

Robust Simple Adaptive Control with Relaxed Passivity and PID control of a Helicopter Benchmark

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IFAC World Congress / Milano / August 28 - September 2, 2011

■ Considered problem:

● Stabilization with simple adaptive control

$$u = Ky, \quad \dot{K} = -Gyy^T\Gamma + \phi$$

● For MIMO LTI systems

■ Assumptions:

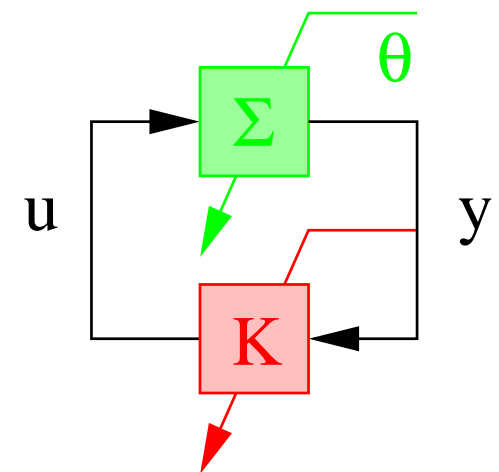
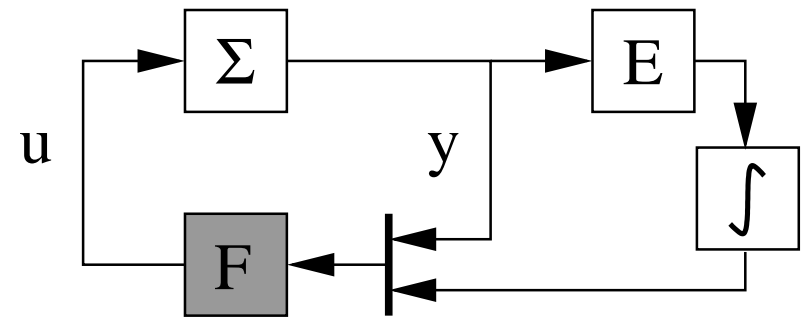
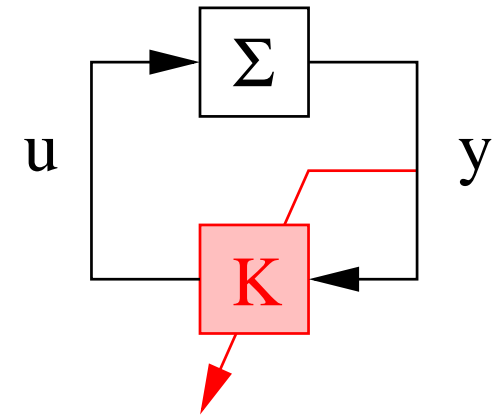
● There exists a (given) stabilizing PI control

$$u = F_P y + F_I \int E y$$

■ Why simple adaptive control?

● Expected to be more robust

● No need for estimation $K \neq F(\hat{\theta})$



- PI structure of the adaptive controller
- Adaptation that stops as the output reaches the reference

- Design of the adaptive law: $\dot{K} = -Gyy^T\Gamma + \phi$
- Virtual feedthrough D & barrier function ϕ for bounding K
- LMI based results \Rightarrow guaranteed robustness

- Preliminary tests on a 3D helicopter Benchmark
- Adaptive PID

PI structure of Adaptive Control

Assumptions:

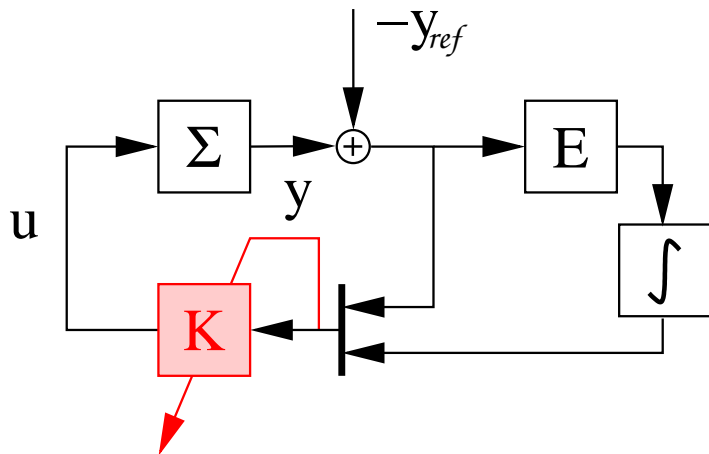
- There exists a (given) stabilizing PI control

