

Integral Quadratic Separators for performance analysis

Dimitri PEAUCELLE , Lucie Baudouin, Frédéric Gouaisbaut

LAAS-CNRS - Université de Toulouse - FRANCE

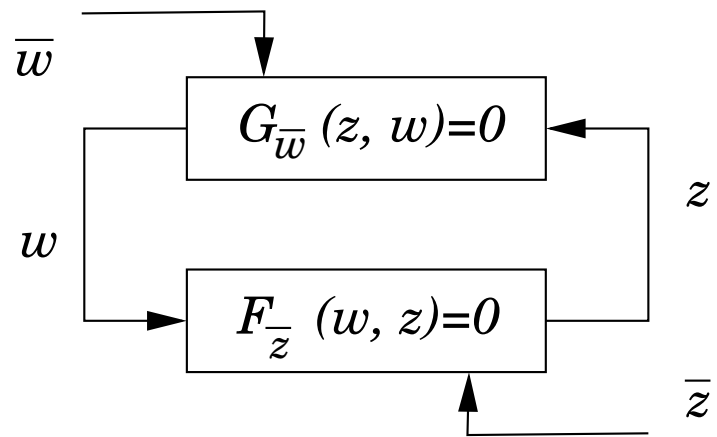
The logo for LAAS-CNRS features the text "LAAS-CNRS" in a bold, blue, sans-serif font. The text is centered between two horizontal lines: a red line above and a yellow line below.

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- ① Topological separation & Integral Quadratic Separation
- ② Norm-to-norm performance in IQS framework
- ③ Impulse-to-norm performance in IQS framework
- ④ Impulse-to-peak performance in IQS framework
- ⑤ Conclusions & The Romuald toolbox

Well-posedness & topological separation



Well-Posedness:

Bounded (\bar{w}, \bar{z})

$\Rightarrow \exists!(w, z)$, $\exists \gamma$:

$$\begin{pmatrix} \|w\| \\ \|z\| \end{pmatrix} \leq \gamma \begin{pmatrix} \|\bar{w}\| \\ \|\bar{z}\| \end{pmatrix}$$

● [Safonov 80] $\exists \theta$ topological separator:

$$\mathcal{G}^I(\bar{w}) = \{(w, z) : G_{\bar{w}}(z, w) = 0\} \subset \{(w, z) : \theta(w, z) \leq \phi_2(\|\bar{w}\|)\}$$

$$\mathcal{F}(\bar{z}) = \{(w, z) : F_{\bar{z}}(w, z) = 0\} \subset \{(w, z) : \theta(w, z) > -\phi_1(\|\bar{z}\|)\}$$

- For dynamic systems $\dot{x} = f(x)$, topological separation \equiv Lyapunov theory

$$\overbrace{z(t) = f(w(t)) + \bar{z}(t)}^F, \quad \overbrace{w(t) = \int_0^t \underbrace{z(\tau)}_{\dot{x}(t)} d\tau + \bar{w}(t)}^G$$

- ▲ \bar{w} : contains information on initial conditions ($x(0) = 0$ by convention)
- Well-posedness \Rightarrow for zero initial conditions and zero perturbations :
 $w = z = 0$ (equilibrium point).
- Well-posedness (global stability)
 \Rightarrow whatever bounded perturbations the state remains close to equilibrium

1 Topological separation

■ For dynamic systems $\dot{x} = f(x)$, topological separation \equiv Lyapunov theory

$$\overbrace{z(t) = f(w(t)) + \bar{z}(t)}^F, \quad \overbrace{w(t) = \int_0^t \underbrace{z(\tau)}_{\dot{x}(t)} d\tau + \bar{w}(t)}^G$$

● Assume a Lyapunov function $V(0) = 0$, $V(x) > 0$, $\dot{V}(x) < 0$

▲ Topological separation w.r.t. $\mathcal{G}^I(\bar{w})$ is obtained with

$$\theta(w = x, z = \dot{x}) = \int_0^\infty -\frac{\partial V}{\partial x}(x(\tau)) \dot{x}(\tau) d\tau = \lim_{t \rightarrow \infty} -V(x(t)) < \gamma_1 \|\bar{w}\|$$

▲ Topological separation w.r.t. $\mathcal{F}(\bar{z})$ does hold as well

$$\theta(w, z = f(w)) = \int_0^\infty -\dot{V}(w(\tau)) d\tau > -\gamma_2 \|\bar{z}\|$$

