Robust analysis in a quadratic separation framework and application to Demeter satellite attitude control system

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ThalesAlenia
A Thales / Finmeccanica Company
Space

CCT SCA & MOSAR - Toulouse - October 14th, 2009

- Results are part of a joint project involving

  CNES, LAAS-CNRS and Thales Alenia Space

- Flexible satellite attitude control benchmark Demeter developed at CNES
- Robust analysis methodology developed at LAAS-CNRS

  and coded in a Matlab toolbox : RoMulOC

  `www.laas.fr/OLOCEP/romuloc`

- Dedicated codes for Demeter and tests realized at LAAS-CNRS
- Further software developments done at Thales Alenia Space
- Large scale tests done at CNES and Thales Alenia Space

1. Demeter satellite
2. Integral Quadratic Separation (IQS)
3. Heuristic algorithm for optimization of stable domains
4. Application to Demeter - 1 axis, 1 flexible mode, 3 uncertainties
5. Conclusions

M-C-K uncertain model

$$M(\delta_J)\begin{pmatrix}\delta\ddot{\theta}\\ \ddot{\eta}\end{pmatrix} = \begin{bmatrix}0 & 0\\ 0 & C_S^-(\delta_\omega,\delta_\xi)\end{bmatrix}\begin{pmatrix}\delta\dot{\theta}\\ \dot{\eta}\end{pmatrix} + \begin{bmatrix}0 & 0\\ 0 & K_S^-(\delta_\omega)\end{bmatrix}\begin{pmatrix}\delta\theta\\ \eta\end{pmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}U$$

- θ, $\dot{\theta}$: satellite orientation (3D)
- η, $\dot{\eta}$: states of the flexible modes (up to 4 for each axis)
- δ_J: 6 scalar uncertainties on the inertia
- δ_ω, δ_ξ: scalar uncertainties on the frequency and damping of flexible modes

- State space model: $\dot{X} = A(\delta)X + B(\delta)U$

$A(\delta)$, $B(\delta)$ are rational w.r.t. uncertainties.

LFT model

$$\begin{aligned} \dot{X} &= AX + B_\Delta w_\Delta + B_u u \\ z_\Delta &= C_\Delta X + D_{\Delta\Delta} w_\Delta + D_{\Delta u} u \quad , \quad w_\Delta = \Delta z_\Delta \\ y &= C_y X + D_{y\Delta} w_\Delta + D_{yu} u \end{aligned}$$

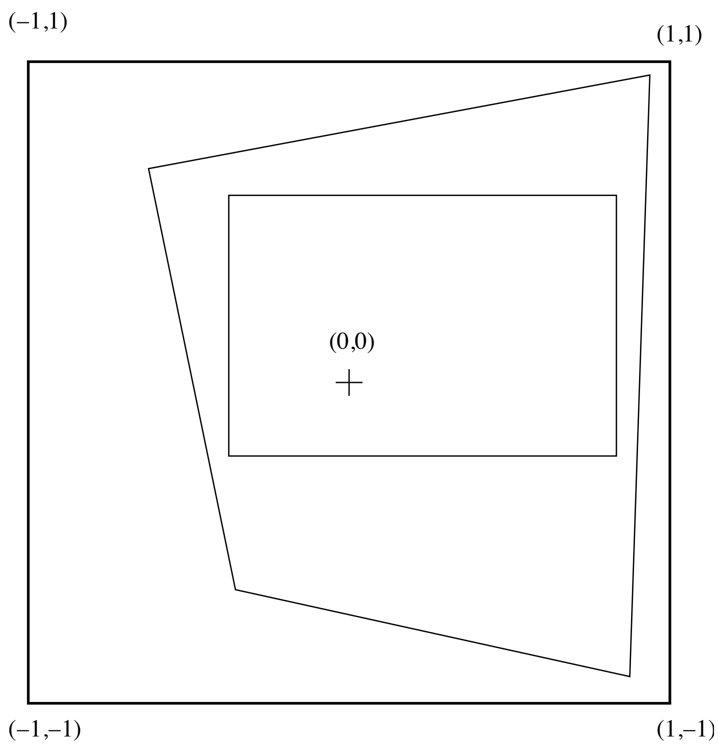
- Δ: diagonal matrix with δ_J, δ_ω and δ_ξ elements
- Some elements δ are repeated
- The problem is normalized: $\delta \in [\,-1\ 1\,]$
- Modeling is made possible in the following toolboxes

`Control` (Matlab©) `LFR` (J.F. Magni) `RoMulOC` (LAAS)

```
>> Ax = [1]; Fm = 1; model_type = 2;
>> usys = demeter2romuloc(Ax,Fm,model_type)
Uncertain model : LFT
-------- WITH --------
               n=4     md=5     mu=1
 n=4    dx  =   A*x +  Bd*wd +  Bu*u
pd=5    zd  =  Cd*x + Ddd*wd + Ddu*u
py=1     y  =  Cy*x
continuous time ( dx : derivative operator )
-------- AND  --------
diagonal structured uncertainty
size: 5x5 | nb blocks: 5 | independent blocks: 3
 wd = diag( #1 #1 #2 #2 #3 ) * zd
index   size    constraint                      name
#1      1x1     interval 1 param        real    dJ11
#2      1x1     interval 1 param        real    dW1
#3      1x1     interval 1 param        real    dX1
```

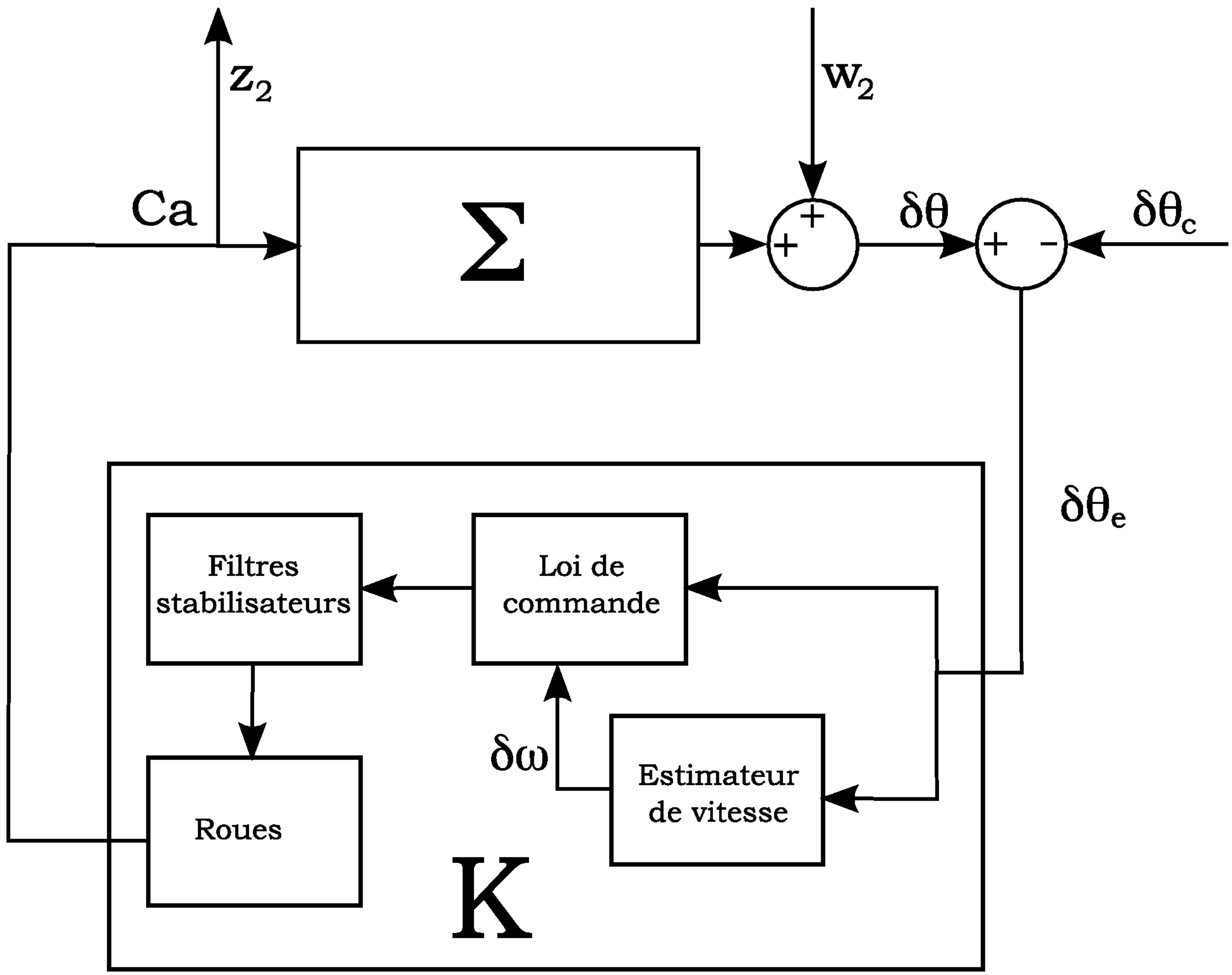
- RoMulOC allows also polytopic models

```
size: 5x5 | nb blocks: 1 | independent blocks: 1
 wd = diag( #4 ) * zd
index    size    constraint                     name
#4       5x5     polytope 8 vertices    real    dJ11,dW1,dX1
```

