Simple Adaptive Control without Passivity Assumptions and Experiments on Satellite Attitude Control

DEMETER Benchmark

Dimitri Peaucelle
Adrien Drouot

Christelle Pittet
Jean Mignot

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Considered problem:

Stabilization with simple adaptive control

\[ \dot{K} = -Gyy^T \Gamma + \phi, \quad u = Ky \]

For MIMO LTI systems

Assumptions:

There exists a (given) stabilizing static output feedback

\[ u = Fy \]

Why simple adaptive control?

Expected to be more robust

No need for estimation \( K \neq F(\hat{\theta}) \)
Outline

- Design of the adaptive law
- Virtual feedthrough $D$ & barrier function $\phi$ for bounding $K$
- LMI based results

- Preliminary tests on a satellite Benchmark
- Attitude control
- Adaptive PD gains

Let \( \Sigma \sim (A, B, C, D) \) be a MIMO system with \( m \) inputs \( p \geq m \) outputs. If \( \exists (G, F) \in (\mathbb{R}^{p \times m})^2 \) such that the following system is passive

\[
\begin{align*}
\dot{K} &= -Gyy^T \Gamma, \quad u = Ky
\end{align*}
\]

then the following adaptive law stabilizes the system for all \( \Gamma > 0 \)
Design of the adaptive law

- Underlying properties
  - Passivity implies that for all $\Delta + \Delta^* \geq 0$ the following system is stable

\[
\begin{align*}
\dot{z} & = \Sigma y_v + u - \Delta F \\
\dot{y} & = F y_v \\
\dot{z} & = G y_v
\end{align*}
\]

- i.e. all gains $(F - \Delta G)$ stabilize the system, for $\Delta + \Delta^* \geq 0$, possibly large
- $\dot{K} = -Gyy^T \Gamma$ “pushes” the gains in that direction until stability is reached

- In practice: Need to limit growth of $K$. Modifications of adaptive law

\[
\dot{K} = -Gyy^T \Gamma + \phi(K) \quad (\text{eg. } \phi(K) = -\sigma K)
\]
Design of the adaptive law

- What if $\Sigma$ is not passifiable by $(G, F)$?
- $\exists S$ a feedthrough (or Shunt) such that the following system is passive

\[ G \exists S \text{ a feedthrough (or Shunt) such that the following system is passive} \]

- then the adaptive law stabilizes the system $\Sigma + S$.

- In practice: $S$ should be small for tracking issues ($u = K(y + Su)$)

- The actual gain is bounded

\[ \hat{K} = K(1 - KS)^{-1} \]
Design of the adaptive law

Proposed result

Stability proof based on a modified feedthrough scheme:

\[ \dot{K} = -G y y^T \Gamma - \psi_D(K) \cdot (K - F) \cdot (K - F) \cdot (K - F), \quad u = K y \]

\( \psi_D \) is a deadzone: no modification when \( K \) is close to \( F \)

\[ \psi_D(K) = 0 \quad \text{if} \quad \| K - F \|_2^2 \leq \nu \]

\( \psi_D \) is a barrier: goes to infinity when \( K \) reaches border of accepted region

\[ \psi_D(K) \rightarrow +\infty \quad \text{if} \quad \| K - F \|_2^2 \rightarrow \nu \beta \quad (\beta > 1) \]
Design of the adaptive law

- LMI-based design of \((G, D = \frac{\mu}{2} 1, \nu)\) assuming a given stabilizing \(F\)

- Step 1 (LMI): minimize \(\mu\) such that following system is passive

  \[
  \begin{bmatrix}
  A^T(F)P + PA(F) & PB - C^T G^T \\
  B^T P - GC & -\mu 1
  \end{bmatrix} < 0
  \]