1 Topological separation

- For linear systems : quadratic Lyapunov function, *i.e. quadratic separator*

$$\underbrace{z(t) = Aw(t) + \bar{z}(t)}_{F_{\bar{z}}(z,w)} \quad , \quad \underbrace{w(t) = \int_0^t \underbrace{z(\tau)}_{\dot{x}(t)} d\tau + \bar{w}(t)}_{G_{\bar{w}}(z,w)}$$

- A possible separator based on quadratic Lyapunov function $V(x) = x^T P x$

$$\theta(w, z) = \int_0^\infty \begin{pmatrix} z^T(\tau) & w^T(\tau) \end{pmatrix} \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} \begin{pmatrix} z(\tau) \\ w(\tau) \end{pmatrix} d\tau$$

- ▲ Quadratic separation w.r.t. $\mathcal{G}^I(\bar{w})$:

$$\lim_{t \rightarrow \infty} -x^T(t) P x(t) \leq \gamma_1 \|\bar{w}\| \quad , \quad i.e. \quad P > 0$$

- ▲ Quadratic separation w.r.t. $\mathcal{F}(\bar{z})$ guaranteed if

$$\forall t > 0 \quad , \quad -2w^T(t) P A w(t) > -\gamma_2 \|\bar{z}(t)\| \quad , \quad i.e. \quad A^T P + P A < 0$$

1 Topological separation

■ Topological separation : alternative to Lyapunov theory

▲ Needs to manipulate systems in a new form

● Suited for systems described as feedback connected blocs

Any linear system with rational dependence w.r.t. parameters writes as such

$$\dot{x} = (A + B_{\Delta} \Delta (1 - D_{\Delta} \Delta)^{-1} C_{\Delta}) x \quad \xleftrightarrow{\text{LFT}} \quad \begin{cases} \dot{x} = Ax + B_{\Delta} w_{\Delta} \\ z_{\Delta} = C_{\Delta} x + D_{\Delta} w_{\Delta} \\ w_{\Delta} = \Delta z_{\Delta} \end{cases}$$

▲ Finding a topological separator is *a priori*

as complicated as finding a Lyapunov function

● Allows to deal with several features simultaneously in a unified way

■ Quadratic separation [Iwasaki & Hara 1998]

- If $F(w) = Aw$ is a linear transformation and $G = \Delta$ is an uncertain operator defined as $\Delta \in \mathbb{\Delta}$ convex set it is necessary and sufficient to look for a quadratic separator

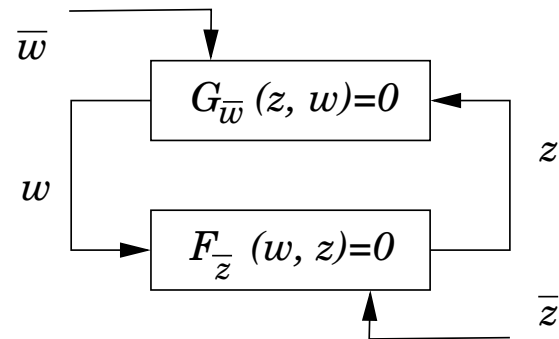
$$\theta(z, w) = \int_0^\infty \begin{pmatrix} z^T & w^T \end{pmatrix} \Theta \begin{pmatrix} z \\ w \end{pmatrix} d\tau$$

- If $F(w) = A(\omega)w$ is a linear parameter dependent transformation and $G = \Delta$ is an uncertain operator defined as $\Delta \in \mathbb{\Delta}$ convex set necessary and sufficient to look for a parameter-dependent quadratic separator

$$\theta(z, w) = \int_0^\infty \begin{pmatrix} z^T & w^T \end{pmatrix} \Theta(\omega) \begin{pmatrix} z \\ w \end{pmatrix} d\tau$$

