Conditions LMI de synthèse de commandes adaptatives directes pour les systèmes linéaires ‘presque stables’

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Cooperation program between CNRS, RAS and RFBR:

A. Fradkov, B. Andrievsky

Application to Demeter satellite with CNES: A. Drouot, Ch. Pittet, J. Mignot

Application to ‘helicopter’ benchmark: B. Andrievsky, V. Mahout

Simple adaptive control (SAC)

For a system $y(t) = [\Sigma u](t)$ to follow reference y_r

$$u(t) = K(t)e(t) \ , \ \ \dot{K}(t) = -Gy(t)e^T(t)\Gamma \ , \ \ e(t) = y(t) - y_r(t)$$

- K is driven to minimize the square of the error $J(t) = e^T(t)e(t)$
- In the scalar case

$$\dot{k} = -\gamma \frac{\partial (y - y_r)}{\partial k}(y - y_r) \simeq -\gamma g y e$$

- Gy: approximation of the gradient of J with respect to K (for the closed-loop)
- $\Gamma > 0$: weight on the adaptation speed
- [Fradkov, Kaufman et al, Ioannou, Barkana]

G is chosen with respect to closed-loop passivity conditions

- Choice of G depends of the systems model
- Adaptive control intended for uncertain systems: robust

Outline

❶ Passivity conditions for Simple Adaptive Control (SAC)

- LMI formulas for SAC - robustness issues

❷ Robustness to noise on measurements & to parametric uncertainty

- Barrier type corrective term
- Passivity with respect to an output with feedthrough gain

❸ Examples and some other features of SAC

- L_2 performance
- Stable neighborhood of the origin in case of time-varying systems

SAC for LTI systems

- Let a linear system $\dot{x} = Ax + Bu \ , \ y = Cx \quad (u \in \mathrm{R}^m, y \in \mathrm{R}^p, m \leq p)$
- and SAC $\quad u = Ky \ , \ \dot{K} = -Gyy^T\Gamma$
- Closed-loop stability is guaranteed if

$$\exists F \ : \ \dot{x} = (A + BFC)x + Bw, \ \ z = GCx \ \text{ strictly passive}$$

or equivalenty if

$$\exists F, P > \mathbf{O} \ : \ \underbrace{(A + BFC)^T P + P(A + BFC)}_{\Upsilon} < \mathbf{O} \ , \ PB = C^T G^T$$

- Proof using Lyapunov function

$$V(x, K) = x^T P x + \mathrm{Tr}((K - F)\Gamma^{-1}(K - F)^T).$$

$$\dot{V} = x^T \Upsilon x + 2x^T PB(K - F)y + 2\mathrm{Tr}(\dot{K}\Gamma^{-1}(K - F)^T) = x^T \Upsilon x$$

SAC versus SOF

■ Closed-loop stability with SAC is guaranteed if system is 'almost passive'

$$\exists F, P \;:\; \underbrace{(A+BFC)^T P + P(A+BFC)}_{\Upsilon} < 0\;,\; PB = C^T G^T$$

▲ Stability with SAC proved by existence of some stabilizing SOF ($u = Fy$)

- Why complicating the control ?

● The condition happens to be LMI+E (for given G):

$$\exists F, P \;:\; A^T P + PA + C^T (G^T F + F^T G) C < 0\;,\; PB = C^T G^T$$

● Any $F = -kG$ with k large enough stabilizes the system (high gain)

▲ Not all SOF stabilizable systems will satisfy such constraints

▲ The SAC design problem is to find G: non convex problem.

Robustness of SAC

- Let an uncertain LTI system $\dot{x} = A(\Delta)x + B(\Delta)u \ , \ \ y = C(\Delta)x$
- and SAC $\quad u = Ky \ , \ \ \dot{K} = -Gyy^T\Gamma$
- Closed-loop robust stability with SAC is guaranteed if $\exists F(\Delta), P(\Delta)$:

$$A^T(\Delta)P(\Delta) + P(\Delta)A(\Delta) + C^T(\Delta)(G^T F(\Delta) + F^T(\Delta)G)C(\Delta) < 0$$

$$P(\Delta)B(\Delta) = C^T(\Delta)G^T$$

- Robustness techniques may be applied to the LMI (given G)
- Equality constraint almost impossible to guarantee robustly

$$P(\Delta)B(\Delta) = C^T(\Delta)G^T \ , \ \ \forall\Delta \in \mathbf{\Delta} \quad !!!$$

Divergence of SAC due to noise

- Assume noisy measurements $y(t) = Cx(t) + d(t)$

$$\dot{K} = -Gyy^T\Gamma = -G(Cx+d)(x^TC^T + d^T)\Gamma$$

- $K(t)$ will diverge even if $x \to 0$ (if d does not go to zero).
- Not acceptable in practice
- Most often corrective terms are added such as

$$\dot{K} = -Gyy^T\Gamma - \mu(K - F_0)$$

- But then $K(t) \to F_0$:

  the closed-loop characteristics tend to those with SOF $u = F_0 y$

  - Why complicating the control ?

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Dead-zone + barrier corrective term

- Usual corrective term $\dot{K} = -GyyT^T\Gamma - \mu(K - F_0)$
- ▲ Corrective term always active even if K does not diverge
- ▲ Corrective term does not guarantee K to be bounded in given set

- Proposed corrective term $\dot{K} = -Gyy^T\Gamma - \phi(\underbrace{K - F_0}_{\hat{K}})\Gamma$

$$\phi(\hat{K}) = \psi(\|\hat{K}\|^2)\hat{K}$$

$$\psi(0 \leq k \leq \nu) = 0 \quad , \quad \frac{d\psi}{dk}(\nu \leq k \leq \beta\nu) > 0 \quad , \quad \psi(\nu\beta) = +\infty$$

- Example: weighted Frobenius norm $\|\hat{K}\|^2 = \mathrm{Tr}(\hat{K}\hat{D}\hat{K}^T)$ and

$$\psi(\nu \leq k \leq \beta\nu) = \exp(\mu k - \log(\beta\nu - k))$$

Dead-zone + barrier corrective term

- Proposed corrective term $\dot{K} = -Gyy^T\Gamma - \phi(\underbrace{K - F_0}_{\hat{K}})\Gamma$

$$\phi(\hat{K}) = \psi(\|\hat{K}\|^2)\hat{K}$$

$$\psi(0 \leq k \leq \nu) = 0 \quad , \quad \frac{d\psi}{dk}(\nu \leq k \leq \beta\nu) > 0 \quad , \quad \psi(\nu\beta) = +\infty$$

- Corrective term active only when $\|K - F_0\| > \nu$
- Guarantees that $K(t)$ is bounded around F_0: $\|K - F_0\| < \nu\beta$

- ⚠ $\beta > 1$ can be chosen arbitrarily based on practical considerations

- $\hat{D}$ defines the geometry of the set $\|\hat{K}\| = \text{Tr}(\hat{K}\hat{D}\hat{K}^T) \leq \nu$
- ν defines the dead-zone and barrier levels

- ⚠ Best to maximize the set $\|\hat{K}\| \leq \nu$, *i.e.* maximize ν and minimize Tr$\hat{D}$

Feedthrough gain for robust passivity

$$\dot{x} = Ax + Bu \ , \ \ y = Cx \quad \text{with SAC} \quad u = Ky \ , \ \ \dot{K} = -Gyy^T\Gamma$$

