Robust passification via static output feedback - LMI results

Dimitri PEAUCELLE[†] & Alexander FRADKOV[‡] & Boris ANDRIEVSKY[‡]

† LAAS-CNRS - Toulouse, FRANCE

‡ IPME-RAS - St Petersburg, RUSSIA

Problem statement

Passification and Passivity-based techniques :

→ linear and nonlinear control

→ simplicity and physical meaning

→ robustness

→ applications to adaptive control, control of partially linear composite systems, flight control, process control...

Passification of LTI systems :

→ SISO and MIMO

→ SOF for Strict Positive Real ⇔ hyper-minimum-phaseness

→ Proof of robustness w.r.t. parametric uncertainty (norm-bounded)

→ Passification of non square systems: G-passification

Problem statement

Let an LTI uncertain system:

$$W(s,\Delta) \sim \begin{cases} \dot{x} = A(\Delta)x + Bu \\ y = Cx \end{cases}$$

rational with respect to Δ

$$A(\Delta) = A + B_\Delta \Delta (\mathrm{I} - D_\Delta \Delta)^{-1} C_\Delta$$

uncertain constant real or complex norm-bounded:

$$\boldsymbol{\Delta}^{\mathbb{C}} = \{\Delta \in \mathbb{C}^{m_\Delta \times l_\Delta} \ : \ \Delta^* \Delta \leq \mathrm{I}\} \,,$$
$$\boldsymbol{\Delta}^{\mathbb{R}} = \{\Delta \in \mathbb{R}^{m_\Delta \times l_\Delta} \ : \ \Delta^T \Delta \leq \mathrm{I}\} \,.$$

And let $G \in \mathbb{C}^{m \times p}$ be given, where $B \in \mathbb{C}^{n \times m}$ and $C \in \mathbb{C}^{p \times n}$

Robust G-Hyper-Minimum-Phaseness

The system is robustly G-HMP if $\forall \Delta \in \mathbf{\Delta}$

$$\phi(s,\Delta) = \det(s\mathsf{I} - A(\Delta)) \det GW(s,\Delta) = \det \begin{bmatrix} s\mathsf{I} - A(\Delta) & -B \\ GC & \mathsf{O} \end{bmatrix}$$

is Hurwitz and the high-frequency gain of $GW(s,\Delta)$ is a square symmetric positive definite matrix: $GCB = B^*C^*G^* > \mathsf{O}$.

→ Generalizes HMP to non-square systems.

→ Robustness: infinite number of conditions to test.

Problem statement

Parameter-dependent and unique static output-feedback

PD-SOF : $u = \mathbf{K}(\Delta)y + v$ SOF : $u = \mathbf{K}y + v$

Robust G-Passive control

The closed-loop system is robustly strictly G-passive if $\forall \Delta \in \mathbf{\Delta}$ there exists a quadratic PD storage function $V(x,\Delta) = x^*\mathbf{H}(\Delta)x > 0$ and a scalar $\rho(\Delta) > 0$ such that

$$V(x(t),\Delta) \leq V(x(0),\Delta) + \int_0^t \left[v(\theta)^* G y(\theta) - \rho(\Delta)|x(\theta)|^2 \right] d\theta$$

→ Generalizes strict passivity for non-square systems

→ G-passification : find $\mathbf{K}$ that makes the closed-loop G-passive

→ G-passification of $W(s) \neq$ Passification of $GW(s)$

Theorem 1 : [Fradkov 1976-2003] Equivalence of

① $W(s,\Delta)$ is robustly G-HMP

② $W(s,\Delta)$ is robustly G-passifiable by PD-SOF $\mathbf{K}(\Delta)$

③ $\exists\mathbf{K}$ unique that robustly G-passificates $W(s,\Delta)$

Proof (Sketch)

G-HMP $\Rightarrow$ High gain control for any Δ:

$$\mathbf{K}(\Delta) = -\mathbf{k}(\Delta)G \;\; : \;\; \mathbf{k}(\Delta) > 0 \;\; , \;\; \text{sufficiently large}$$

Well-posedness of uncertain modeling: $\mathbf{K} = -\max_{\Delta\in\boldsymbol{\Delta}} \mathbf{k}(\Delta)G$

Outline

① LMI results for robust G-HMP analysis

③ LMI results for robust G-passifying SOF design

→ Numerical example : cruise missile model.