1 Topological separation

■ A well-known example : the Lur'e problem

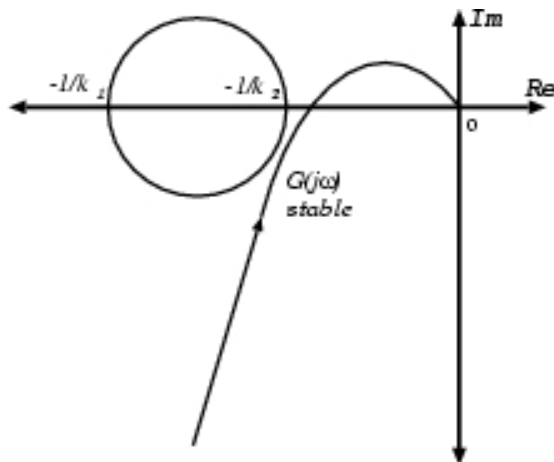


▲ $F = T(j\omega)$ is a transfer function

▲ $G(z)/z \in [-k_1, -k_2]$ is a sector-bounded gain

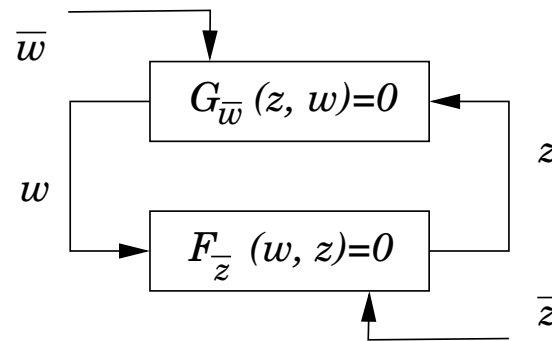
(i.e. the inverse graph of G is in $[-1/k_1, -1/k_2]$)

● Circle criterion : exists a quadratic separator (circle) for all ω



1 Topological separation

Another example : parameter-dependent Lyapunov function



▲ $F = A(\delta)$ parameter-dependent LTI state-space model ($\dot{\delta}$)

▲ $G = \mathcal{I}$ is an integrator

● Necessary and sufficient to have

$$\Theta(\delta) = \begin{bmatrix} 0 & -P(\delta) \\ -P(\delta) & 0 \end{bmatrix}$$

■ Direct relation with the IQC framework

▲ $F = T(j\omega)$ is a transfer matrix

▲ $G = \Delta$ is an operator known to satisfy an Integral Quadratic Constraint (IQC)

$$\int_{-\infty}^{+\infty} \begin{bmatrix} \mathbf{1} & \Delta^*(j\omega) \end{bmatrix} \Pi(\omega) \begin{bmatrix} 1 \\ \Delta(j\omega) \end{bmatrix} d\omega \leq 0$$

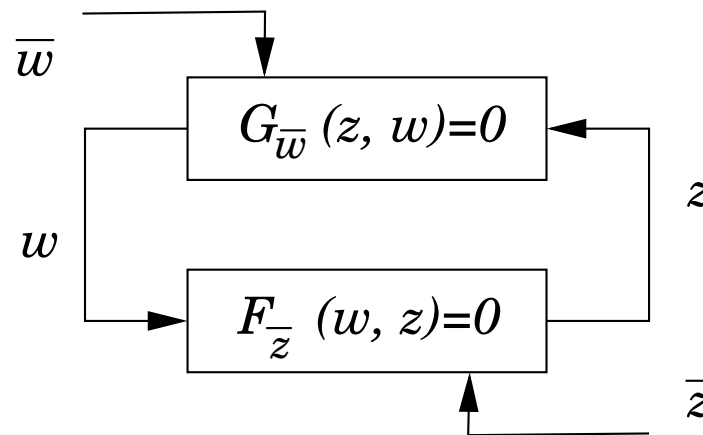
● Stability of the closed-loop is guaranteed if for all ω

$$\begin{bmatrix} T^*(j\omega) & \mathbf{1} \end{bmatrix} \Pi(\omega) \begin{bmatrix} T(j\omega) \\ 1 \end{bmatrix} > 0$$

▲ Knowing Δ the set of Δ how to choose $\Pi = \Theta$?

(i.e. the quadratic separator)

1 Topological separation



- Linear implicit application in feedback loop with an uncertain operator

$$\underbrace{\mathcal{E}z(t) = \mathcal{A}w(t)}_F, \quad \underbrace{w(t) = [\nabla z](t)}_G \quad \nabla \in \mathbb{W}$$

- ∇ is bloc-diagonal contains scalar, full-bloc, LTI and LTV uncertainties and other operators such as integrator etc.