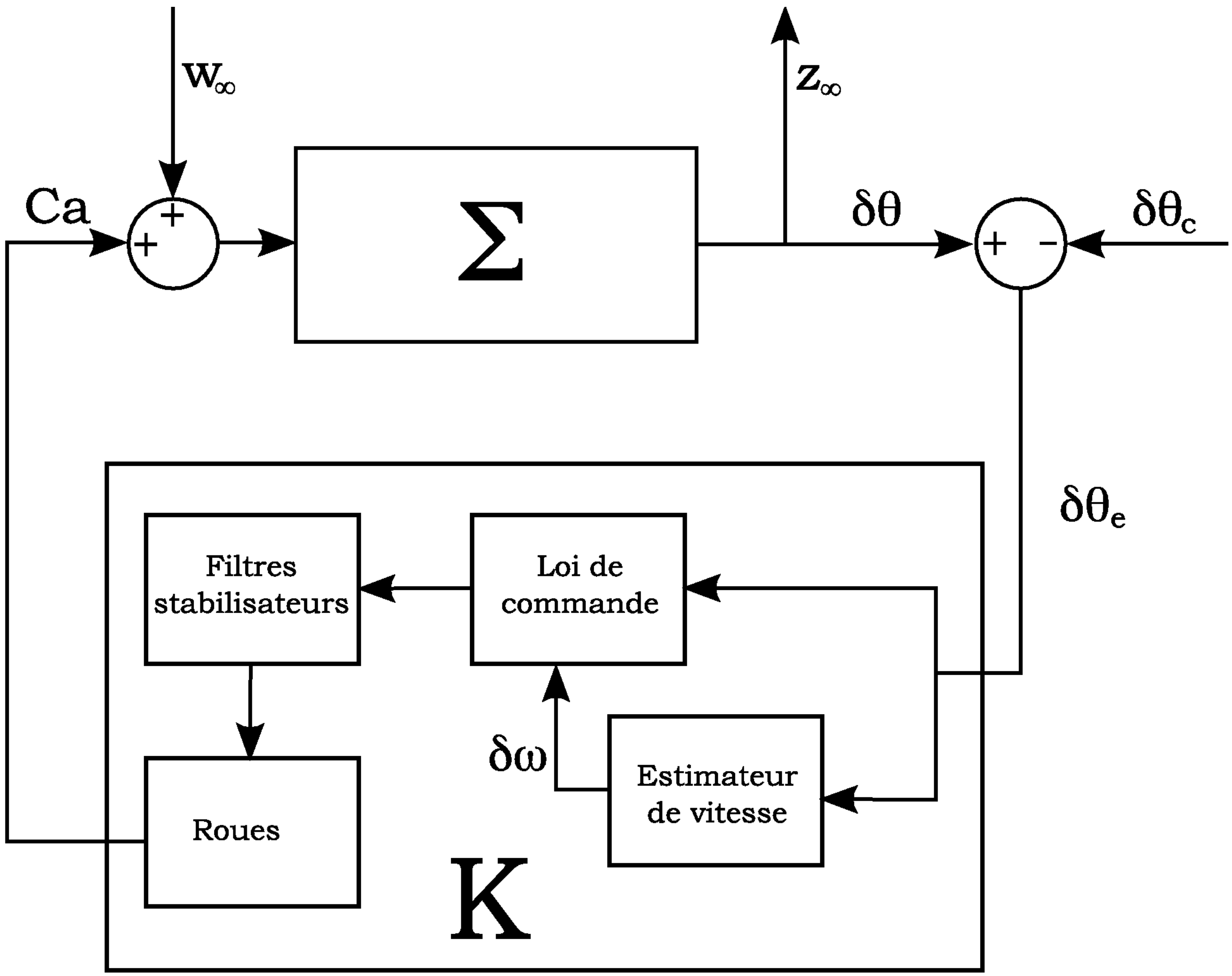


▲ Aim: Guaranteed closed-loop robust stability for the ”biggest” polytope

Secondary problem: Robust guaranteed H_2 norm (consumption)

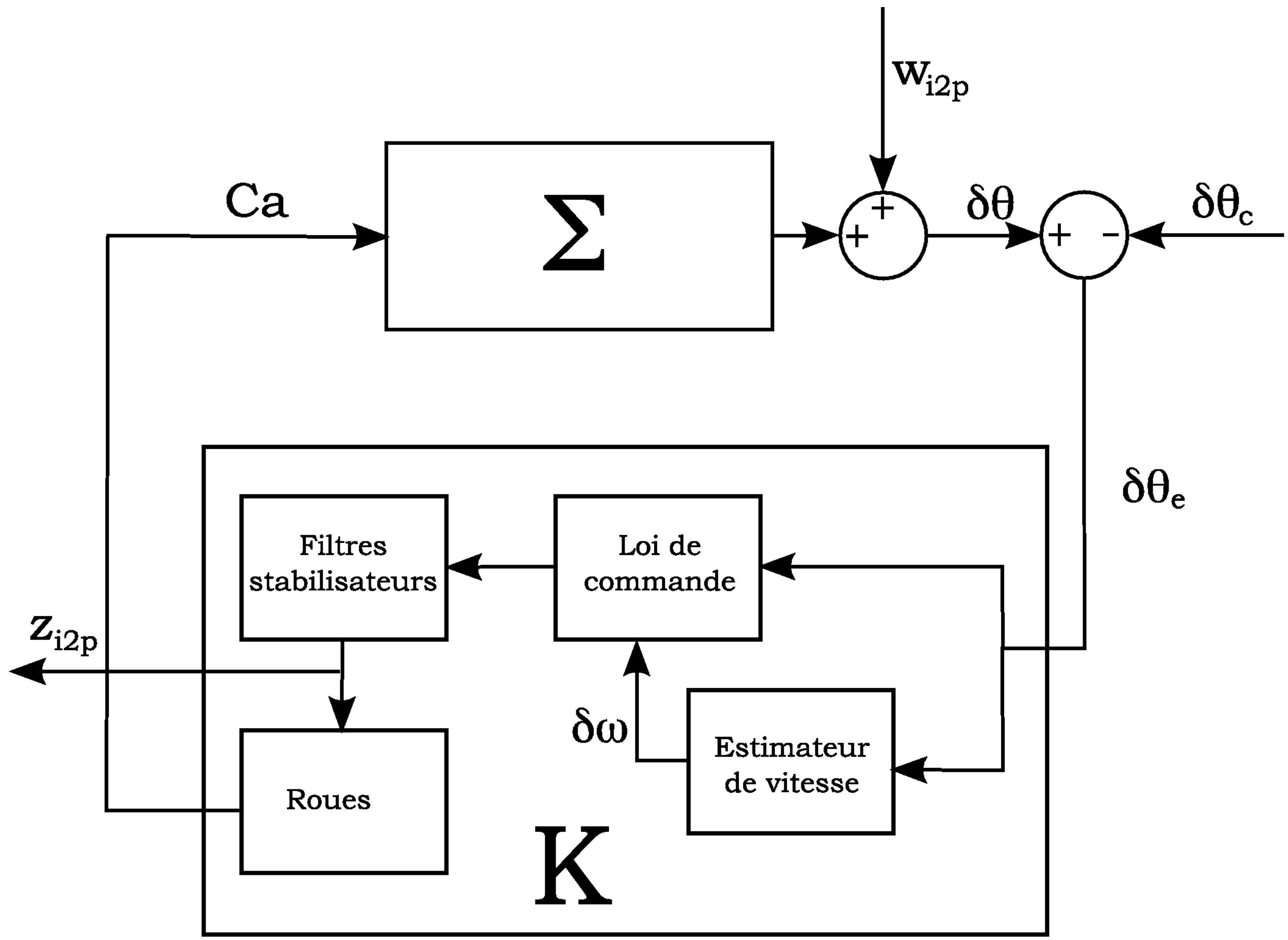


▲ Secondary problem: Robust guaranteed H_∞ norm (robustness to unmodeled dynamics)

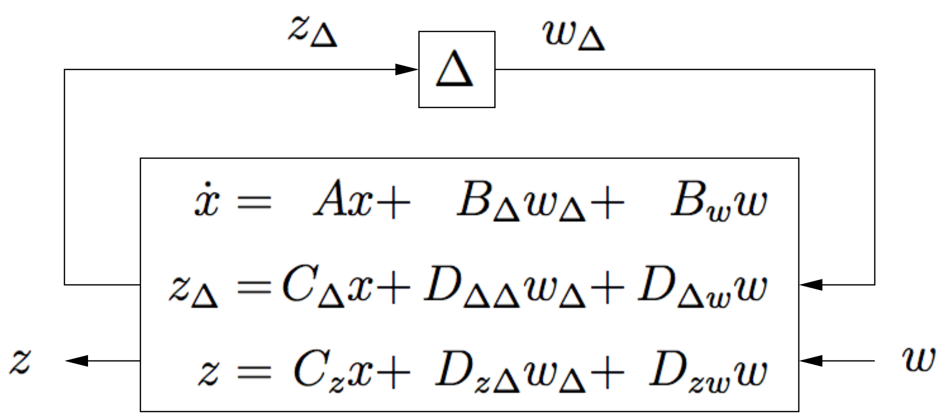


▲ Secondary problem: Robust guaranteed impulse-to-peak performance

(control input saturation w.r.t. to initial depointing)



❶ Demeter satelite




```
Uncertain model : closed-loop satellite with performances
-------- WITH --------
               n=4     md=5     mw=1
 n=4     dx   =    A*x +  Bd*wd +  Bw*w
pd=5     zd   =   Cd*x + Ddd*wd + Ddw*w
pz=1      z   =   Cz*x
continuous time ( dx : derivative operator )
-------- AND  --------
 wd = diag( #4 ) * zd
index   size   constraint                 name
#4      5x5    polytope 8 vertices    real    dJ11,dW1,dX1
```

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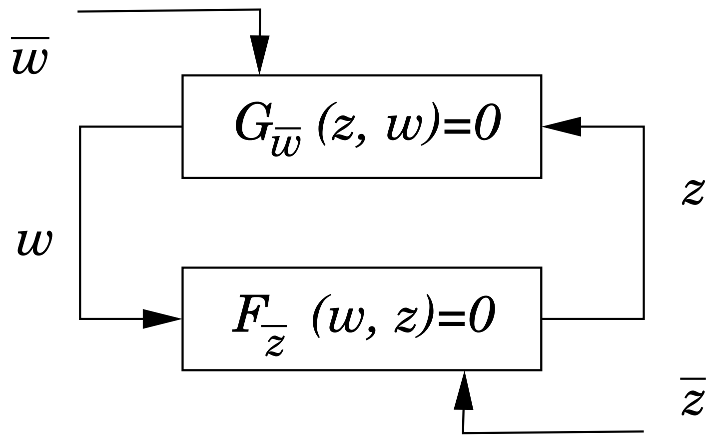
❷Integral Quadratic Separation (IQS)

Well-posedness

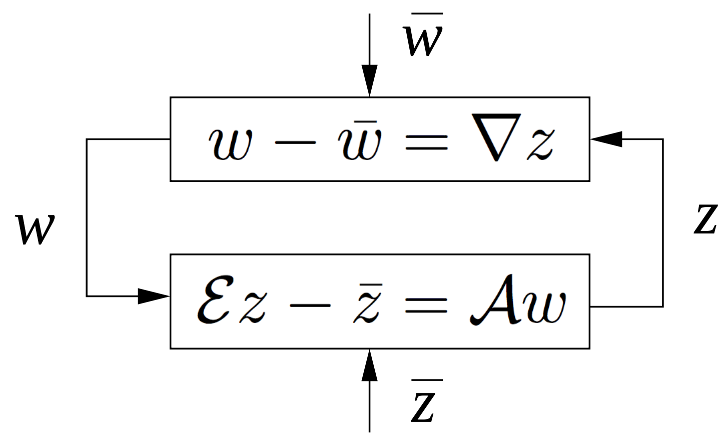


Bounded $(\bar{w}, \bar{z})$

$\Rightarrow$ unique bounded (w, z)

Considered case

- Linear implicit application: $\mathcal{E}$ and $\mathcal{A}$ are matrices (possibly not square)
- $\nabla \in \mathbb{\nabla}$ is bloc-diagonal. Contains scalar, full-bloc, LTI and LTV uncertainties



- For $\mathcal{E} = 1$ and $\mathcal{A} = H(j\omega)$ one recovers IQC framework

❷Integral Quadratic Separation (IQS)

- For dynamic systems $\dot{x} = Ax$: well posedness $\equiv$ internal stability

$$\overbrace{z(t) = Aw(t) + \bar{z}(t)}^{F} \quad , \quad \overbrace{\underbrace{w(t)}_{x(t)} = \int_0^t \underbrace{z(\tau)}_{\dot{x}(t)} d\tau + \underbrace{\bar{w}(t)}_{x(0)}}^{G}$$

- $\bar{w}$ contains information on initial conditions
- Well-posedness

$$\Leftrightarrow \forall(\bar{w}, \bar{z}) \in L_2 \, , \;\; \exists!(w, z) \in L_2 \;\; : \;\; \left\| \begin{array}{c} w \\ z \end{array} \right\| \leq \gamma \left\| \begin{array}{c} \bar{w} \\ \bar{z} \end{array} \right\|$$

$\Rightarrow$ for zero initial conditions and zero perturbations:

$w = z = 0$ is the unique solution (equilibrium point).