$$u = F_P(y - y_{ref}) + F_I \int E(y - y_{ref})$$

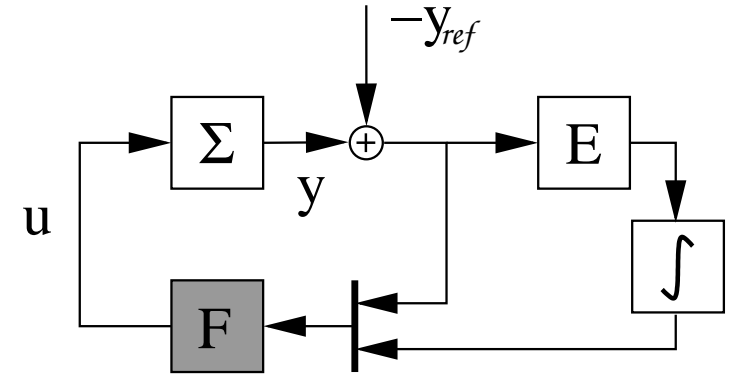
- Integral term: precision to constant reference

Impossible to apply the following adaptive control:

$$u = K\eta, \quad \dot{K} = -G\eta\eta^T\Gamma + \phi, \quad \eta = \begin{pmatrix} y - y_{ref} \\ \int E(y - y_{ref}) \end{pmatrix}$$



NO! because $\int E(y - y_{ref})$ does not go to 0



■ Proposed PI adaptive control law: $u = K_P(y - y_{ref}) + K_I \int E(y - y_{ref})$

$$\dot{K}_P = -G_P \begin{pmatrix} y - y_{ref} \\ \int E(y - y_{ref}) \end{pmatrix} (y - y_{ref})^T \Gamma_P + \phi_P$$

$$\dot{K}_I = -G_I (y - y_{ref}) \left(\int E(y - y_{ref}) \right)^T \Gamma_I + \phi_I$$

● When $(y - y_{ref})$ goes to zero adaptation stops

■ Results applicable to any controller structure of the type

$$u = K_1 y_1 + K_2 y_2, \quad \begin{aligned} \dot{K}_1 &= -G_1 z_1 y_1^T \Gamma_1 + \phi_1 \\ \dot{K}_2 &= -G_2 z_2 y_2^T \Gamma_2 + \phi_2 \end{aligned}$$

as long as stability and/or tracking imply that y_1 and z_2 go to zero.

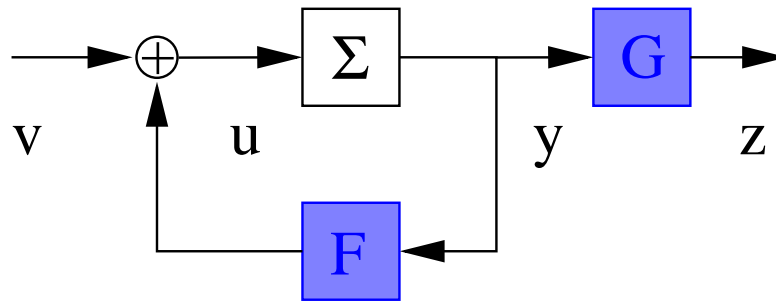
● Can also be extended to more than 2 terms ($u = K_1 y_1 + K_2 y_2 + K_3 y_3 + \dots$)

■ Passivity-Based Adaptive Control [Fradkov 1974, 2003]

& Simple Adaptive Control [Kaufman, Barkana, Sobel 94]

● Let $\Sigma \sim (A, B, C, D)$ be a MIMO system with m inputs / $p \geq m$ outputs.

