S-переменные для получения результатов в виде матричных неравенств

и несколько результатов для робастного анализа дускретных систем

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Slack variables for deriving LMI results

and some examples of such results for robust analysis of discrete-time systems

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Since year 2000 many results in the LMI-framework tend to introduce some additional (slack) variables that are apparently unnecessary from the Lyapunov theory. Such variables are related to Finsler lemma that can also be seen as a variant of the S-procedure. The methodology producing these variables is sometimes called the descriptor form approach and results sometimes designated as dilated, or extended, LMIs. In this talk we shall explain the rationale of all these appellations. We shall show that the slack variables are useful for robust analysis and are not if all system parameters are known. Issues of numerical complexity induced by the slack variables will also be discussed. Finally, if we have time, some new results for robust analysis of switching discrete-time systems will be exposed.

- Positive definite matrices $M \in \mathbb{R}^{n \times n}$

$$\begin{aligned} M = M^T \succ 0 \\ \Leftrightarrow \quad \forall x \neq 0 \, , \; x^T M x > 0 \\ \Leftrightarrow \quad \lambda(M) > 0 \end{aligned}$$

- $\mathbb{S}^n_+ = \{ M \in \mathbb{R}^{n \times n} \;:\; M = M^T \succ 0 \}$ is an open convex cone

■ Linear matrix inequality constraints [BGFB94, NN94, EGN00]

▲ Representation with scalar decision variables

$$M(y) = M_0 + \sum y_i M_i \succ 0 \ , \ \ y_i \in \mathbb{R}$$

▲ Representation with matrix decision variables

$$M(Y_s, Y_f) = M_0 + (\sum N_{si}^T Y_{si} N_{si}) + (\sum N_{1j}^T Y_{fj} N_{2j} + N_{2j}^T {Y_{fj}}^T N_{1j})$$
$$Y_{si} = {Y_{si}}^T \in \mathbb{R}^{n_{si} \times n_{si}} \ , \ \ Y_{fj} \in \mathbb{R}^{m_{fj} \times p_{fj}}$$

- About LMIs

- Convex constraints
- Exist efficient solvers (Semi-Definite Programming) for (polynomial-time) optimization
- Recommended (free) tool in Matlab : YALMIP users.isy.liu.se/johanl/yalmip
- A nice lecture about LMIs homepages.laas.fr/henrion/courses/lmi13/

- Any “LMI representable" problem is considered as “solved"
- Numerical burden grow very fast with size of problem: $O(n^6)$

- Example: Global optimization over polynomials is (almost) “solved"
- See results on SOS and the moment problem

[Las01, Las02, Las06, HL03], [Par03, PPSP04], [SH06]

- A.M. Lyapunov - “grand father of LMIs"

- Asymptotic stability of a linear system proved with existence of V s.t.

$$V = x^T P x > 0\,,\ \ {x_{k+1}}^T P x_{k+1} - {x_k}^T P x_k < 0\,,\ \forall x_{k+1} \neq 0\ :\ \ x_{k+1} = A x_k$$

$$\Updownarrow$$

$$x^T P x > 0\ ,\ \ x^T(A^T P A - P)x < 0\ ,\ \forall x \neq 0$$

$$\Updownarrow$$

$$P \succ 0\ ,\ \ A^T P A - P \prec 0$$

- V.A. Yakubovich - “father of LMIs"

- S-procedure

$$x^T Q_0 x < 0 \,,\ \forall x \neq 0 \ : \ x^T Q_1 x \leq 0 \quad \Leftrightarrow \quad \exists \tau > 0 \ : \ Q_0 \prec \tau Q_1$$

- (τ denoted s in first publication using this technique [E.N. Rozenvasser 1963])

- KYP lemma

$$\begin{pmatrix} x \\ u \end{pmatrix}^* M \begin{pmatrix} x \\ u \end{pmatrix} < 0 \,,\ \forall u \neq 0 \,,\ \forall \omega \in \mathbb{R} \ : \ (e^{\jmath\omega} I - A) x = B u$$

$$\Leftrightarrow \quad \exists P = P^T \ : \ M \prec \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & -P \end{bmatrix} \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}$$