$\Rightarrow$ whatever bounded perturbations the state remains close to equilibrium (global stability)

■ Integral Quadratic Separation [Automatica'08, CDC'07, ROCOND'09, ECC'09]

- For the case of linear application with uncertain operator

$$\mathcal{E} z(t) = \mathcal{A} w(t) \ , \ \ w(t) = [\nabla z](t) \quad \nabla \in \mathbb{W}$$

where $\mathcal{E} = \mathcal{E}_1 \mathcal{E}_2$ with $\mathcal{E}_1$ full column rank,

- Integral Quadratic Separator (IQS) : $\exists \Theta$, matrix, solution of LMI

$$\begin{bmatrix} \mathcal{E}_1 & -\mathcal{A} \end{bmatrix}^{\perp *} \Theta \begin{bmatrix} \mathcal{E}_1 & -\mathcal{A} \end{bmatrix}^{\perp} > 0$$

and Integral Quadratic Constraint (IQC) $\forall \nabla \in \mathbb{W}$

$$\int_0^\infty \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix}^* \Theta \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix} dt \leq 0$$

❷Integral Quadratic Separation (IQS)

- For some given $\nabla\!\!\!\nabla$, $\exists$ LMI conditions for Θ solution to IQC

$$\int_0^\infty \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix}^* \Theta \begin{pmatrix} \mathcal{E}_2 z(t) \\ [\nabla z](t) \end{pmatrix} dt \leq 0$$

- ▲ Θ is build out of IQS for elementary blocs of ∇
- ▲ Improved DG-scalings, full-bloc S-procedure, vertex separators...
- ▲ Building Θ and related LMIs is tedious but can be automatized (`RoMulOC`)
- ▲ It is conservative except in few special cases [Meinsma et al., 1997].

Robust analysis in IQS framework:

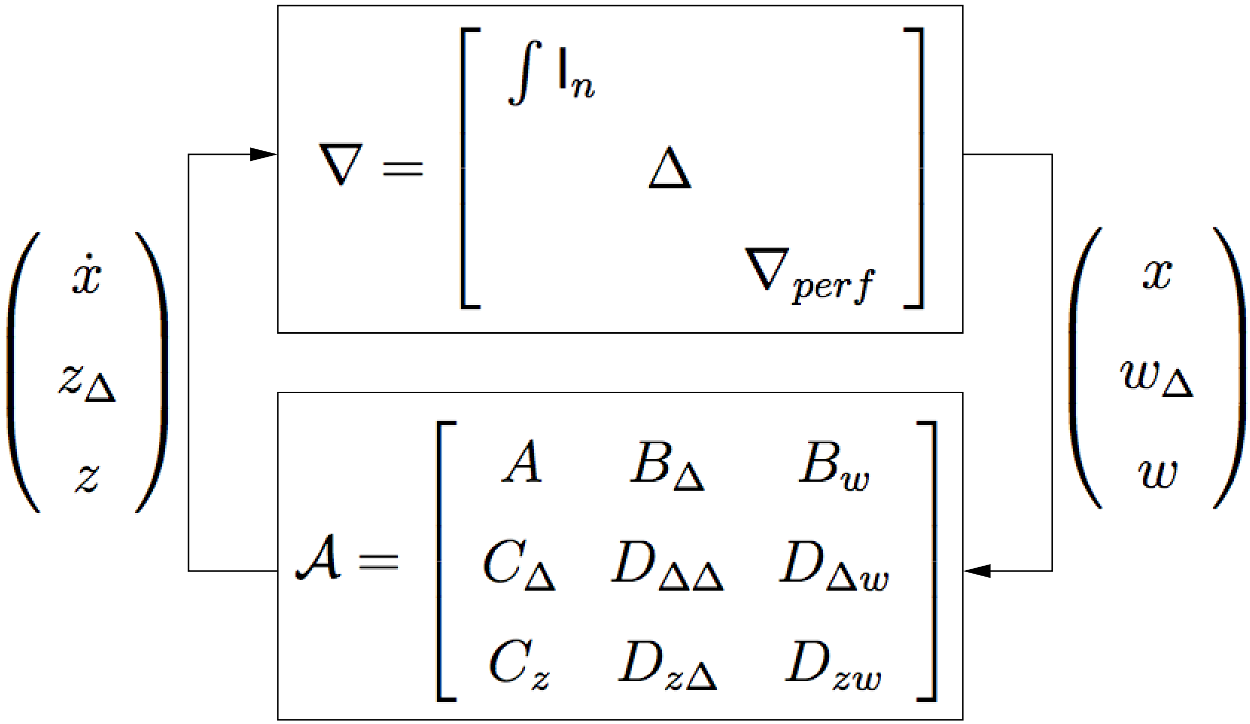
- 1- Write the robust analysis problem as a well-posedness problem

$$\mathcal{E}z = \mathcal{A}w \ , \ \ w = \nabla z$$

- 2- Build Integral Quadratic Separators for each elementary bloc of ∇
- 3- Apply the IQS results to get (conservative) LMIs

■ <u>Demeter analysis problems:</u>

● Well-posedness of



- $\int 1_n$ integrator
- Δ matrix of uncertainties
- ∇_{perf} operator related to performances

❷Integral Quadratic Separation (IQS)

Other problem modeling produce other LMI conditions

- Dual system: $z_d = \mathcal{A}^T w_d \;,\; w_d = \nabla^* z_d$
- System augmentation (produces systems in descriptor form)

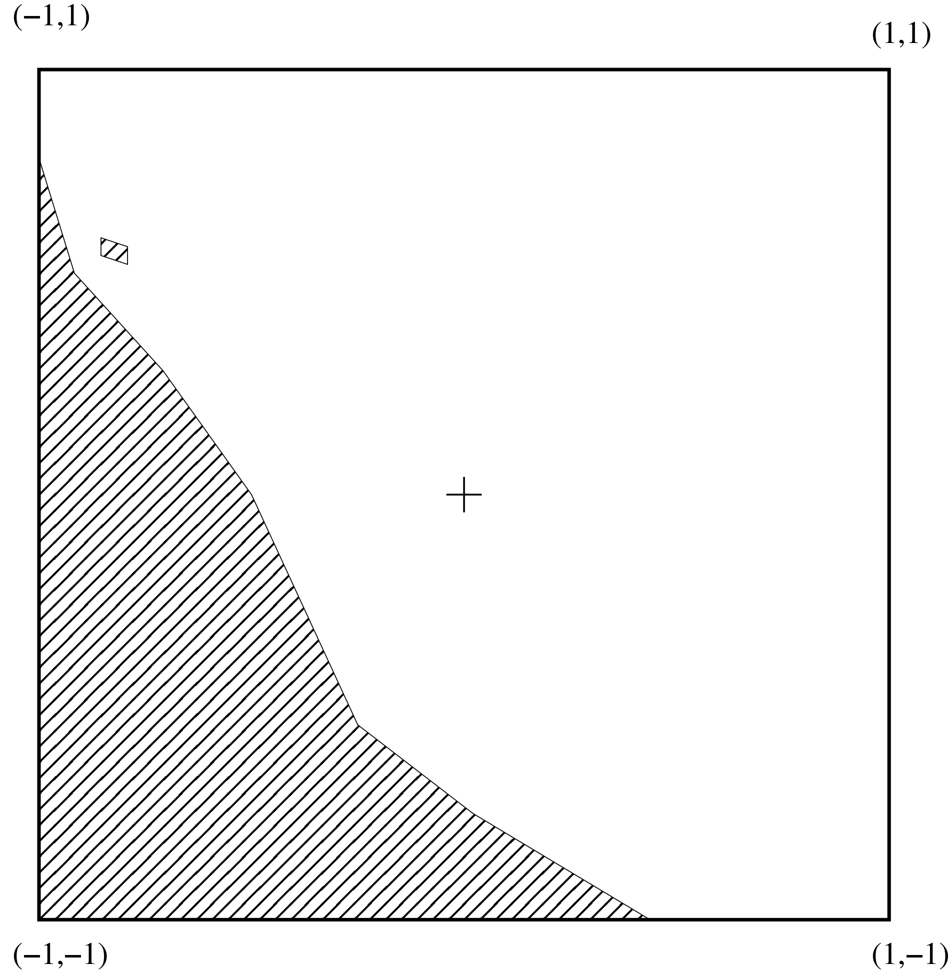
$$\left\{\begin{array}{rl} \dot{x} = & Ax + \; B_\Delta w_\Delta \\ z_\Delta = & C_\Delta x + D_{\Delta\Delta} w_\Delta \end{array}\right. \quad \Rightarrow \quad \left\{\begin{array}{rl} \dot{x} = & Ax + \; B_\Delta w_\Delta \\ z_\Delta = & C_\Delta x + D_{\Delta\Delta} w_\Delta \\ \dot{z}_\Delta = & C_\Delta \dot{x} + D_{\Delta\Delta} \dot{w}_\Delta \end{array}\right.$$

$$w_\Delta = \Delta z_\Delta \qquad\qquad \left\{\begin{array}{l} w_\Delta = \Delta z_\Delta \\ \dot{w}_\Delta = \Delta \dot{z}_\Delta + \overbrace{\dot{\Delta} z_\Delta}^{=0} \end{array}\right.$$

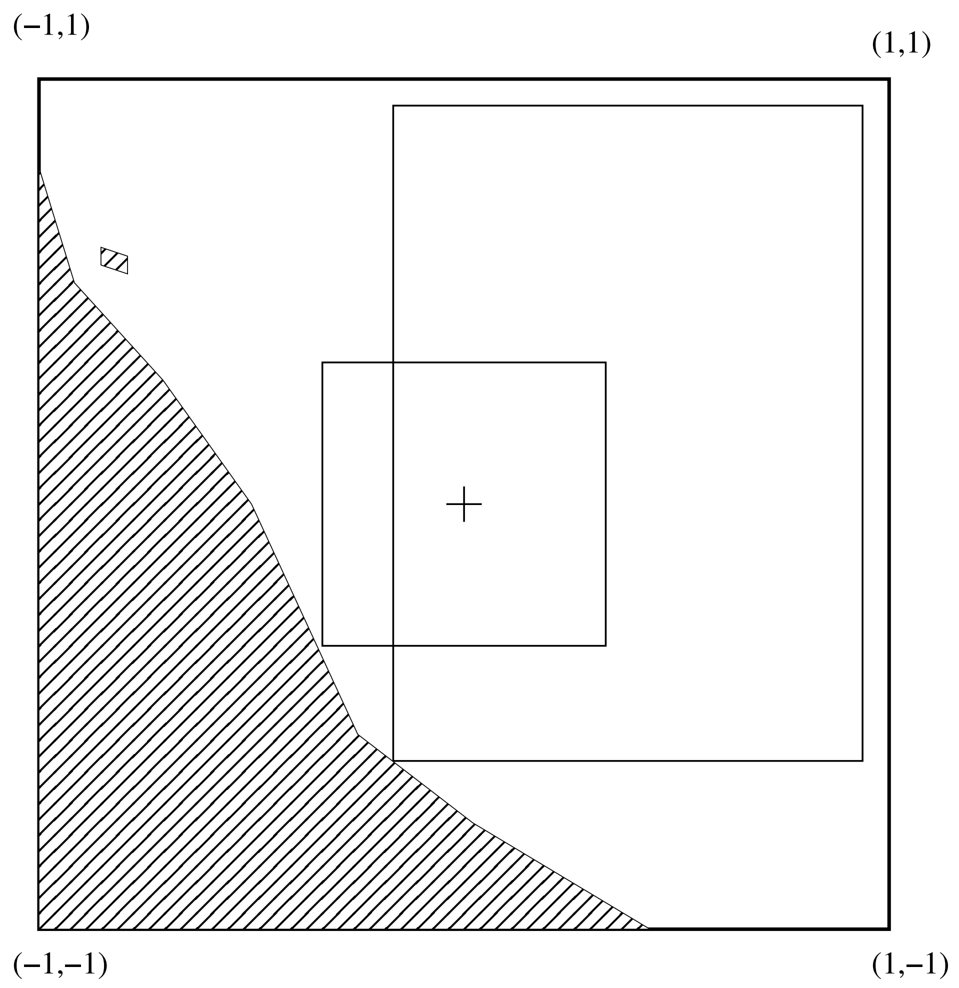
Equivalent to increasing the dependency of the (implicit) Lyapunov function

$$V_0(x) = x^* P x \;\Rightarrow\; V_1(x, \Delta) = \begin{pmatrix} x^* & z_\Delta^* \end{pmatrix} \hat{P} \begin{pmatrix} x^* & z_\Delta^* \end{pmatrix}^*$$