Theorem 1 *Let the following matrices*

$$N = (GC)^{\perp} \;,\;\; M = (NN^* + BB^*)^{-1} \;,\;\; \tilde{A} = N^* M A N \;.$$

$W(s, \Delta)$, $\Delta \in \mathbb{\Delta}^{\mathsf{C}}$ *is robustly G-HMP*

if and only if $GCB > 0$ *and* $\exists \mathbf{P} > 0 \;\in \mathsf{C}$

$$\begin{bmatrix} \mathbf{P}\tilde{A} + \tilde{A}^*\mathbf{P} & \mathbf{P}N^*MB_\Delta \\ B_\Delta^* M N \mathbf{P} & -\mathsf{I} \end{bmatrix} + \begin{bmatrix} N^* C_\Delta^* \\ D_\Delta^* \end{bmatrix} \begin{bmatrix} N^* C_\Delta^* \\ D_\Delta^* \end{bmatrix}^* < 0$$

where $N = (GC)^{\perp}$ *and* $M = (NN^* + BB^*)^{-1}$.

In case $\Delta \in \mathbb{\Delta}^{\mathsf{R}}$, $\mathbf{P} \in \mathsf{R}$ *; LMI conditions are only sufficient.*

Proof

Robust G-HMP is reformulated as

the robust Hurwitz stability of a reduced order system.

Theorem 2 $W(s,\Delta)$ *is uniformly robustly strictly G-passifiable via SOF if and only if* $\exists \mathbf{H} > 0 \in \mathsf{C}$, $\exists \mathbf{K} \in \mathsf{C}$:

$$\mathbf{H}B = C^*G^*$$

$$\begin{bmatrix} \mathbf{H}A + A^*\mathbf{H} + C^*(G^*\mathbf{K} + \mathbf{K}^*G)C & \mathbf{H}B_\Delta \\ B_\Delta^*\mathbf{H} & -\mathbf{I} \end{bmatrix} + \begin{bmatrix} C_\Delta^* \\ D_\Delta^* \end{bmatrix} \begin{bmatrix} C_\Delta^* \\ D_\Delta^* \end{bmatrix}^* < 0$$

Proof

Classical LMI results for 'quadratic' stability

➞ Uniform storage function $V(x,\Delta) = V(x) = x^*\mathbf{H}x$.

Theorem 3 $W(s, \Delta)$ *is uniformly robustly strictly* G*-passifiable via SOF if and only if* $\exists \mathbf{H} > 0 \in \mathcal{C}$, $\exists \mathbf{K} \in \mathcal{C}$:

$$\mathbf{H}B = C^* G^*$$

$$\begin{bmatrix} \mathbf{H}A + A^*\mathbf{H} + C^*(G^*\mathbf{K} + \mathbf{K}^*G)C & \mathbf{H}B_\Delta \\ B_\Delta^*\mathbf{H} & -\mathrm{I} \end{bmatrix} + \begin{bmatrix} C_\Delta^* \\ D_\Delta^* \end{bmatrix} \begin{bmatrix} C_\Delta^* \\ D_\Delta^* \end{bmatrix}^* < 0$$

Remarks

- Thm 2 $\Rightarrow$ Thm 1 with $\mathbf{P} = N^*\mathbf{H}N$ (conjecture : converse also holds)
- PB : design $\mathbf{K}$ and G simultaneously ?

  LMI problem if $\exists \mathbf{S}$ such that $\mathbf{P}B = B\mathbf{S}$ (conservative)
- Always possible to take $\mathbf{K} = -\mathbf{k}G$ if feasible.
- Possible to add LMI constraints on $\mathbf{K}$, *e.g.* find $\mathbf{K}$ with minimum norm.

Numerical example : cruise missile

Model definiion

→ 4th order model of lateral dynamics for cruise missile + actuator dynamics

→ Dynamics depend on altitude $h \in [\underline{h} \;\; \bar{h}] \subset \mathbb{R}^+$ (converted into $\Delta \in \mathbf{\Delta}^R$)

→ Measured outputs:

yaw angle $\varphi(t)$, yaw angular rate $r(t)$ and the rudder deflection angle $\delta_r(t)$

