Robust Analysis using RoMuLOC for the Longitudinal Control of a Civil Aircraft

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Introduction

- Test robust analysis tools on aerospace industrial application
- LMIs for parameter-dependent Lyapunov functions results
- Two type of results based on two different uncertain models
- Stability and performances (pole location, $H_\infty$, $H_2$, impulse-to-peak)

RoMulOC
- Tests performed using the RoMulOC toolbox
- LMIs in YALMIP format, solved using SeDuMi and SDPT3
- Indications on the numerical performances of the toolbox

Aircraft motion in the vertical plane (longitudinal)
- LTI uncertain modeling of the non-linear aircraft and the control
- Models that cover the flight envelope
Outline

1. Uncertain modeling
2. LMIs for parameter-dependent Lyapunov functions results
3. RoMuLOC toolbox
4. Numerical results
5. Conclusions
Uncertain modeling

Aircraft motion in the vertical plane (longitudinal)

Actuators: elevators

Dynamics: angle of attack + pitch rate

Sensors: modeled as first order

Control: gain scheduled dynamic

Closed-loop system of order 9

Non-linear model + controller are linearized at 633 flight configurations

6 parameters:

weight, balance, speed,

Mach nb, altitude, motor thrust.
Analysis of each 633 LTI models

gives small information on robustness for the total flight envelope

LFT model can be built to have a parameter-dependent LTI representation

of the whole flight envelope: uncertainty blocks of size 150!

Adopted strategy: build uncertain models valid around each flight configuration

Union of local uncertain models covers the flight envelope

Robust analysis gives upper bounds on performances achievable locally
Uncertain modeling

- Adopted strategy: build uncertain models valid around each flight configuration
- For a given flight configuration $\theta_i$

  algorithm gives its neighbors in parametric space $\theta_j \in N(i)$.

- Heuristic algorithm combines

  Euclidian distance in the 6D space $\theta$ + search along parametric directions.

- Tuned to provide 8 to 12 neighbors with a mean value of 11.19.

- Uncertain model around $\theta_i$ is defined as the convex hull of models at $\theta_j \in N(i)$

\[
\begin{pmatrix}
\dot{x} \\
z
\end{pmatrix} = 
\begin{bmatrix}
A_i(\zeta) & B_i(\zeta) \\
C_i(\zeta) & A_i(\zeta)
\end{bmatrix}
\begin{pmatrix}
x \\
w
\end{pmatrix} = 
\sum_{j \in N(i)} \zeta_j
\begin{bmatrix}
A_j & B_j \\
C_j & D_j
\end{bmatrix}
\begin{pmatrix}
x \\
w
\end{pmatrix}
\]

\[\sum \zeta_j = 1, \quad \zeta_j \geq 0\]
Uncertain modeling

Uncertain model around $\theta_i$ is defined as the convex hull of models at $\theta_j \in N(i)$

$$
\begin{align*}
\begin{pmatrix}
\dot{x} \\
z
\end{pmatrix} &= 
\begin{bmatrix}
A_i(\zeta) & B_i(\zeta) \\
C_i(\zeta) & A_i(\zeta)
\end{bmatrix}
\begin{pmatrix}
x \\
w
\end{pmatrix} = 
\sum_{j \in N(i)} \zeta_j
\begin{bmatrix}
C[j] & D[j]
\end{bmatrix}
\begin{pmatrix}
x \\
w
\end{pmatrix} \\
\text{such that} & \\
\sum \zeta_j &= 1, \, \zeta_j \geq 0
\end{align*}
$$

Each uncertain model is also converted in LFT form

$$
\begin{align*}
\begin{pmatrix}
\dot{x} \\
\dot{z}_\Delta \\
z
\end{pmatrix} &= 
\begin{bmatrix}
A_i & B_{\Delta i} & B_i \\
C_{\Delta i} & 0 & D_{\Delta wi} \\
C_i & D_{z\Delta i} & D_{\Delta i}
\end{bmatrix}
\begin{pmatrix}
x \\
w_\Delta \\
w
\end{pmatrix} = 
\sum_{j \in N(i)} \zeta_j \Delta[j] z_\Delta \\
\text{such that} & \\
\sum \zeta_j &= 1, \, \zeta_j \geq 0
\end{align*}
$$
Performances to be tested