- Step 2 (LMI): maximize \(\nu\), the size of admissible adaptive gains

  \[
  \begin{bmatrix}
  T & (\hat{F} - F)^T \\
  (\hat{F} - F)^T & \mu^{-1} 1
  \end{bmatrix} \geq 0, \quad \text{Tr}(T) \leq \nu \mu,
  \]

  \[
  \begin{bmatrix}
  A^T(F)Q + QA(F) + \nu \mu \beta C^T C \\
  + R + C^T (G^T (\hat{F} - F) + (\hat{F} - F)^T G) C \\
  R & QB - C^T G^T \\
  B^T Q - GC & \mu 1
  \end{bmatrix} < 0.
  \]
Design of the adaptive law

- LMI-based design of \((G, D = \frac{\mu}{2} 1, \nu)\) assuming a given stabilizing \(F\)

- Procedure guaranteed to succeed if \(F\) stabilizes the system

- \(K\) remains in a convex set around \(F\) (appreciated by engineers)

- \(\nu\) may be small, i.e. small admissible adaptation \((K \simeq F)\)

- LMI results can be easily extended to uncertain systems
  \[\Rightarrow\] proof of robustness of adaptive control for given uncertainty set

- In the robust case, step 2 is based on the existence of a PDSOF \(\hat{F}(\theta)\)

  Compared to \(\hat{F}(\theta)\), the adaptive gain \(K\) needs not the estimation of \(\theta\)

- Lyapunov function for global stability of PBAC

\[
V(x, K) = x^T Q x + \text{Tr}((K - \hat{F}) \Gamma^{-1}(K - \hat{F})^T)
\]
DEMETER satellite attitude control

- Disturbances
- Reaction wheels
- Flexible Satellite
- Filter
- Speed Bias
- PD Controller
- Velocity Estimator
- Star Tracker
- Ground Guidance
- Flight software

\[ \theta_r \quad \omega_r \]

\[ \delta\theta \quad \delta\omega \]
First experiment: adaptive tuning of PD gains of second axis with given filter

Corresponds to OF for $2 \times 1$ system

\[
\begin{bmatrix}
1 \\
\Sigma_{\text{estim}}(s)
\end{bmatrix} \Sigma_{\text{sat},2}(s) \Sigma_{\text{filter},2}(s)
\]

\[
\Sigma_{\text{estim}}(s) = \frac{s}{s+0.5}
\]

\[
\Sigma_{\text{sat},2}(s) = \frac{0.04736s^2+0.0006546s+0.2991}{s^2(s^2+0.01387s+6.338)}
\]

\[
\Sigma_{\text{filter},2}(s) = \frac{0.5411s^4-3.678s^3-4.99s^2-1.747s-0.1241}{s(0.25s^5+1.961s^4+5.094s^3+5.722s^2+3.068s+0.5784)}
\]

Given stabilizing SOF $F = \begin{bmatrix} 0.1 & 2 \end{bmatrix}$. 
First experiment: adaptive tuning of PD gains of second axis with given filter

Comparison of given SOF (dotted) and obtained PBAC

Engineers have well chosen the PD and filter gains
Second experiment: same model but modified initial stabilizing $F$

- Modified SOF $F = \begin{bmatrix} 0.3 & 2 \end{bmatrix}$
- PBAC design gives

$$G = \begin{bmatrix} 28.13 & -165.56 \end{bmatrix}, \quad \mu = 196.49, \quad \nu = 0.0244$$
Evolution of $K$ during the simulation

- Remains in largest circle (region such that $\|K - F\| \cdot \leq \nu \beta$)
- When the system settles it converges in the smaller circle ($\|K - F\| \cdot \leq \nu$)
Trird experiment: Same adaptive law but applied to first axis of satellite

(robustness test)

PBAC not affected by the uncontrollable oscillating mode
Conclusions

- LMI-based method that guarantees stability of PBAC
- Applies to any stabilizable LTI MIMO system
- Adaptive gains remain bounded
- Adaptive gains remain close to initial given values

Prospectives
- Enlarge admissible region for $K$: ellipsoids rather than discs [IFAC 2011]
- Structured control (decentralized etc.)
- Guaranteed robustness for time-varying uncertainties
- Take advantage of flexibilities on $G$ for engineering issues (saturations...)
- ...

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