirements and limitations of the control subsystem have prompted several authors to seek for control methods which can deal appropriately with performance requirements, nonlinearities and parametric uncertainty, while maintaining low design and implementation costs. Recent examples of robust control design for dc–dc converters are shown in Guesmi, Essounboui, and Hamzaoui (2008), Liping, Hung, and Nelms (2009), Geyer, Papafotiou, Frasca, and Morari (2008), Beccuti, Mariethoz, Cliquennois, Shu, and Morari (2009), El Fadil and Giri (2009), El Fadil, Giri, El Magueri, and Chaoui (2009), Montagner, Oliveira, Peres, Tarbouriech, and Queinnec (2007), Torres-Pinzon and Leyva (2009), Al-Rabadi and Alsindi (2011). Concretely, in Guesmi et al. (2008) and Liping et al. (2009), a fuzzy-PID control structure is proposed, which allows to consider several operating points depending on the fuzzy membership region. A model predictive control for a nonlinear converter is presented in Geyer et al. (2008) and Beccuti et al. (2009), where the online solution of the optimal control problem is implemented experimentally by means of a lookup-table. Robust nonlinear controllers which take into account the
bilinearity and the uncertainty of the converter are proposed in the rigorous references (El Fadil & Giri, 2009; El Fadil et al., 2009). Time-varying controllers based on linear matrix inequality techniques (LMIs) are presented in Montagner et al. (2007) and Torres-Pinzon and Leyva (2009), while the neural approach of robust control of converters is combined with the advantages of LMI optimization in Al-Rabadi and Alsmadi (2009, 2011). Despite that these approaches allow to deal with the uncertainty and the nonlinearities, they result in nonlinear or time-varying control laws which usually require a complex implementation.

On the other hand, simpler robust control realizations for dc–dc converters can be found in Vidal-Idiarte et al. (2006), Olalla, Leyva, El Aroudi, and García (2009), Olalla, Leyva, El Aroudi, and Queinnec (2009) and Olalla, Leyva, El Aroudi, and Queinnec (2010), where the parameter changes and plant nonlinearities are considered as a form of uncertainty. The $H_{\infty}$ method is treated in Vidal-Idiarte et al. (2006), while a design based on quantitative feedback theory (QFT) is deployed in Olalla, Leyva, El Aroudi and García (2009). Finally, Olalla, Leyva, El Aroudi and Queinnec (2009, Olalla et al., 2010) proposed robust optimal control methods based on LMIs.

This work is concerned with the investigation of these optimal control synthesis methods, and concretely with the investigation of LMI control for dc–dc converters, since it presents several advantages. In the first place, the LMI formulation of the synthesis problem can include several design requirements and optimization criteria, as for example: a $H_{\infty}$ disturbance rejection bound (Chilali & Gahinet, 1996), pole placement restrictions (García, Daafouz, & Bernussou, 1996) and control effort constraints (Tarbouriech, Garcia, & Glattfelder, 2007). Each constraint can be expressed independently as one or several LMI's, and therefore, they can be included or disregarded depending on the requirements and characteristics of the converter, allowing a direct adaptation of the synthesis procedure for different cases. Besides, the synthesis procedure can be applied to one plant case, or it can take into account multiple plants, i.e. it can cope with the uncertainty and the nonlinearities of the converter while maintaining the simplicity of the control problem formulation (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994). Finally, the solution of a synthesis problem described by LMIs can be solved by means of very efficient convex optimization algorithms (Gahinet, Nemirovski, Laub, & Chilali, 1995) that allow to obtain the optimal controller (Rockafellar, 1993) while maintaining the simplicity of the control law. Thus, in order to design dc–dc converters in large with robust specifications, we formulate the control problem in terms of LMIs and we solve it by convex optimization methods.

Our approach differs from previous works in the following points. (i) Unlike other references (Montagner et al., 2007; Torres-Pinzon & Leyva, 2009; Olalla, Leyva, El Aroudi & Queinnec, 2009; Olalla et al., 2010), we propose an appropriate polytopic covering to reduce the conservatism. (ii) The treatment of the bilinear terms in dc–dc converters by LMI techniques, has not been addressed in previous papers. Therefore, this constitutes another contribution of this paper. (iii) Although considering the bilinear terms and a reduction of the conservatism with respect to previously published papers on the subject dealing with uncertainty, the final controller is yet carried out by a mere linear state-feedback, hence, with the same difficulty degree that those used by power electronics practitioners. This is also an additional important contribution with respect to previous nonlinear or time-varying approaches (Guesmi et al., 2008; Leping et al., 2009; Geyer et al., 2008; Becuti et al., 2009; El Fadil & Giri, 2009; El Fadil et al., 2009; Al-Rabadi & Alsmadi, 2009, 2011; Montagner et al., 2007; Torres-Pinzon & Leyva, 2009). In addition (iv) the startup response of the system is investigated, and a condition on the rate of change of the output voltage reference is derived. The last contribution of the paper (v) has to do with the few experimental verifications of the proposed robust controller which corroborate the approaches. The experimental measurements are in perfect agreement with the analytical derivations and the schematics for the practical implementation of the proposed design are shown in detail.

The rest of the paper is outlined as follows. Section 2 describes the general model of a dc–dc converter and introduces the representation of uncertainty and bilinearity. In addition, we present the model in error coordinates, in order to include the rate of change of the voltage reference. In Section 3, an optimal synthesis procedure that takes into account the performances, uncertainties and nonlinearities of the design problem is proposed. The last part of this section treats the conditions of stability for a time-varying reference. In Section 4, we employ the proposed approach to design a robust controller for a boost converter. An experimental set up of the design example is deployed in Section 5, where we demonstrate the validity of this design procedure and we compare the results with the method shown in Olalla et al. (2010), in order to illustrate the advantages of this approach. Section 6 summarizes the key aspects of this paper and presents some conclusions and perspectives.

### 2. Polytopic modeling of power converters

This section deals with the modeling of dc–dc converters. The approach shown here is related to the step-up (boost) converter, but it could be easily adapted for other bilinear converters.

#### 2.1. Averaged model of a PWM boost converter

Fig. 1 shows the circuit diagram of a dc–dc boost converter where $v_{o}(t)$ is the output voltage, $v_{g}(t)$ is the line voltage and $i_{load}(t)$ is the load disturbance. The output voltage must be kept at a given value $v_{ refere }$. The converter load is modeled as a linear resistor $R$. The capacitance of the capacitor and the inductance of the inductor are represented, respectively, by $C$ and $L$. Their equivalent series resistances, $R_{s}$ and $R_{L}$, are considered sufficiently small to be neglected. Thus, the measurable states of the converter are the inductor current $i_{L}(t)$ and the capacitor voltage $v_{C}(t)=v_{o}(t)$. Note that the time dependence of the variables may be omitted to simplify the notation.