1 Topological separation

■ Integral Quadratic Separation [Automatica'08, CDC'08, ROCOND'09]

● For the case of linear application with uncertain operator

$$\mathcal{E}z(t) = \mathcal{A}w(t) \quad , \quad w(t) = [\nabla z](t) \quad \nabla \in \mathbb{W}$$

where $\mathcal{E} = \mathcal{E}_1 \mathcal{E}_2$ with \mathcal{E}_1 full column rank,

● Integral Quadratic Separator (IQS) : $\exists \Theta$, matrix, solution of LMI

$$\begin{bmatrix} \mathcal{E}_1 & -\mathcal{A} \end{bmatrix}^{\perp*} \Theta \begin{bmatrix} \mathcal{E}_1 & -\mathcal{A} \end{bmatrix}^{\perp} > 0$$

and Integral Quadratic Constraint (IQC) $\forall \nabla \in \mathbb{W}$

$$\int_0^{\infty} \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix}^* \Theta \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix} dt \leq 0$$

- ① Topological separation & Integral Quadratic Separation
 - Rich framework for robust stability analysis
 - ▲ Can input-output performances be treated in the same framework ?
- ② Norm-to-norm performance in IQS framework
- ③ Impulse-to-norm performance in IQS framework
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- Integral Quadratic Separator : all signals are assumed L_2 : $\|z\|^2 < \infty$

$$\|z\|^2 = \text{Trace} \int_0^{\infty} z^*(t)z(t)dt \quad , \quad \langle z|w \rangle = \text{Trace} \int_0^{\infty} z^*(t)w(t)dt$$

▲ Notation

$$\|z\|_T^2 = \text{Trace} \int_0^T z^*(t)z(t)dt \quad , \quad \langle z|w \rangle_T = \text{Trace} \int_0^T z^*(t)w(t)dt$$

- Induced L_2 norm (H_∞ in the LTI case)

$$E\dot{x} = Ax + Bv, \quad g = Cx + Dv$$

- ▲ Prove that system is asymptotically stable
- ▲ and $\|g\| < \gamma\|v\|$ for zero initial conditions $x(0) = 0$
(strict upper bound on the L_2 gain attenuation)

- Equivalent to well-posedness with respect to

Integrator with zero initial conditions $x(t) = [\mathcal{I}_1 \dot{x}](t) = \int_0^t \dot{x}(\tau) d\tau$

and signals such that $\|v\| \leq \frac{1}{\gamma} \|g\|$

3 Norm-to-norm performance in quadratic separation framework

■ Induced L_2 norm

$$E\dot{x} = Ax + Bv, \quad g = Cx + Dv$$

▲ Define ∇_{n2n} the fictitious non-causal uncertain operator such that

$$v = \nabla_{n2n}g \quad \text{iff} \quad \|v\| \leq \frac{1}{\gamma} \|g\|$$

● Induced L_2 norm problem is equivalent to well-posedness of

$$\underbrace{\begin{bmatrix} E & 0 \\ 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{pmatrix} \dot{x} \\ g \end{pmatrix}}_z = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{pmatrix} x \\ v \end{pmatrix}}_w, \quad \nabla = \begin{bmatrix} \mathcal{I}_1 & 0 \\ 0 & \nabla_{n2n} \end{bmatrix}$$

3 Norm-to-norm performance in quadratic separation framework

■ Induced L_2 norm

$$\underbrace{\begin{bmatrix} E & 0 \\ 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{pmatrix} \dot{x} \\ g \end{pmatrix}}_z = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_A \underbrace{\begin{pmatrix} x \\ v \end{pmatrix}}_w, \quad \nabla = \begin{bmatrix} \mathcal{I}_1 & 0 \\ 0 & \nabla_{n2n} \end{bmatrix}$$

● Elementary IQS for bloc \mathcal{I}_1 is

$$\Theta_{\mathcal{I}_1} = \begin{bmatrix} 0 & -P \\ -P & 0 \end{bmatrix} : P > 0$$

Indeed (recall $x(t) = [\mathcal{I}_1 \dot{x}](t) = \int_0^t \dot{x}(\tau) d\tau$ and $x(0) = 0$)

$$\left\langle \begin{pmatrix} \dot{x} \\ \mathcal{I}_1 \dot{x} \end{pmatrix} \middle| \Theta_{\mathcal{I}_1} \begin{pmatrix} \dot{x} \\ \mathcal{I}_1 \dot{x} \end{pmatrix} \right\rangle_T = -x^*(T) P x(T) \leq 0$$