- P. Finsler
- Finsler's lemma

$$y^T M y < 0 \;,\; \forall y \neq 0 \;:\; By = 0$$
$$\Leftrightarrow B^{\perp T} M B^{\perp} \prec 0 \;:\; BB^{\perp} = 0 \;,\; \text{rank}(B) = \dim(\ker(B))$$
$$\Leftrightarrow \exists \tau \;:\; M \prec \tau B^T B$$
$$\Leftrightarrow \exists F \;:\; M \prec FB + B^T F$$

- Example: S-procedure

$$y^T M y < 0 \;,\; \forall y \neq 0 \;:\; By = 0$$
$$\Leftrightarrow y^T M y < 0 \;,\; \forall y \neq 0 \;:\; y^T B^T B y \leq 0$$
$$\Leftrightarrow \exists \tau \;:\; M \prec \tau B^T B$$

- P. Finsler

$$y^T M y < 0 \ , \ \forall y \neq 0 \ : \ By = 0$$
$$\Leftrightarrow B^{\perp T} M B^{\perp} \prec 0 \ : \ BB^{\perp} = 0 \ , \ \text{rank}(B) = \dim(\ker(B))$$
$$\Leftrightarrow \exists \tau \ : \ M \prec \tau B^T B$$
$$\Leftrightarrow \exists F \ : \ M \prec FB + B^T F$$

- Example: The Lyapunov result $y = \begin{pmatrix} x_{k+1}{}^T & x_k{}^T \end{pmatrix}^T$

$$V_{k+1} - V_k = y^T \begin{bmatrix} P & 0 \\ 0 & -P \end{bmatrix} y < 0 \ : \ \forall y \neq 0 \ : \ \begin{bmatrix} I & -A \end{bmatrix} y = 0$$
$$\Leftrightarrow \ A^T P A - P = B^{\perp T} \begin{bmatrix} P & 0 \\ 0 & -P \end{bmatrix} B^{\perp} \prec 0 \ : \ B^{\perp} = \begin{bmatrix} A \\ I \end{bmatrix}$$

- Notice the descriptor-like representation of the model: $\begin{bmatrix} I & -A \end{bmatrix} y = 0.$

- Finsler's lemma

$$y^T M y < 0 \ , \ \forall y \neq 0 \ : \ By = 0$$
$$\Leftrightarrow B^{\perp T} M B^{\perp} \prec 0 \ : \ BB^{\perp} = 0 \ , \ \text{rank}(B) = \text{dim}(\text{ker}(B))$$
$$\Leftrightarrow \exists \tau \ : \ M \prec \tau B^T B$$
$$\Leftrightarrow \exists F \ : \ M \prec FB + B^T F$$

- What if introducing F ? Approach known as:

  "Finsler based" - "Slack variables" - "dilated LMI" - "extended LMI" - "descriptor"

- For robust analysis results (LTI, LTV, TDS, Periodic...):

  [PABB00, DOS01, OG05, EPAH05, PDSV09, EPA09, PS09, TPAE13]...

- For robust state-feedback and filter design:

  [OBG99, OGH99, AP00, GdOB02, FS02, EH02, EH04, PG05, EPA11]...

- For output feedback and anti-windup design:

  [APT00, AHP02, PA01, ACP06, GKB07, AGPP10, TGGdSJQ11]...

