Passification-Based Adaptive Control: Robustness Issues

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Passification-based adaptive control [Fradkov 1974]

Let an LTI $\Sigma$ and assume there exists $F$ and $G$ such that $\Sigma \star F$ is $G$-passive i.e.

\[
\begin{align*}
\dot{V}(x(t)) &< V(x(0)) \\
+ \int_0^t [u(\theta) \ast Gy(\theta)] \, d\theta
\end{align*}
\]

then whatever positive $\Gamma_i > 0$, $\Sigma$ is $G$-passified by the PBAC

\[
u(t) = K(t)y(t), \quad \dot{K}_i(t) = -y_i^*(t)\Gamma_i Gy(t).
\]

Technical remark

→ One can always take $F = -kG$ with $k$ sufficiently large

→ $F$ exists if $G\Sigma$ is hyper-minimum-phase
Introduction

Known Advantages of the PBAC
- Good observed behavior w.r.t. uncertainties and non-linearities
- Simple to design
- Based on physical meaning

Drawbacks
- Need to prove the robustness properties
- Need for numerical methods for choosing $G$
- Divergence of $K(t)$ due to disturbances

Outline
1. Modified PBAC that bounds $K(t)$ + robustness conditions
2. BMI design of $G$ for the nominal system
3. LMI design of Parameter-Dependent $F(\Delta)$ that fulfills robustness conditions
4. Example: autonomous aircraft
Bounded Passification Based Adaptive Algorithm

→ Let $\mathcal{B}$ a bounded set of $C^m$ and let a penalty function $\phi : C^m \mapsto C^m$:  

$$\phi(K) = 0 \quad \forall K \in \mathcal{B}$$  

$$(K - F)^* \phi(K) \geq 0 \quad \forall F \in \mathcal{B}$$

→ If there exists $F(\Delta) \in \mathcal{B}$ and $G$ such that $\Sigma(\Delta) \star F(\Delta)$ is $G$-passive for all $\Delta \in \Delta$, then whatever positive $\Gamma_i > 0$, $\Sigma(\Delta)$ is robustly $G$-passified by the BPBAC

$$u(t) = K(t)y(t) \; , \; \dot{K}_i(t) = -y_i^*(t)\Gamma_i G y(t) - \Gamma_i \phi(K_i(t)) \; .$$

→ Convergence of the BPBAC is such that

$$x(\infty) = 0 \; , \; K(\infty) \in \mathcal{B} \; , \; \Sigma(\Delta) \star K(\infty) \text{ is } G\text{-passive}$$
Bounded PBAC

Proof of BPBAC properties

→ The considered LTI uncertain models $\Sigma(\Delta)$ are such that:

$$\dot{x} = A(\Delta)x + Bu \ , \ y = Cx .$$

→ $x = 0$ and $K = F(\Delta)$ are stable equilibrium points.

→ $\Sigma(\Delta) \ast F(\Delta)$ being $G$-passive implies the existence of $H(\Delta)$:

$$H(\Delta) = H^*(\Delta) > 0 \ , \ H(\Delta)B = C^*G^*$$

$$H(\Delta)A(\Delta, F(\Delta)) + A^*(\Delta, F(\Delta))H(\Delta) < 0$$

and the BPBAC closed-loop system has a storage function of the class

$$V(x, K, \Delta) = \frac{1}{2}x^*H(\Delta)x + \frac{1}{2} \sum_{i=1}^{l}(K_i - F_i(\Delta))^*\Gamma_i^{-1}(K_i - F_i(\Delta))$$
BMI design of $G$ for the nominal system

Conditions for BPBAC passification

Existence of $F(\Delta) \in \mathcal{B}$ and $G$ such that
$\Sigma(\Delta) \star F(\Delta)$ is $G$-passive for all $\Delta \in \Delta$.

Conservative procedure for the BPBAC conditions

→ Design $G$ such that $\Sigma(\Delta = 0)$ is $G$-passifiable via static output feedback.
→ For a given $G$, prove the existence of $F(\Delta)$ such that
$\Sigma(\Delta) \star F(\Delta)$ is $G$-passive for all uncertainties.
BMI design of $G$ for the nominal system

$G$-passification of $\Sigma(0)$ [Fradkov 1976]

\[
H = H^* > 0 \quad , \quad HB = C^*G^*
\]

\[
H(A(0) + BFC) + (A(0) + BFC)^*H < 0
\]

Along solutions, there always exists $F = -kG$ with $k$ sufficiently large.