- Closed-loop stability is guaranteed if

$$\exists F \ : \ \dot{x} = (A + BFC)x + Bw, \ \ z = GCx \ \ \text{strictly passive}$$

or equivalenty if

$$\exists F, P \ : \ (A + BFC)^T P + P(A + BFC) < 0 \ , \ \ PB = C^T G^T$$

⚠ Need for conditions without equality constraints

$\Rightarrow$ need for a feedthrough gain ($z = GCx + Dw$)

❷ Robustness to noise on measurements & to parametric uncertainty

Feedthrough gain for robust passivity

- Passivity conditions without equality constraints

$$\dot{x} = (A + BFC)x + Bw, \quad z = GCx + Dw \ \text{ strictly passive}$$

if and only if

$$\exists P \ : \ \begin{bmatrix} (A+BFC)^T P + P(A+BFC) & PB - C^T G^T \\ B^T P - GC & -D - D^T \end{bmatrix} < 0$$

- Feedthrough gain D always exists if system is SOF stabilizable
- If D is small, then conditions are close to original ones

- ⚠ Choice of $F = -kG$ with $k \gg 1$ no more valid
- ⚠ Gains should be bounded

- Gains are bounded thanks to corrective term ϕ

Main result - part 1

- Let F_0 be a stabilizing SOF for $\dot{x} = Ax + Bu \ , \ y = Cx$
- There exists $(P > 0, G, \hat{D})$ solution to

$$\begin{bmatrix} (A+BF_0C)^T P + P(A+BF_0C) & PB - C^T G^T \\ B^T P - GC & -\hat{D} \end{bmatrix} < 0$$

- minimize Tr$\hat{D}$ and choose
  - G for the adaptation gain $\dot{K} = -Gyy^T\Gamma - \phi(\underbrace{K - F_0}_{\hat{K}})\Gamma$
  - $\|\hat{K}\|^2 = \text{Tr}(\hat{K}\hat{D}\hat{K}^T)$ for the norm in the corrective term ϕ
  - F_0 as the center of the set around which the adaptation is performed

❷ Robustness to noise on measurements & to parametric uncertainty

Main result - part 2

- $(F_0, G, \hat{D})$ being chosen, there exist $(Q > 0, R, F, T, \nu)$ solution to

$$\begin{bmatrix} R & QB - C^T G^T \\ B^T Q - GC & \hat{D} \end{bmatrix} \geq 0$$

$$\begin{bmatrix} T & (F - F_0)^T \\ (F - F_0) & \hat{D}^{-1} \end{bmatrix} \geq 0 \;, \quad \mathrm{Tr} T \leq \nu$$

$$(A + BF_0C)^T Q + Q(A + BF_0C) + \nu\beta C^T C + R + C^T(G^T(F - F_0) + (F - F_0)^T G)C < 0$$

- maximize ν and take it for the levels in the corrective term ϕ

- SAC defined by $(G, F_0, \hat{D}, \nu)$ stabilizes the system. Proof with

$$V(x, K) = x^T Q x + \mathrm{Tr}((K - F)\Gamma^{-1}(K - F)^T).$$

Characteristic of the results

- ‘Almost passive’ conditions extended to ‘almost stable’

  SAC can be applied to all SOF stabilizable systems

- Stability is proved for SAC with the corrective barrier function

  Moreover, $K(t)$ is strictly bounded, even w.r.t. perturbations and noise

- The gain $K(t)$ remains ‘close’ to initial SOF guess F_0

  Interesting feature for practitioners: keep close to a ‘safe’ situation

  Benefit of adaptation expected to be improved if domain is large

[Submitted to IFAC World Congress 2011, Milano]

Guaranteed robustness

- Results formulated as LMIs:

  Can be extended to uncertain models $A(\Delta), B(\Delta), C(\Delta)$

- Procedure for robust SAC design

  $1\bullet$ Choose an SOF F_0 (stabilizes nominal system $A(0), B(0), C(0)$)

  $2\bullet$ Solve first LMI problem (robust version) to get G, $\hat{D}$

  $3\bullet$ Solve second LMI problem (robust version) to get ν

- Stability is proved with a parameter-dependent Lypunov function

$$V(x,K) = x^T Q(\Delta) x + \mathrm{Tr}((K - F(\Delta))\Gamma^{-1}(K - F(\Delta))^T).$$

- SAC and parameter-dependent $u = F(\Delta)y$ stablilize the system
- SAC does it without measure/estimation of Δ