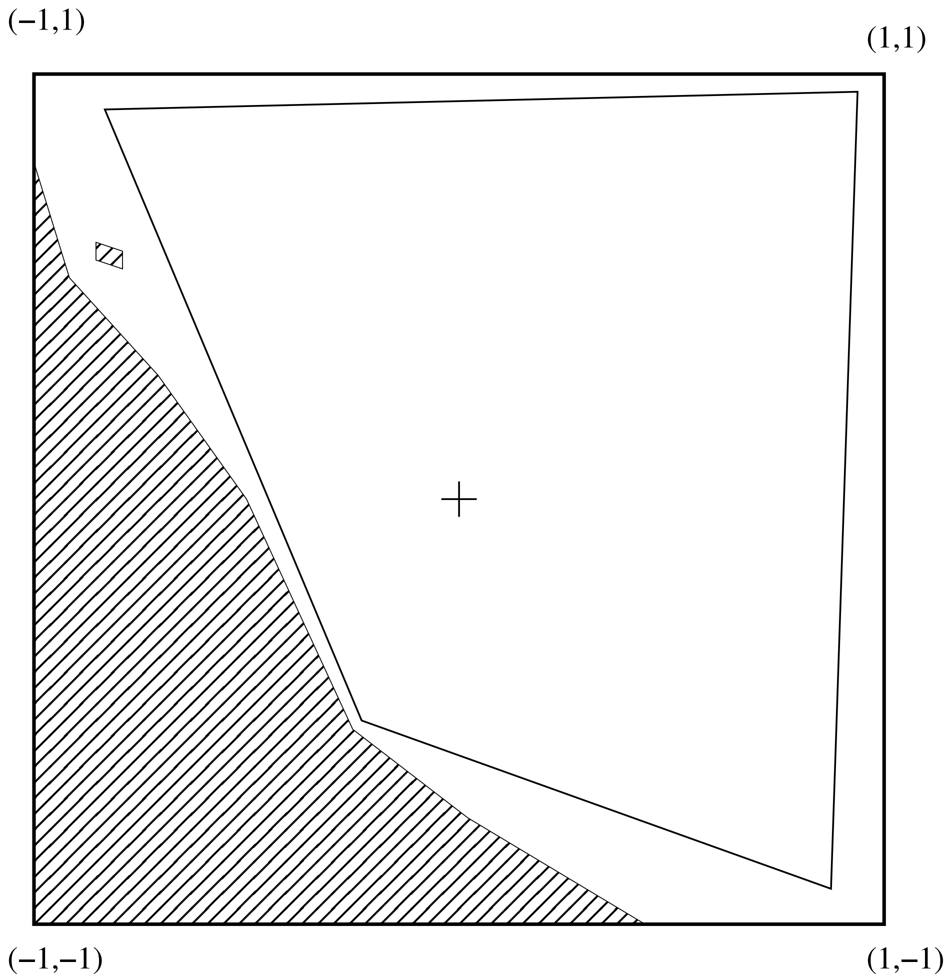
- Values of parameters making the system stable and unstable (unkown)



- Biggest squares and rectangles possibly obtained

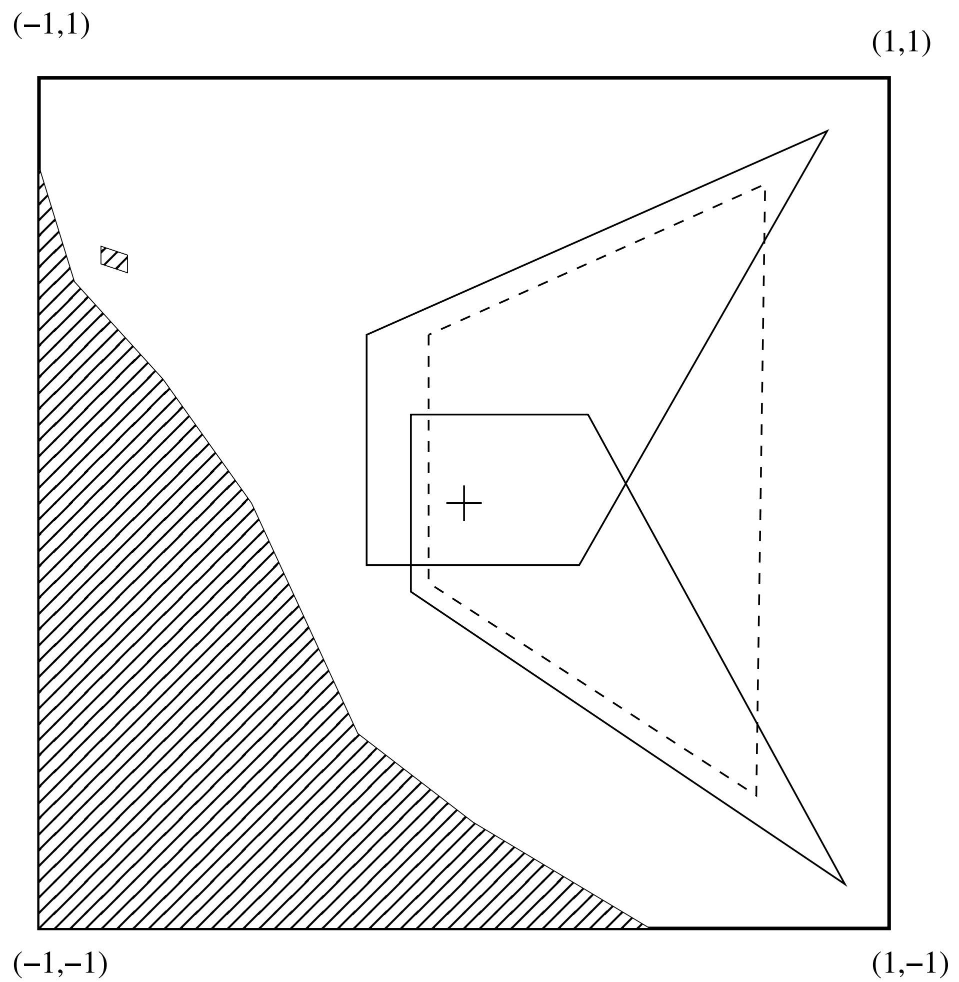


- Biggest polytope possibly obtained (if LMIs are not conservative)



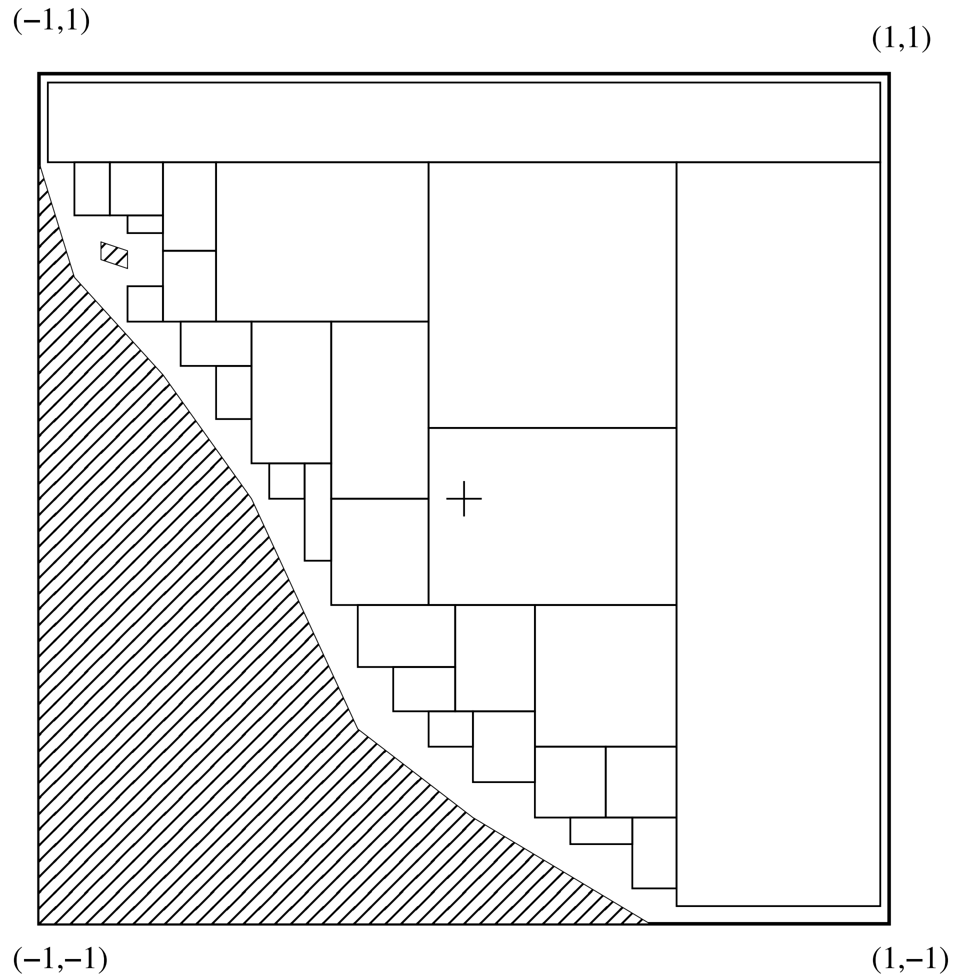
❸ Heuristic algorithm for optimization of stable domains

- Due to conservatism some polytopes may give feasible LMIs (full), others not (dotted)



❸ Heuristic algorithm for optimization of stable domains

- One solution: pave the feasible set



- Proposed algorithm - Step 1
- Find by bisection a initial hyper-cube of safe parameters

