Integral Quadratic Separators for performance analysis

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Outline

1. Topological separation & Integral Quadratic Separation
2. Norm-to-norm performance in IQS framework
3. Impulse-to-norm performance in IQS framework
4. Impulse-to-peak performance in IQS framework
5. Conclusions & The Romuald toolbox
Well-posedness & topological separation

Well-Posedness:

Bounded \((\bar{w}, \bar{z})\)

\[\exists \bar{w}, \bar{z}, \exists \gamma : \|w\| \leq \gamma \|\bar{w}\|, \|z\| \leq \gamma \|\bar{z}\|\]

[Safonov 80] \(\exists \theta\) topological separator:

\[G^I(\bar{w}) = \{(w, z) : G_{\bar{w}}(z, w) = 0\} \subset \{(w, z) : \theta(w, z) \leq \phi_2(\|\bar{w}\|)\}\]

\[F(\bar{z}) = \{(w, z) : F_{\bar{z}}(w, z) = 0\} \subset \{(w, z) : \theta(w, z) > -\phi_1(\|\bar{z}\|)\}\]
For dynamic systems $\dot{x} = f(x)$, topological separation $\equiv$ Lyapunov theory

$$z(t) = f(w(t)) + \tilde{z}(t), \quad w(t) = \int_{0}^{t} z(\tau) d\tau + \bar{w}(t)$$

$\bar{w}$ : contains information on initial conditions ($x(0) = 0$ by convention)

Well-posedness $\Rightarrow$ for zero initial conditions and zero perturbations :

$$w = z = 0 \text{ (equilibrium point)}.$$  

Well-posedness (global stability)

$\Rightarrow$ whatever bounded perturbations the state remains close to equilibrium
For dynamic systems \( \dot{x} = f(x) \), topological separation \( \equiv \) Lyapunov theory

\[
\begin{align*}
F & : z(t) = f(w(t)) + \bar{z}(t) \\
G & : w(t) = \int_0^t z(\tau) d\tau + \bar{w}(t)
\end{align*}
\]

Assume a Lyapunov function \( V(0) = 0 \), \( V(x) > 0 \), \( \dot{V}(x) < 0 \)

Topological separation w.r.t. \( G^I(\bar{w}) \) is obtained with

\[
\theta(w = x, z = \dot{x}) = \int_0^\infty -\frac{\partial V}{\partial x}(x(\tau)) \dot{x}(\tau) d\tau = \lim_{t \to \infty} -V(x(t)) < \gamma_1 \|\bar{w}\|
\]

Topological separation w.r.t. \( F(\bar{z}) \) does hold as well

\[
\theta(w, z = f(w)) = \int_0^\infty -\dot{V}(w(\tau)) d\tau > -\gamma_2 \|\bar{z}\|
\]
For linear systems: quadratic Lyapunov function, *i.e. quadratic separator*

\[
F_{\bar{z}}(z,w)
\]

\[
z(t) = Aw(t) + \bar{z}(t)
\]

\[
G_{\bar{w}}(z,w)
\]

\[
w(t) = \int_0^t z(\tau) \, d\tau + \bar{w}(t)
\]

A possible separator based on quadratic Lyapunov function \( V(x) = x^T P x \)

\[
\theta(w,z) = \int_0^\infty \begin{pmatrix} z^T(\tau) & w^T(\tau) \end{pmatrix} \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} \begin{pmatrix} z(\tau) \\ w(\tau) \end{pmatrix} \, d\tau
\]

Quadratic separation w.r.t. \( G^I(\bar{w}) \):

\[
\lim_{t \to \infty} -x^T(t) P x(t) \leq \gamma_1 \|\bar{w}\| , \text{ i.e. } P > 0
\]

Quadratic separation w.r.t. \( F(\bar{z}) \) guaranteed if

\[
\forall t > 0 , \quad -2w^T(t) PAw(t) > -\gamma_2 \|\bar{z}(t)\| , \text{ i.e. } A^T P + PA < 0
\]
Topological separation

