From passivity-based adaptive control to LMI tuned adaptive control

or how Alexander Fradkov convinced me that direct adaptive control can be robust

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α^{70}! FRADCON 2018

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Robust | Direct Adaptive

[Peaucelle 2000]

$$\mathcal{L}(P) \prec 0$$

$$\Downarrow$$

$\dot{x} = A(\Delta)x$ is robustly stable

[Fradkov 1974]

$$\left\{ \begin{array}{l} \dot{x} = Ax + Bu \\ y = Cx \end{array} \right\}$$ is Hyper Minimum Phase

$$\Downarrow$$

Closed-loop x-strictly passive with

$$u = Ky,\ \dot{K} = -yy^T\Gamma,\ \forall \Gamma \succ 0$$

LMIs for passivity

■ $\dot{x} = A_c x + Bu$, $y = Cx$ strictly passive iff

$$\exists P \succ 0 \quad : \quad A_c^T P + P A_c \prec -\epsilon I, \;\; PB = C^T$$

● F static output feecback ($A_c = A + BFC$), closed loop strictly passive **iff**

$$\exists \begin{array}{l} P \succ 0 \\ \epsilon > 0 \\ F \end{array} \quad : \quad \begin{array}{l} (A + BFC)^T P + P(A + BFC) \\ = A^T P + C^T F^T C + PA + C^T F C \prec -\epsilon I \end{array} \;, \;\; PB = C^T$$

▲ SOF design for strict passivity is an LMI problem

▲ $\dot{x} = Ax + Bu$, $y = Cx$ is Hyper Minimum Phase

iff it is "almost passive" (ie $\exists F$ s.t. closed-loop strictly passive)

LMIs for passivity based adaptive control

■ $\dot{x} = Ax + Bu$, $y = Cx$ HMP, then it is x-strictly passive with adaptive control :

$$u = Ky,\ \dot{K} = -yy^T\Gamma,\ \forall \Gamma \succ 0$$

● Proof with Lyapunov function $V(x, K) = x^T P x + 2\text{Tr}(K - F)^T \Gamma^{-1}(K - F)$

▲ LMI implies along trajectories with $u = Ky$

$$\frac{d}{dt}(x^T P x) + 2y^T(F - K)y < -\epsilon\|x\|^2$$

▲ and $\frac{d}{dt}(2\text{Tr}(K - F)^T\Gamma^{-1}(K - F)) = -2y^T(F - K)y$

▲ Hence $V(x, K) < -\epsilon\|x\|^2$, i.e. asymptotic convergence of x to 0

Robust & Direct Adaptive ?

$$\mathcal{L}(P, F) \prec 0$$

$$\Downarrow$$

$$\left\{ \begin{array}{l} \dot{x} = A(\Delta)x + B(\Delta)u \\ y = C(\Delta)x \end{array} \right\} \text{ is robustly HMP}$$

$$\Downarrow$$

Closed-loop robustly stable with $u = Ky$, $\dot{K} = -yy^T\Gamma$, $\forall \Gamma \succ 0$

▲ LMI-based results to prove robustness of adaptive control.

Robust & Direct Adaptive ?

Closed-loop robustly stable with $u = Fy$

$$\Uparrow$$

$$\mathcal{L}(P, F) \prec 0$$

$$\Downarrow$$

$$\left\{ \begin{array}{l} \dot{x} = A(\Delta)x + B(\Delta)u \\ y = C(\Delta)x \end{array} \right\} \text{ is robustly HMP}$$

$$\Downarrow$$

Closed-loop robustly stable with $u = Ky$, $\dot{K} = -yy^T\Gamma$, $\forall \Gamma \succ 0$

- Purpose of non-linear adaptive control if static output feedback (SOF) does the job ?
  - K is unbounded and shall diverge due to noise on measurements
    - Special case where robust SOF design is LMI !

Robust & Direct Adaptive ?

Closed-loop robustly stable with $u = Fy$

$$\Uparrow$$

$$\mathcal{B}(P, F, G) \prec 0$$

$$\Downarrow$$

$$\left\{ \begin{array}{l} \dot{x} = A(\Delta)x + B(\Delta)u \\ y = C(\Delta)x \\ z = Gy \end{array} \right\} \text{ is robustly HMP w.r.t. } (u, z)$$

$$\Downarrow$$

Closed-loop robustly stable with $u = Ky$, $\dot{K} = -zy^T\Gamma$, $\forall \Gamma \succ 0$

- ▼ BMI conditions (but with heuristic to solve it locally ▲)
- ▼ K is unbounded and shall diverge due to noise on measurements
- ▼ Purpose of non-linear adaptive control if SOF does the job ?