The binary signal $u_{b}(t)$, which turns on and off the switches, is controlled by means of a fixed-frequency PWM (see Fig. 2). The constant switching frequency is $1/T_{s}$, where $T_{s}$ is the switching period which is equal to the sum of $t_{on}$ and $t_{off}$. The ratio $t_{on}/(t_{on}+t_{off})$ represents the duty-cycle of the converter which, in the averaging context, is equivalent to the control signal $d(t)$ if it is compared with a sawtooth signal $v_{o}(t)$ of amplitude $V_{sat}=1$. We assume that the converter operates in continuous conduction mode (CCM).

---

**Fig. 1.** Schematic of the boost converter.
The state-space averaged model of a dc–dc converter, as described in Leyva et al. (2006), can be written as

\[
\dot{x}_a = (A_{off} + (A_{on} - A_{off})U)X_a + (B_{off} + (B_{on} - B_{off})U)[1 \ 0]' W
\]

\[
+ (A_{off} + (A_{on} - A_{off})U)\dot{x}_a + (B_{off} + (B_{on} - B_{off})U)[0 \ 1]' \tilde{w}
\]

\[
+ ((A_{on} - A_{off})X_a + (B_{on} - B_{off})[1 \ 0]' W)\dot{u}
\]

\[
+ ((A_{on} - A_{off})\dot{x}_a + (B_{on} - B_{off})[0 \ 1]' \tilde{w})\dot{u},
\]

(1)

where \(A_{on}\) and \(B_{on}\) are the state-space matrices during \(t_{on}\) and \(A_{off}\) and \(B_{off}\) are the state-space matrices during \(t_{off}\). The incremental and equilibrium input vectors are \(\dot{u}(t)\) and \(U\), while the incremental and equilibrium state vectors are \(\dot{x}_a(t)\) and \(X_a\), respectively. In addition, the incremental and steady-state vector of disturbance inputs, which corresponds to the output current disturbance \(i_{load}(t)\) and the input voltage \(V_{g_p}\), are noted as \(\tilde{w}(t)\) and \(W\). These vectors and state-space matrices are as follows:

\[
U = D, \quad X_a = \begin{bmatrix} \frac{V_{g_p}}{\rho n} & 0 \\ \frac{V_{g_p}}{\rho n} & \frac{V_{g_p}}{\rho n} \end{bmatrix}, \quad W = V_{g_p}.
\]

\[
\dot{u}(t) = \tilde{d}(t), \quad \dot{x}_a(t) = \begin{bmatrix} \dot{i}_a(t) \\ \dot{v}_a(t) \end{bmatrix}, \quad \tilde{w}(t) = i_{load}(t),
\]

\[
A_{on} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{\rho n} \end{bmatrix}, \quad A_{off} = \begin{bmatrix} 0 & -\frac{1}{\rho n} \\ \frac{1}{\rho n} & \frac{1}{\rho n} \end{bmatrix},
\]

\[
A_{on} - A_{off} = \begin{bmatrix} 0 & \frac{1}{\rho n} \\ -\frac{1}{\rho n} & 0 \end{bmatrix},\quad B_{on} = B_{off} = \begin{bmatrix} \frac{1}{\rho n} & 0 \\ 0 & -\frac{1}{\rho n} \end{bmatrix}.
\]

(2)

where \(D = 1 - D\) is the complementary steady-state duty-cycle. Since, for the boost converter, \(A_{on} \neq A_{off}\), its averaged model is bilinear.

In order to obtain zero steady-state error between the voltage reference \(V_{ref}(t) = V_{ref} + \dot{V}_{ref}(t)\) and the output voltage \(v_{o}(t)\), and to investigate the stability of the converter during startup, the model is augmented with an additional state variable \(x_{int}(t)\), which stands for the integral of the output voltage error, i.e. \(x_{int}(t) = -\int (V_{ref}(t) - v_{o}(t))dt\). Also, we introduce an error coordinate representation as in Krikelis and Barkas (1984), the aim being to obtain a model in which the rate of change of the reference appears explicitly. Considering that the error state is \(e(t) = v_{o}(t) - V_{ref}(t)\), the state vector of the new model is then written as

\[
\begin{bmatrix} \dot{i}(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} \dot{i}(t) \\ v_{ref}(t) \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} V_{ref}(t) \\ x_{int}(t) \end{bmatrix}.
\]

(3)

Since the steady-state part is \(AX + BuW + B_{ref}R_s = 0\), the averaged model of (1) can then be written as the following augmented model:

\[
\dot{x}(t) = AX(t) + B_u\tilde{w}(t) + B_{ref}\tilde{u}(t) + B_{r}\hat{f}_r(t) + B_{int}\hat{f}_r(t),
\]

(4)

where

\[
A = \begin{bmatrix} 0 & -\frac{D}{\rho n} \\ \frac{D}{\rho n} & 0 \end{bmatrix}, \quad B_u = \begin{bmatrix} \frac{V_{ref}}{\rho n} \\ \frac{V_{ref}}{\rho n} \end{bmatrix}, \quad B_{ref} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.
\]

(5)

This model involves a linear part \(AX(t) + B_u\tilde{w}(t) + B_{ref}\tilde{u}(t)\) and a bilinear part \(B_u\hat{f}_r(t)\tilde{w}(t) + B_{int}\hat{f}_r(t)\tilde{u}(t)\). The dimensions of the system matrices are defined as \(A, B_u, B_{ref} \in \mathbb{R}^{n \times n}, B_u, B_{ref} \in \mathbb{R}^{n \times 1}\) and \(R_s, B_{int} \in \mathbb{R}^{n \times \nu}\), where \(n = 3, m = 1, l = 1\) and \(\nu = 2\).