● If $\exists (G, F) \in (\mathbb{R}^{p \times m})^2$ such that the following system is passive

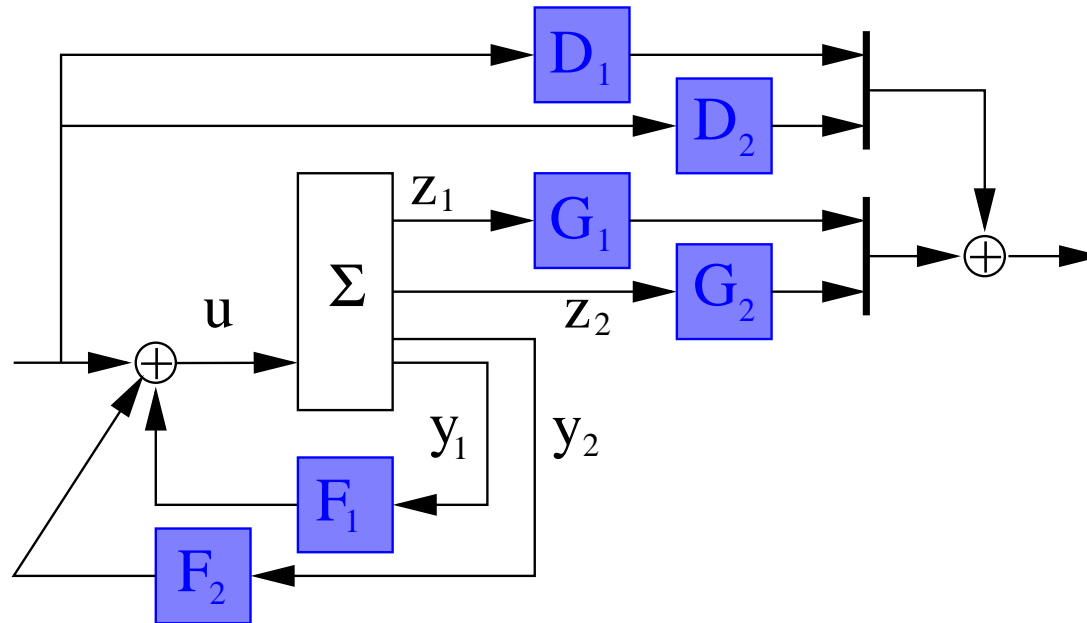


● then the following adaptive law stabilizes the system for all $\Gamma > 0$

$$\dot{K} = -Gyy^T\Gamma, \quad u = Ky$$

■ Proposed result

● If $\exists (G_1, G_2, F_1, F_2, D_1, D_2)$ such that the system below is passive



● then the following adaptive law stabilizes the system for all $\Gamma_1 > \mathbf{0}, \Gamma_2 > \mathbf{0}$

$$u = K_1 y_1 + K_2 y_2, \quad \begin{aligned} \dot{K}_1 &= -G_1 z_1 y_1^T \Gamma_1 - \phi_1(K_1) \\ \dot{K}_2 &= -G_2 z_2 y_2^T \Gamma_2 - \phi_2(K_2) \end{aligned}$$

where $\phi_1(K_1)$ and $\phi_2(K_2)$ are to be determined (depend of D_1 and D_2).

■ 2 step LMI design procedure

- Step 1: Given stabilizing F_1, F_2 solve LMI problem \mathcal{L}_1 to get (G_1, G_2, D_1, D_2) :

$$\mathcal{L}_1 : \begin{bmatrix} A^T(F_1, F_2)P + PA(F_1, F_2) & PB - C_1^T G_1^T & PB - C_2^T G_2^T \\ B^T P - G_1 C_1 & -2D_1 & \mathbf{0} \\ B^T P - G_2 C_2 & \mathbf{0} & -2D_2 \end{bmatrix} < \mathbf{0}.$$

- Step 2: Given $(G_1, G_2, F_1, F_2, D_1, D_2)$ solve LMI problem \mathcal{L}_2 (see paper)

to get α_1 and α_2 that define the functions ϕ_1, ϕ_2 :

dead-zone: $\phi_i(K_i) = 0$ if $\text{Tr}((K_i - F_i)^T D_i (K_i - F_i)) \leq \alpha_i$

barrier: $\phi_i(K_i) \rightarrow +\infty$ if $\text{Tr}((K_i - F_i)^T D_i (K_i - F_i)) \rightarrow \alpha_i \beta$