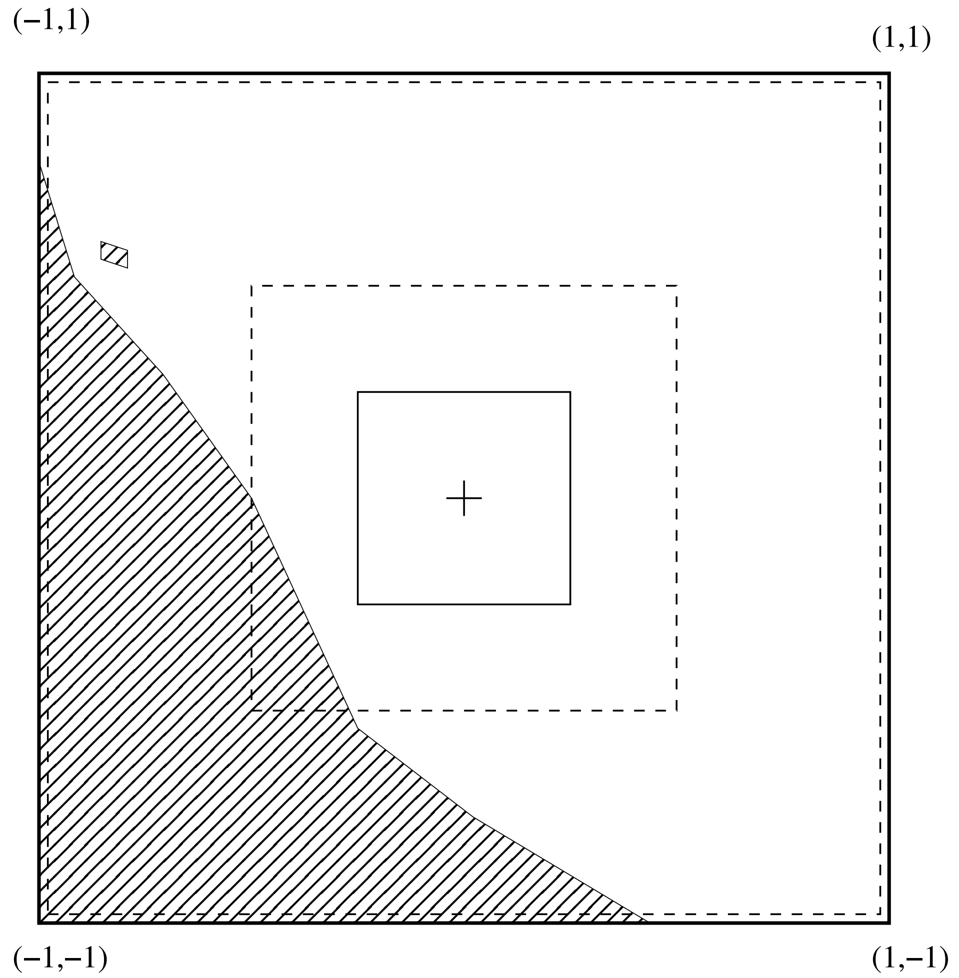
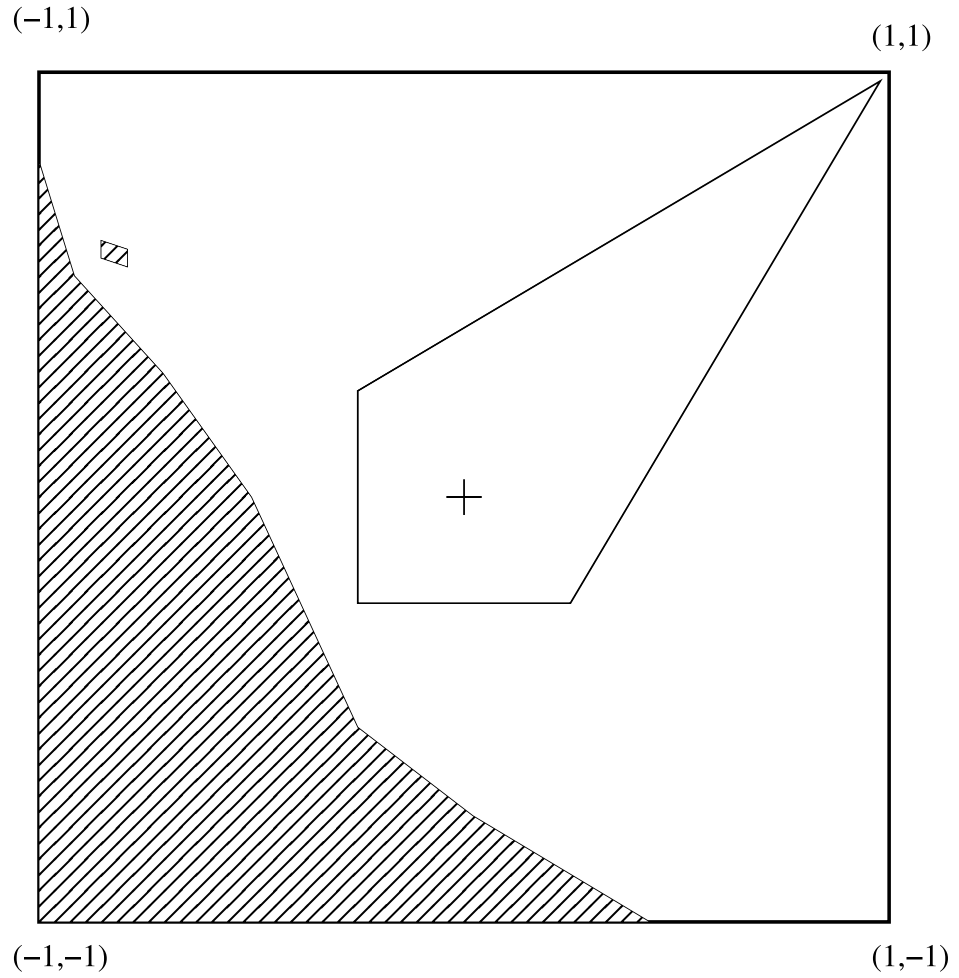


Figure 1: Recherche par bissection d'un carr faisable

- Proposed algorithm - Step 2
- Pull one after the other the vertices towards the corners



❸ Heuristic algorithm for optimization of stable domains

- Proposed algorithm - Step 2
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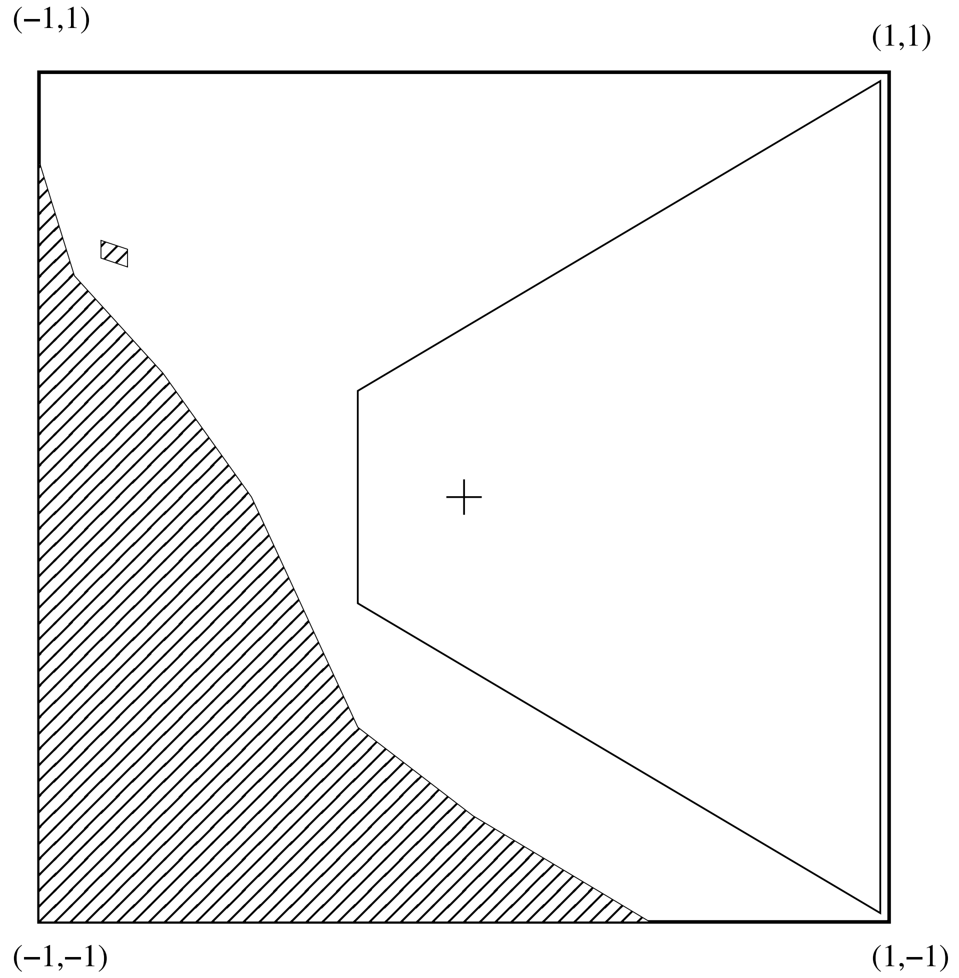
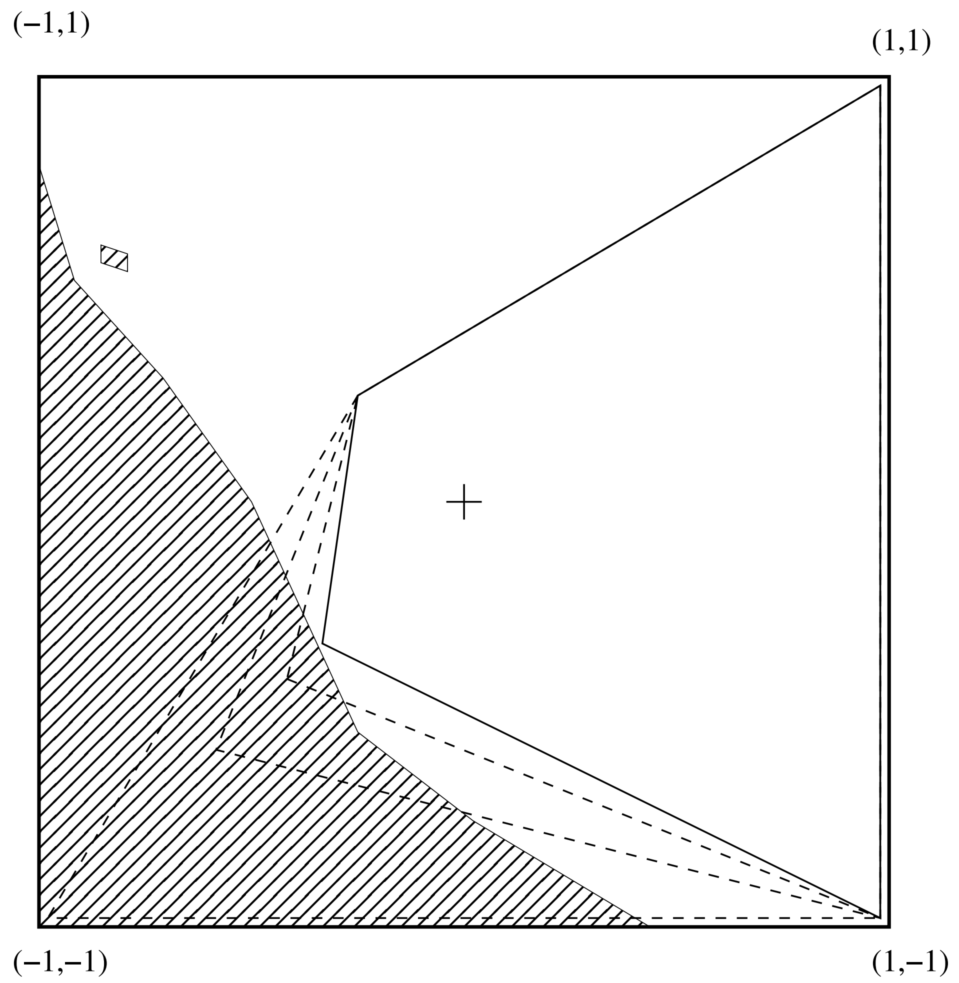