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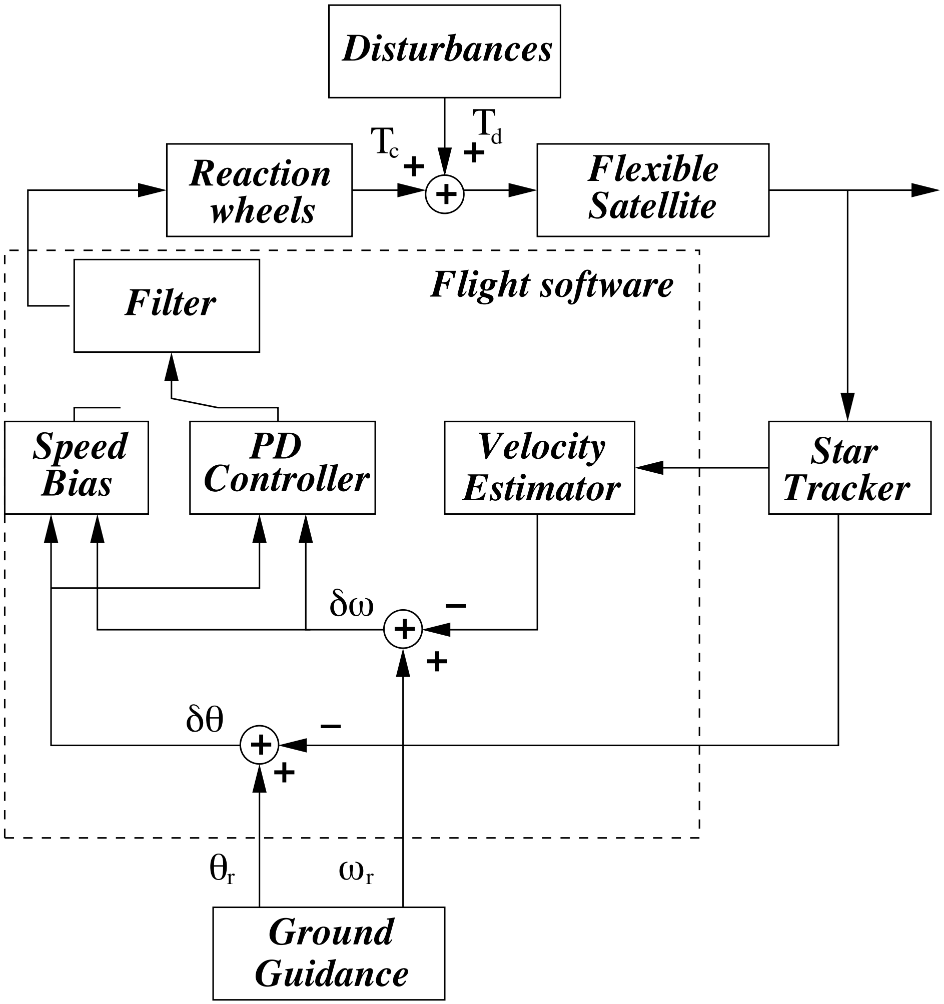
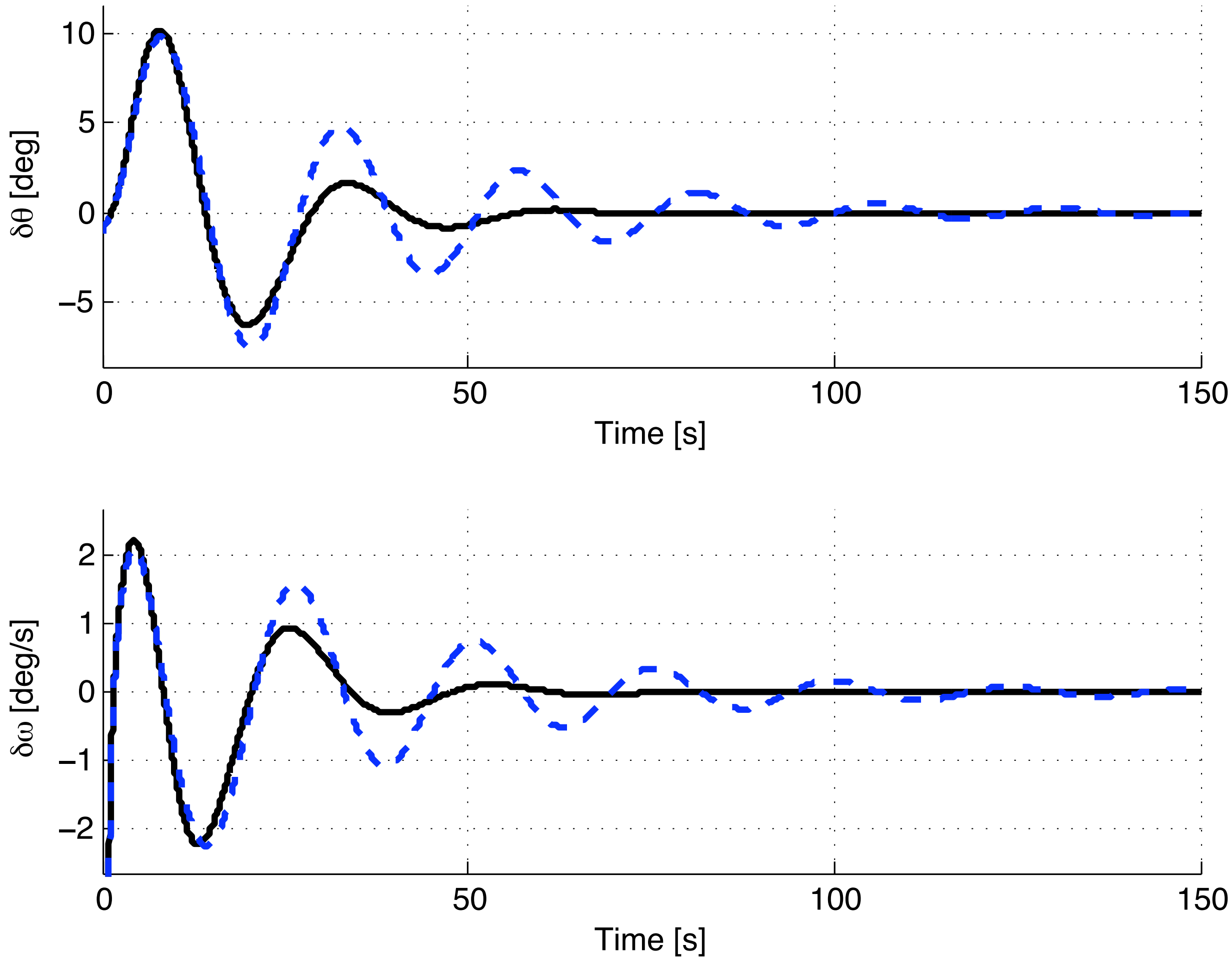
Demeter satellite

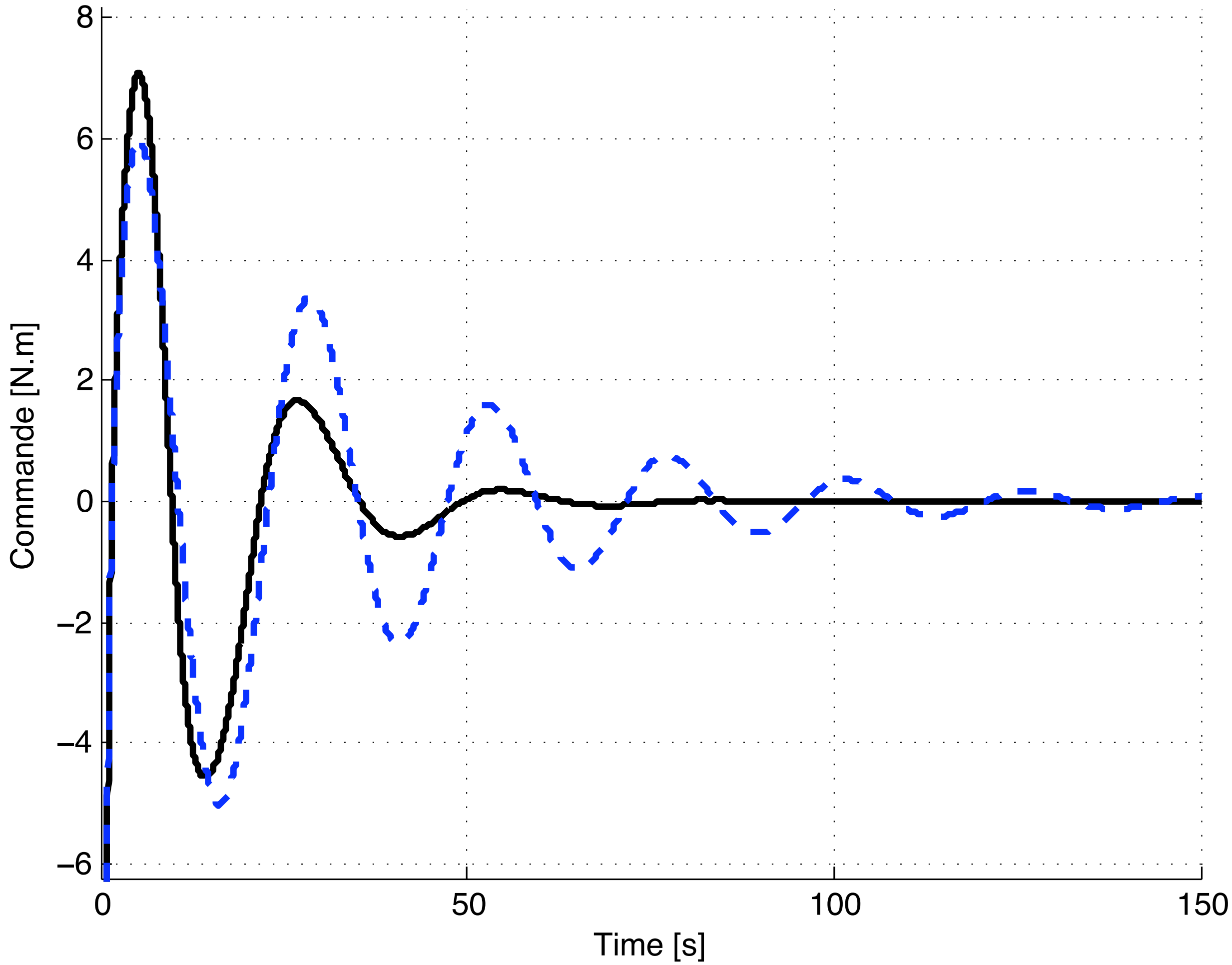


- Given stabilizing PD gains (F_0) replaced by adaptive gains
- LMIs solved on LTI model design of G and corrective term