■ Properties of the LMI design procedure

- Applicable as soon as there is a given stabilizing control F_1, F_2
- The TV gains K_1, K_2 are guaranteed to be bounded in

$$\text{Tr} \left((K_i(t) - F_i)^T D_i (K_i(t) - F_i) \right) < \alpha_i \beta$$

- It is possible to maximize the domain of admissible adaptation values by
minimizing $\text{Tr}(D_i)$ in the first LMI step
maximizing α_i in the second LMI step
- Proof of stability with adaptive control using Lyapunov function

$$V(x, K_1, K_2) = x^T Q x + \sum_{i=1}^2 \text{Tr} \left((K_i - \hat{F}_i) \Gamma_i^{-1} (K_i - \hat{F}_i)^T \right)$$

where Q, \hat{F}_1 and \hat{F}_2 are solutions to the LMI problem \mathcal{L}_2

- Robustness properties of the LMI design procedure

- In case of polytopic uncertainties

$$\begin{bmatrix} A(\zeta) & B(\zeta) \end{bmatrix} = \sum_{j=1}^N \zeta_j \begin{bmatrix} A^{[j]} & B^{[j]} \end{bmatrix}, \quad \sum_{j=1}^N \zeta_j = 1, \quad \zeta_j \geq 0$$

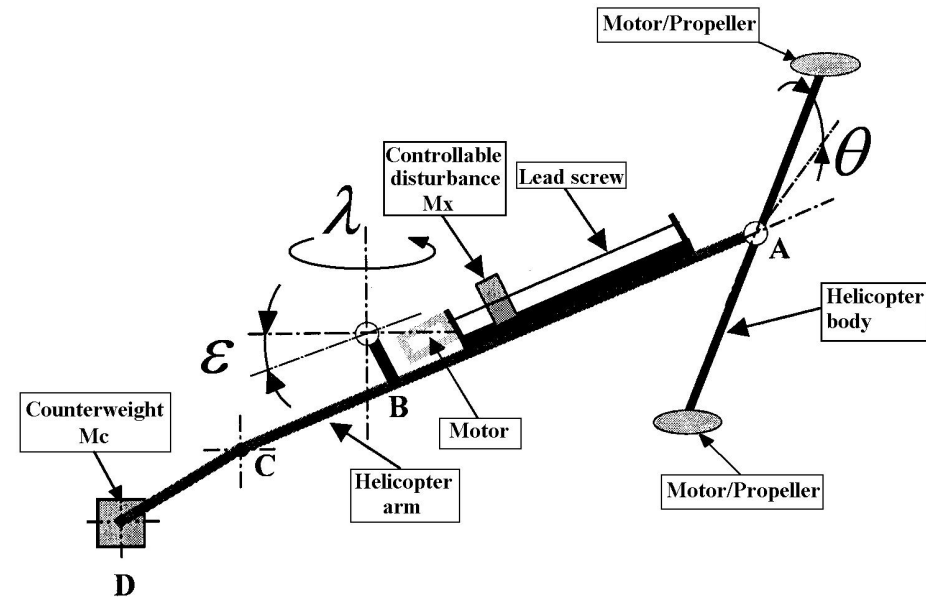
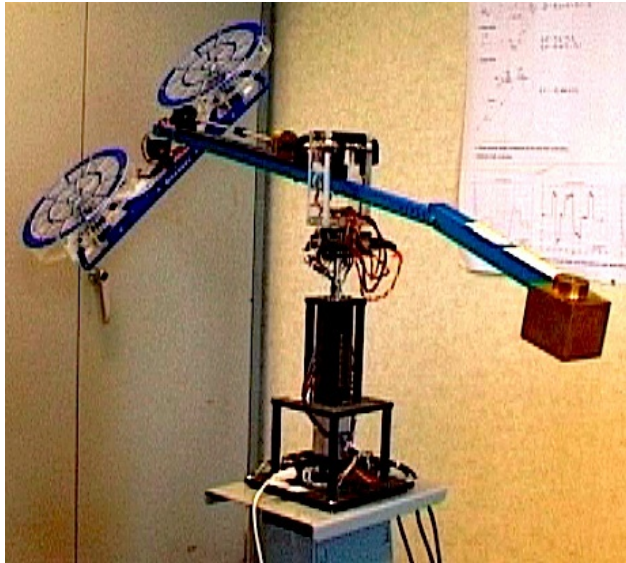
- Robust LMI version of second step (based on slack variables technique)
proves stability with parameter-dependent Lyapunov function

$$V(x, K_1, K_2, \zeta) = x^T Q(\zeta)x + \sum_{i=1}^2 \text{Tr} \left((K_i - \hat{F}_i(\zeta)) \Gamma_i^{-1} (K_i - \hat{F}_i(\zeta))^T \right)$$

where $Q(\zeta) = \sum_{j=1}^N \zeta_j Q^{[j]}$ and $\hat{F}_i(\zeta) = \sum_{j=1}^N \zeta_j \hat{F}_i^{[j]}$.