3 Norm-to-norm performance in quadratic separation framework

■ Induced L_2 norm

$$\underbrace{\begin{bmatrix} E & 0 \\ 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{pmatrix} \dot{x} \\ g \end{pmatrix}}_z = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_A \underbrace{\begin{pmatrix} x \\ v \end{pmatrix}}_w, \quad \nabla = \begin{bmatrix} \mathcal{I}_1 & 0 \\ 0 & \nabla_{n2n} \end{bmatrix}$$

● Elementary IQS for bloc ∇_{n2n} is (small gain theorem)

$$\ominus_{\nabla_{n2n}} = \begin{bmatrix} -\tau \mathbf{1} & 0 \\ 0 & \tau \gamma^2 \mathbf{1} \end{bmatrix} : \tau > 0$$

Indeed (recall $v = \nabla_{n2n} g$ iff $\|v\| \leq \frac{1}{\gamma} \|g\|$)

$$\left\langle \begin{pmatrix} g \\ \nabla_{n2n} g \end{pmatrix} \middle| \ominus_{\nabla_{n2n}} \begin{pmatrix} g \\ \nabla_{n2n} g \end{pmatrix} \right\rangle = \tau (-\|g\|^2 + \gamma^2 \|v\|^2) \leq 0$$

③ Norm-to-norm performance in quadratic separation framework

- Apply IQS and get (for non-descriptor case $E = 1$)

$$P > 0, \quad \tau > 0$$
$$\begin{bmatrix} A^*P + PA + \tau C^*C & PB + \tau C^*D \\ B^*P + \tau D^*C & -\tau\gamma^2\mathbf{1} + \tau D^*D \end{bmatrix} < 0$$

which is the classical H_∞ result.

- No difficulty to generate LMIs for descriptor case
& if there are more blocs in ∇ such as uncertainties ...

- ① Topological separation & Integral Quadratic Separation
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3 Impulse-to-norm performance in quadratic separation framework

- Impulse to norm performance (H_2 in the LTI case if $D = 0$)

$$E\dot{x} = Ax + Bv, \quad g = Cx + Dv$$

- ▲ Prove that system is asymptotically stable
- ▲ and $\|g\| < \gamma$ if $v = \alpha\delta(t)\mathbf{1}_m$, $|\alpha| \leq 1$ and zero initial conditions $x(0) = 0$
- ! The Dirac delta function $\delta(t)$ is not in L_2
- Impulse inputs define jumps of the state

3 Impulse-to-norm performance in quadratic separation framework

- Impulse to norm performance (H_2 in the LTI case if $D = 0$)

$$E\dot{x} = Ax + Bv, \quad g = Cx + Dv$$

- ▲ Prove that system is asymptotically stable
- ▲ and $\|g\| < \gamma$ if $v = \alpha\delta(t)\mathbf{1}_m$, $|\alpha| \leq 1$ and zero initial conditions $x(0) = 0$
- Redefinition of the problem :

$$Ex(0) = \alpha B, \quad g(0) = \alpha D$$

$$E\dot{x}(t > 0) = Ax(t > 0), \quad g(t > 0) = Cx(t > 0)$$

- ▲ Prove that system is asymptotically stable
- ▲ and $\|g\| < \gamma$ for all $\alpha \leq 1$
- Need to describe initial conditions as signals in L_2

3 Impulse-to-norm performance in quadratic separation framework

■ Square-root of the shifted delta function φ_θ :
$$\begin{cases} L_2 \longrightarrow L_2 \\ x \longmapsto \varphi_\theta x \end{cases}$$

with properties that φ_θ is linear, and whatever x, y in L_2 and whatever P :

$$[\varphi_\theta y]^*(t)P[\varphi_\theta x](t) = \delta(t - \theta)y^*(t)Px(t)$$

$$[\varphi_{\theta_1} y]^*(t)P[\varphi_{\theta_2} x](t) = 0 \quad \text{if } \theta_1 \neq \theta_2$$

● A formal definition:

$[\varphi_\theta x](t) = \varphi(t - \theta)x(t)$ where φ is the limit of complex valued functions

$$\varphi(t) = \lim_{\epsilon \rightarrow 0} \frac{\sqrt{\epsilon/\pi}}{t + j\epsilon} \quad \left(\lim_{\epsilon \rightarrow 0} \frac{\epsilon/\pi}{(t - j\epsilon)(t + j\epsilon)} = \delta(t) \right)$$

● $\varphi_0 x$ is an L_2 signal that contains the information $x(0)$.