2.2. Polytopic covering of non-affine uncertainties

In a boost converter, it can be considered that the load \(R\) and the complementary duty-cycle at the operating point \(D^*\) are the most relevant uncertain terms. Matrices \(A, B_u, B_{int}\) depend on these parameters, which can be grouped in a vector \(p = (p_1, \ldots, p_5)\). In this context, Olalla, Leyva, El Aroudi and Queinnec (2009) and Olalla et al. (2010) presented an uncertain model based on a convex polytope (Boyd et al., 1994; Gahinet et al., 1995; Bernardus et al., 1989). Such a polytope, defined by its vertices \((g_1, \ldots, g_n)\), is a convex envelope containing all the admissible values of \([A(p), B_u(p), B_{int}(p)]\):

\[
[A(p), B_u(p), B_{int}(p)] = \{\sum_{i=1}^{N} \lambda_i g_i, \lambda_i \geq 0, \sum_{i=1}^{N} \lambda_i = 1\}.
\]

(6)

In general, if the system matrices depend linearly on \(p\), all the admissible values of vector \(p\) are constrained in a hypersubset of \(N = 2^m\) vertices, with no additional conservatism. This is the case of the buck converter, in which the duty-cycle is not present in the system matrices. However, in other converter topologies, such as the boost or the buck-boost converter, there exist non-affine relations between the duty-cycle and the elements of the system matrices.

Considering that \(R\) and \(D^*\) are the uncertain parameters, the uncertainty vector \(p\) is equal to \(p = [D^*/1/R]\) for the uncertain \(B_{int}(p)\). However, the polytope presented in Olalla, Leyva, El Aroudi and Queinnec (2009) and Olalla et al. (2010), noted \(\mathcal{P}_{std}\), introduced two fictitious variables \(1/D^*/1/(D^*/2)R\) in order to remove the non-affine dependence of matrices \(A\) and \(B_u\) on the original \(p\). Hence, an uncertainty vector of four parameters: \(p = [D^*/1/D^*/1/(D^*/2)R, 1/R]\) was proposed, although such an approach was already presented as a potentially conservative solution in the mentioned references.

Since \(D^*\) and \(1/R\) are independent, no additional uncertainty reduction can be carried out on them, and our objective is to
reduce the conservatism with respect to the dependent terms. Fig. 3 depicts the three dependent uncertain parameters:

\[
f(D') = \left\{ \left( \frac{D'}{D''}, \frac{1}{D''} \right) : D' \in [0, D'] \right\}
\]

(7)

for the range of acceptable values of \(D'\), which are also taken from Olalla et al. (2010):

\[D' \in [0, D'] \rightarrow D' \in [0, 3].\]

(8)

In the figure, it can be observed the nonlinearity of \(f(D')\) and how it was included in the original polytope \(\mathcal{P}_{\text{old}}\).

In order to obtain a less conservative model, we carry out a polytopic covering (examples in Biannic, Roos, & Knauf, 2006) of the nonlinear function \(f(D')\). The approach presented here depends on the range of uncertainty of \(D'\), but it remains of interest due to its simplicity. This method can be applied systematically to other uncertainty sets and different topologies of converters. For general iterative algorithms on polytopic covering, see Amato, Garofalo, Glielmo, and Pironti (1995) and the references therein.

Fig. 4 depicts the projection of \(f(D')\) on the \((x-z)\) plane, which presents the largest nonlinear relation. This projection can be covered optimally by the plane formed by three vertices \((x_a, y_a), (x_b, y_b), (x_c, y_c)\), using the tangent lines at the extremes of \(f(D')\). Similarly, the projection of \(f(D')\) on the \((x,y)\) and \((y,z)\) axes is also covered using two other triangles whose vertices are \((x_a, y_a), (x_b, y_b), (x_c, y_c)\) and \((y_a, z_a), (y_b, z_b), (y_c, z_c)\), respectively. Nevertheless, these vertices are not optimal with respect to the tangent lines, and are determined by the \(x\) and \(z\) coordinates of the first triangle, in order to ensure that \(f(D')\) is inside the final polytope. The coordinates of these vertices are given in Table 1.

The faces of the new polytope are derived as a combination of the three projections in each plane. Fig. 5 shows the projections of the three triangles in each plane and the new polytopic model, noted \(\mathcal{P}_{\text{new}}\), formed by vertices \(v_1, v_2, v_3\) and \(v_4\).

\[
v_1 = (x_a, y_a), v_2 = (x_b, y_b), v_3 = (x_b, y_b), v_4 = (x_c, y_c).
\]

(9)

It can be observed that the new polytope covers a smaller volume than the original polytope shown in Fig. 3 and, consequently, it is potentially less conservative. In addition, the new polytope consists of half the vertices of the old one, which results in a reduced numerical complexity.

![Fig. 3. Plot of nonlinear uncertainty function \(f(D')\) (solid line) and original polytope \(\mathcal{P}_{\text{old}}\) (dashed line).](image)

![Fig. 4. Projection of \(f(D')\) (solid line) and the new reduced polytope (dashed line) in the \((x-z)\) plane.](image)

![Fig. 5. Plot of nonlinear uncertainty function \(f(D')\) (solid line) and new reduced polytope \(\mathcal{P}_{\text{new}}\) (dashed line). The projections of the polytope are also shown in each respective plane.](image)

The polytope \(\mathcal{P}_{\text{new}}\) is combined with the remaining uncertainty term \(1/R\),

\[
\frac{1}{R} = \begin{bmatrix} 1 & 1 \\ 1 & R \end{bmatrix}
\]

(10)

to yield a polytope of \(N=8\) vertices:

\[
\mathcal{G}_1 = [A(v_1, R), B_1(v_1, R)], \quad \mathcal{G}_2 = [A(v_2, R), B_2(v_2, R)], \quad \mathcal{G}_3 = [A(v_3, R), B_3(v_3, R)], \quad \mathcal{G}_4 = [A(v_4, R), B_4(v_4, R)],
\]

\[
\mathcal{G}_5 = [A(v_1, R), B_5(v_1, R)], \quad \mathcal{G}_6 = [A(v_2, R), B_6(v_2, R)].
\]
\[ G_1 = [A(v_2, R), B_0, (v_1, R)], \quad G_0 = [A(v_4, R), B_1, (v_3, R)]. \] (11)

Note that there exists a multiplication between the terms \(1/R^2\) and \(1/R\) in the input matrix \(B_n\). Nevertheless, since both functions are strictly decreasing, all the possible values of the multiplication will be included in the hyperrectangle formed by the combination of the extremes. Also note that a more general polytope could also be constructed with right-angle triangles to cover the nonlinearities of \(f(D)\), but at the cost of an increased potential conservatism.

### 2.3. Polytopic model of nonlinear terms

Another aspect in the robust control of the dc–dc converters, which is usually overlooked, is the appearance of nonlinear terms as, for example, \(B_0\tilde{x}(t)u(t)\) and \(B_1\tilde{v}(t)u(t)\) in the boost converter. These bilinear terms can destabilize the closed-loop system out of the considered equilibrium points.