- Stability
- Pole location

- $H_2$ norm - measure of control effort due to noise
  ($w$ additive noise on measurements, $z = u$ control signal)

- $H_\infty$ norm - stability margin w.r.t. dynamic uncertainty
  ($w$ additive signal on control $u$, $z = y$ measurements)

- Impulse-to-peak - control peak to initial conditions
  ($w$ impulse on state vector, $z = u$ control signal)
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2 results for polytopic models

- ‘Quadratic stability’ - $V(x) = x^T P x$ independent of uncertain parameters
  
  $$A[j]^T P + PA[j] < 0, \quad P > 0$$

- Polytopic PDLF - $V(x) = x^T \left( \sum \zeta_j P[j] \right) x$
  
  ‘Slack variable’ approach [SCL 00]
  
  $$\begin{bmatrix} 0 & P[j] \\ P[j] & 0 \end{bmatrix} < F \begin{bmatrix} A[j] & -1 \end{bmatrix} + \begin{bmatrix} A[j]^T \\ -1 \end{bmatrix} F^T, \quad P[j] > 0$$
1 result for LFT models

- Quadratic PDLF - $V(x) = x^T \begin{bmatrix} 1 & \Delta^T \end{bmatrix} \hat{P} \begin{bmatrix} 1 \\ \Delta \end{bmatrix} x$, $\Delta = \sum \zeta_j \Delta[j]$

Quadratic separation’ approach [Iwasaki 01]

\[
\mathcal{L}(\hat{P}, \Theta) < 0 \quad \Theta \begin{bmatrix} 1 \\ \Delta[j] \end{bmatrix} \leq 0 \quad \hat{P} > 0
\]

Results of all three methods are extended to deal with the performance criteria (pole location, $H_2$, $H_\infty$ and impulse-to-peak)
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Robust Multi-Objective Control toolbox

Freely distributed at www.laas.fr/LOCEP/romuloc

Includes uncertain modeling features

```matlab
>> usys_h2
Uncertain model : polytope 11 vertices
    n=9   mw=2   mu=1
    n=9   dx = A*x + Bw*w + Bu*u
    pz=1   z = Cz*x + Dzw*w
    py=2   y = Cy*x + Dyu*u
continuous time ( dx: derivative )
```
**Robust Multi-Objective Control toolbox**

- Freely distributed at [www.laas.fr/OLOCEP/romuloc](http://www.laas.fr/OLOCEP/romuloc)
- Includes uncertain modeling features

```matlab
>> usys_hinf

Uncertain model: LFT

```

```
-------- WITH --------

n=9     md=6     mw=1     mu=1

n=9     dx = A*x + Bd*wd + Bw*w + Bu*u

pd=7     zd = Cd*x + Ddw*w + Ddu*u

pz=3     z = Cz*x + Dzd*wd + Dzw*w

py=2     y = Cy*x

continuous time (dx: derivative operator)

-------- AND --------

wd = #1 * zd

<table>
<thead>
<tr>
<th>index</th>
<th>size</th>
<th>constraint</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>6x7</td>
<td>polytope 11 vertices</td>
<td>real</td>
</tr>
</tbody>
</table>
```
RoMulOC toolbox

- Robust Multi-Objective Control toolbox
- Freely distributed at www.laas.fr/OLCEP/romuloc
- LMI formulas pre-coded - easy to generate

```matlab
quiz = ctrpb('a',LyapType)+ h2(usys_h2)
```

LyapType defines the method to be applied

- **h2** or **stability**, **dstability**, **hinfty**, **i2p**: performance to test

quiz contains the LMI constraints and variables in YALMIP format

- Solve the LMI problem with any solver

```matlab
result = solvesdp(quiz, sdpsettings(...))
```
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Table 1: LMI sizes and times for stability tests