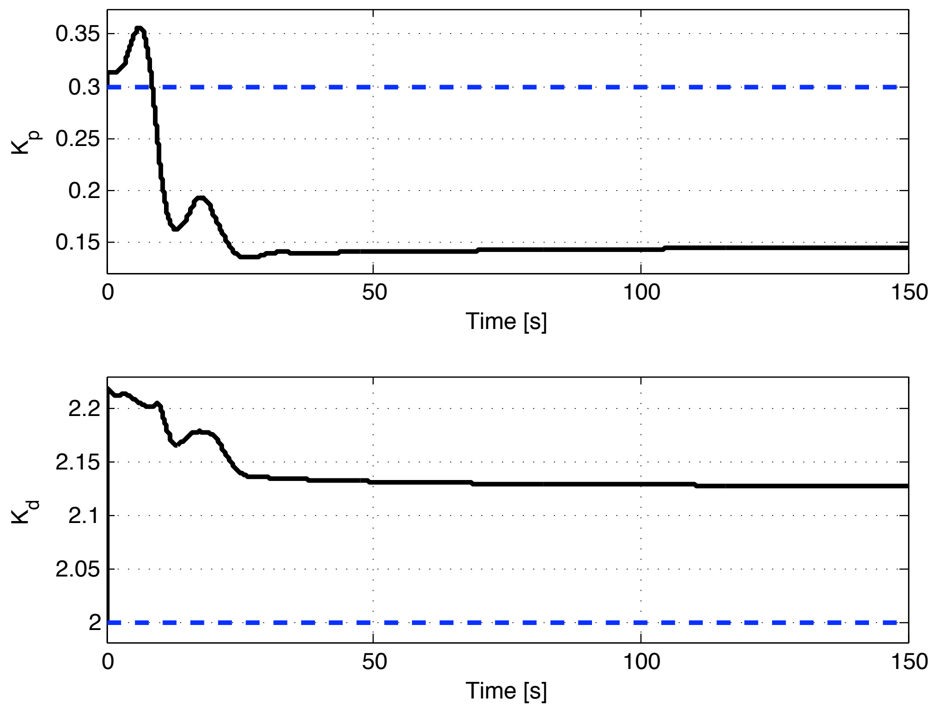
- Outputs of closed-loop system with F_0 (dotted) and SAC

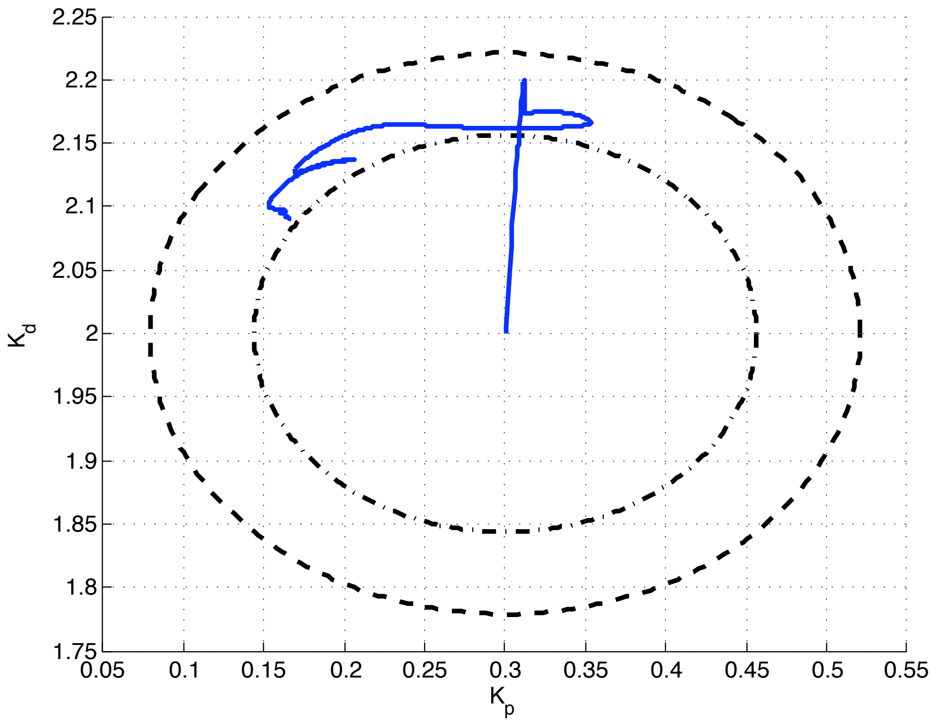


- Input of closed-loop system with F_0 (dotted) and SAC



- Control gains of SAC





From LTI to non-linear systems

- Parametric uncertain LTI systems $\simeq$ slowly TV systems
- ⚠ Will properties be lost for non-linear or rapidly time-varying systems ?
- LMI-based results for:
  - L_2 norm minimization: robustness to norm bounded non-linearities
  - LTV systems with bounded rates of variations (classical LPV hyp)

L_2-gain performance

- Lur'e type modeling of a close-to-linear non-linear system

$$\begin{aligned} \dot{x}(t) &= A(\Delta)x(t) + B_\Phi(\Delta)w_\Phi(t) + B(\Delta)u(t) \\ z_\Phi(t) &= C_\Phi(\Delta)x(t) + D_\Phi(\Delta)w_\Phi(t) \\ y(t) &= C(\Delta)x(t) \end{aligned} \quad , \quad \begin{aligned} w_\Phi(t) &= [\Phi z_\Phi](t) \\ \|w_\Phi\|_2 &\leq \gamma \|z_\Phi\|_2 \end{aligned}$$

- Small gain theorem: guarantee input/output performance

$$\begin{aligned} \dot{x}(t) &= A(\Delta)x(t) + B_\Phi(\Delta)w_\Phi(t) + B(\Delta)u(t) \\ z_\Phi(t) &= C_\Phi(\Delta)x(t) + D_\Phi(\Delta)w_\Phi(t) \\ y(t) &= C(\Delta)x(t) \end{aligned} \quad , \quad \begin{aligned} \|z_\Phi\|_2 &\leq \tfrac{1}{\gamma} \|w_\Phi\|_2 \\ \|\Sigma(\Delta)\|_2 &\leq \tfrac{1}{\gamma} \end{aligned}$$

L_2-gain performance

- LMI results that give a PD-SOF $F(\Delta)$ used to prove L_2 performance of SAC:

$$\|\Sigma(\Delta) \star K(t)\|_2 \leq \|\Sigma(\Delta) \star F(\Delta)\|_2$$

- Guaranteed L_2 performance of SAC
- Not worse than the PD-SOF
- Result explained by the fact that SAC is conceived to minimize square of error
- No similar result expected for other criteria (convergence time, etc.)