- Topological separation: alternative to Lyapunov theory

- Needs to manipulate systems in a new form
- Suited for systems described as feedback connected blocks

Any linear system with rational dependence w.r.t. parameters writes as such:

\[
\dot{x} = (A + B\Delta \Delta (1 - D\Delta \Delta)^{-1} C\Delta)x
\]

\[
\begin{align*}
\dot{x} &= Ax + B\Delta w\Delta \\
z\Delta &= C\Delta x + D\Delta w\Delta \\
w\Delta &= \Delta z\Delta
\end{align*}
\]

- Finding a topological separator is a priori as complicated as finding a Lyapunov function
- Allows to deal with several features simultaneously in a unified way
Topological separation

Quadratic separation [Iwasaki & Hara 1998]

- If $F(w) = Aw$ is a linear transformation and $G = \triangle$ is an uncertain operator defined as $\triangle \in \Delta$ convex set it is necessary and sufficient to look for a quadratic separator

$$\theta(z, w) = \int_{0}^{\infty} \left( z^T \ w^T \right) \Theta \left( \begin{bmatrix} z \\ w \end{bmatrix} \right) \ d\tau$$

- If $F(w) = A(\omega)w$ is a linear parameter dependent transformation and $G = \triangle$ is an uncertain operator defined as $\triangle \in \Delta$ convex set necessary and sufficient to look for a parameter-dependent quadratic separator

$$\theta(z, w) = \int_{0}^{\infty} \left( z^T \ w^T \right) \Theta(\omega) \left( \begin{bmatrix} z \\ w \end{bmatrix} \right) \ d\tau$$
A well-known example: the Lur’e problem

\[ F = T(j\omega) \] is a transfer function

\[ G(z)/z \in [-k_1, -k_2] \] is a sector-bounded gain

(i.e. the inverse graph of \( G \) is in \([-1/k_1, -1/k_2]\))

Circle criterion: exists a quadratic separator (circle) for all \( \omega \)
Another example: parameter-dependent Lyapunov function

\[ G(z, w) = 0 \]

\[ F(w, z) = 0 \]

\[ F = A(\delta) \] parameter-dependent LTI state-space model \((\dot{\delta})\)

\[ G = I \] is an integrator

Necessary and sufficient to have

\[ \Theta(\delta) = \begin{bmatrix} 0 & -P(\delta) \\ -P(\delta) & 0 \end{bmatrix} \]
Topological separation

Direct relation with the IQC framework

\( F = T(j\omega) \) is a transfer matrix

\( G = \Delta \) is an operator known to satisfy an Integral Quadratic Constraint (IQC)

\[
\int_{-\infty}^{+\infty} \begin{bmatrix} 1 & \Delta^*(j\omega) \end{bmatrix} \Pi(\omega) \begin{bmatrix} 1 \\ \Delta(j\omega) \end{bmatrix} d\omega \leq 0
\]

Stability of the closed-loop is guaranteed if for all \( \omega \)

\[
\begin{bmatrix} T^*(j\omega) & 1 \end{bmatrix} \Pi(\omega) \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix} > 0
\]

Knowing \( \Delta \) the set of \( \Delta \) how to choose \( \Pi = \Theta \)?

(\textit{i.e.} the quadratic separator)
Topological separation

Linear implicit application in feedback loop with an uncertain operator

\[ \mathcal{E}z(t) = Aw(t) , \quad w(t) = [\nabla z](t) , \quad \nabla \in \mathcal{W} \]

\( \nabla \) is bloc-diagonal contains scalar, full-bloc, LTI and LTV uncertainties and other operators such as integrator etc.
Topological separation

**Integral Quadratic Separation** [Automatica’08, CDC’08, ROCOND’09]

- For the case of linear application with uncertain operator

\[ \mathcal{E} z(t) = A w(t) , \quad w(t) = [\nabla z](t) \quad \nabla \in \mathbb{W} \]

where \( \mathcal{E} = \mathcal{E}_1 \mathcal{E}_2 \) with \( \mathcal{E}_1 \) full column rank,