The polytopic modeling of bilinear terms presented in Tarbouriech, Queinnec, Calliero, and Peres (2009) has been adapted to the converter model in order to cope with the effect of such bilinear terms on the stability of the closed-loop system. With this method, the possible values of the term \(B_n\tilde{x}(t)\) are included in a convex polytope \(X(\tilde{x})\):

\[ X(\tilde{x}) = C_0(v_j, j = 1, \ldots, N_b). \] (12)

where \(N_b = 2^n\) is the number of vertices of \(X(\tilde{x})\).

In a general case, for any \(\tilde{x} \in X(\tilde{x})\), one has

\[ [B_n, \tilde{x}(t) \ldots B_n, \tilde{x}(t)] = \sum_{j=1}^{N_b} \beta_j [B_n, v_j \ldots B_n, v_j] \]

\[ = \sum_{j=1}^{N_b} \beta_j \beta_j = \mathbb{B}(\beta) \] (13)

with \(\beta \in \mathbb{R}^{N_b}\) belonging to the convex set

\[ \mathbb{U} = \left\{ \beta \in \mathbb{R}^{N_b}; \sum_{j=1}^{N_b} \beta_j = 1, \beta_j \geq 0, j = 1, \ldots, N_b \right\}. \] (14)

The vertices of \(X(\tilde{x})\) are set up in a symmetric space:

\[ X(\tilde{x}) = \{ \tilde{x} \in \mathbb{R}^n; -\mu \leq \tilde{x} \leq \mu \}, \] (15)

where \(\mu \in \mathbb{R}^n\) represents a set of possible variations of the state vector. Thus, vertices \(v_j\) are obtained from \(\mu\) by the linear combination

\[ v_j = \Delta_j \mu, \quad j = 1, \ldots, N_b, \] (16)

where \(\Delta_j\) are diagonal matrices formed with all possible combinations of \(\pm 1\).

For instance, the term \(B_n\tilde{x}(t)\) can be modeled for the possible values of \(\tilde{x}(t)\) in \(X(\tilde{x})\), with \(B_n = B_0\), as \(\mathbb{B}(\beta)\). It is worth to point out that the third column of matrix \(B_0\) is zero, which in practice means that the state \(x_{out}\) does not affect the bilinear dynamics of the boost converter. Consequently, the vertices \(v_j\) do not require to consider the third state of the converter, i.e., they are obtained with \(N_b = 2^{n-1}) = 4\) diagonal matrices \(\Delta_j\) as follows:

\[ \Delta_j = \begin{bmatrix} \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \] (17)

Therefore, for the boost converter, considering the states \(\mu = [\tilde{i}_L, \tilde{v}_C, \tilde{x}_{out}]^T\), vertices \(v_j\) are defined as

\[ v_1 = [\tilde{i}_L, \tilde{v}_C, 0]^T, \quad v_2 = [-\tilde{i}_L, -\tilde{v}_C, 0]^T, \]

\[ v_3 = [\tilde{i}_L, -\tilde{v}_C, 0]^T, \quad v_4 = [-\tilde{i}_L, \tilde{v}_C, 0]^T. \] (18)

**Remark 2.1.** The same modeling approach is applied to the term \(B_n\tilde{x}(t)u(t)\) as soon as the control gain \(K\) \((\hat{u} = \hat{K}\hat{x})\) is given. Then, the bilinear term \(B_n\tilde{x}(t)u(t)\) can be included in a convex polytope as a function of the state space defined by \(\mu\) since \(B_n\tilde{x}(t)u(t) = B_n\tilde{x}(t)\hat{K}(\hat{x})u(t) = B_nJ(t)\hat{K}(\hat{x})u(t)\).

### 3. Proposed optimal control synthesis method

This section proposes a method to derive an optimal control law for robust control of a boost converter. The method considers the stabilization of the closed-loop system by a state-feedback controller \(K\), as shown in Fig. 6. As mentioned above, the main advantage of this formulation is that uncertainty can be included systematically and that the solution can be automatically found by efficient standard numerical algorithms (Gahinet et al., 1995) with convenient interfaces (Löfberg, 2004).

The following subsections present the LMI conditions that assure the closed-loop performance, and that guarantee the stability of the converter despite uncertainty and bilinear dynamics. Besides, the conditions to analyze the converter stability for a varying reference, based on the results shown in Boyd et al. (1994), are also presented. Each LMI condition is briefly discussed. For detailed proofs of each constraint, see the mentioned references.

#### 3.1. Performance constraints

Our approach ensures some performance characteristics, taken from the LMI \(\mathcal{H}_\infty\) control framework (Gahinet & Apkarian, 1994), which was already used in dc–dc converters in Olalla et al. (2010). With this method, the designer can obtain the control gain \(K\) that guarantees an optimal \(\mathcal{H}_\infty\) norm between the disturbance input \(\hat{I}_{load}\) and the regulated output \(\hat{v}_{out}\) assuring a minimum disturbance rejection level \(\lambda\). Considering the transfer function \(H(s)\) from disturbance \(\hat{I}_{load}\) to output \(\hat{v}_{out}\), the \(\mathcal{H}_\infty\) norm of such system is equal to

\[ \|H(s)\|_{\mathcal{H}_\infty} \triangleq \sup_{\omega \in \mathbb{R}} \|H(j\omega)\|_2 \] (19)

where \(\| \cdot \|_{\mathcal{H}_\infty}\) and \(\| \cdot \|_2\) stand for the infinity and the Euclidian norms, respectively. It represents the worst case of transmission of the disturbance \(\hat{I}_{load}(t)\) to the output \(\hat{v}_{out}(t)\).

Besides, several transient performances, as for example, the decay rate and overshoot of the output voltage are also guaranteed. Such transient performances, which are of critical interest in dc–dc converters, are specified by a set of constraints (Chilali & Gahinet, 1996; Garcia et al., 1996), which ensure the location of the closed-loop poles in the region \(S(\alpha, \beta, \rho)\). This region, for the \(V_{out}\) block in the linear state-feedback system.