❸ Heuristic algorithm for optimization of stable domains

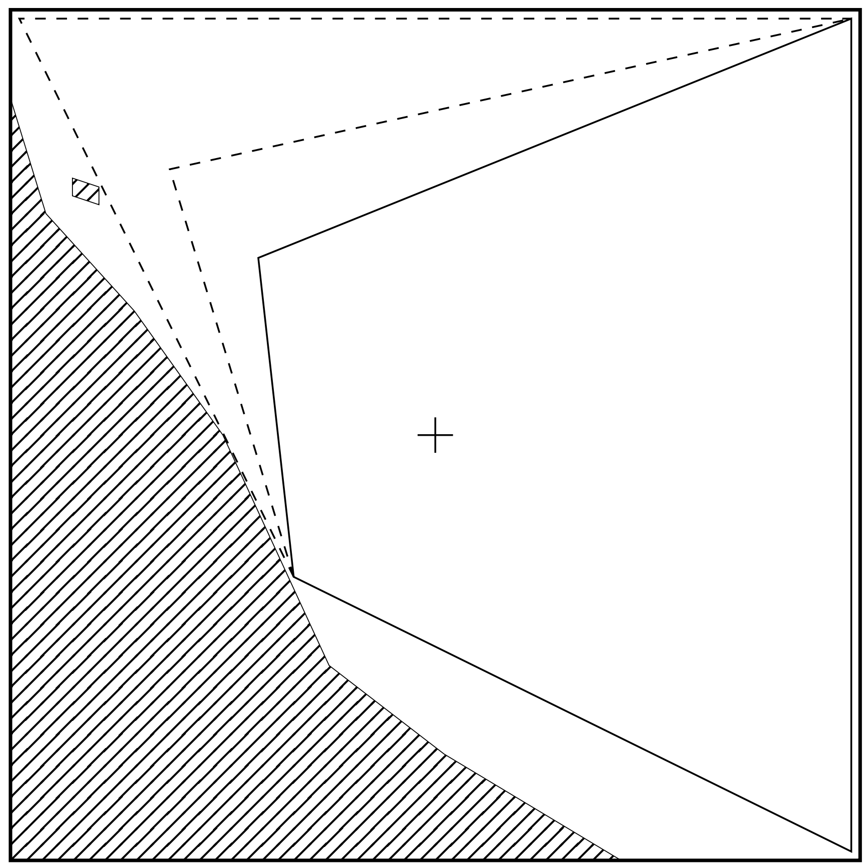
■ Proposed algorithm - Step 2

● Pull one after the other the vertices towards the corners

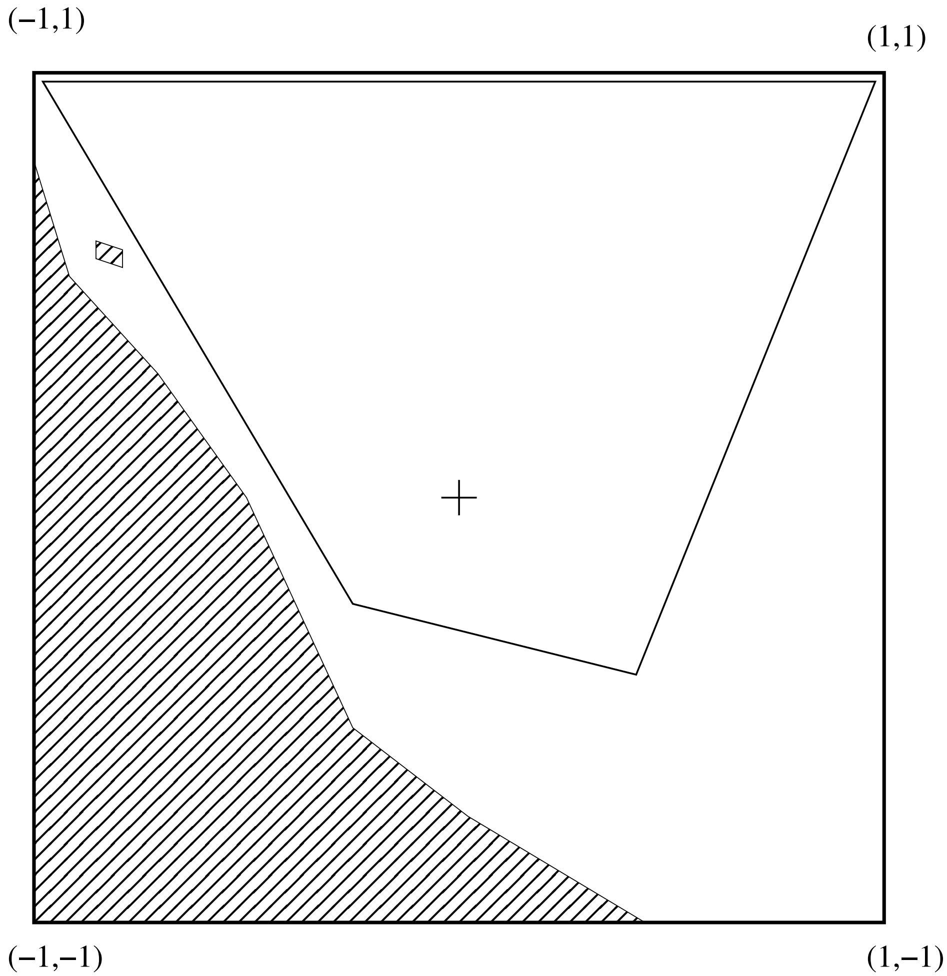


❸ Heuristic algorithm for optimization of stable domains

- Proposed algorithm - Step 2
- Pull one after the other the vertices towards the corners



- Due to the conservatism of the LMIs, the result depends of the ordering
- An other possible solution.



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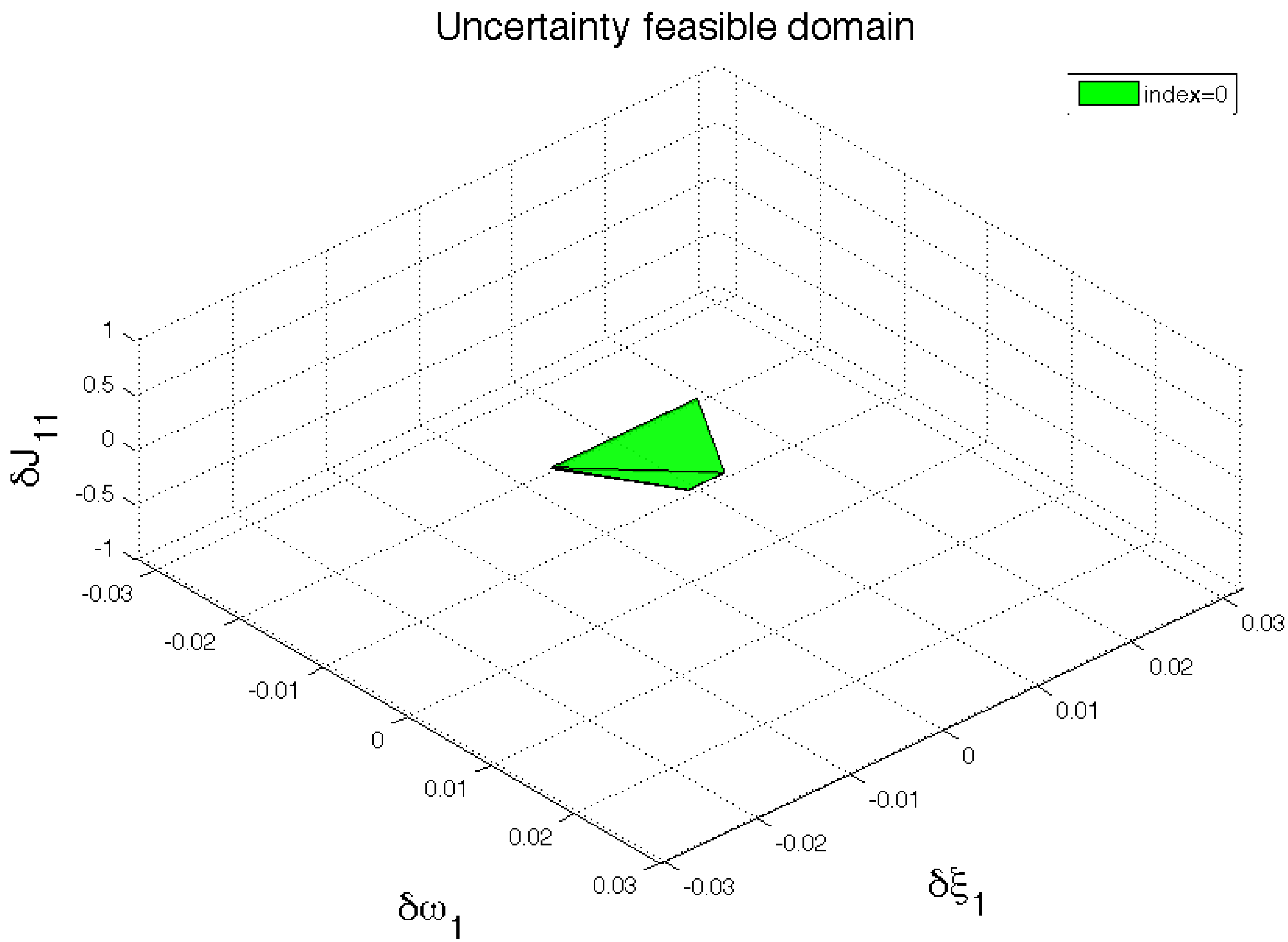
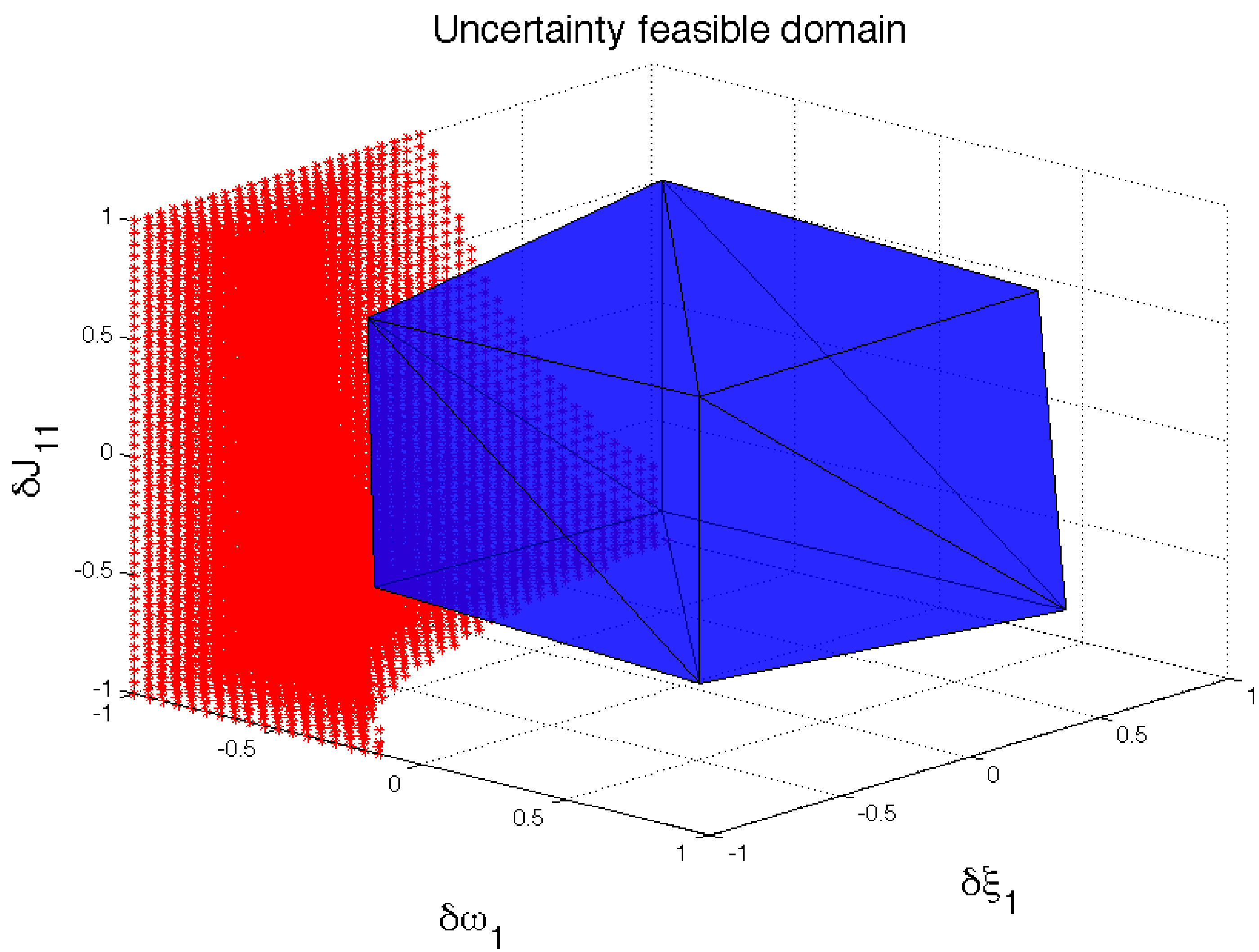
- X axis, one flexible mode (3 uncertainties)
- IQS result applied to the original modeling

