Passification-Based Adaptive Control : Robustness Issues

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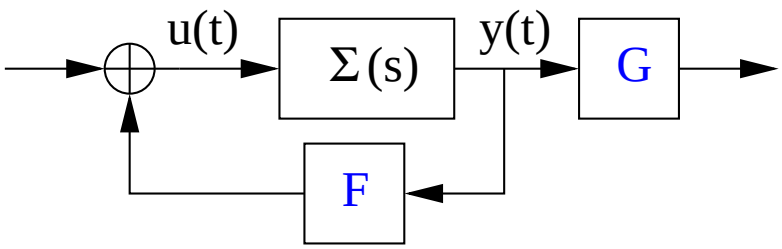
Passification-based adaptive control [Fradkov 1974]

Let an LTI Σ and assume there exists F and G such that $\Sigma \star F$ is G-passive

i.e.



is passive $\left(\begin{array}{c} V(x(t)) < V(x(0)) \\ + \int_0^t \left[u(\theta)^* G y(\theta) \right] \, d\theta \end{array} \right)$

then whatever positive $\Gamma_i > 0$, Σ is G-passified by the PBAC

$$u(t) = K(t)y(t) \ , \ \dot{K}_i(t) = -y_i^*(t)\Gamma_i G y(t) \ .$$

Technical remark

→ One can always take $F = -kG$ with k sufficiently large

→ F exists if $G\Sigma$ is hyper-minimum-phase

Known Advantages of the PBAC

- ↗ Good observed behavior w.r.t. uncertainties and non-linearities
- ↗ Simple to design
- ↗ Based on physical meaning

Drawbacks

- ↘ Need to prove the robustness properties
- ↘ Need for numerical methods for choosing G
- ↘ Divergence of $K(t)$ due to disturbances

Outline

① Modified PBAC that bounds $K(t)$ + robustness conditions

② BMI design of G for the nominal system

③ LMI design of Parameter-Depenent $F(\Delta)$ that fulfills robustness conditions

④ Example : autonomous aircraft

Bounded Passification Based Adaptive Algorithm

→ Let $\mathcal{B}$ a bounded set of C^m and let a penalty function $\phi : \mathsf{C}^m \mapsto \mathsf{C}^m$:

$$\begin{aligned} \phi(K) &= 0 && \forall K \in \mathcal{B} \\ (K - F)^* \phi(K) &\geq 0 && \forall F \in \mathcal{B} \end{aligned}$$

→ If there exists $F(\Delta) \in \mathcal{B}$ and G such that $\Sigma(\Delta) \star F(\Delta)$ is G-passive for all $\Delta \in \mathbf{\Delta}$, then whatever positive $\Gamma_i > 0$, $\Sigma(\Delta)$ is robustly G-passified by the BPBAC

$$u(t) = K(t)y(t) \ , \ \dot{K}_i(t) = -y_i^*(t)\Gamma_i G y(t) - \Gamma_i \phi(K_i(t)) \ .$$

→ Convergence of the BPBAC is such that

$$x(\infty) = 0 \ , \ K(\infty) \in \mathcal{B} \ , \ \Sigma(\Delta) \star K(\infty) \text{ is } G\text{-passive}$$

Proof of BPBAC properties

→ The considered LTI uncertain models $\Sigma(\Delta)$ are such that:

$$\dot{x} = A(\Delta)x + Bu \ , \ \ y = Cx \ .$$

→ $x = 0$ and $K = F(\Delta)$ are stable equilibrium points.

→ $\Sigma(\Delta) \star F(\Delta)$ being G-passive implies the existence of $H(\Delta)$:

$$H(\Delta) = H^*(\Delta) > 0 \ \ , \ \ H(\Delta)B = C^*G^*$$

$$H(\Delta)A(\Delta, F(\Delta)) + A^*(\Delta, F(\Delta))H(\Delta) < 0$$

and the BPBAC closed-loop system has a storage function of the class

$$V(x, K, \Delta) = \frac{1}{2}x^*H(\Delta)x + \frac{1}{2}\sum_{i=1}^{l}(K_i - F_i(\Delta))^*\Gamma_i^{-1}(K_i - F_i(\Delta))$$

② BMI design of G for the nominal system

Conditions for BPBAC passification

Existence of $F(\Delta) \in \mathcal{B}$ and G such that
$\Sigma(\Delta) \star F(\Delta)$ is G-passive for all $\Delta \in \mathbf{\Delta}$.

Conservative procedure for the BPBAC conditions

→ Design G such that $\Sigma(\Delta = 0)$ is G-passifiable via static output feedback.