- Finsler's lemma

$$y^T M y < 0 \ , \ \ \forall y \neq 0 \ \ : \ \ By = 0$$
$$\Leftrightarrow B^{\perp T} M B^{\perp} \prec 0 \ \ : \ \ BB^{\perp} = 0 \ , \ \ \text{rank}(B) = \text{dim}(\text{ker}(B))$$
$$\Leftrightarrow \exists \tau \ \ : \ \ M \prec \tau B^T B$$
$$\Leftrightarrow \exists F \ \ : \ \ M \prec FB + B^T F$$

- What if introducing F ?
- Example: stability of $x_{k+1} = Ax_k$:

$$\exists P \succ 0, F \ \ : \ \ \begin{bmatrix} P & 0 \\ 0 & -P \end{bmatrix} \prec F \begin{bmatrix} I & -A \end{bmatrix} + \begin{bmatrix} I \\ -A^T \end{bmatrix} F^T$$

- $2n \times 2n$ LMIs with $\frac{n(n+1)}{2} + 2n^2$ variables !! ($n \times n$ and $\frac{n(n+1)}{2}$ in original problem)
- Why using such a numerically expensive condition ?

- Discrete-time system with uncertainties $a \in [1\,,\,2]$, $b \in [-0.5\,,\,\beta]$.

$$a y_{k+2} + b^2 y_{k+1} + a b y_k = 0.$$

- By hand: Robust stability is guaranteed for $\beta < 1$.
- Can we build an LMI problem that guarantees robust stability for fixed β?

- Discrete-time system with uncertainties $a \in [1\,,\,2]$, $b \in [-0.5\,,\,\beta]$.

$$a y_{k+2} + b^2 y_{k+1} + a b y_k = 0.$$

- State-space representation $x_{k+1} = \begin{bmatrix} -b^2/a & -b \\ 1 & 0 \end{bmatrix} x_k = A(a,b) x_k$.

- Interval arithmetics: $b^2/a \in [0\,,\,\beta^2]$ (assuming $\beta \geq 0.5$)

$$\left\{ A(a,b)\,, \quad \begin{array}{l} a \in [1\,,\,2] \\ b \in [-0.5\,,\,\beta] \end{array} \right\} \subset CO \left\{ \begin{array}{cc} \begin{bmatrix} 0 & 0.5 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & -\beta \\ 1 & 0 \end{bmatrix} \\ \begin{bmatrix} -\beta^2 & 0.5 \\ 1 & 0 \end{bmatrix} & \begin{bmatrix} -\beta^2 & -\beta \\ 1 & 0 \end{bmatrix} \end{array} \right\}$$

- Enters the general formulation of polytopic systems $x_{k+1} = A(\theta) x_k$

$$A(\theta) \in CO\left\{ A^{[1]}, A^{[2]}, \dots A^{[\bar{v}]} \right\}$$

- I.e. $A(\theta) = \sum_{v=1}^{\bar{v}} \theta_v A^{[v]}$ where $\theta \in \Xi_{\bar{v}} = \{\theta_v \geq 0\ ,\ \ \sum_{v=1}^{\bar{v}} \theta_v = 1\}$.

- Robust stability condition: existence of $P(\theta)$ s.t.

$$P(\theta) \succ 0 \ , \ \ A(\theta)^T P(\theta) A(\theta) - P(\theta) \prec 0 \ , \ \ \forall \theta \in \Xi_{\bar{v}}$$

- Conservative assumption: $P(\theta) = P$ unique Lyapunov Matrix $\forall \theta$

$$P \succ 0 \ , \ \ A(\theta)^T P A(\theta) - P \prec 0 \ , \ \ \forall A(\theta) \in CO\left\{A^{[1]}, A^{[2]}, \dots A^{[\bar{v}]}\right\}$$

- Convexity of $\mathbb{S}_+$ allows to conclude that

$$\Leftrightarrow \ \ P \succ 0 \ , \ \ A^{[v]T} P A^{[v]} - P \prec 0 \ , \ \ \forall v = 1 \dots \bar{v}$$

- For the example, LMIs feasible up to $\beta = 0.7057$ (far from the actual upper bound)
- Actually, vertex $\begin{bmatrix} -\beta^2 & 0.5 \\ 1 & 0 \end{bmatrix}$ is unstable as soon as $\beta = 0.7071$
- Need for better representations of the model with uncertainties

$$a y_{k+2} + b^2 y_{k+1} + ab y_k = 0.$$

- Equivalent model, affine in the uncertainties

$$\begin{bmatrix} a & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} x_{k+1} + \begin{bmatrix} b \\ 0 \\ 1 \end{bmatrix} \pi_k = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ b & a \end{bmatrix} x_k.$$

