Robust passification via static output feedback - LMI results

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Problem statement

Passification and Passivity-based techniques:
- linear and nonlinear control
- simplicity and physical meaning
- robustness
- applications to adaptive control, control of partially linear composite systems, flight control, process control...

Passification of LTI systems:
- SISO and MIMO
- SOF for Strict Positive Real $\Leftrightarrow$ hyper-minimum-phaseness
- Proof of robustness w.r.t. parametric uncertainty (norm-bounded)
- Passification of non square systems: G-passification
Problem statement

Let an LTI uncertain system:

\[
W(s, \Delta) \sim \begin{cases}
\dot{x} = A(\Delta)x + Bu \\
y = Cx
\end{cases}
\]

rational with respect to \( \Delta \)

\[
A(\Delta) = A + B_\Delta \Delta(I - D_\Delta \Delta)^{-1}C_\Delta
\]

uncertain constant real or complex norm-bounded:

\[
\Delta^C = \{ \Delta \in \mathbb{C}^{m_\Delta \times l_\Delta} : \Delta^* \Delta \leq I \} ,
\Delta^R = \{ \Delta \in \mathbb{R}^{m_\Delta \times l_\Delta} : \Delta^T \Delta \leq I \} .
\]

And let \( G \in \mathbb{C}^{m \times p} \) be given, where \( B \in \mathbb{C}^{n \times m} \) and \( C' \in \mathbb{C}^{p \times n} \)
Robust $G$-Hyper-Minimum-Phaseness

The system is robustly $G$-HMP if $\forall \Delta \in \Delta$

$$
\phi(s, \Delta) = \det(sl - A(\Delta)) \det GW(s, \Delta) = \det \begin{bmatrix}
    sl - A(\Delta) & -B \\
    GC' & 0
\end{bmatrix}
$$

is Hurwitz and the high-frequency gain of $GW(s, \Delta)$ is a square symmetric positive definite matrix: $GCB = B^*C^*G^* > 0$.

$\rightarrow$ Generalizes HMP to non-square systems.

$\rightarrow$ Robustness: infinite number of conditions to test.
Problem statement

Parameter-dependent and unique static output-feedback

PD-SOF : \( u = K(\Delta)y + v \)  
SOF : \( u = Ky + v \)

Robust \( G \)-Passive control

The closed-loop system is robustly strictly \( G \)-passive if \( \forall \Delta \in \mathbb{D} \)
there exists a quadratic PD storage function \( V(x, \Delta) = x^*H(\Delta)x > 0 \)
and a scalar \( \rho(\Delta) > 0 \) such that

\[
V(x(t), \Delta) \leq V(x(0), \Delta) + \int_0^t \left[ v(\theta)^*Gy(\theta) - \rho(\Delta)|x(\theta)|^2 \right] d\theta
\]

→ Generalizes strict passivity for non-square systems
→ \( G \)-passification : find \( K \) that makes the closed-loop \( G \)-passive
→ \( G \)-passification of \( W(s) \) ≠ Passification of \( GW(s) \)
Problem statement

Theorem 1: [Fradkov 1976-2003] Equivalence of

① $W(s, \Delta)$ is robustly $G$-HMP
② $W(s, \Delta)$ is robustly $G$-passifiable by PD-SOF $K(\Delta)$
③ $\exists K$ unique that robustly $G$-passificates $W(s, \Delta)$

Proof (Sketch)

$G$-HMP $\Rightarrow$ High gain control for any $\Delta$:

$$K(\Delta) = -k(\Delta)G : k(\Delta) > 0$$, sufficiently large

Well-posedness of uncertain modeling: $K = -\max_{\Delta \in \Delta} k(\Delta)G$

Outline

① LMI results for robust $G$-HMP analysis
③ LMI results for robust $G$-passifying SOF design

→ Numerical example: cruise missile model.
**Theorem 1** Let the following matrices

\[
N = (GC)^\perp, \quad M = (NN^* + BB^*)^{-1}, \quad \tilde{A} = N^*MAM.
\]

\[
W(s, \Delta), \quad \Delta \in \Delta^C \text{ is robustly } G\text{-HMP}
\]

if and only if \(GC\) \(B > 0\) and \(\exists P > 0 \in \mathbb{C}\)

\[
\begin{bmatrix}
P\tilde{A} + \tilde{A}^*P & PN^*MB\Delta \\
B\Delta^*MNP & -I
\end{bmatrix} + \begin{bmatrix}
N^*C^*_\Delta \\
D^*_\Delta
\end{bmatrix} \begin{bmatrix}
N^*C^*_\Delta \\
D^*_\Delta
\end{bmatrix}^* < 0
\]

where \(N = (GC)^\perp\) and \(M = (NN^* + BB^*)^{-1}\).