→ For a given G, prove the existence of $F(\Delta)$ such that
$\Sigma(\Delta) \star F(\Delta)$ is G-passive for all uncertainties.

② BMI design of G for the nominal system

G-passification of $\Sigma(0)$ [Fradkov 1976]

$$H = H^* > 0 \quad , \quad HB = C^* G^*$$

$$H(A(0) + BFC) + (A(0) + BFC)^* H < 0$$

Along solutions, there always exists $F = -kG$ with k sufficiently large.

BMI (YALMIP+PENBMI) design

→ Choose a (large) value of k

→ Choose an upper bound on H (we took $\bar{h} = 1$) for scaling the solutions

→ Declare in YALMIP the following BMI problem

$$\bar{h}1 > H > 0 \ , \ HB = C^* G^* \ , \ t > -1$$

$$HA(0) + A^*(0)H - kC^* G^* GC < t1$$

→ Minimize t using PenBMI, if it returns $t < 0$, the procedure succeeded.

③ LMI design of $F(\Delta)$

"Quadratic stability", norm-bounded result

For a given G, if there exists H, F and ρ such that

$$H > 0 \ , \ \ HB = C^*G^* \ , \ \ F \in \mathcal{B}$$

$$\begin{bmatrix} \langle HA(0) + C^*G^*FC \rangle & HB_\Delta \\ B_\Delta^* H & -1 \end{bmatrix} + \rho \begin{bmatrix} C_\Delta^* \\ D_\Delta^* \end{bmatrix} \begin{bmatrix} C_\Delta^* \\ D_\Delta^* \end{bmatrix}^* < 0$$

then $\Sigma(\Delta) \star F$ is G-passive for all uncertainties $\Delta^*\Delta \leq \rho 1$ where $\Sigma(\Delta)$:

$$\dot{x} = [A(0) + B_\Delta \Delta (1 - D_\Delta \Delta)^{-1} C_\Delta] x + Bu \ , \ \ y = Cx \ .$$