**Fig. 6.** Block diagram of a linear state-feedback system.
The performances are considered in the vicinity of the operating points and consequently the varying reference and the bilinear terms are not taken into account. The LMIs that correspond to these performance constraints are expressed in the following proposition:

Proposition 1. The uncertain system (4) where \( \dot{r}_i(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) and where the bilinear term is neglected, with the output vector \( y(t) = Cx(t) + D_0u(t) \), is stabilizable by state-feedback \( \dot{u} = Kx \), with \( \mathcal{H}_\infty \) norm \( \|y(t)\|_2/\|w(t)\|_2 \) lower than \( \lambda \) and with the closed-loop poles inside the region \( S(\tau, \theta, \rho) \), if there exist a symmetric definite positive matrix \( W \in \mathbb{R}^{n\times n} \) and a matrix \( Y \in \mathbb{R}^{m\times n} \) such that the following inequalities hold:

\[
\begin{bmatrix}
A_iW + W A_i' + B_iY + Y B_i' & B_iW + WC_i' + Y D_i' \\
B_i' - \lambda I & 0 \\
-C_iW + D_iY & -\lambda I
\end{bmatrix} < 0,
\]

(21)

\[
A_iW + W A_i' + B_iY + Y B_i' + 2xW < 0,
\]

(22)

\[
\begin{bmatrix}
\cos(\lambda (W + W A_i' + B_iY + Y B_i')) & \cos(\lambda (A_iW + W A_i' + B_iY + Y B_i')) \\
\sin(\lambda (A_iW + W A_i' + B_iY + Y B_i')) & \sin(\lambda (A_iW + W A_i' + B_iY + Y B_i'))
\end{bmatrix} < 0,
\]

(23)

\[
\begin{bmatrix}
-\rho W & W A_i' + Y B_i' \\
A_iW + B_iY & -\rho W
\end{bmatrix} < 0.
\]

(24)

For all vertices of the uncertainty model: \( \forall i = 1, \ldots, N \). The controller is recovered by \( K = YW^{-1} \).

The disturbance rejection bound is complied if inequality (21) holds, while the poles are placed in the region \( S(\tau, \theta, \rho) \) if inequalities (22)–(24) are verified.

3.2. Bilinear dynamics

In addition to the pole placement and disturbance rejection constraints of Olalla et al. (2010), our approach considers additional constraints to handle the bilinear dynamics. During the changes of operating point it is assumed that references and disturbances are equal to zero. The conditions ensuring system stability despite the nonlinear term have been adapted from Tarbouriech et al. (2009) in the following proposition:

Proposition 2. Given a bound on the state space \( \mu \), the bilinear system (4), where \( \dot{r}_i(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) and \( \dot{w}(t) = 0 \), is stabilizable by state-feedback \( \dot{u} = Kx \), with a guaranteed region of stability \( \alpha(P,1) \) including specific initial conditions \( x_0 \), if there exist a symmetric definite positive matrix \( W \in \mathbb{R}^{n\times n} \) and a matrix \( Y \in \mathbb{R}^{m\times n} \) such that the following inequalities hold:

\[
\begin{bmatrix}
A_iW + W A_i' + B_iY + Y B_i' \\
A_iW + B_iY
\end{bmatrix} < 0,
\]

(25)

\[
\begin{bmatrix}
\mu_{(r)} & W \\
1_{m} & \mu_{(r)}
\end{bmatrix} \geq 0,
\]

(26)

\[
\begin{bmatrix}
\sum_{r=1}^{n} \mu_{(r)}x_0 \\
x_0
\end{bmatrix} \geq 0.
\]

(27)

For all vertices of the uncertainty model: \( \forall k = 1, \ldots, N_k \), for all vertices of the bilinearity model: \( \forall j = 1, \ldots, N_b \), and for all states of the converter: \( \forall r = 1, \ldots, n \). The controller is recovered by \( K = YW^{-1} \).

The quadratic Lyapunov function \( V(x) = \dot{x}^TP\dot{x} = x^TW^{-1}x \) can be used to define an ellipsoid in \( \mathbb{R}^n \):

\[
\alpha(P,1) : = \{x \in \mathbb{R}^n; x^TW^{-1}x \leq 1 \}.
\]

(28)

By inequality (26), this ellipsoid is included in the polyhedral region describing the bilinear term, i.e. \( \alpha(P,1) \subseteq \mathcal{X}(\dot{x}) \). Since inequality (25) ensures that \( V(x) \) is strictly negative inside \( \mathcal{X}(\dot{x}) \), the trajectories of the system that belong to \( \alpha(P,1) \) will remain inside this contractive region despite the bilinear term.

Finally, in order to include a specific initial condition \( x_0 \) in \( \alpha(P,1) \), inequality (27) can be added (Boyd et al., 1994). Usually, each initial condition \( x_0 \) is associated with a plant model \( (A_i,B_i) \). If the vertices \( (A_i,B_i) \) are the vertices of the polytopic model \( (A_k,B_k) \), then \( N_k \) is equal to \( N \), and we can find a shared region of stability valid for all the operating points of the uncertainty model. In that case, the left-hand side of (25) can substitute the first block of the left-hand side of (21). Obviously, the number of constraints is increased, potentially to the detriment of the achievable performance.

3.3. Controller synthesis algorithm

Given the previous LMI conditions, we propose a synthesis procedure in which the objective is to find the smallest possible \( \mathcal{H}_\infty \) norm (\( \lambda \)) from disturbance to output, while assuring that the region of stability is large enough to contain the trajectories of the converter between specific operation points \( x_0 \) given beforehand.

It is worth to point out that the state vector \( \mu \) determines the size of the stability region \( \alpha(P,1) \), since \( \alpha(P,1) \subseteq \mathcal{X}(\dot{x}) \). The control objective being to find the smallest possible \( \mathcal{H}_\infty \) norm, the synthesis algorithm consists in fixing \( \mu \) while searching for the rest of variables \( W,Y,\lambda \). Since the optimization of the \( \mathcal{H}_\infty \) performance can result in a very small region of attraction, the inclusion of \( x_0 \) in the region of stability allows to find an ellipsoid with an appropriate size and the best possible performance. Therefore, the initial choice of \( \mu \) must be large enough so that \( \alpha(P,1) \) can contain \( x_0 \) but small enough so that the performance requirements can be achieved. Since the system is stable in open-loop, a sufficiently large \( \mu \) can always be found, at the expense of worse performances. Then, in order to solve LMI problems at each step, the synthesis of the optimal controller can be accomplished with the following algorithm, adapted from Tarbouriech et al. (2009):

Algorithm 1. Step 1: Initialization. Choose \( \mu \) large enough so that \( \alpha(P,1) \) can contain the points \( x_0 \), but small enough so that the performance constraints are feasible.

Step 2: Compute \( W,Y,\lambda \) solutions to

\[
\min_{W,Y} \text{subject to } \sum_{r=1}^{n} \mu_{(r)}x_0 \
\text{LMI (26)}.
\]

Step 3: Fix \( W,Y,\lambda \). Compute \( \mu_{(r)} \), \( r = 1, \ldots, n \) solution to

\[
\min_{\sum_{r=1}^{n} \mu_{(r)}} \text{subject to } \text{LMI (26)}.
\]

Step 4: Return to Step 1 until the reduction of \( \lambda \) is not significative.