- General descriptor model to be considered here:

$$E_x(\theta) x_{k+1} + E_\pi(\theta) \pi_k = F(\theta) x_k$$

- Assumption: $E(\theta) = \begin{bmatrix} E_x(\theta) & E_\pi(\theta) \end{bmatrix}$ square invertible $\forall \theta \in \Xi_{\bar{v}}$
- System is causal, without impulsive modes, π_k well defined for all $k \geq 0$.

- General descriptor model:

$$M(\theta)y_k = \left[\begin{array}{ccc} E_x(\theta) & E_\pi(\theta) & -F(\theta) \end{array}\right] \begin{pmatrix} x_{k+1} \\ \pi_k \\ x_k \end{pmatrix} = 0$$

- Robust stability if exists $V(k,\theta) = {x_k}^T P(\theta) x_k$ such that

$$\begin{array}{r} V(k+1,\theta) \\ -V(k,\theta) \end{array} = {y_k}^T \left[\begin{array}{ccc} P(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P(\theta) \end{array}\right] y_k < 0\,,\ \ \forall y_k \neq 0 \ :\ M(\theta)y_k = 0$$

- Finsler lemma: equivalent condition (should hold $\forall\theta \in \Xi_{\bar{v}}$).

$$\begin{array}{l} \exists P(\theta) \succ 0 \\ \exists F(\theta) \end{array} \ :\ \left[\begin{array}{ccc} P(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P(\theta) \end{array}\right] < F(\theta)M(\theta) + M^T(\theta)F^T(\theta)$$

- Finsler lemma based robust stability condition (should hold $\forall\theta \in \Xi_{\bar{v}}$).

$$\begin{array}{l} \exists P(\theta) \succ 0 \\ \exists F(\theta) \end{array} : \begin{bmatrix} P(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P(\theta) \end{bmatrix} < F(\theta)M(\theta) + M^T(\theta)F^T(\theta)$$

- Conservative assumption: $F(\theta) = F$ unique $\forall\theta$.

$$\begin{array}{l} \exists P(\theta) \succ 0 \\ \exists F \end{array} : \begin{bmatrix} P(\theta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P(\theta) \end{bmatrix} < FM(\theta) + M^T(\theta)F^T \ , \ \ \forall\theta \in \Xi_{\bar{v}}$$

- Convexity of $\mathbb{S}_+$ allows to conclude that

$$\Leftrightarrow \begin{array}{l} \exists P^{[v]} \succ 0 \\ \exists F \end{array} : \begin{bmatrix} P^{[v]} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P^{[v]} \end{bmatrix} < FM^{[v]} + M^{[v]T}F^T \ , \ \ \forall v = 1 \ldots \bar{v}$$

Slack-variables LMI result for stability analysis

$$\begin{array}{l}\exists P^{[v]} \succ 0 \\ \exists F\end{array} : \begin{bmatrix} P^{[v]} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P^{[v]} \end{bmatrix} < FM^{[v]} + M^{[v]T}F^T \,, \quad \forall v = 1 \ldots \bar{v}$$

- Stability proved with $P(\theta) = \sum_{v=1}^{\bar{v}} \theta_v P^{[v]}$ (consequence of the choice of F unique)

- LMIs of very large dimensions:
  - Number of variables: $\bar{v}\frac{n(n+1)}{2} + (n+p)(2n+p)$
  - Number of rows of LMIs: $\bar{v}(3n+p)$

- For the considered example
  - LMI feasible up to $\beta = 0.9805$
  - Number of variables =27 ; number of rows =28

■ Slack-variables LMI result for stability analysis

$$\begin{array}{l}\exists P^{[v]} \succ 0 \\ \exists F\end{array} \; : \; \left[\begin{array}{ccc} P^{[v]} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P^{[v]} \end{array}\right] < FM^{[v]} + M^{[v]T}F^T \;,\; \forall v = 1 \dots \bar{v}$$