["Robust adaptive L2-gain control of polytopic MIMO LTI systems - LMI results", D. Peaucelle, A.L. Fradkov, System and Control Letters, Volume 57, Issue 11, November 2008, Pages 881-887]

UAV Example

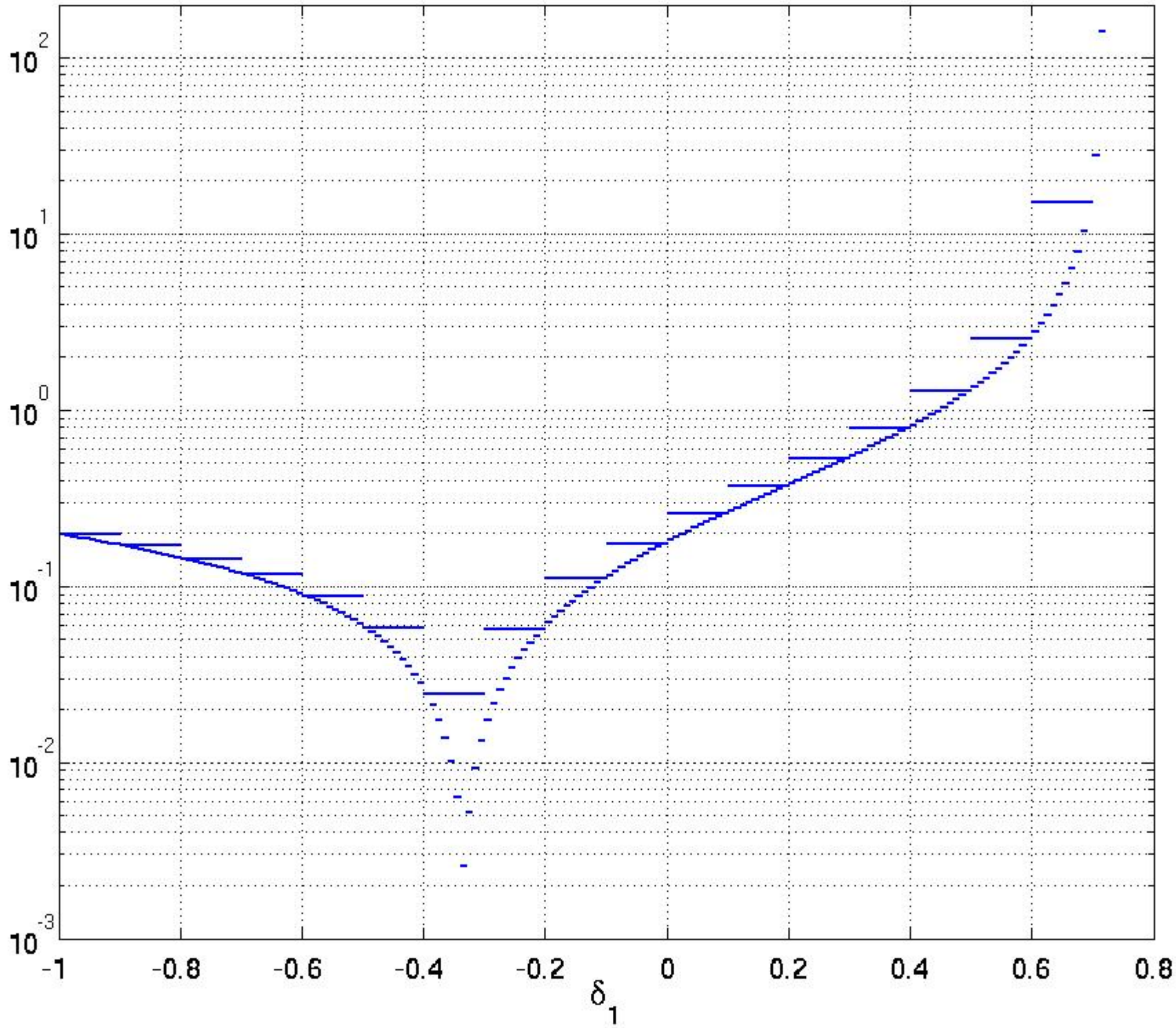
4 states, 2 scalar uncertainties, $\delta_2 \in [\,0\ \ 2.5\,]$

Tests on large intervals of δ_1

δ_1	min γ	δ_1	min γ	δ_1	min γ
$[\,-1\ \ 0\,]$	0.2	$[\,0.7\ \ 0.72\,]$	141	$[\,0.72\ \ 0.722\,]$	1001
$[\,-1\ \ 0.7\,]$	24	$[\,0.7\ \ 0.73\,]$	infeas.	0.723	infeas.
$[\,-1\ \ 0.72\,]$	infeas.				