Once the lowest achievable \( \mathcal{H}_\infty \) norm is found, the solution \( W,Y,\lambda \) is used to recover the optimal controller \( K = YW^{-1} \).
3.4. Analysis of stability for varying reference

The stability and the performance of the converter in presence of uncertainty and bilinear dynamics can be ensured with the proposed synthesis algorithm. However, the trajectories of the system during a step change of the voltage reference (\(\dot{V}_{\text{ref}}(t) = \infty\)), as during startup, can be too large to be enclosed in the ellipsoid of attraction found in the synthesis step. On the other hand, the step signal of the voltage reference can be controlled or filtered to maintain the closed-loop system in the region of stability. The following conditions allow to find the affordable changes of initial conditions inside the region of stability.

**Proposition 3.** The states of the closed-loop bilinear system (4) for initial conditions inside the region of stability \(u(P,1)\), with a given state-feedback \(\hat{u} = \hat{K}\hat{x}\), and with inputs \((\hat{r}_t \in \mathbb{R}^2)\) contained in \(a(R,1) \times \hat{r}_t, R, f_r \leq 1\), remain inside the region of stability \(u(P,1)\), if there exist a symmetric definite positive matrix \(W \in \mathbb{R}^{n \times n}\) and a matrix \(R \in \mathbb{R}^{n \times n}\) such that the following inequalities hold:

\[
\begin{bmatrix}
   W(A + \hat{B}_1\hat{K})(A + \hat{B}_1\hat{K})W + \lambda W & \hat{B}_1 \\
   \hat{B}_1^T & -\lambda R
\end{bmatrix} < 0, \tag{29}
\]

**Proof.** In absence of disturbances, the bilinear system (4) can be rewritten in closed-loop for \(\hat{u} = \hat{K}\hat{x}\) as follows:

\[
\hat{x}(t) = (A + \hat{B}_1\hat{K})\hat{x}(t) + \hat{B}_1\hat{r}_t(t), \tag{31}
\]

where \(\hat{B}_1 = B_1 + B_4(f_r)\) (see Remark 2.1). Considering a quadratic Lyapunov function \(V(\hat{x}) = \hat{x}^T P\hat{x}\), the stability of the system (31) is guaranteed if

\[
V(\hat{x}) = \hat{x}^T [(A + \hat{B}_1\hat{K})P + P(A + \hat{B}_1\hat{K})]\hat{x} + 2\hat{x}^T \hat{B}_1^T \hat{r}_t < 0
\]

for all \(\hat{x}\) and \(\hat{r}_t\), such that \(\hat{x}^T P\hat{x} \geq 1\) and \(\hat{r}_t^T R\hat{r}_t \leq 1\).

Thanks to the S-procedure, the previous condition can be tested if there exists a \(\sigma > 0\) such that the following inequality is satisfied:

\[
V(\hat{x}) = \hat{x}^T [(A + \hat{B}_1\hat{K})P + P(A + \hat{B}_1\hat{K})]\hat{x} + 2\hat{x}^T \hat{B}_1^T \hat{r}_t + \sigma \hat{x}^T P\hat{x} - \sigma \hat{x}^T (\hat{r}_t^T R\hat{r}_t) < 0
\]

which can be rewritten as the following LMI:

\[
\begin{bmatrix}
   (A + \hat{B}_1\hat{K})P + P(A + \hat{B}_1\hat{K}) + \sigma P & \hat{B}_1^T \\
   \hat{B}_1 & -\sigma R
\end{bmatrix} < 0, \tag{34}
\]

that corresponds to (29) after pre- and post-multiplication by \([W \quad 0] [0 \quad 1]^T\), where \(W = P^{-1}\). \(\square\)

4. Design example

In this section, we apply the proposed control synthesis procedure to the uncertain and nonlinear model of the boost converter shown in Section 2, whose parameter set is shown in Table 2. The first part of the section deals with the controller synthesis initialization and results, while the second part presents the analysis of the region of stability during startup.

4.1. Synthesis procedure

The pole region constraints for parameters \(\varphi, \rho, \theta\) are specified in Table 3, following the guidelines of Olalla et al. [2010]: in order to limit the poles inside the valid frequency range of the averaged model, \(\rho\) is set to 1/10 of the switching frequency. For a minimum damping ratio of \(\zeta = 0.4\), \(\theta\) is set to 25°. Finally, the maximum value of \(\varphi\) for the required region of stability is equal to 1600, which corresponds to a maximum settling time of 2.5 ms.

The stability and the mentioned performances are ensured for the set of loads and operation points expressed in the uncertainty description. However, when moving from one operation point to another one, it is not ensured that the trajectory remains within the dynamic models described by the uncertainty set, and therefore the stability of the converter is not guaranteed. For this reason we include the bilinear dynamics in the controller synthesis step, in order to ensure that the trajectory between different operation points will be kept in the region of attraction.

In our case, the uncertainty description considers that the load \(R\) belongs to the set \([10,50]\)Ω. In order to guarantee the stability when switching between the maximum and the minimum load at a certain operation point, we can include specific initial conditions \(x_0\) in the ellipsoidal region of stability. For the nominal duty-cycle \((D = 0.5)\), the inductor current drops from 4.8 to 0.96 A when the load switches between 10 and 50Ω, which corresponds to a current step of ±3.84 A. Assuming that the converter is in steady-state before the load change, the system trajectories after such a change of operation point are included in the region of attraction if inequalities (25)–(27) are satisfied for \(A(p)\) and \(B(p)\) at the nominal duty-cycle and \(x_0\) equal to ±3.84,0\(x_{\text{int}}\) (Table 4).