- Integral Quadratic Separator (IQS) : \( \exists \Theta \), matrix, solution of LMI

\[
\left[ \begin{array}{cc} \mathcal{E}_1 & -A \\ \end{array} \right] \perp^* \Theta \left[ \begin{array}{cc} \mathcal{E}_1 & -A \\ \end{array} \right] \perp > 0
\]

and Integral Quadratic Constraint (IQC) \( \forall \nabla \in \mathbb{W} \)

\[
\int_0^\infty \left( \begin{array}{c} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{array} \right)^* \Theta \left( \begin{array}{c} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{array} \right) dt \leq 0
\]
Outline

1. Topological separation & Integral Quadratic Separation
   - Rich framework for robust stability analysis
   - Can input-output performances be treated in the same framework?

2. Norm-to-norm performance in IQS framework

3. Impulse-to-norm performance in IQS framework

4. Impulse-to-peak performance in IQS framework

5. Conclusions & The Romuald toolbox
Integral Quadratic Separator: all signals are assumed $L_2$: $\|z\|^2 < \infty$

$$\|z\|^2 = \text{Trace} \int_0^\infty z^*(t)z(t)\,dt \quad <z|w> = \text{Trace} \int_0^\infty z^*(t)w(t)\,dt$$

Notation

$$\|z\|^2_T = \text{Trace} \int_0^T z^*(t)z(t)\,dt \quad <z|w>_T = \text{Trace} \int_0^T z^*(t)w(t)\,dt$$
Induced $L_2$ norm ($H_\infty$ in the LTI case)

$$Ex = Ax + Bu, \ g = Cx + Du$$

▲ Prove that system is asymptotically stable

▲ and $\|g\| < \gamma \|v\|$ for zero initial conditions $x(0) = 0$

(strict upper bound on the $L_2$ gain attenuation)

○ Equivalent to well-posedness with respect to

Integrator with zero initial conditions $x(t) = [I_1 \dot{x}](t) = \int_0^t \dot{x}(\tau) d\tau$

and signals such that $\|v\| \leq \frac{1}{\gamma} \|g\|$
Induced $L_2$ norm

$$E \dot{x} = Ax + Bv, \quad g = Cx + Dv$$

Define $\nabla_{n2n}$ the fictitious non-causal uncertain operator such that

$$v = \nabla_{n2n} g \quad \text{iff} \quad \|v\| \leq \frac{1}{\gamma} \|g\|$$

Induced $L_2$ norm problem is equivalent to well-posedness of

$$
\begin{bmatrix}
E & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
g \\
z
\end{bmatrix}
= 
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
x \\
v \\
w
\end{bmatrix},
\nabla = 
\begin{bmatrix}
I_1 & 0 \\
0 & \nabla_{n2n}
\end{bmatrix}
$$
Norm-to-norm performance in quadratic separation framework

Induced $L_2$ norm

\[
\begin{bmatrix}
E & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
g
\end{bmatrix}
= \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
x \\
v
\end{bmatrix},
\nabla = \begin{bmatrix}
\mathcal{I}_1 & 0 \\
0 & \nabla_{n2n}
\end{bmatrix}
\]

Elementary IQS for bloc $\mathcal{I}_1$ is

\[
\Theta_{\mathcal{I}_1} = \begin{bmatrix}
0 & -P \\
-P & 0
\end{bmatrix} : P > 0
\]

Indeed (recall $x(t) = [\mathcal{I}_1 \dot{x}] (t) = \int_0^t \dot{x}(\tau) d\tau$ and $x(0) = 0$)

\[
\left\langle \begin{bmatrix}
\dot{x} \\
\mathcal{I}_1 \dot{x}
\end{bmatrix} | \Theta_{\mathcal{I}_1} \begin{bmatrix}
\dot{x} \\
\mathcal{I}_1 \dot{x}
\end{bmatrix} \right\rangle_T = -x^*(T) P x(T) \leq 0
\]
Induced $L_2$ norm

$$\begin{pmatrix} E & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} \\ g \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix}, \quad \nabla = \begin{pmatrix} \mathcal{I}_1 & 0 \\ 0 & \nabla_{n2n} \end{pmatrix}$$