(corresponds to the use of a unique Lyapunov function for all parameters)



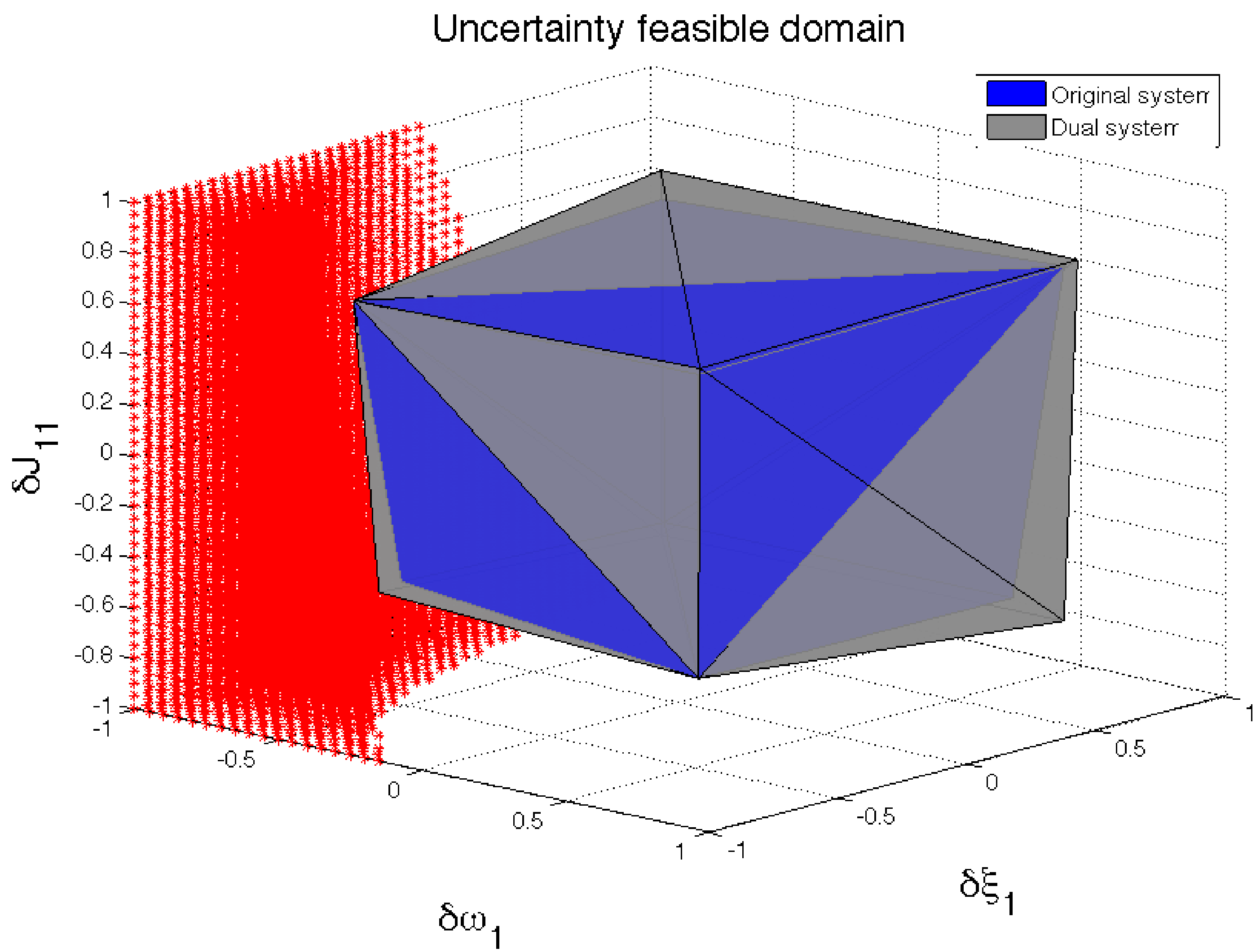
- IQS result applied to the augmented system (added information on $\dot{\Delta} = 0$)

(corresponds to Lyapunov function $V(x) = x^T P(\Delta) x$ with $P(\Delta)$ quadratic)