UAV Example

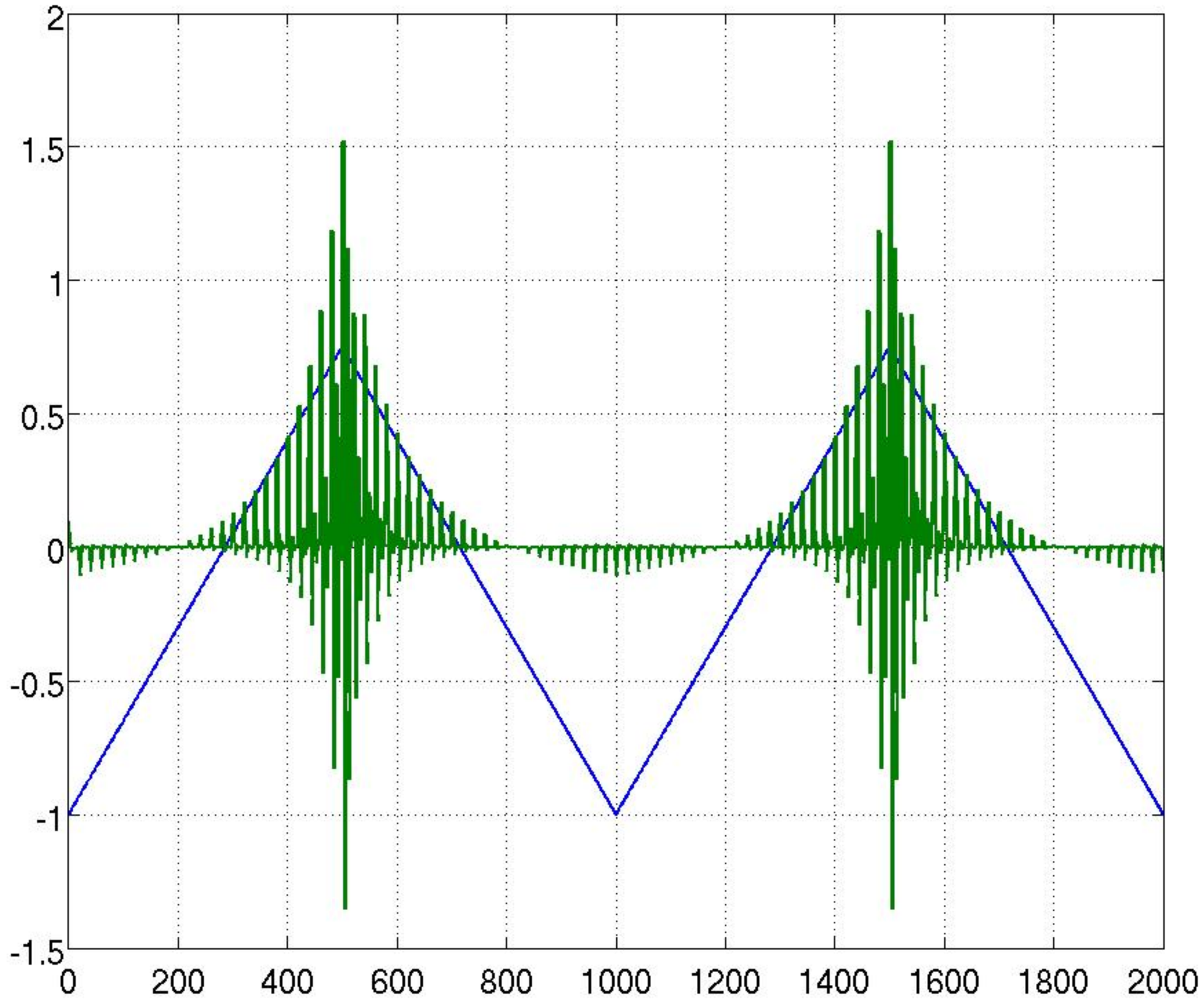
Tests on small intervals of δ_1



❸ Examples and some other features of SAC

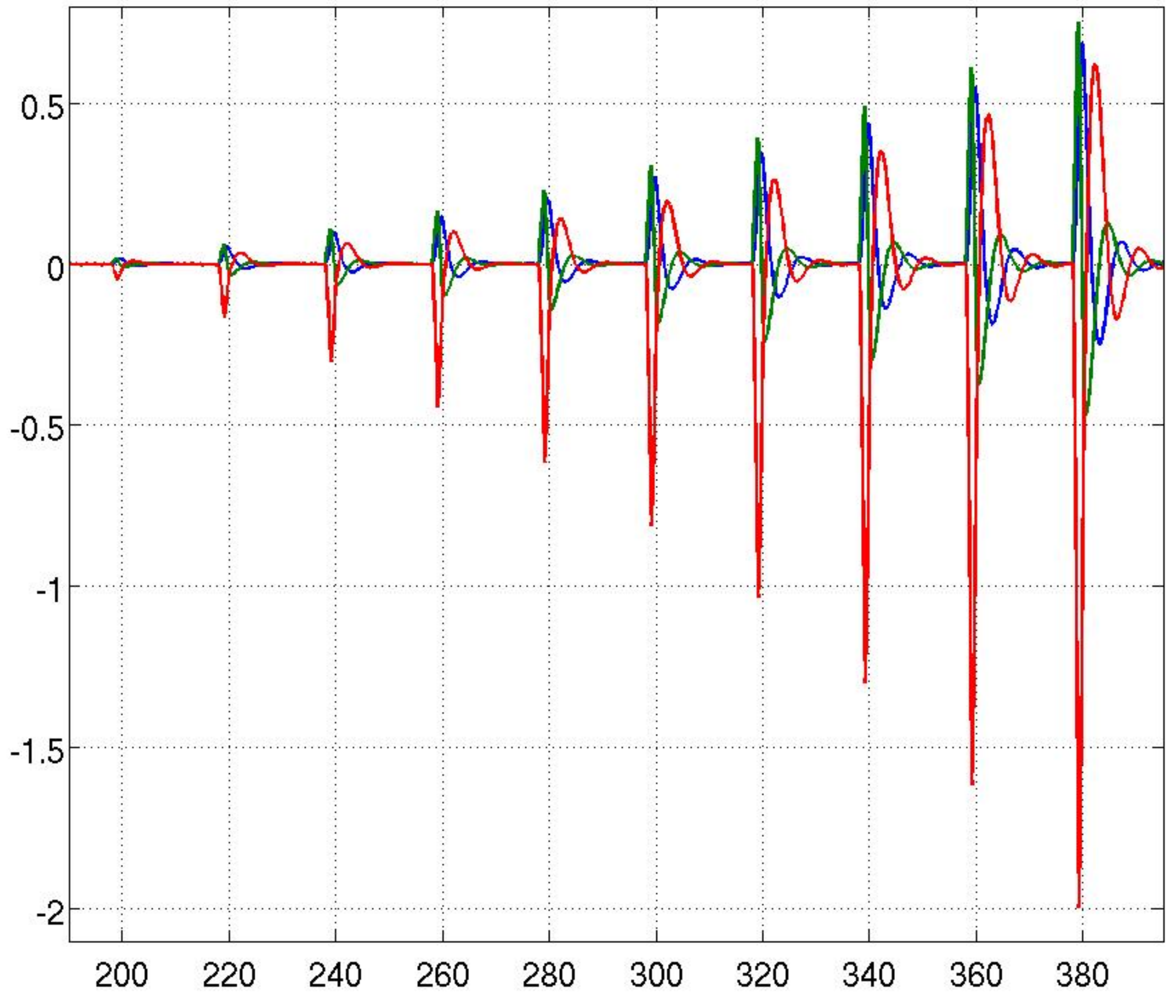
SAC simulations with impulse disturbances w_L (every 20s)

and slowly varying δ_1 (beyond proved stable values).