Elementary IQS for bloc $\nabla_{n2n}$ is (small gain theorem)

$$\Theta_{\nabla_{n2n}} = \begin{pmatrix} -\tau & 0 \\ 0 & \tau \gamma^2 1 \end{pmatrix} : \quad \tau > 0$$

Indeed (recall $v = \nabla_{n2n} g$ iff $\|v\| \leq \frac{1}{\gamma} \|g\|$)

$$\left\langle \begin{pmatrix} g \\ \nabla_{n2n} g \end{pmatrix} \right| \Theta_{\nabla_{n2n}} \begin{pmatrix} g \\ \nabla_{n2n} g \end{pmatrix} \right\rangle = \tau (-\|g\|^2 + \gamma^2 \|v\|^2) \leq 0$$
Apply IQS and get (for non-descriptor case $E = 1$)

$$P > 0 \ , \ \tau > 0$$

$$\begin{bmatrix}
A^*P + PA + \tau C^*C & PB + \tau C^*D \\
B^*P + \tau D^*C & -\tau \gamma^2 I + \tau D^*D
\end{bmatrix} < 0$$

which is the classical $H_\infty$ result.

No difficulty to generate LMIs for descriptor case

& if there are more blocs in $\nabla$ such as uncertainties ...
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Impulse-to-norm performance ($H_2$ in the LTI case if $D = 0$)

\[
E \dot{x} = Ax + Bv, \quad g =Cx + Dv
\]

△ Prove that system is asymptotically stable
△ and $\|g\| < \gamma$ if $v = \alpha \delta(t) 1_m, |\alpha| \leq 1$ and zero initial conditions $x(0) = 0$

▲ The Dirac delta function $\delta(t)$ is not in $L_2$

▲ Impulse inputs define jumps of the state
Impulse-to-norm performance \((H_2\) in the LTI case if \(D = 0\))

\[
E\dot{x} = Ax + Bv, \quad g = Cx + Dv
\]

Prove that system is asymptotically stable

and \(\|g\| < \gamma\) if \(v = \alpha \delta(t)1_m, \quad |\alpha| \leq 1\) and zero initial conditions \(x(0) = 0\)

Redefinition of the problem:

\[
Ex(0) = \alpha B, \quad g(0) = \alpha D
\]

\[
E\dot{x}(t > 0) = Ax(t > 0), \quad g(t > 0) = Cx(t > 0)
\]

Prove that system is asymptotically stable

and \(\|g\| < \gamma\) for all \(\alpha \leq 1\)

Need to describe initial conditions as signals in \(L_2\)
Square-root of the shifted delta function $\varphi_\theta$:

$$\begin{cases} L_2 \rightarrow L_2 \\ x \mapsto \varphi_\theta x \end{cases}$$

with properties that $\varphi_\theta$ is linear, and whatever $x, y$ in $L_2$ and whatever $P$:

$$[\varphi_\theta y]^*(t) P[\varphi_\theta x](t) = \delta(t - \theta) y^*(t) P x(t)$$

$$[\varphi_{\theta_1} y]^*(t) P[\varphi_{\theta_2} x](t) = 0 \text{ if } \theta_1 \neq \theta_2$$

A formal definition:

$$[\varphi_\theta x](t) = \varphi(t - \theta) x(t)$$

where $\varphi$ is the limit of complex valued functions

$$\varphi(t) = \lim_{\epsilon \to 0} \frac{\sqrt{\epsilon/\pi}}{t + j\epsilon} \left( \lim_{\epsilon \to 0} \frac{\epsilon/\pi}{(t - j\epsilon)(t + j\epsilon)} = \delta(t) \right)$$

$\varphi_0 x$ is an $L_2$ signal that contains the information $x(0)$. 
**Impulse to norm performance in quadratic separation framework**