Remarks

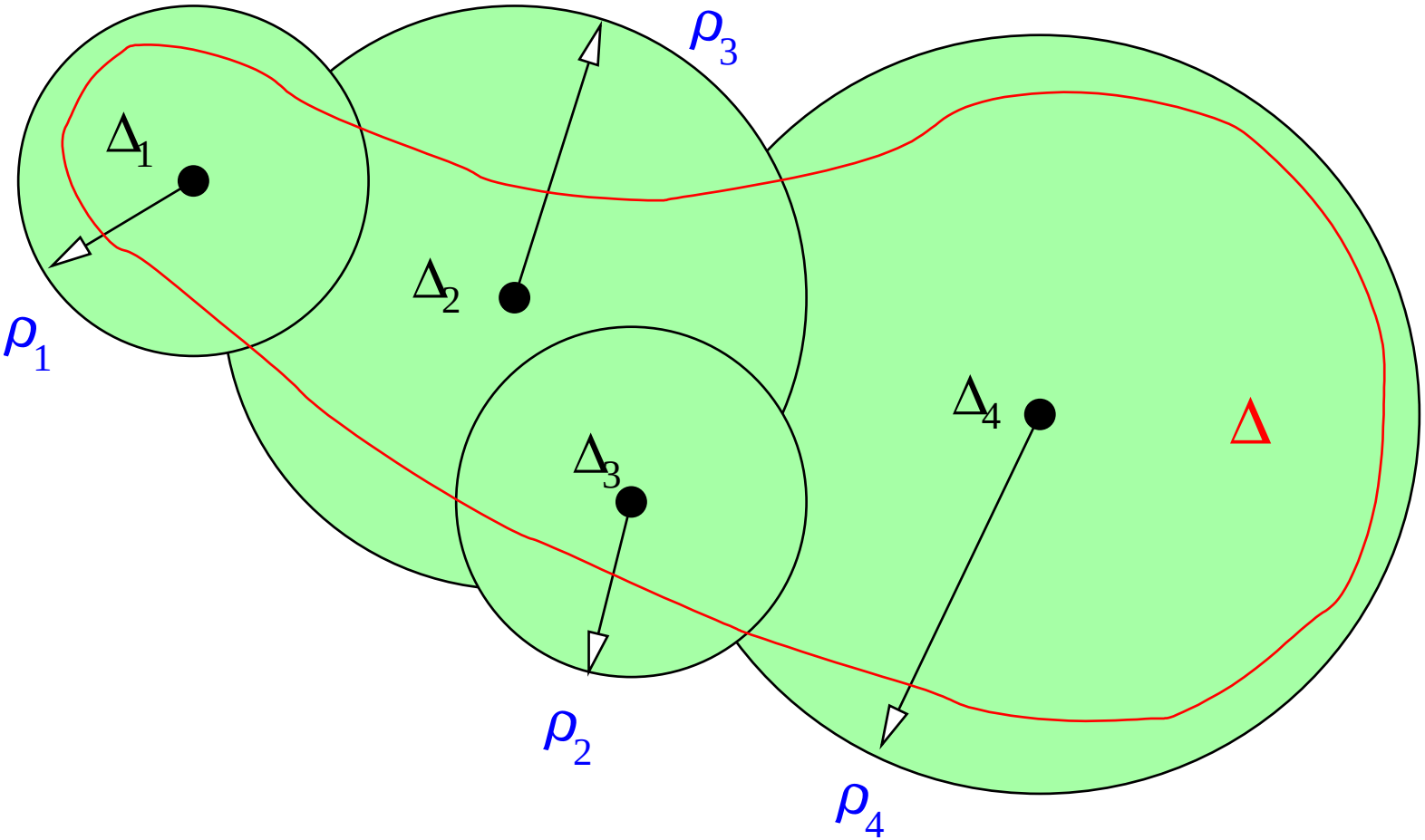
→ Maximization of ρ is LMI

→ If $\boldsymbol{\Delta} \subset \boldsymbol{\Delta}_\rho = \{\Delta^*\Delta \leq \rho 1\}$ the problem is solved with $F(\Delta) = F$

→ Replacing in the LMIs $A(0)$ by $A(\Delta_i)$ for some Δ_i, gives a couple (F_i, ρ_i) such that $\Sigma(\Delta) \star F_i$ is robustly G-passive w.r.t. $\{\Delta_i + \Delta \ : \ \Delta \in \boldsymbol{\Delta}_{\rho_i}\}$.

③ LMI design of $F(\Delta)$

Solving the LMI conditions for some finite sequence $\{\Delta_i\}$ gives sequences $\{F_i\}$, $\{\rho_i\}$ that defines an $F(\Delta)$ for $\bigcup\{\Delta_i + \Delta \ : \ \Delta \in \mathbf{\Delta}_{\rho_i}\}$.



If $\mathbf{\Delta} \subset \bigcup\{\Delta_i + \Delta \ : \ \Delta \in \mathbf{\Delta}_{\rho_i}\}$

the robustness BPABC G-passification conditions are fulfilled.

④ Example : autonomous aircraft

Linearized fourth-order model of lateral dynamics for an autonomous aircraft including model of actuator dynamics. Uncertainty is the flight altitude ($\Delta = h$). System has 3 outputs, one input.

→ BMI design of G for the nominal system $h = 5$km.

For two choices of high gain k

$$G_{1\text{PenBMI}} = 10^{-2} \begin{bmatrix} 4.5404 & 2.8436 & 1.7107 \end{bmatrix}$$

$$G_{2\text{PenBMI}} = 10^{-3} \begin{bmatrix} 8.4000 & 5.4505 & 3.0961 \end{bmatrix}$$

and after simple analysis step:

$$G_1 = \begin{bmatrix} 4 & 3 & 2 \end{bmatrix} \ , \ G_2 = \begin{bmatrix} 8 & 5 & 3 \end{bmatrix}$$

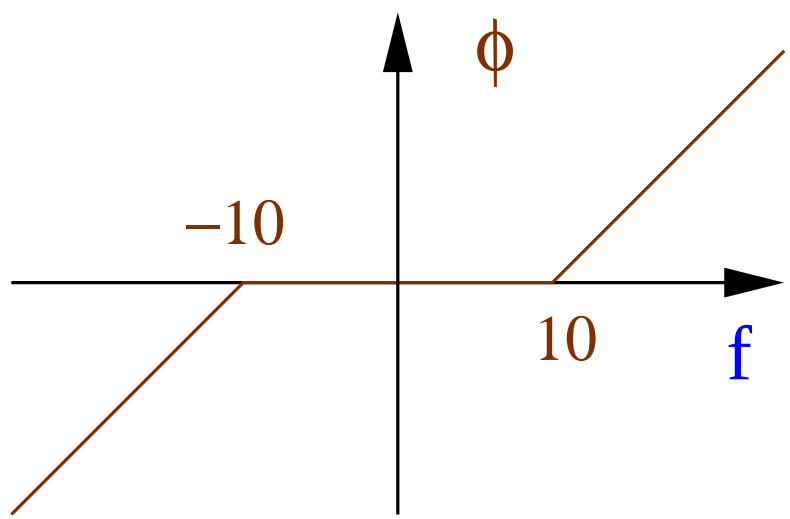
④ Example : autonomous aircraft

→ Choice of LMI representable $\mathcal{B}$ (bounded set of control gains)

$$F = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix} \ , \quad -10 \leq f_i \leq 10$$