- LMIs of very large dimensions:
  - Can the size be reduced when $P^{[v]} = P$ (“quadratic stability" case) ? YES
  - Can the size be reduced when some components $M_{ij}^{[v]} = M_{ij}$? YES
- Results are conservative ($F(\theta) = F$), can conservatism be reduced ? YES

■ Slack-variables LMI result for stability analysis

$$\begin{array}{l}\exists P^{[v]} \succ 0\\ \exists F\end{array} : \left[\begin{array}{ccc} P^{[v]} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P^{[v]} \end{array}\right] < F M^{[v]} + M^{[v]T} F^T , \ \forall v = 1 \dots \bar{v}$$

▲ Can the size be reduced when $P^{[v]} = P$ (“quadratic stability" case) ? YES

● Example for the case when $M(\theta) = \left[\begin{array}{cc} I & -A(\theta) \end{array}\right]$

▲ Alternative LMI (more conservative because $P^{[v]} = P$)

$$\exists P \succ 0 , \ A^{[v]T} P A^{[v]} - P \prec 0 , \ \forall v = 1 \dots \bar{v}$$

● This can be generalized to all parameter-independent columns of $M(\theta)$ associated to positive semi-definite diagonal elements in the left-hand side matrix.

▲ Not applicable for systems where $M(\theta) = \left[\begin{array}{cc} E(\theta) & -A \end{array}\right]$

■ Slack-variables LMI result for stability analysis

$$\begin{array}{l}\exists P^{[v]} \succ 0 \\ \exists F\end{array} : \begin{bmatrix} P^{[v]} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P^{[v]} \end{bmatrix} < FM^{[v]} + M^{[v]T}F^T , \ \forall v = 1 \dots \bar{v}$$

▲ Can the size be reduced when some rows $M_{i:}^{[v]} = M_{i:}$? YES

● Assume $M(\theta) = \begin{bmatrix} M_1(\theta) \\ M_2 \end{bmatrix}$, then equivalent LMI (no conservatism)

$$\begin{array}{l}\exists P^{[v]} \succ 0 \\ \exists \hat{F}\end{array} : M_2^{\perp T} \begin{bmatrix} P^{[v]} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -P^{[v]} \end{bmatrix} M_2^{\perp} < \hat{F} M_1^{[v]} M_2^{\perp} + M_2^{\perp T} M_1^{[v]T} \hat{F}^T$$

▲ Size of LMIs and number of decision variables reduced by rank(M_2)

▲ For considered example: Number of variables =20 ; number of rows =24

- Results are conservative ($F(\theta) = F$), can conservatism be reduced ? YES

- Purely mathematical approach [Sch05, SH06, Sch06, OP06, PS09]...

  Solve the parameter-dependent LMIs for polynomial choices of $P(\theta)$ and $F(\theta)$

- Alternative, “model augmentation" technique.

- Illustration on the example

$$a y_{k+3} + b^2 y_{k+2} + ab y_{k+1} = 0$$
$$a y_{k+2} + b^2 y_{k+1} + ab y_k = 0$$

- Augmented model has increased size descriptor modeling to which results apply
- LMIs are of augmented size: Number of variables =48 ; number of rows =32
- Conservatism is reduced: $\beta = 0.99519$
- Equivalent to searching for implicitly defined forms of $P(\theta)$ and $F(\theta)$

- “Slack-variables" framework
  - Extends Lyapunov, S-procedure type LMI results
  - Easy to manipulate, even for descriptor systems
  - Contributes to conservatism reduction
  - Numerical complexity is increased
  - ... but can be controlled
  - In particular: non need for slack variables if system without uncertainties
- Non discussed issues
  - Design of state-feedback, filter etc.
  - Continuous-time systems
  - Performances: H_∞, H_2 etc
  - Other than LTI systems: switching, time-delay, periodic etc.
- Springer monograph by Y. Ebihara & D. Peaucelle to be published in 2014

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