<table>
<thead>
<tr>
<th>Method</th>
<th>No. of vars</th>
<th>No. of rows</th>
<th>Mean time</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad-poly</td>
<td>45</td>
<td>110</td>
<td>0.25s</td>
</tr>
<tr>
<td>PDLF-poly</td>
<td>676</td>
<td>215</td>
<td>0.93s</td>
</tr>
<tr>
<td>PDLF-LFT</td>
<td>456</td>
<td>221</td>
<td>1.08s</td>
</tr>
</tbody>
</table>
Numerical results

Table 2: Results for settling time criterion

<table>
<thead>
<tr>
<th></th>
<th>$\sigma%$</th>
<th>Mean time per LMIs</th>
<th>Mean nb iter</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad-poly</td>
<td>15.27%</td>
<td>0.35s</td>
<td>7.29</td>
</tr>
<tr>
<td>PDLF-poly</td>
<td>2.38%</td>
<td>1.35s</td>
<td>1.95</td>
</tr>
<tr>
<td>PDLF-LFT</td>
<td>2.38%</td>
<td>1.45s</td>
<td>1.96</td>
</tr>
</tbody>
</table>

- Robust upper bound on $\sigma$ optimized by bisection (iterative LMI algorithm)
- $\sigma\%$: Gap between robust upper bound and worst case on vertices
### Numerical results

#### Table 3: Results for damping criterion

<table>
<thead>
<tr>
<th></th>
<th>$\psi_%$</th>
<th>Mean time per LMIs</th>
<th>Mean nb iter</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad-poly</td>
<td>11.40%</td>
<td>0.46s</td>
<td>6.45</td>
</tr>
<tr>
<td>PDLF-poly</td>
<td>1.44%</td>
<td>1.76s</td>
<td>1.25</td>
</tr>
<tr>
<td>PDLF-LFT</td>
<td>1.62%</td>
<td>1.52s</td>
<td>1.75</td>
</tr>
</tbody>
</table>

#### Table 4: Damping criterion for two particular flight points

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\psi_m(i)$</th>
<th>$\psi^*(i)$</th>
<th>quad-poly</th>
<th>PDLF-poly</th>
<th>PDLF-LFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.7286</td>
<td>0.5408</td>
<td>0.7213</td>
<td>0.6650</td>
<td></td>
</tr>
<tr>
<td>517</td>
<td>0.4978</td>
<td>0.4200</td>
<td>0.4735</td>
<td>0.4766</td>
<td></td>
</tr>
</tbody>
</table>
Numerical results

Table 5: Results for robust $\mathcal{H}_\infty$ cost

<table>
<thead>
<tr>
<th>Method</th>
<th>$\gamma_\infty$%</th>
<th>Mean time</th>
<th>Less conservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad-poly</td>
<td>39.64%</td>
<td>0.55s</td>
<td></td>
</tr>
<tr>
<td>PDLF-poly</td>
<td>0.19%</td>
<td>2.38s</td>
<td>52</td>
</tr>
<tr>
<td>PDLF-LFT</td>
<td>0.26%</td>
<td>9.04s</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6: Results for robust impulse-to-peak criterion

<table>
<thead>
<tr>
<th>Method</th>
<th>$\gamma_{i2p}$%</th>
<th>Mean time</th>
<th>Less conservative</th>
</tr>
</thead>
<tbody>
<tr>
<td>quad-poly</td>
<td>43.59%</td>
<td>0.81s</td>
<td></td>
</tr>
<tr>
<td>PDLF-poly</td>
<td>27.98%</td>
<td>2.66s</td>
<td>500</td>
</tr>
<tr>
<td>PDLF-LFT</td>
<td>30.16%</td>
<td>6.39s</td>
<td>0</td>
</tr>
</tbody>
</table>
Outline

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Parameter-dependent Lyapunov type results tested on an industrial application

Overall test over 633 points takes 3 hours on a PC
(negligible compared to Monte Carlo tests on high order non-linear model)

May be used at the control design phase to pre-validate (or not) a control law

Gives information on robust stability and performances

Can be used to retune LPV controllers in regions of the flight domain.

PDLF results show very low conservatism

PDLF-Poly always better than PDLF-LFT (can it be proved?)

No severe numerical problem - Validates the coding of LMIs in RoMuLOC

www.laas.fr/OLOCEP/romuloc