3DOF Helicopter control

■ 3DOF helicopter from Quanser© - LAAS configuration

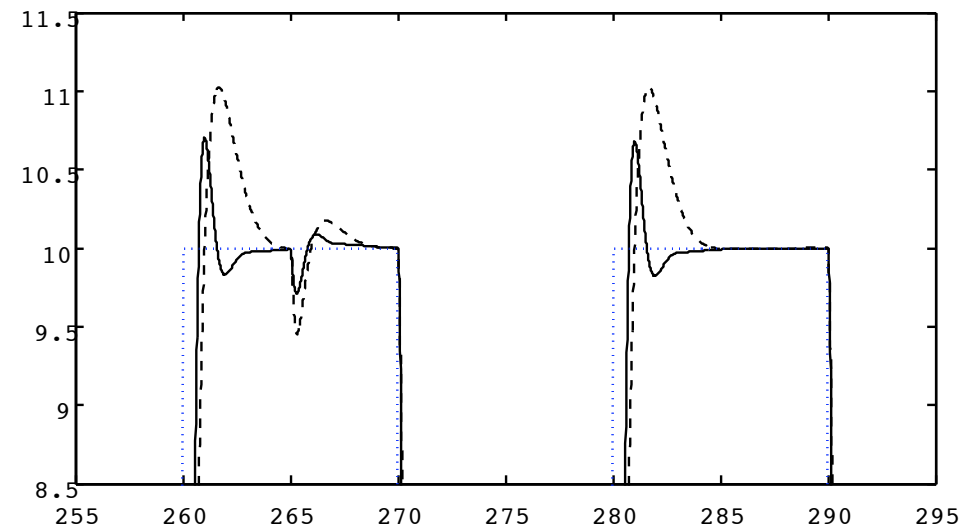
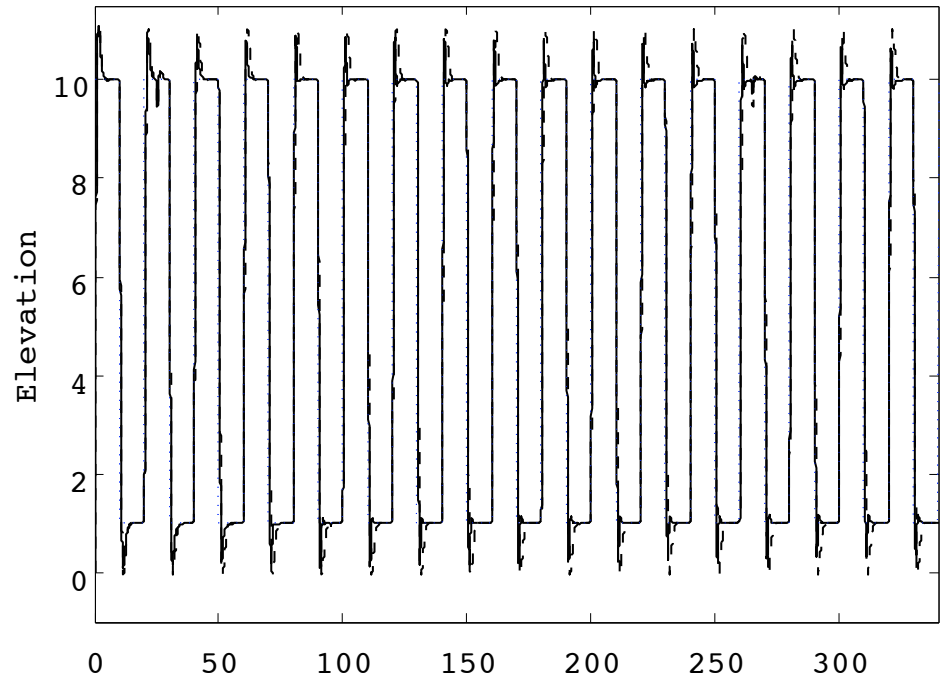
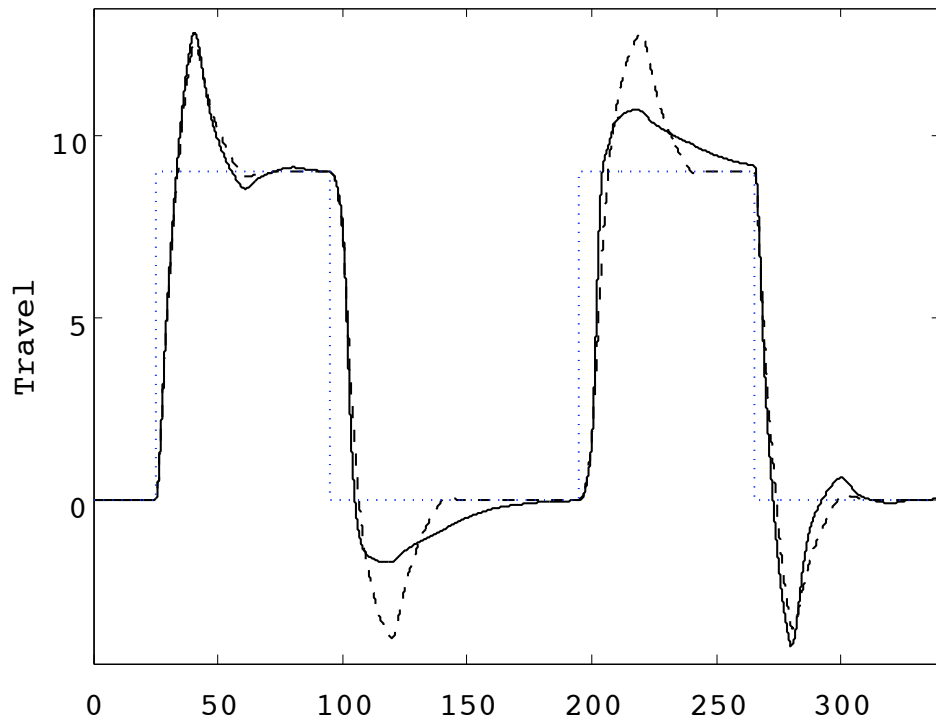