3 Impulse-to-norm performance in quadratic separation framework

■ Impulse to norm performance equivalent to well-posedness of

$$\underbrace{\begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{pmatrix} \varphi_0 x \\ \dot{x} \\ \varphi_0 g \\ g \end{pmatrix}}_z = \underbrace{\begin{bmatrix} 0 & B \\ A & 0 \\ 0 & D \\ C & 0 \end{bmatrix}}_A \underbrace{\begin{pmatrix} x \\ v \end{pmatrix}}_w, \quad \nabla = \begin{bmatrix} \mathcal{I}_2 & 0 \\ 0 & \nabla_{i2n} \end{bmatrix}$$

▲ \mathcal{I}_2 is the integrator with non-zero initial conditions

$$x(t) = \begin{bmatrix} \mathcal{I}_2 \\ \end{bmatrix} \begin{pmatrix} \varphi_0 x \\ \dot{x} \end{pmatrix} (t) = x(0) + \int_0^t \dot{x}(\tau) d\tau$$

$$v = \nabla_{i2n} \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} : v = \alpha \varphi_0 \mathbf{1}_m, \quad |\alpha| \leq \frac{1}{\gamma} \left\| \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} \right\|$$

3 Impulse-to-norm performance in quadratic separation framework

- Elementary IQS for bloc \mathcal{I}_2 is

$$\Theta_{\mathcal{I}_2} = \left[\begin{array}{cc|c} -P & 0 & 0 \\ 0 & 0 & -P \\ \hline 0 & -P & 0 \end{array} \right] : P > 0$$

Indeed (recall $x(t) = [\mathcal{I}_2 \begin{pmatrix} \varphi_0 x \\ \dot{x} \end{pmatrix}](t) = x(0) + \int_0^t \dot{x}(\tau) d\tau$)

$$\left\langle \begin{pmatrix} \varphi_0 x \\ \dot{x} \\ \hline x \end{pmatrix} \middle| \Theta_{\mathcal{I}_2} \begin{pmatrix} \varphi_0 x \\ \dot{x} \\ \hline x \end{pmatrix} \right\rangle_T = -\text{Trace}(x^*(T) P x(T)) \leq 0$$

3 Impulse-to-norm performance in quadratic separation framework

- Elementary IQS for bloc ∇_{i2n} is

$$\Theta_{\nabla_{i2n}} = \left[\begin{array}{cc|c} -\tau \mathbf{1} & 0 & 0 \\ 0 & -\tau \mathbf{1} & 0 \\ \hline 0 & 0 & Q \end{array} \right] : \text{Trace}(Q) < \tau \gamma^2$$

Indeed (recall $v = \nabla_{i2n} \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} : v = \alpha \varphi_0 \mathbf{1}_m$, $|\alpha| \leq \frac{1}{\gamma} \left\| \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} \right\|$)

$$\left\langle \begin{pmatrix} \varphi_0 g \\ g \\ v \end{pmatrix} \middle| \Theta_{\nabla_{i2n}} \begin{pmatrix} \varphi_0 g \\ g \\ v \end{pmatrix} \right\rangle = -\tau \left\| \begin{pmatrix} \varphi_0 g \\ g \end{pmatrix} \right\|^2 + \alpha^2 \text{Trace}(Q) \leq 0$$

③ Impulse-to-norm performance in quadratic separation framework

- Apply IQS and get (for non-descriptor case $E = 1$)

$$P > 0 \quad , \quad \tau > 0 \quad , \quad \text{Trace}(Q) \leq \tau\gamma^2$$
$$A^*P + PA + \tau C^*C < 0 \quad , \quad Q > B^*PB + \tau D^*D$$

which is the classical H_2 result (when $D = 0$) as expected.

- No difficulty to generate LMIs for descriptor case
& if there are more blocs in ∇ such as uncertainties ...

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3 Impulse-to-peak performance in quadratic separation framework

■ Impulse to peak performance

$$E\dot{x} = Ax + Bv, \quad g = Cx + Dv$$

▲ Prove that system is asymptotically stable

▲ and $\max_{t \geq 0} \|g(t)\| < \gamma$ if $v = \delta(t)\alpha$, $\|\alpha\| \leq 1$ and $x(0) = 0$

● Redefinition of the problem :

▲ Let $\theta = \arg \max_{t \geq 0} \|g(t)\|$ (unknown positive or zero)

$$Ex(0) = B\alpha, \quad g(0) = D\alpha$$

$$E\dot{x}(\theta > t > 0) = Ax(\theta > t > 0), \quad g(\theta) = Cx(\theta)$$

▲ Prove that system is asymptotically stable

▲ and $\|g(0)\| < \gamma$, $\|g(\theta)\| < \gamma$ for all $\|\alpha\| \leq 1$

■ Need to describe final conditions.