Note that, in the previous paragraph, the point \(x_0\) has not been completely defined. The value of \(x_{\text{int}}\) depends on the control gain \(K\), and therefore, it cannot be specified a priori. We propose the following procedure: \(x_{\text{int}}\) is set to zero at the first iteration of the synthesis algorithm. Then, \(x_0\) is updated to contain \(x_{\text{int}} = (-3.84K_1)/K_3\) at each iteration. This approach guarantees

**Table 2 Boost converter parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Nominal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>([10, 50]) Ω</td>
<td>10 Ω</td>
</tr>
<tr>
<td>(D)</td>
<td>([0.3, 1.0])</td>
<td>0.5</td>
</tr>
<tr>
<td>(v_c(V_{\text{ref}}))</td>
<td>24 V</td>
<td></td>
</tr>
<tr>
<td>(V_d)</td>
<td>12 V</td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td>200 μF</td>
<td></td>
</tr>
<tr>
<td>(L)</td>
<td>100 μH</td>
<td></td>
</tr>
<tr>
<td>(T_s)</td>
<td>5 μs</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3 Pole placement parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi)</td>
<td>1600</td>
</tr>
<tr>
<td>(\theta)</td>
<td>25</td>
</tr>
<tr>
<td>(\rho)</td>
<td>(2\pi/10T_s)</td>
</tr>
</tbody>
</table>

**Table 4 Bilinear dynamics.**

<table>
<thead>
<tr>
<th>(x_0)</th>
<th>Value</th>
<th>(A_{01}B_{01})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_01)</td>
<td>±([3.84,0,x_{\text{int}}])</td>
<td>(A_{01}B_{01}(D = 0.5,R = 10) Ω)</td>
</tr>
<tr>
<td>(x_02)</td>
<td>±([3.84,0,x_{\text{int}}])</td>
<td>(A_{01}B_{01}(D = 0.5,R = 50) Ω)</td>
</tr>
</tbody>
</table>
that the trajectory of the converter states will be bounded in the region of stability.

Another parameter that must be initialized is the vector \( \mu \), which must be chosen large enough to obtain a region of stability that contains the point \( x_0 \). In our case, an initial state vector of \( \mu = [8, 4, \mu_3] \)

\[
\text{(35)}
\]

has provided a sufficiently large \( \lambda(x) \) to contain \( x_0 = [3.84, 0, 0] \). As explained in Section 2.3, the value of \( \mu \) does not affect the synthesis results and it must only be kept large enough not to limit the volume of \( \mathbf{K}(\mathbf{P}, 1) \) in the direction of the third state.

Once defined \( \mu, x_0, p, 0 \) and \( x_0 \), Algorithm 1 yields, after 12 iterations, the following gain vector

\[
\mathbf{K}_{\text{bil}} = [-0.36, -1.07, -1.9228].
\]

This controller ensures a minimum \( H_\infty \) performance factor of \( \lambda_{\text{bil}} = 1.69(4.55 \text{ dB}) \). The final values of \( \mu \) and \( x_0 \) are \([6.75, 2.48, \mu_3]\) and \([3.84, 0, 0.72 \times 10^{-3}] \), respectively.

The properties of the proposed controller are illustrated in Fig. 7, which presents a simulation of the switched-mode circuit of Fig. 1 with the proposed controller \( \mathbf{K}_{\text{bil}} \) using PSIM (POWER SIM, 2003). The upper waveform of the figure shows the output voltage signal \( v_o \), while the lower waveform depicts the current being delivered to the load \( i_o \). The converter load is initially the minimum value (\( R = 50 \Omega \)), and the steady-state duty-cycle is \( D = 0.5 \). At \( t = 1 \text{ ms} \), the load changes to its maximum value (\( R = 10 \Omega \)) and at \( t = 6 \text{ ms} \), it returns to its original value. As expected, it can be observed that the performance requirements are satisfied, since the settling time is approximately \( 2 \text{ ms} \) and the damping ratio is larger than 0.4. Furthermore, although such a large disturbance of the inductor current could excite the bilinear dynamics of the converter, the trajectories of the system remain inside the region of stability. This region of stability is depicted in Fig. 8. The figure shows the ellipsoid of attraction \( \mathbf{K}(\mathbf{P}, 1) \) and its section on the plane \( (\tilde{t}_i, \tilde{v}_o) \) for \( \lambda_{\text{bil}} \) equal to zero. It can be observed that this ellipsoid is included in the polyhedral set \( \lambda(x) \). As expected, the trajectories of the system remain inside the region of attraction and converge to the equilibrium point.

**Remark 4.1.** As noted above, the stability of the system can be ensured for additional initial conditions at different operation points, as for example, for other load switches at non-nominal duty-cycles. However, the larger is the region of stability that

\[
\text{(36)}
\]

must be ensured, the weaker are the performances that can be achieved. The proposed method has the advantage that it allows to deal with the trade-off between specifications (the required region of stability) and the achievable performances.

In order to illustrate this point, let us consider an example with a different set of trajectories, corresponding to the same load switches between \( R = 10 \) and \( 50 \Omega \) but for different duty-cycles, i.e. \( D = 0.35 \) and 0.9. Such trajectories are represented by the initial conditions shown in Table 5.

Since these points are farther from the origin than the points of Table 4, the required region of stability must be larger. The increase in the robustness specifications must be accompanied by an enlargement of the stability region and therefore by a relaxation on the performance requirements. In order to obtain a feasible solution, the decay rate specification has been changed to \( \alpha \approx 500 \), i.e. the settling time is approximately twice. The application of Algorithm 1 under these conditions results in

\[
\mathbf{K}_{\text{bil}} = [-0.38, -0.44, -488].
\]

Simulations are shown in Figs. 9 and 10. The achieved \( H_\infty \) performance factor with this controller is equal to \( \lambda_{\text{bil}} = 2.52(8.02 \text{ dB}) \).

**4.2. Analysis of convergence during startup**

The control design of the boost converter ends with the set up of the voltage reference which ensures the convergence of the system during startup. Considering the control gain \( \mathbf{K}_{\text{bil}} \), we analyze the closed-loop stability of the system considering the rate of change of the voltage reference, using the error model presented in Section 2.1 and the stability conditions of Section 3.4. With this approach, the maximum slope of \( v_{\text{ref}} \), for which the

![Fig. 7. Simulated transient of the boost converter under a load step transient \((R = 50 \rightarrow R = 10 \rightarrow R = 50)\) at nominal duty-cycle with the robust controller \( \mathbf{K}_{\text{bil}} \). Top waveform: output voltage; bottom waveform: output current.](image)

![Fig. 8. Simulated transient of the trajectories of the boost converter around the ellipsoid of stability, with controller \( \mathbf{K}_{\text{bil}} \) for load switchings between 10 and 50 \( \Omega \).](image)

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Bilinear dynamics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_0 )</td>
<td>Value</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>( \pm [3.84, 0, x_{\text{int}}] )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( \pm [3.84, 0, x_{\text{int}}] )</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>( \pm [5.48, 0, x_{\text{int}}] )</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>( \pm [5.48, 0, x_{\text{int}}] )</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>( \pm [2.13, 0, x_{\text{int}}] )</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>( \pm [2.13, 0, x_{\text{int}}] )</td>
</tr>
</tbody>
</table>
converter stability is ensured, can be obtained. Such a value can be used to filter the voltage reference step accordingly.