- All states measured: elevation ϵ , pitch θ , travel λ and their derivatives
- Two inputs: drag forces due to the propellers
- Non linear model:
 - ▲ linearized at operating point: allows to design an initial PID controller
(state-feedback problem)
 - ▲ non-linearities taken into account through an uncertain linear model

Robust adaptive PID control design applied

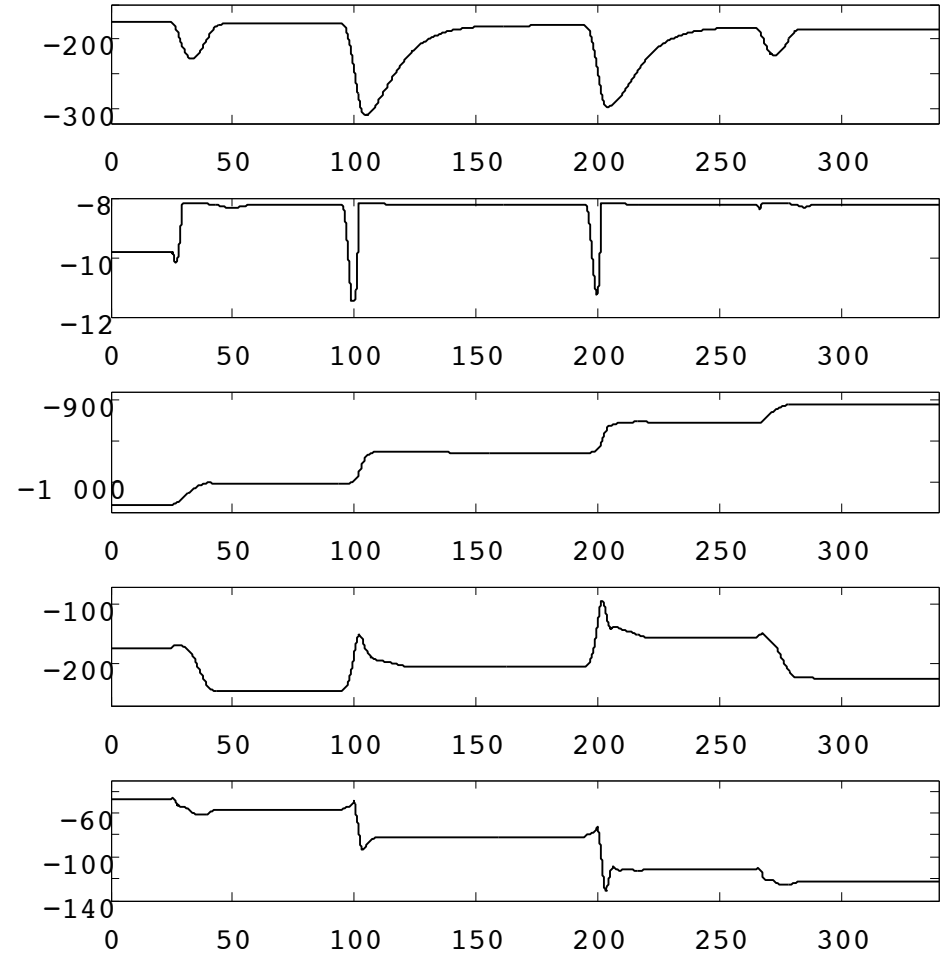
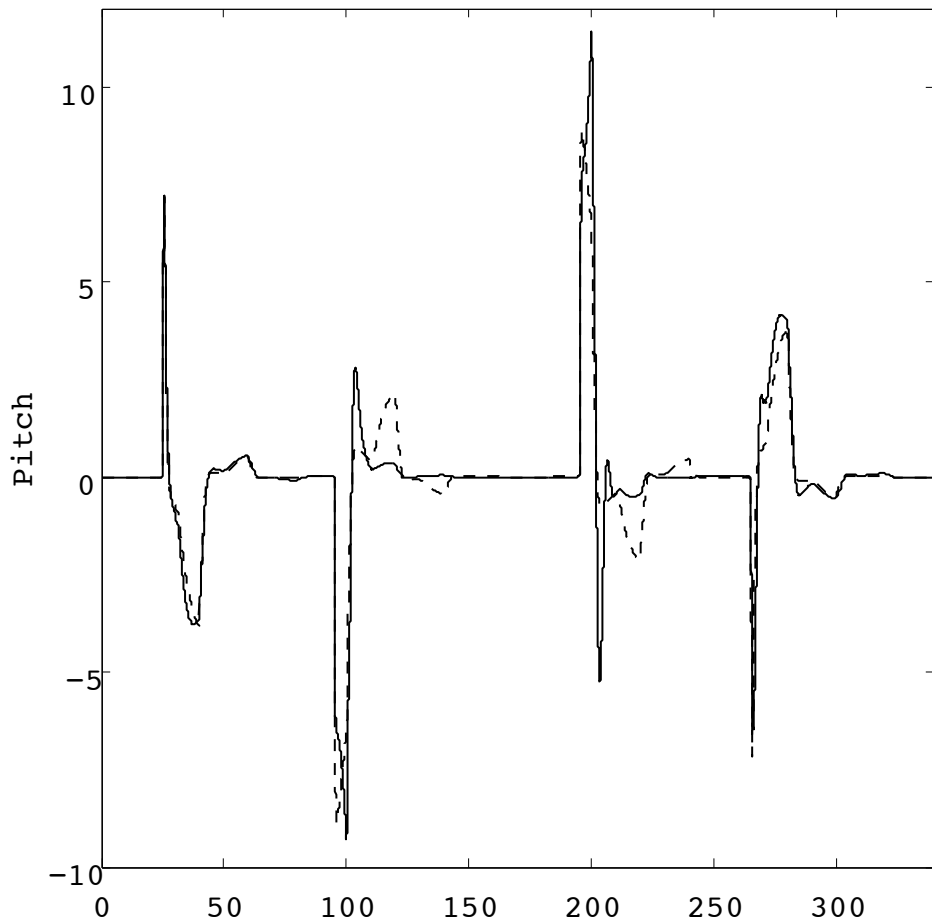
3DOF Helicopter control

■ Simulation results: travel and elevation (dotted: PID - solid: Adaptive)



3DOF Helicopter control

■ Simulation results: pitch and Adaptive PID gains



Conclusions

■ LMI-based method that guarantees robust stability of Adaptive control

● Applies to any stabilizable LTI MIMO system

● Allows to keep some structure such as PID

● Adaptive gains are bounded and remain close to initial given values

■ Prospectives

● More structured control (decentralized etc.)

● Guaranteed robustness for time-varying uncertainties

● Take advantage of flexibilities on G for engineering issues (saturations...)

● Apply to the actual helicopter benchmark