Impulse to norm performance equivalent to well-poedness of

\[
\begin{bmatrix}
E & 0 & 0 & 0 \\
0 & E & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\varphi_0 x \\
\dot{x} \\
\varphi_0 g \\
g
\end{bmatrix}
= \begin{bmatrix}
0 & B \\
A & 0 \\
0 & D \\
C & 0
\end{bmatrix}
\begin{bmatrix}
x \\
v \\
w
\end{bmatrix},
\nabla = \begin{bmatrix}
I_2 & 0 \\
0 & \nabla_{i2n}
\end{bmatrix}
\]

\[\Delta I_2\] is the integrator with non-zero initial conditions

\[x(t) = \left[ I_2 \begin{bmatrix}
\varphi_0 x \\
\dot{x}
\end{bmatrix}\right] (t) = x(0) + \int_0^t \dot{x}(\tau) d\tau
\]

\[v = \nabla_{i2n} \begin{bmatrix}
\varphi_0 g \\
g
\end{bmatrix} : v = \alpha \varphi_0 1_m , \quad \left| \alpha \right| \leq \frac{1}{\gamma} \left\| \begin{bmatrix}
\varphi_0 g \\
g
\end{bmatrix}\right\|\]
Elementary IQS for bloc $\mathcal{I}_2$ is

$$
\Theta_{\mathcal{I}_2} = \begin{bmatrix}
-P & 0 & 0 \\
0 & 0 & -P \\
0 & -P & 0
\end{bmatrix} : \quad P > 0
$$

Indeed (recall $x(t) = [\mathcal{I}_2 \begin{pmatrix} \varphi_0 x \\ \dot{x} \end{pmatrix}] \right) = x(0) + \int_0^t \dot{x}(\tau) d\tau$)

$$
\left\langle \begin{pmatrix} \varphi_0 x \\ \dot{x} \\ x \end{pmatrix} \right|_{\Theta_{\mathcal{I}_2}} \left( \begin{pmatrix} \varphi_0 x \\ \dot{x} \\ x \end{pmatrix} \right)^T_{\mathcal{I}_2} = -\text{Trace}(x^*(T)P x(T)) \leq 0
$$
Elementary IQS for block $\nabla_{i2n}$ is

$$
\Theta \nabla_{i2n} = \begin{bmatrix}
-\tau 1 & 0 \\
0 & -\tau 1 \\
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix} Q \end{bmatrix} : \text{Trace}(Q) < \tau \gamma^2
$$

Indeed (recall $v = \nabla_{i2n} \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix}$ : $v = \alpha \varphi_0 1_m$, $|\alpha| \leq \frac{1}{\gamma}$)

$$
\left\langle \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} \right| \Theta \nabla_{i2n} \left( \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} \right) \rangle = -\tau \left\| \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} \right\|^2 + \alpha^2 \text{Trace}(Q) \leq 0
$$
Apply IQS and get (for non-descriptor case $E = 1$)

$$P > 0 \ , \ \tau > 0 \ , \ \text{Trace}(Q) \leq \tau \gamma^2$$

$$A^*P + PA + \tau C^*C < 0 \ , \ Q > B^*PB + \tau D^*D$$

which is the classical $H_2$ result (when $D = 0$) as expected.

No difficulty to generate LMIs for descriptor case

& if there are more blocs in $\nabla$ such as uncertainties ...
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Impulse-to-peak performance in quadratic separation framework

Impulse to peak performance

\[ \dot{E}x = Ax + Bu, \quad g = Cx + Du \]

\( \Delta \) Prove that system is asymptotically stable

\( \Delta \) and \( \max_{t \geq 0} \|g(t)\| < \gamma \) if \( v = \delta(t)\alpha, \|\alpha\| \leq 1 \) and \( x(0) = 0 \)

\( \bullet \) Redefinition of the problem :