BMI (YALMIP+PENBMI) design

→ Choose a (large) value of $k$

→ Choose an upper bound on $H$ (we took $\bar{h} = 1$) for scaling the solutions

→ Declare in YALMIP the following BMI problem

\[
\bar{h}1 > H > 0 \quad , \quad HB = C^*G^* \quad , \quad t > -1
\]

\[
HA(0) + A^*(0)H - kC^*G^*GC < t1
\]

→ Minimize $t$ using PenBMI, if it returns $t < 0$, the procedure succeeded.
"Quadratic stability", norm-bounded result

For a given $G$, if there exists $H$, $F$ and $\rho$ such that

$$
\begin{align*}
H &> 0 , \quad HB = C^*G^* \quad , \quad F \in \mathcal{B} \\
\begin{bmatrix}
\langle HA(0) + C^*G^*FC \rangle & HB_\Delta \\
B^*_\Delta H & -1
\end{bmatrix} + \rho
\begin{bmatrix}
C^*_\Delta \\
D^*_\Delta
\end{bmatrix}
\begin{bmatrix}
C^*_\Delta \\
D^*_\Delta
\end{bmatrix}^* < 0
\end{align*}
$$

then $\Sigma(\Delta) \star F$ is $G$-passive for all uncertainties $\Delta^*\Delta \leq \rho 1$ where $\Sigma(\Delta)$:

$$
\dot{x} = [A(0) + B_\Delta \Delta (1 - D_\Delta \Delta)^{-1} C_\Delta]x + Bu \quad , \quad y = Cx .
$$

Remarks

→ Maximization of $\rho$ is LMI

→ If $\Delta \subset \Delta_\rho = \{\Delta^*\Delta \leq \rho 1\}$ the problem is solved with $F(\Delta) = F$

→ Replacing in the LMIs $A(0)$ by $A(\Delta_i)$ for some $\Delta_i$, gives a couple $(F_i, \rho_i)$ such that $\Sigma(\Delta) \star F_i$ is robustly $G$-passive w.r.t. $\{\Delta_i + \Delta : \Delta \in \Delta_{\rho_i}\}$. 
Solving the LMI conditions for some finite sequence \( \{ \Delta_i \} \) gives sequences \( \{ F_i \}, \{ \rho_i \} \) that defines an \( F(\Delta) \) for \( \bigcup \{ \Delta_i + \Delta : \Delta \in \Delta_{\rho_i} \} \).

If \( \Delta \subset \bigcup \{ \Delta_i + \Delta : \Delta \in \Delta_{\rho_i} \} \)
the robustness BPABC \( G \)-passification conditions are fulfilled.
Example: autonomous aircraft

Linearized fourth-order model of lateral dynamics for an autonomous aircraft including model of actuator dynamics. Uncertainty is the flight altitude ($\Delta = h$). System has 3 outputs, one input.

BMI design of $G$ for the nominal system $h = 5$ km.

For two choices of high gain $k$

$$G_{1\text{PenBMI}} = 10^{-2} \begin{bmatrix} 4.5404 & 2.8436 & 1.7107 \end{bmatrix}$$

$$G_{2\text{PenBMI}} = 10^{-3} \begin{bmatrix} 8.4000 & 5.4505 & 3.0961 \end{bmatrix}$$

and after simple analysis step:

$$G_1 = \begin{bmatrix} 4 & 3 & 2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 8 & 5 & 3 \end{bmatrix}$$
Example: autonomous aircraft

→ Choice of LMI representable $\mathcal{B}$ (bounded set of control gains)

$$F = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix}, \quad -10 \leq f_i \leq 10$$

and $\phi$ dead-zone penalty function of the BPBAC:

$$\phi(f_i) = f_i - \text{sat}_{10}(f_i)$$
Example: autonomous aircraft

→ LMI design of admissible $h$ intervals

$\Delta \in \mathbb{C}$ for $G_1$

Result is $h \in [0 \ 9.2976]$

$\Delta \in \mathbb{C}$ for $G_2$

Result is $h \in [0 \ 9.6213]$

Simulations are done with the choice of $G = G_2$. 
Example: autonomous aircraft

Simulations with $\Gamma_i = 1$

$h = 5\text{km}$

$h = 9.6\text{km}$

$h = 8\text{km}$ with noisy measurements and saturated input ($\pm 20$)
Conclusion

- LMI-based proof of robustness
- Easy to generalize to other uncertain modeling
- BMI design of $G$
- $K(t)$ constrained to converge to a bounded set

- Conservative step:
  design of $G$ a priori before checking the LMI robustness conditions

- Need for $\Gamma_i$ design results: convergence speed, disturbance rejections ...