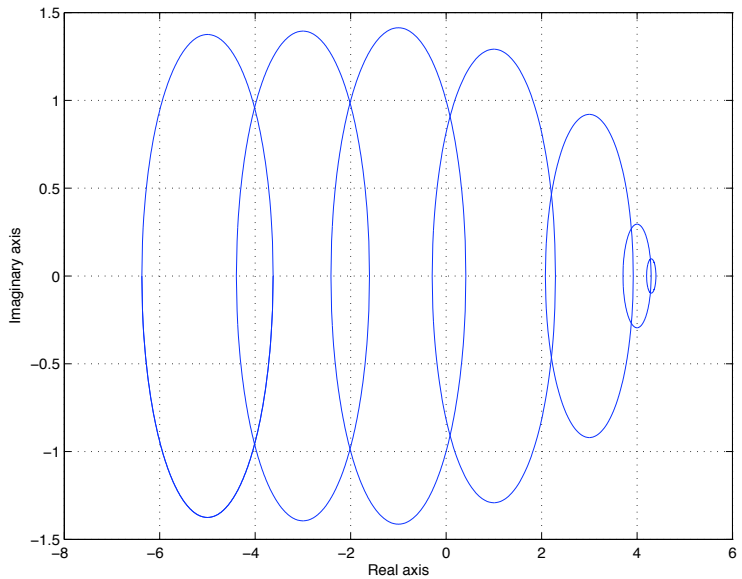
and ϕ dead-zone penalty function of the BPBAC:

$$\phi(f_i) = f_i - \text{sat}_{10}(f_i)$$



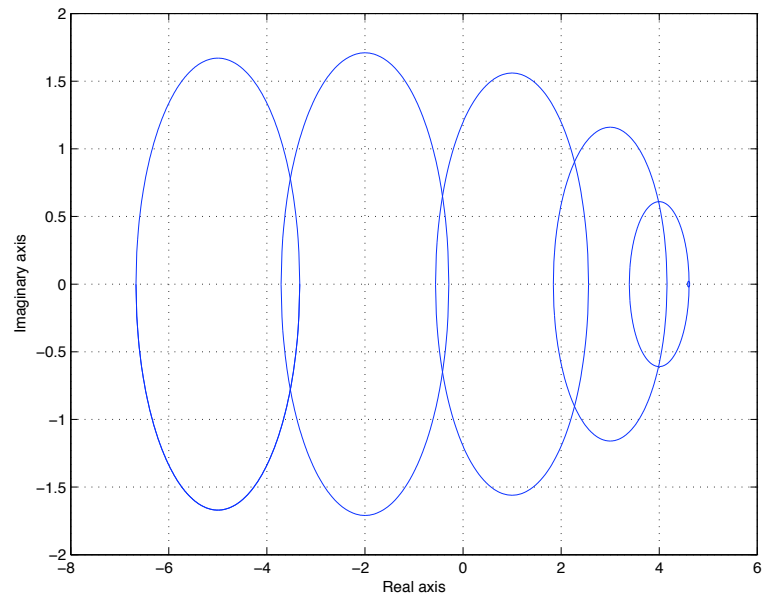
④ Example : autonomous aircraft

→ LMI design of admissible h intervals



Admissible $\Delta \in \mathsf{C}$ for G_1

Result is $h \in [0 \;\; 9.2976]$



Admissible $\Delta \in \mathsf{C}$ for G_2

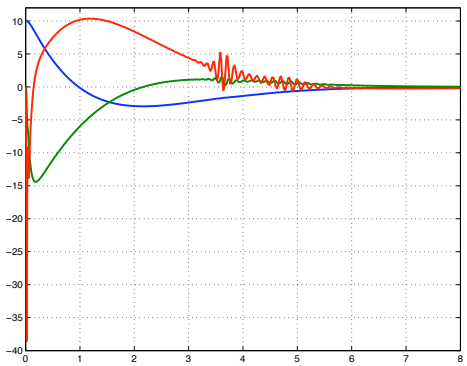
Result is $h \in [0 \;\; 9.6213]$

Simulations are done with the choice of $G = G_2$.