3 Impulse-to-peak performance in quadratic separation framework

■ Truncation operator \mathbb{T}_θ :
$$\begin{cases} L_2 \longrightarrow L_2 \\ x \longmapsto \mathbb{T}_\theta x \end{cases}$$

with properties
$$\begin{cases} [\mathbb{T}_\theta x](t) = x(t) & \forall t \in [0, \theta] \\ [\mathbb{T}_\theta x](t) = 0 & \forall t > \theta \end{cases}$$

▲ Integration \mathcal{I}_3 maps
$$\begin{pmatrix} \varphi_0 x \\ \mathbb{T}_\theta \dot{x} \end{pmatrix} \text{ to } \begin{pmatrix} \mathbb{T}_\theta x \\ \varphi_\theta x \end{pmatrix}$$

$$\left[\begin{array}{l} \mathcal{I}_3 \\ \mathcal{I}_3 \end{array} \begin{pmatrix} \varphi_0 x \\ \mathbb{T}_\theta \dot{x} \end{pmatrix} \right] (t) = x(0) + \int_0^t \dot{x} d\tau = x(t) = \mathbb{T}_\theta x(t) , \quad \forall t \in [0, \theta]$$
$$\left[\begin{array}{l} \mathcal{I}_3 \\ \mathcal{I}_3 \end{array} \begin{pmatrix} \varphi_0 x \\ \mathbb{T}_\theta \dot{x} \end{pmatrix} \right] (t) = x(0) + \int_0^\theta \dot{x} d\tau = x(\theta) , \quad \forall t > \theta .$$

3 Impulse-to-peak performance in quadratic separation framework

■ Impulse to peak performance equivalent to well-poedness of

$$\underbrace{\begin{bmatrix} E & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathcal{E}} \underbrace{\begin{pmatrix} \varphi_0 x \\ \frac{\mathbb{T}_\theta \dot{x}}{\varphi_0 g} \\ \frac{\varphi_\theta g}{\varphi_\theta g} \end{pmatrix}}_z = \underbrace{\begin{bmatrix} 0 & 0 & 0 & B \\ A & 0 & 0 & 0 \\ 0 & 0 & D & 0 \\ 0 & C & 0 & 0 \end{bmatrix}}_A \underbrace{\begin{pmatrix} \mathbb{T}_\theta x \\ \frac{\varphi_\theta x}{v_0} \\ \frac{v_\theta}{v_\theta} \end{pmatrix}}_w$$

$$\nabla = \begin{bmatrix} \mathcal{I}_3 & 0 & 0 \\ 0 & \nabla_{i2p,0} & 0 \\ 0 & 0 & \nabla_{i2p,\theta} \end{bmatrix}$$

where $v_\theta = \nabla_{i2p,\theta} \varphi_\theta g : v = \varphi_0 \bar{v}$, $\bar{v}^* \bar{v} \leq \frac{1}{\gamma^2} \langle \varphi_\theta g | \varphi_\theta g \rangle$

... LMIs can be produced in the same way as for other performances...

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- Well-posedness and topological separation extended to performance analysis
- ▲ Known results for LTI systems recovered
- New extensions for descriptor systems
- New results made possible for uncertain systems :
 - advanced parameter-dependent Lyapunov functions
- Expected extensions for some non-linear systems

5 The Romuald toolbox

■ Freely distributed software to test the theoretical results

● Existing software : RoMulOC

`www.laas.fr/OLOCEP/romuloc`

▲ Contains some of the analysis results plus some state-feedback features

● Currently developed software : Romuald

▲ Dedicated to analysis of descriptor systems

▲ Fully coded using the quadratic separation theory

▲ Polynomially parameter-dependent Lyapunov functions of any order

▲ First preliminary tests currently done for satellite and plane applications

```
>> quiz = ctrpb( OrderOfLyapunovFunction ) + i2n (usys);  
>> result = solvesdp( quiz )
```

Conclusions