It is considered that, before the regulation subsystem is connected, the boost converter is in steady-state with an input duty-cycle of \( D = 0 \) \( (D' = 1) \). As soon as the conditions of stability (29) and (30) are satisfied despite the bilinear dynamics, for the whole uncertainty set of Section 2.1, a unique region of stability is obtained for all the operation points contained in the polytope. This means that the converter will be able to startup from the initial state to any output voltage in the range \([V_L/D, V_g/D]\).

Hence, the optimization program that yields the affordable rate of change of \( \dot{V}_{ref} \) is set up as follows:

\[
\min_{W, K; \gamma, \delta} \gamma + \delta \quad \text{subject to}
\]

LMIs (29), (30),

\[
\begin{bmatrix}
\gamma & d_w \\
d_w & W
\end{bmatrix} > 0,
\]

\[d_g R_d < \delta.\]

The result of the above-mentioned optimization program for \( \mu = [2, 4, 0.8, \mu_1] \) and the controller \( K_{bil} \) yields a maximum reference rate of change of \( |\dot{V}_{ref}| \leq 554 \). According to such a maximal slope, for the nominal duty-cycle \( D = 0.5 \), \( \dot{V}_{ref} \) must rise up to 12 V \( (V_{ref} + \dot{V}_{ref} = 24 \text{ V}) \), and the voltage reference has to be filtered with a first-order low-pass filter with a time constant of \( \tau = 12/554 \). Fig. 11 shows the transient simulation waveforms of the output voltage \( V_d(t) \) and the duty-cycle \( d(t) \) under these conditions. The converter closely follows the voltage reference up to the desired value, with no overshoot and with a settling time of approximately 80 ms. Obviously, this value can be reduced up to approximately 25 ms with a higher order low-pass filter.

5. Control implementation and experimental results

In order to verify the results derived in the previous sections, we have built a 100 W boost converter. The structure of the converter with the proposed controller can be seen in Fig. 12, where the component values are those given in Table 2. We have used a shunt resistance \( R_{shunt} = 25 \text{ m}\Omega \) and an INA139 differential amplifier to measure the inductor current. The load change experiments have been carried out by means of a voltage-controlled switch. A detail of
the implementation of controller $K_{lin}$ is given in Fig. 13. Note that the proposed implementation requires two additional operational amplifiers with respect to the circuits shown in Olalla, Levey, El Aroudi and Queinnec (2009) and Olalla et al. (2010).

In order to compare the proposed synthesis method with other approaches, we have implemented two additional versions of state-feedback controllers. These controllers have been synthesized using the controller synthesis method proposed in Olalla et al. (2010). Such a method presents the same optimization criteria, i.e. it obtains the controller with the lowest possible $H_{\infty}$ norm between disturbance and output voltage. Other standard procedures usually employed by practitioners, such as PID control, were also evaluated in Olalla et al. (2010), which indirectly provides a comparison of the current approach with various control strategies.

The first controller, noted $K_{lin1}$, was synthesized with the conservative uncertainty model proposed in the mentioned work, and the achieved performance factor was $\lambda_{lin1} = 4.81$ (13.64 dB).

In Fig. 14 we show transient waveforms of the converter with controllers $K_{lin}$ (Fig. 14a) and $K_{lin1}$ (Fig. 14b) under the load disturbances simulated in the previous section. It can be observed in Fig. 14a, a perfect agreement with respect to the simulation results of Figs. 7 and 8. It can be pointed out that with the proposed synthesis method, the output voltage presents a deviation of approximately 1 V ($\lambda = 0.57$, -4.83 dB) while with controller $K_{lin1}$ the deviation is approximately two times larger ($\lambda = 1.30$, 2.29 dB). Furthermore, it can be noted that the settling time after each disturbance is approximately four times smaller with the proposed approach.

Thus, our approach allows to obtain better performances while maintaining the desired robustness properties. This point is verified in Fig. 15, which illustrates the performance of the proposed regulator $K_{bil}$ during the transient startup. Again, the experimental result matches accurately the solid lines of the simulation plotted in Fig. 11. Besides, our approach is compared with the controller that disregards the bilinear dynamics $K_{lin2}$. It can be observed that with such controller the trajectories of the closed-loop system present an undesirable behavior involving duty-cycle saturation and an output voltage overshoot of approximately 12 V.

Finally, the trajectories of the converter in closed-loop with controller $K_{bil}$ are verified under the load transients of Fig. 8. By comparing Fig. 8 with Fig. 16, it can be concluded that the experimental results and numerical simulations are in good agreement.

6. Conclusions and future research

The proposed controller synthesis approach allows to consider the converter uncertainty and to ensure its stability between different operation points, thanks to the inclusion of the bilinear...
dynamics. Besides, once the controller has been found, the stability during startup can also be ensured despite the uncertainty and the bilinear dynamics by setting up a bound on the rate of change of the voltage reference. It is worth to point out that this approach does not solve the trade-off between the achievable performance and the robustness of the controller, but it allows to obtain a bound on the $H_1$ performance for the set of uncertain terms and considered trajectories, that are established by the designer regarding the converter requirements.

It is also worth to mention that we have not considered the possible saturation of the control effort signal, which does not appear in the set of trajectories considered. Nevertheless, such nonlinear dynamics may strongly affect the converter dynamics, and they should be taken into account in future works. From a practical point of view, other future works can also consider other sources of uncertainty as inductive loads (for dc motors), or constant power loads (automotive systems). Besides, the approach used here can be extended to other more complex converters such as parallel converters, multi-level converters, AC–DC power factor correction circuits and DC–AC inverters.

**Fig. 14.** Experimental transient of the boost converter under a load step transient of 1.92 A at nominal duty-cycle $D_d = 0.5$. Upper traces are output voltage. Lower traces are output current. (a) With the proposed controller $K_{bil}$; (b) with the controller synthesis method proposed in Olalla et al. (2010).

**Fig. 15.** Experimental transient of the boost converter during startup. Upper traces are output voltage. Lower traces are duty-cycle. (a) With the proposed controller $K_{bil}$; (b) with the controller synthesis method proposed in Olalla et al. (2010).

**Fig. 16.** Experimental trajectories of the boost converter under a load step transient of 1.92 A at nominal duty-cycle $D_d = 0.5$. Channel 1 is inductor current $i_l$. Channel 2 is output voltage.
Other future works can deal with the application of the proposed approach to this kind of systems.

Acknowledgments

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