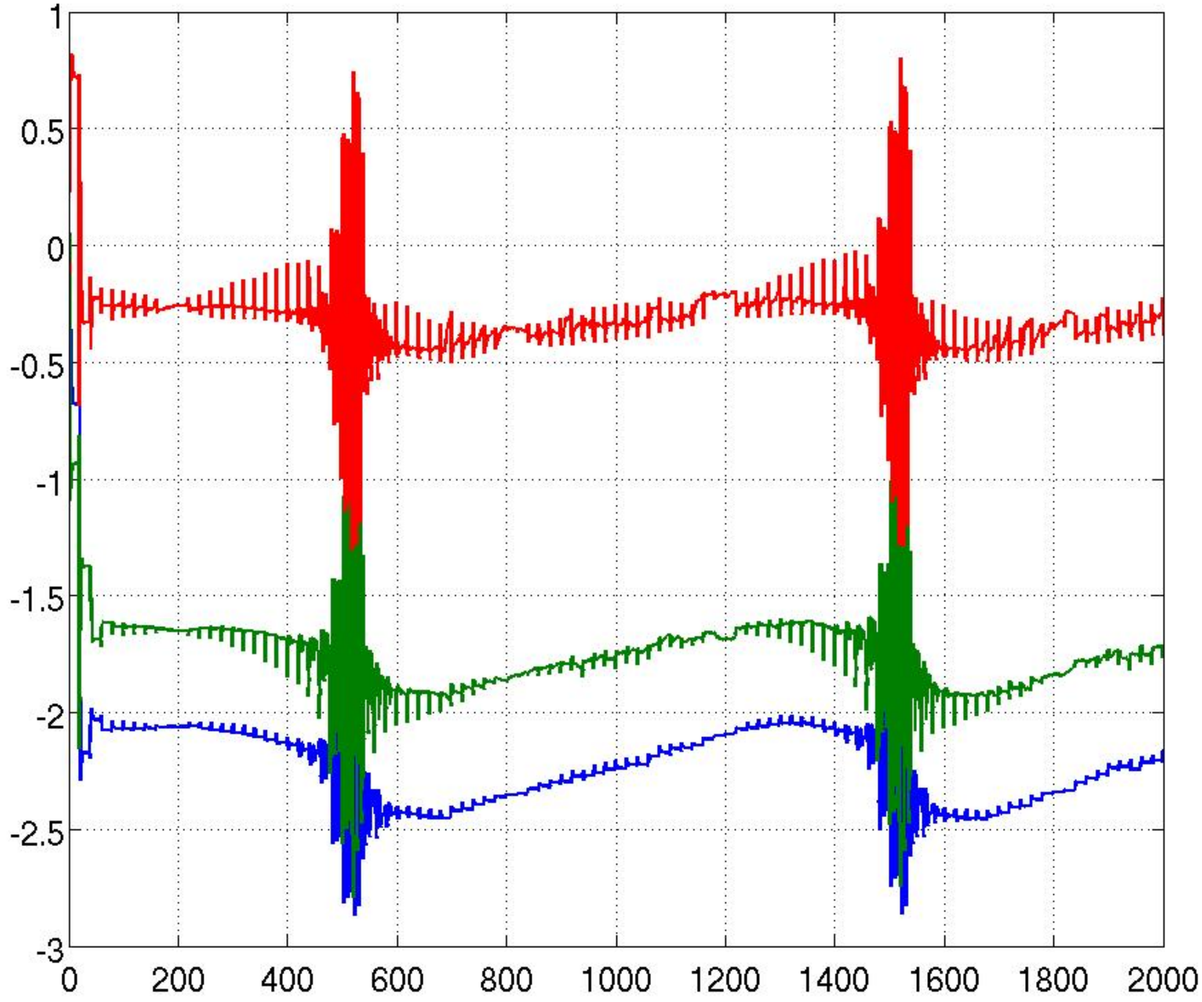
UAV Example

Zoom on the output responses.



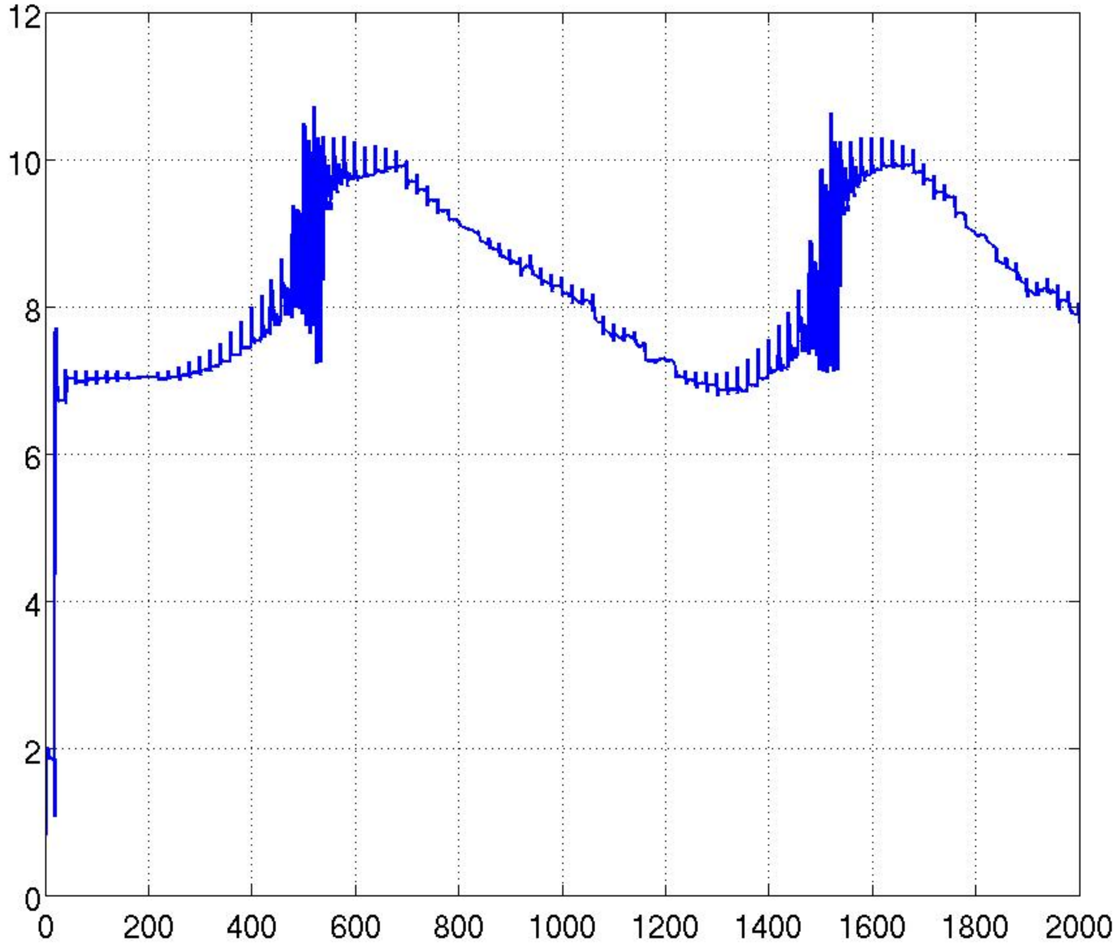
UAV Example

Time histories of the SAC gains



UAV Example

$\nu = 10$, $\beta = 1.2$: the gains are bounded Tr$(K^T K) \leq \nu\beta$.



❹ Robust stability in case of time varying uncertainties

Unceratin time-varying linear system

$$\dot{x}(t) = A(\Delta(t))x(t) + B(\Delta(t))u(t) \quad , \quad y = C(\Delta(t))x(t)$$