\( \Delta \) Let \( \theta = \arg\max_{t \geq 0} \|g(t)\| \) (unknown positive or zero)

\[ Ex(0) = B\alpha, \quad g(0) = D\alpha \]

\[ E\dot{x}(\theta > t > 0) = Ax(\theta > t > 0), \quad g(\theta) = Cx(\theta) \]

\( \Delta \) Prove that system is asymptotically stable

\( \Delta \) and \( \|g(0)\| < \gamma, \|g(\theta)\| < \gamma \) for all \( \|\alpha\| \leq 1 \)

\( \blacksquare \) Need to describe final conditions.
Impulse-to-peak performance in quadratic separation framework

**Truncation operator** $\mathbb{T}_\theta : \begin{cases} L_2 \longrightarrow L_2 \\ x \longmapsto \mathbb{T}_\theta x \end{cases}$

with properties

$$\left\{ \begin{array}{ll} [\mathbb{T}_\theta x](t) = x(t) & \forall t \in [0, \theta] \\ [\mathbb{T}_\theta x](t) = 0 & \forall t > \theta \end{array} \right.$$ 

**Integration** $\mathcal{I}_3$ maps

$$\begin{pmatrix} \varphi_0 x \\ \mathbb{T}_\theta \dot{x} \end{pmatrix}$$

to

$$\begin{pmatrix} \mathbb{T}_\theta x \\ \varphi_\theta x \end{pmatrix}$$

$$\begin{bmatrix} \mathcal{I}_3 & & & & & \end{bmatrix} \begin{pmatrix} \varphi_0 x \\ \mathbb{T}_\theta \dot{x} \end{pmatrix} (t) = x(0) + \int_0^t \dot{x} d\tau = x(t) = \mathbb{T}_\theta x(t), \ \forall t \in [0, \theta]$$

$$\begin{bmatrix} \mathcal{I}_3 & & & & & \end{bmatrix} \begin{pmatrix} \varphi_0 x \\ \mathbb{T}_\theta \dot{x} \end{pmatrix} (t) = x(0) + \int_0^\theta \dot{x} d\tau = x(\theta), \ \forall t > \theta.$$
Impulse-to-peak performance equivalent to well-poisedness of

$$\begin{bmatrix}
E & 0 & 0 & 0 \\
0 & E & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\varphi_0 x \\
\mathbb{T}_\theta \dot{x} \\
\varphi_0 g \\
\varphi_\theta g
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & B \\
A & 0 & 0 & 0 \\
0 & 0 & D & 0 \\
0 & C & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varphi_\theta x \\
\varphi_\theta g \\
v_0 \\
v_\theta
\end{bmatrix}$$

$$\nabla = \begin{bmatrix}
\mathcal{I}_3 & 0 & 0 \\
0 & \nabla_{i2p,0} & 0 \\
0 & 0 & \nabla_{i2p,\theta}
\end{bmatrix}$$

where $v_\theta = \nabla_{i2p,\theta} \varphi_\theta g : v = \varphi_0 \bar{v} , \quad \bar{v}^* \bar{v} \leq \frac{1}{\gamma^2} < \varphi_\theta g | \varphi_\theta g >$

... LMIs can be produced in the same way as for other performances...
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Conclusions

- Well-posedness and topological separation extended to performance analysis
- Known results for LTI systems recovered
- New extensions for descriptor systems
- New results made possible for uncertain systems:
  - advanced parameter-dependent Lyapunov functions
- Expected extensions for some non-linear systems
The Romuald toolbox

- Freely distributed software to test the theoretical results

- Existing software: RoMulOC
  
  www.laas.fr/OLOCEP/romuloc

- Contains some of the analysis results plus some state-feedback features

- Currently developed software: Romuald
  
  ▲ Dedicated to analysis of descriptor systems
  
  ▲ Fully coded using the quadratic separation theory
  
  ▲ Polynomially parameter-dependent Lyapunov functions of any order
  
  ▲ First preliminary tests currently done for satellite and plane applications

```matlab
>> quiz = ctrpb(OrderOfLyapunovFunction) + i2n(usys);
>> result = solvesdp(quiz)
```