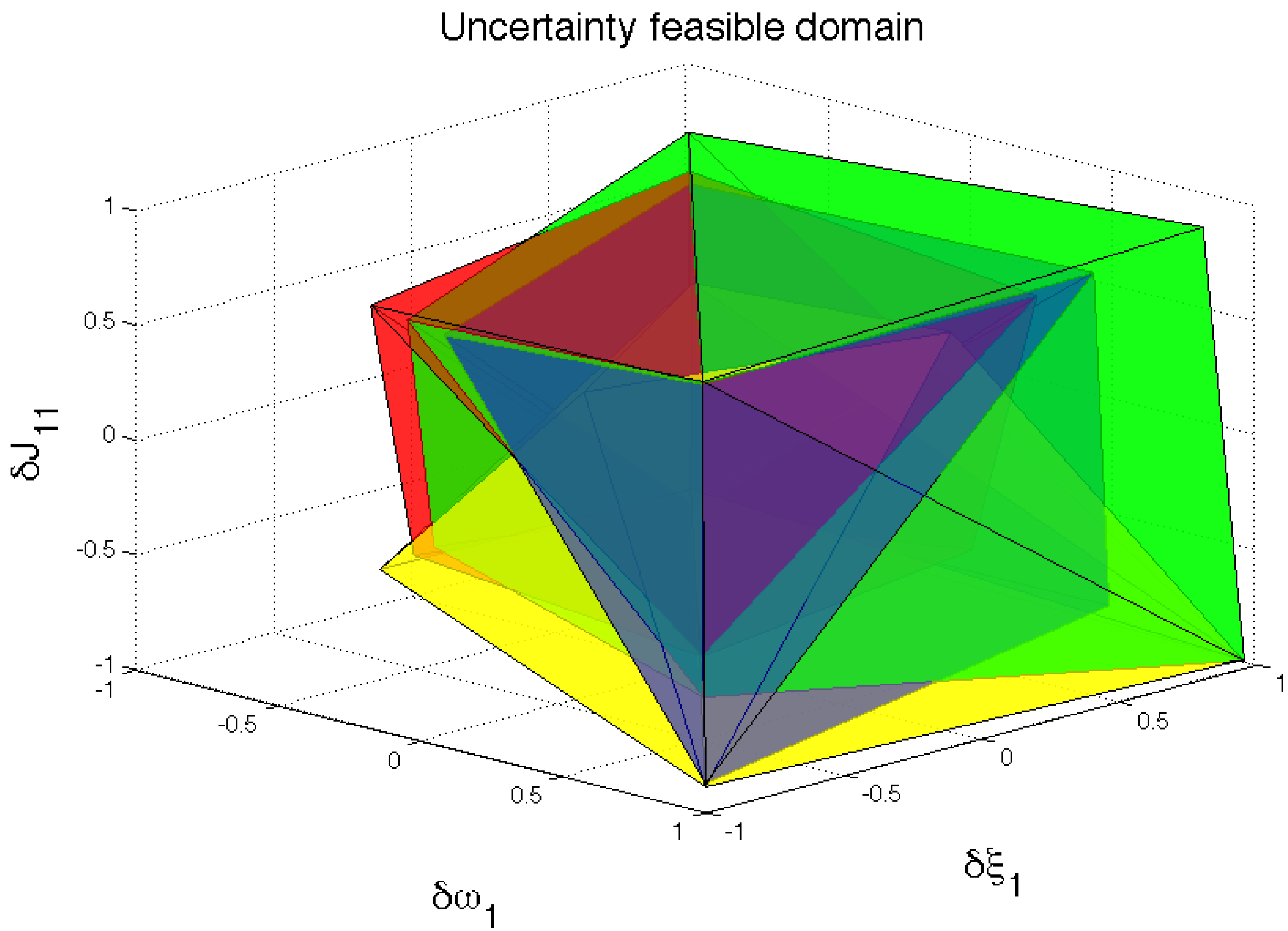


- IQS result applied to the dual of the augmented system

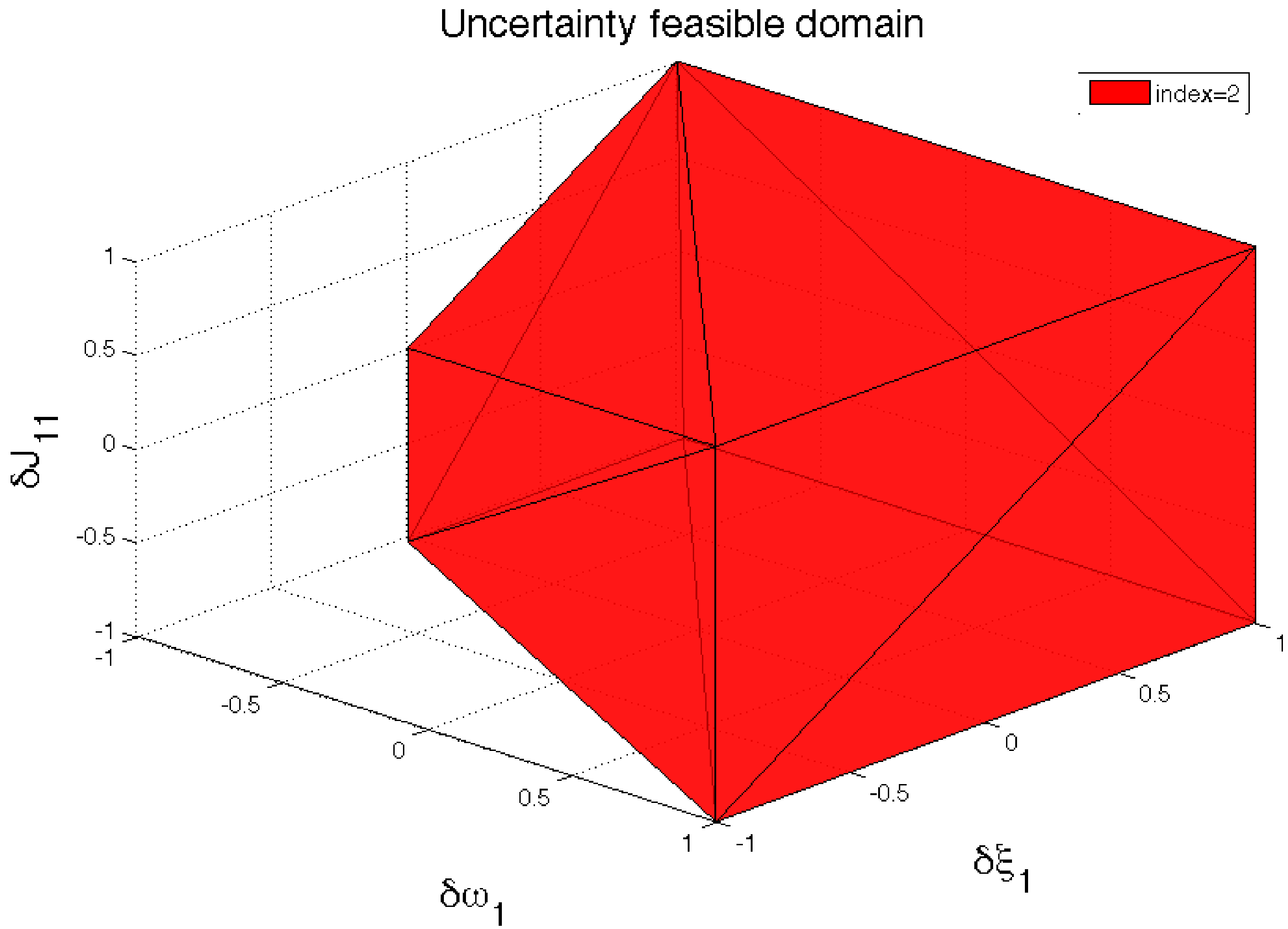
(Lyapunov function $V(x) = x^T P(\Delta)x$ with $X(\Delta) = P^{-1}(\Delta)$ quadratic)



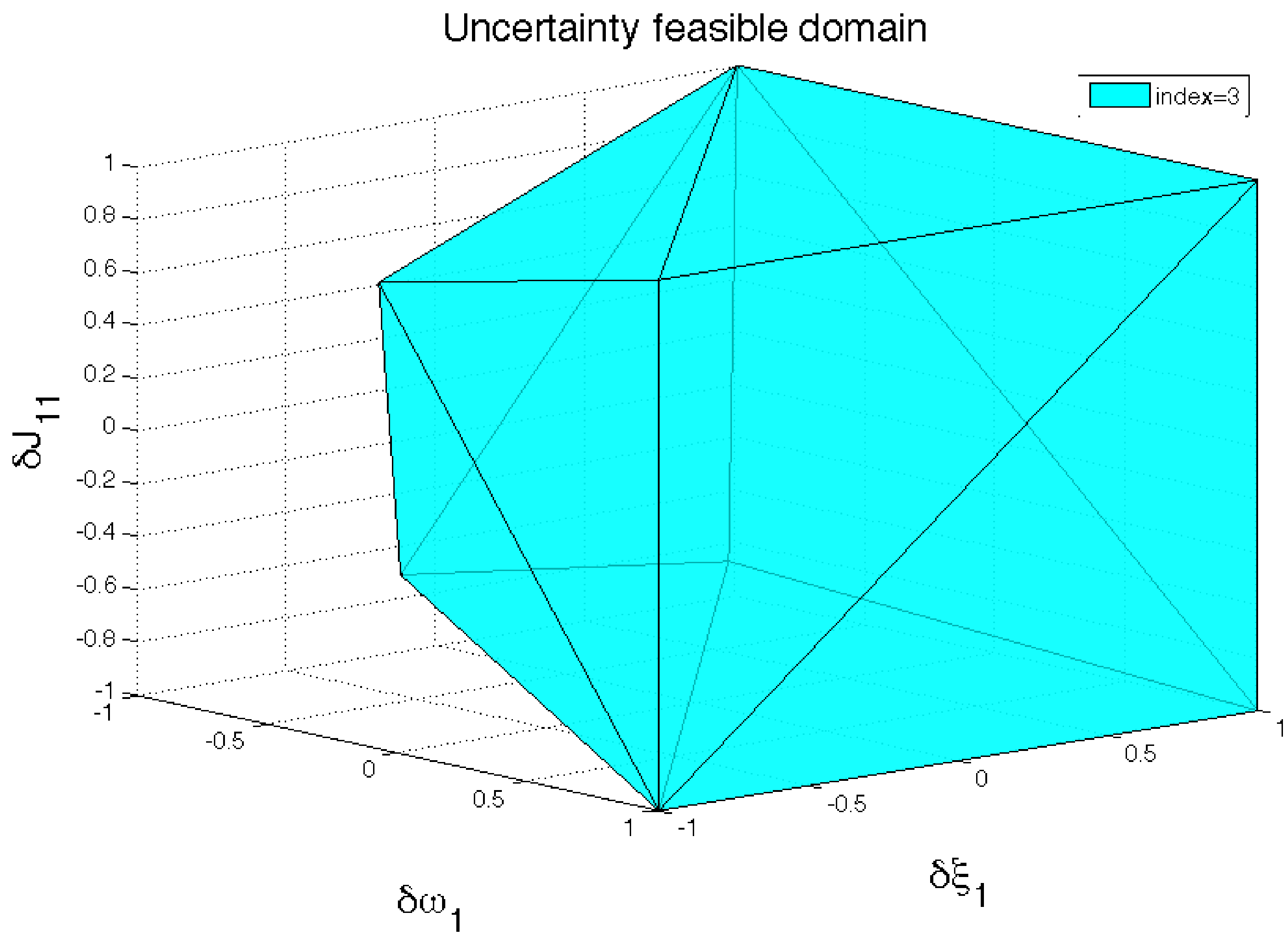
- IQS result applied to the primal of the augmented system
- Bisection is replaced by random search in heuristic algorithm (4 tests)



- IQS result applied to twice augmented system (added information on $\ddot{\Delta} = 0$) (corresponds to Lyapunov function $V(x) = x^T P(\Delta) x$ with $P(\Delta)$ of order 4)



- IQS result applied to augmented system 3 times (added information $\Delta^{(3)} = 0$) (corresponds to Lyapunov function $V(x) = x^T P(\Delta) x$ with $P(\Delta)$ of order 6)



- Vertices obtained for systems augment 2 and 3 times

index=2			index=3		
δJ_{11}	$\delta\omega_1$	$\delta\xi_1$	δJ_{11}	$\delta\omega_1$	$\delta\xi_1$
-0.5156	-0.5156	-0.5156	-0.5312	-0.5312	-0.5312
-0.6562	-0.6562	0.6562	-0.7812	-0.7812	0.7812
-1	1	-1	-1	1	-1
-1	1	1	-1	1	1
0.5156	-0.5156	-0.5156	0.5703	-0.5703	-0.5703
1	-1	1	1	-1	1
1	1	-1	1	1	-1
1	1	1	1	1	1

- Size of LMIs and computation time

index	nb vars	dim LMI	solve LMI	find polytope
0	216	949×949	0.3s	113s
1	646	2754×2754	3s	746s
2	1614	6185×6185	25s	793s
3	3024	11050×11050	204s	3859s

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❺ Conclusions

- IQS framework: Produces new results based on model manipulations

- Adding more equations for higher derivatives of the state:

  Less conservative LMI conditions

- Same technique works for time varying uncertainties

  (if known bounds on derivatives)

- Has been applied successfully to time-delay systems [Gouaisbaut]:

  Gives sequences of LMI conditions with decreasing conservatism

- Related to SOS representations of positive polynomials [Sato 2009]:

  Conservatism decreases as the order of the representation is augmented

- No need to manipulate by hand LMIs (Schur complements etc.), polynomials...

- Does conservatism vanishes? Exactly? Asymptotically?

- Is it possible to cope with non-linearities in the same way?

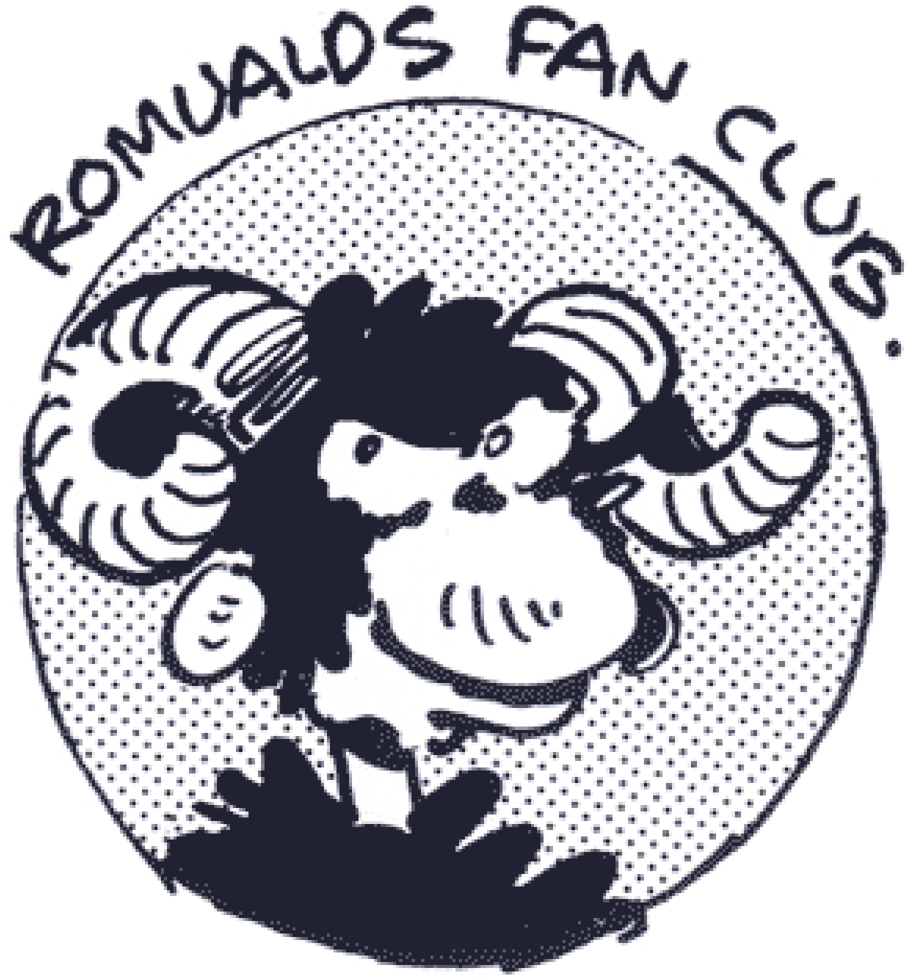
- Applied currently to attitude control robust analysis:
  - Other axis and more flexible modes
  - Allows to reduce significantly the validation time: Guaranteed robustness
  - Tests being done with existing software : RoMulOC

    `www.laas.fr/OLOCEP/romuloc`

  - Contains some analysis (`index = 0,1`) + some state-feedback features

- Currently developed software: Romuald
  - Dedicated to analysis of descriptor systems
  - Fully coded using IQS theory
  - Allows systematic system augmentation

```
>> quiz = ctrpb( OrderOfAugmentation ) + h2( usys );
>> result = solvesdp( quiz )
```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