*In case \(\Delta \in \Delta^R, P \in \mathbb{R}; LMI \text{ conditions are only sufficient}.*

**Proof**

Robust \(G\)-HMP is reformulated as

the robust Hurwitz stability of a reduced order system.
Robust $G$-passifying design

**Theorem 2** $W(s, \Delta)$ is uniformly robustly strictly $G$-passifiable via SOF if and only if $\exists H > 0 \in C, \exists K \in C$:

$$HB = C^*G^*$$

$$\begin{bmatrix}
HA + A^*H + C^*(G^*K + K^*G)C \\
B^*_\Delta H \\
\end{bmatrix}
+ \begin{bmatrix}
C^*_\Delta \\
D^*_\Delta \\
\end{bmatrix}
\begin{bmatrix}
C^*_\Delta \\
D^*_\Delta \\
\end{bmatrix}^* < 0$$

**Proof**

Classical LMI results for 'quadratic' stability

$\rightarrow$ Uniform storage function $V(x, \Delta) = V(x) = x^*Hx$. 

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Theorem 3  
$W(s, \Delta)$ is uniformly robustly strictly $\mathcal{G}$-passifiable via SOF if and only if $\exists H > 0 \in C, \exists K \in C$:

$$HB = C^*G^*$$
$$\begin{bmatrix}
    HA + A^*H + C^*(G^*K + K^*G)C & HB\Delta \\
    B^*_\Delta H & -I
\end{bmatrix} + \begin{bmatrix}
    C^*_\Delta \\
    D^*_\Delta
\end{bmatrix} \begin{bmatrix}
    C^*_\Delta \\
    D^*_\Delta
\end{bmatrix}^* < 0$$

Remarks

✪ Thm 2 $\Rightarrow$ Thm 1 with $P = N^*HN$ (conjecture: converse also holds)

✪ PB: design $K$ and $G$ simultaneously?

   LMI problem if $\exists S$ such that $PB = BS$ (conservative)

✪ Always possible to take $K = -kG$ if feasible.

✪ Possible to add LMI constraints on $K$, e.g. find $K$ with minimum norm.
Numerical example: cruise missile

Model definition

→ 4th order model of lateral dynamics for cruise missile + actuator dynamics
→ Dynamics depend on altitude $h \in [\underline{h}, \bar{h}] \subset \mathbb{R}^+$ (converted into $\Delta \in \mathbb{R}^n$)
→ Measured outputs:
yaw angle $\varphi(t)$, yaw angular rate $\dot{r}(t)$ and the rudder deflection angle $\delta_r(t)$
→ Control input: rudder servo command signal
→ $G$ is chosen a priori such that $GCB > 0$. 
**Numerical example: cruise missile**

**Robust $G$-HMP analysis**

- For $\underline{h} = 0$ and $\overline{h} = 10\text{km}$: feasible
- For $\underline{h} = 9.9925\text{km}$ and $\overline{h} = 10.2105\text{km}$: feasible
- For $\underline{h} = 0$ and $\overline{h} = 10.2105\text{km}$: infeasible
  - $h = 10.1 + 0.5i$ makes system non $G$-HMP.
  - Conservatism for real-valued uncertainty.

- Exists a SOF for $h \in [0 \quad 10.2105]$, cannot be found with Thm 2.
Robust $G$-passifying SOF design

Assume $h \in [0, 10]$

- Thm 2
  - $\Rightarrow K_1 = -\begin{bmatrix} 79.28 & 50.34 & 11.92 \end{bmatrix}$

- Thm 2, $\min \| K \|:$
  - $\Rightarrow K_2 = -\begin{bmatrix} 60.75 & 34.47 & 10.67 \end{bmatrix}$

- Thm 2, $\min_{K=-kG} k,$:
  - $\Rightarrow K_3 = -\begin{bmatrix} 118.53 & 44.45 & 14.82 \end{bmatrix}$


Yaw angle and rudder deflection

for control $K_2$

and for $h = 0.1, 5, 9$
Non-conservative (complex case) LMI conditions of robust strict $G$-passification

Conservative LMI design method

Design simultaneously $K$ and $G$

Design of robust $G$-passifying adaptive control $u(t) = K(t)y(t)$