Stability proof based on the Lyapunov function $V(x, K, \Delta) =$

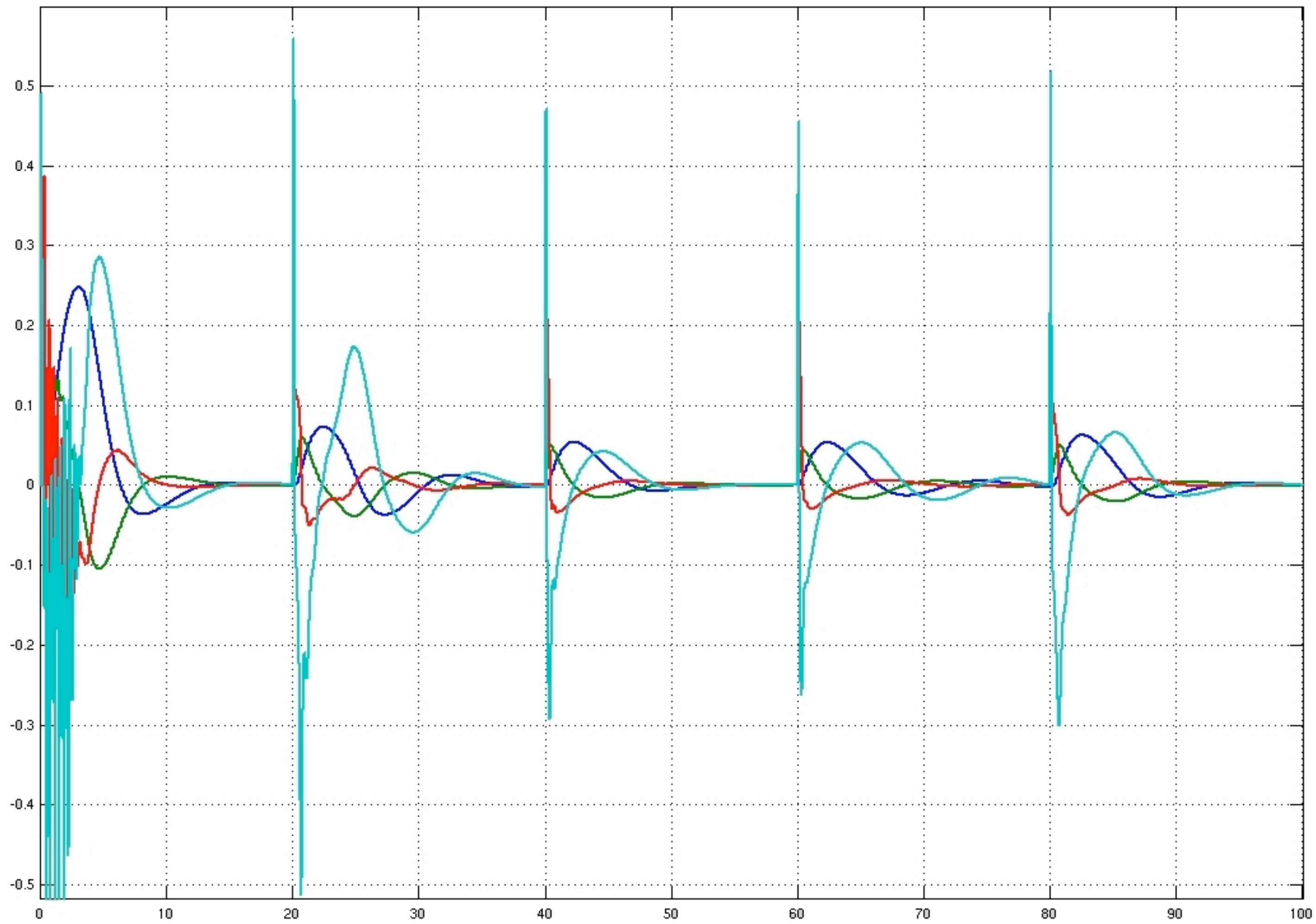
$$x^T(t)P(\Delta(t))x(t) + \text{Tr}(K(t) - F(\Delta(t))\Gamma^{-1}(K(t) - F(\Delta(t)))^T$$

- If $\dot{\Delta}$ is unbounded, then $\dot{V}(x, K, \Delta)$ exists only if:
  $P(\Delta) = P$, $F(\Delta) = F$, are constant
  i.e. the robust stabilisation is solved with constant SOF F.
- If $\dot{\Delta}$ is bounded, then [Auto.R.Ctr'09], LMI conditions for
  $\dot{V}(x, K, \Delta) < 0$ whatever x s.t. $x^T Q x \geq 1$,
  i.e. Lasalle's principle $x^T Q x \leq 1$ attractive set.
- Attractive domain can be made arbitrarily small if $\dot{\Delta} \to 0$ or $\Gamma \to \infty$

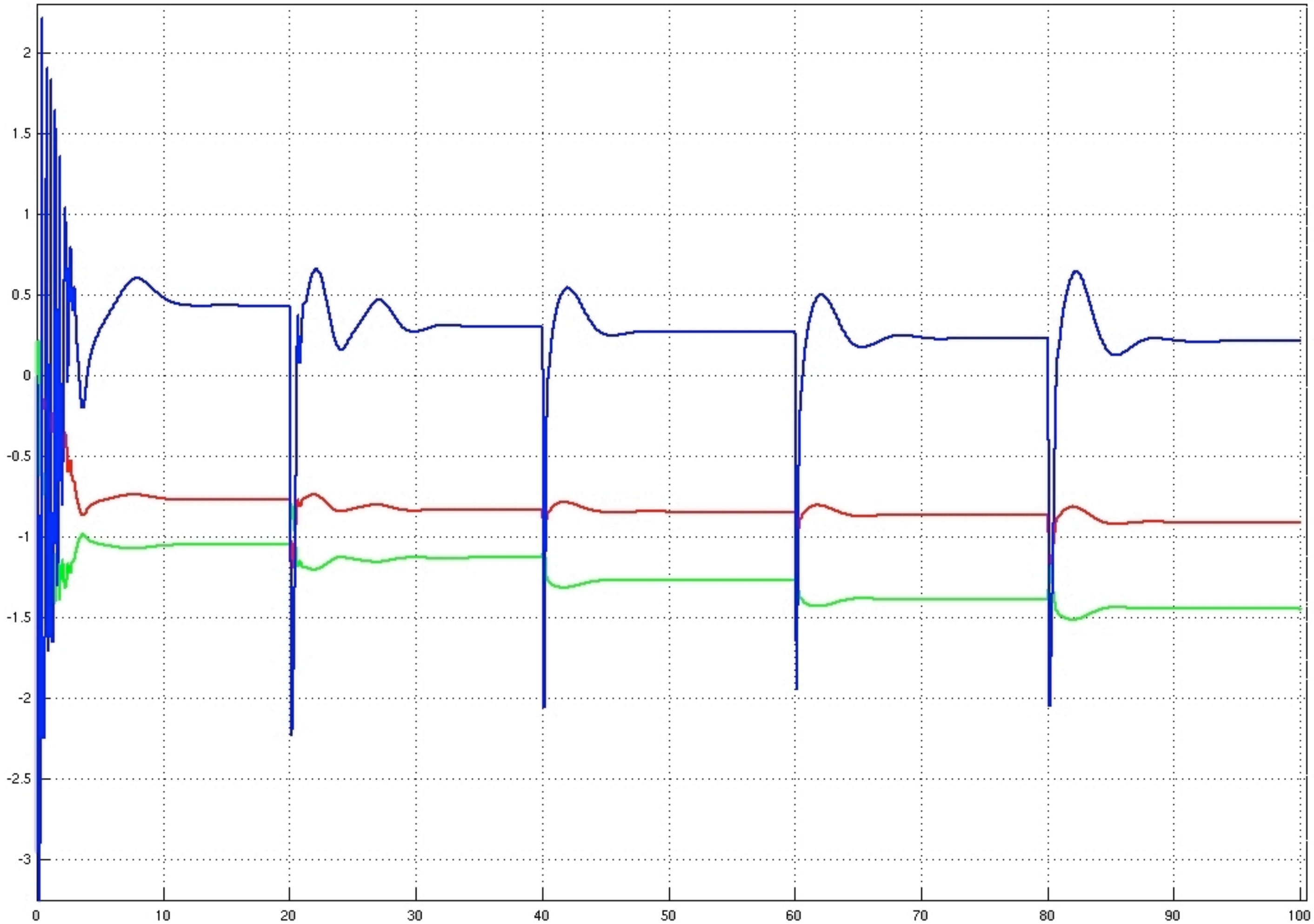
$$u(t) = K(t)y(t) + w(t) \quad , \quad \dot{K}(t) = -Gy(t)y^T(t)\Gamma - \phi(K(t))$$

Example State of the UAV for input impulses every 20s and

$$\delta_1(t) = 0.75\sin(0.125t + 3\pi/2) + 0.1\sin(49t + 3\pi/2) - 0.15 \leq 0.7$$



Example Gains of SAC:



Conclusions

- SAC revisited in LMI-based Robust Control framework

- Guaranteed robustness
- Form 'almost passive' to 'almost stable' systems
- Bounded control gains in given regions

- SAC is intended for non-linear systems

- Implementation not trivial: choosing Γ, ψ...
- Need to validate on real applications
- Can other features such as rapidity, damping, noise rejection performance etc. be treated?