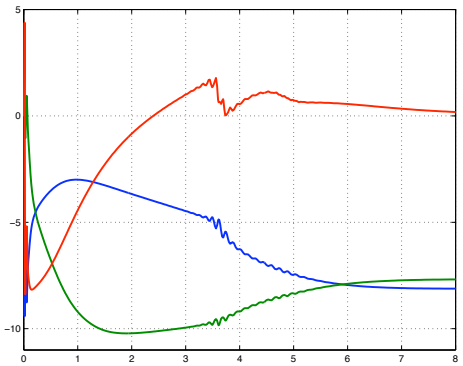
④ Example : autonomous aircraft

Simulations with $\Gamma_i = 1$

$h = 5$km

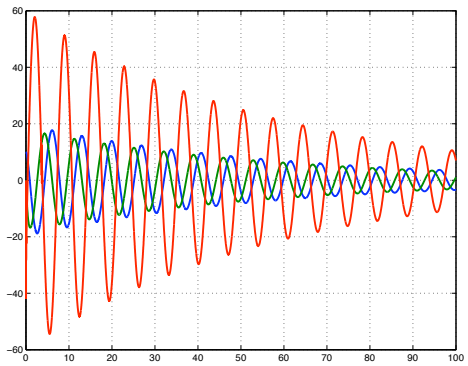


$y(t)$

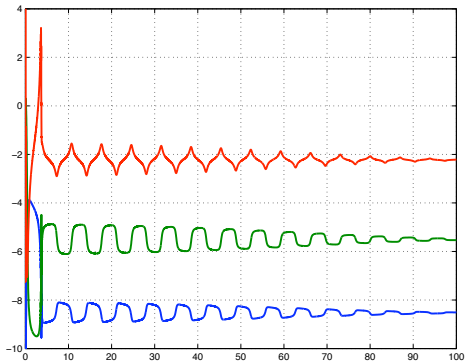


$k(t)$

$h = 9.6$km

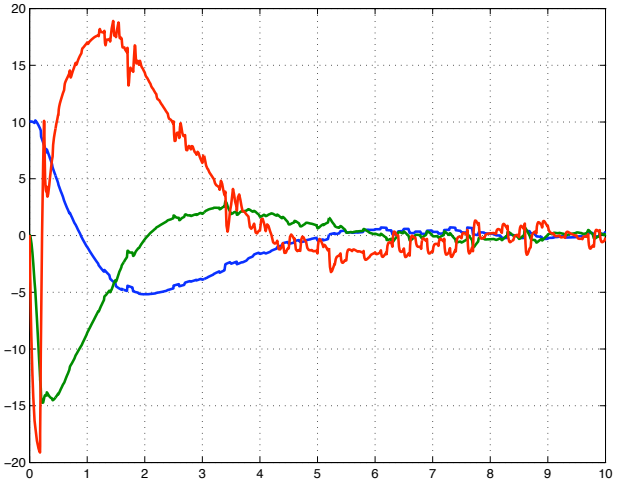


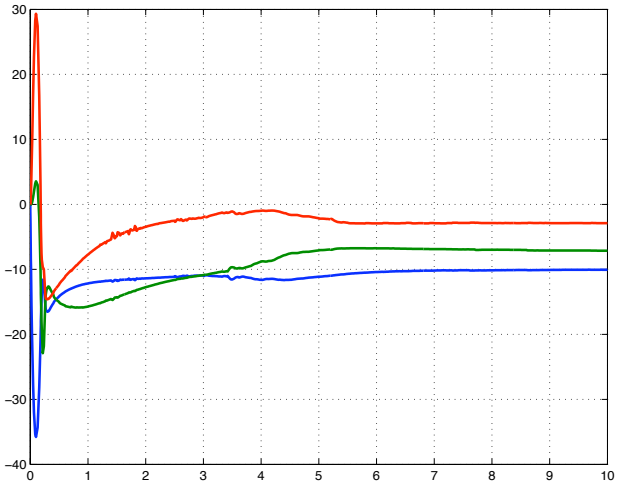
$y(t)$



$k(t)$

$h = 8$km with noisy measurements and saturated input (± 20)





Conclusion

- ➚ LMI-based proof of robustness
- ➚ Easy to generalize to other uncertain modeling
- ➚ BMI design of G
- ➚ $K(t)$ constrained to converge to a bounded set

➘ Conservative step :

design of G *a prioiri* before checking the LMI robustness conditions

✪ Need for Γ_i design results : convergence speed, disturbance rejections ...