Robust & Direct Adaptive ?

Closed-loop robustly stable with $u = Fy_s$

$$\Uparrow$$

$$\mathcal{B}(P, F, G, D) \prec 0$$

$$\Downarrow$$

$$\left\{ \begin{array}{l} \dot{x} = A(\Delta)x + B(\Delta)u \\ y_s = C(\Delta)x + Du \\ z = Gy_s \end{array} \right\} \text{ is robustly HMP w.r.t. } (u, z)$$

$$\Downarrow$$

Closed-loop robustly stable with $u = Ky_s$, $\dot{K} = -zy_s^T\Gamma$, $\forall \Gamma \succ 0$

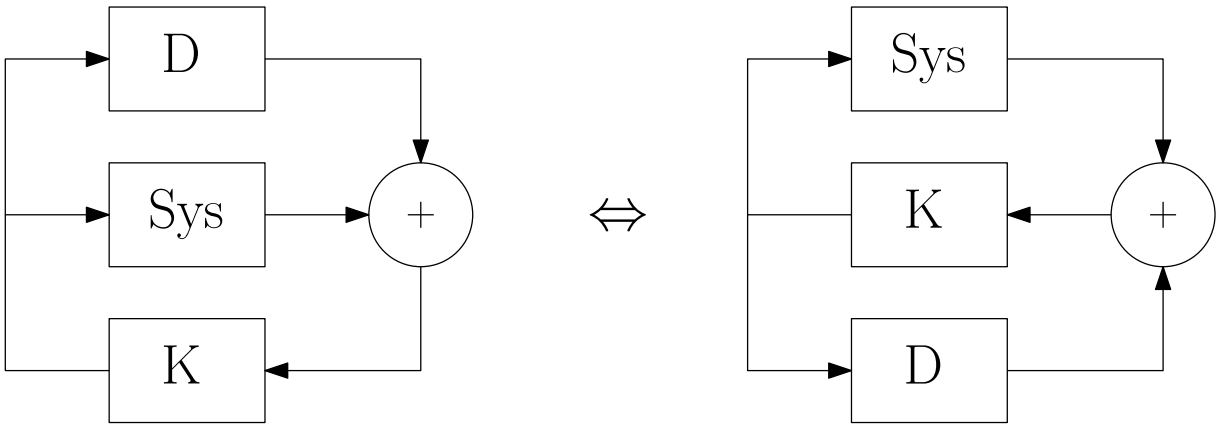
▼ BMI conditions (but with heuristic to solve it locally ▲)

▼ K is unbounded and shall diverge due to noise on measurements

▼ Purpose of non-linear adaptive control if SOF does the job ?

▼ Purpose of controlling y_s ?

Robust & Direct Adaptive ?



The shunt is equivalent to a feedback on the control gain

▲ Even thought K is unbounded, the actual gain seen by the plant $K \star D$ is bounded

■ Justifies the search for adaptive control with gains forced to be bounded

Robust & Direct Adaptive ?

Closed-loop robustly stable with $u = Fy$

$$\Uparrow$$

$$\mathcal{B}(P, F, G, D) \prec 0$$

$$\Downarrow$$

$$\left\{ \begin{array}{l} \dot{x} = A(\Delta)x + B(\Delta)u \\ y = C(\Delta)x \\ z = Gy + Du \end{array} \right\} \text{ is robustly HMP w.r.t. } (u, z)$$

$$\Downarrow$$

Closed-loop robustly stable with $u = Ky$, $\dot{K} = \mathrm{P}_{\mathcal{E}(D)}[-Gyy^T]\Gamma$, $\forall \Gamma \succ 0$

where $\mathrm{P}_{\mathcal{E}(D)}$ is a projection forcing K in a set with size proportional to D^{-1}

▼ Purpose of non-linear adaptive control if SOF does the job ?

▼ BMI conditions (but with heuristic to solve it locally ▲)

▲ Adaptive control with bounded gains

Robust & Direct Adaptive ?