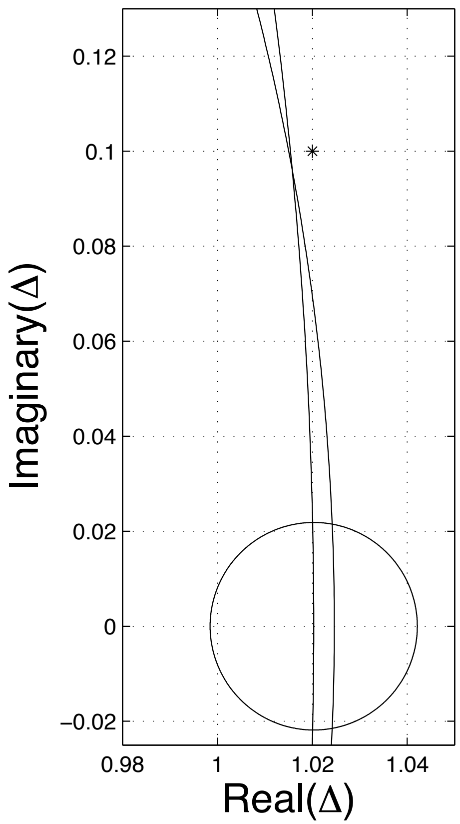
→ Control input: rudder servo command signal

→ G is chosen *a priori* such that $GCB > 0$.

Numerical example : cruise missile

Robust G-HMP analysis

- For $\underline{h} = 0$ and $\bar{h} = 10$km : feasible
- For $\underline{h} = 9.9925$km and $\bar{h} = 10.2105$km : feasible
- For $\underline{h} = 0$ and $\bar{h} = 10.2105$km : infeasible
  - → $h = 10.1 + 0.5i$ makes system non G-HMP.
  - → Conservatism for real-valued uncertainty.



→ Exists a SOF for $h \in [0 \;\; 10.2105]$, cannot be found with Thm 2.

Numerical example : cruise missile

Robust G-passifying SOF design Assume $h \in [0 \;\; 10]$

✪ Thm 2 $\rightarrow \mathbf{K}_1 = -\left[\begin{array}{ccc} 79.28 & 50.34 & 11.92 \end{array} \right]$

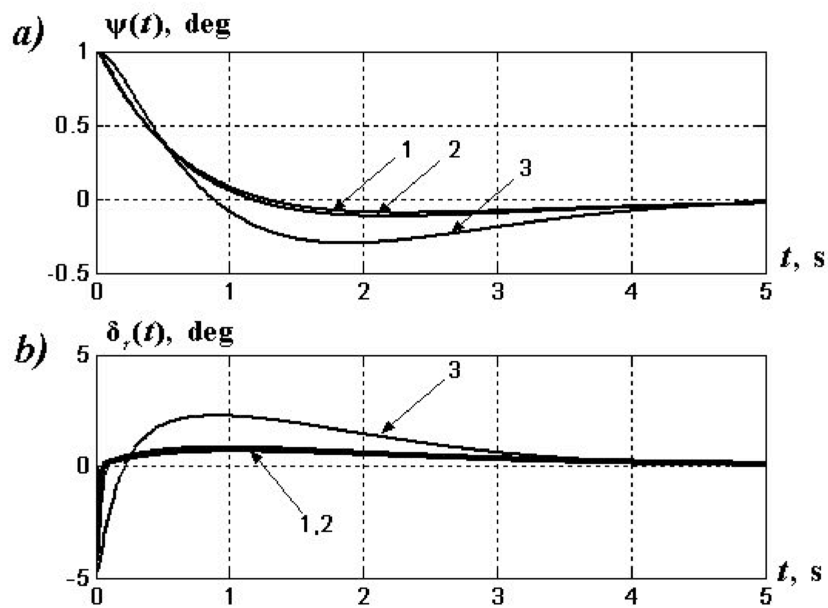
✪ Thm 2, $\min \|\mathbf{K}\|$: $\rightarrow \mathbf{K}_2 = -\left[\begin{array}{ccc} 60.75 & 34.47 & 10.67 \end{array} \right]$

✪ Thm 2, $\min_{\mathbf{K}=-\mathbf{k}G} \mathbf{k}$, : $\rightarrow \mathbf{K}_3 = -\left[\begin{array}{ccc} 118.53 & 44.45 & 14.82 \end{array} \right]$

Yaw angle and rudder deflection

for control $\mathbf{K}_2$

and for $h = 0.1 \;, \;\; 5 \;, \;\; 9$



Conclusion

→ Non-conservative (complex case) LMI conditions of robust strict G-passification

→ Conservative LMI design method

✪ Design simultaneously $\mathbf{K}$ and G

✪ Design of robust G-passifying adaptive control $u(t) = \mathbf{K}(t)y(t)$