Closed-loop robustly stable with $u = F(\Delta)y$

$$\Uparrow$$

$$\mathcal{B}^{[v]}(P^{[v]}, F^{[v]}, S, G, D, K_c) \prec 0$$

$$\Downarrow$$

Closed-loop robustly stable with $u = Ky$, $\dot{K} = \mathsf{P}_{\mathcal{E}(D,K_c)}[-Gyy^T]\Gamma$, $\forall \Gamma \succ 0$

where $\mathsf{P}_{\mathcal{E}(D,K_c)}$ is a projection forcing K in a set with size proportional to D^{-1}

▼ BMI conditions (but with heuristic to solve it locally ▲)

▲ There might not be any SOF that does the job (only a gain scheduled one)

▲ Adaptive control with bounded gains

Sketch of the proof

- Robustness is for linear systems with uncertainties

$$\dot{x} = A_\zeta x + B_\zeta u, \quad y = Cx$$

where affine polytopic uncertainty is assumed for simplicity

$$\begin{bmatrix} A_\zeta & B_\zeta \end{bmatrix} = \sum_{v=1}^{\bar{v}} \zeta_v \begin{bmatrix} A^{[v]} & B^{[v]} \end{bmatrix}, \quad \zeta_v \geq 0, \quad \sum_{v=1}^{\bar{v}} \zeta_v = 1.$$

Other types of uncertainties easily handled with appropriate LMI-based robustness tools

- Matrix inequalities $\mathcal{B}^{[v]}(P^{[v]}, F^{[v]}, S, G, D, K_c) \prec 0$

  assumed to hold for all vertices $v = 1 \ldots \bar{v}$

- Thanks to S-variable type result,

  inequalities hold true for all $\zeta \in \{\zeta_v \geq 0, \quad \sum_{v=1}^{\bar{v}} \zeta_v = 1\}$.

$$\mathcal{B}_\zeta(X_\zeta, F_\zeta, S, G, D, K_c) \prec 0$$

Sketch of the proof

$$\mathcal{B}_\zeta(P_\zeta, F_\zeta, S, G, D, K_c) \prec 0$$

Implies the following inequalities along closed-loop trajectories

$$2\dot{x}^T P_\zeta x + \epsilon x^T x + 2y^T(D - (K_c - K)^T(K_c - K) - G^T(K - F_\zeta))y \leq 0$$

$$(K_c - F_\zeta)^T(K_c - F_\zeta) \preceq D$$

Adaptive law $\dot{K} = \mathsf{P}_{\mathcal{E}(D,K_c)}[-Gyy^T]\Gamma$ is conceived in order to satisfy

$$(K_c - K)^T(K_c - K) \preceq D$$

$$\mathsf{Tr}(\dot{K}\Gamma^{-1} + Gyy^T)(K - F) \leq 0$$

whatever F such that $(K_c - F)^T(K_c - F) \preceq D$.

With these properties one gets

$$2\dot{x}^T P_\zeta x + \epsilon x^T x + 2\mathsf{Tr}\dot{K}^T\Gamma^{-1}(F_\zeta - K) \leq 0$$

Sketch of the proof

■ Stability is proved with the parameter-dependent Lyapunov function

$$V_\zeta(x,K) = x^T P_\zeta x + \mathrm{Tr}(F_\zeta - K)^T \Gamma^{-1}(F_\zeta - K), \qquad \dot{V}_\zeta(x,K) \leq -\epsilon x^T x$$

▲ Both P_ζ and F_ζ are parameter-dependent

▲ F_ζ is a stabilyzing static output feedback

cannot be implemented without knowledge of ζ

▲ Adaptive control achieves the same robustness property without knowledge of ζ

▲ Methodology applies without difficulty for the design of structured adaptive gains

e.g. K can be constrained to be diagonal

▲ Result extend to descriptor uncertain systems

thus applicable to systems rational in the uncertainties ζ

▼ The results are in terms of BMIs : Non-convex

Heuristic method for adaptive control design

- Design a controller $C(s)$ for the nominal plant $H_{\zeta=0}(s)$

- Convert the loop $H_\zeta(s) \star C(s)$ into $G_\zeta(s) \star K_o$ where

  K_o is diagonal and contains the gains to be adapted

  $G_\zeta(s)$ has a descriptor state-space representation, affine in the uncertainties ζ

- Starting from $K_c = K_o$ and a small set of uncertainties $\zeta \in Z_o$

  solve iteratively the BMIs while increasing the size of the set of uncertainties Z

▲ The algorithm converges on examples to sets Z

containing values ζ for which $H_\zeta(s) \star C(s)$ is unstable

▼ Does not mean that no robustly stabilizing LTI control exists

Applied to a satellite attitude control example

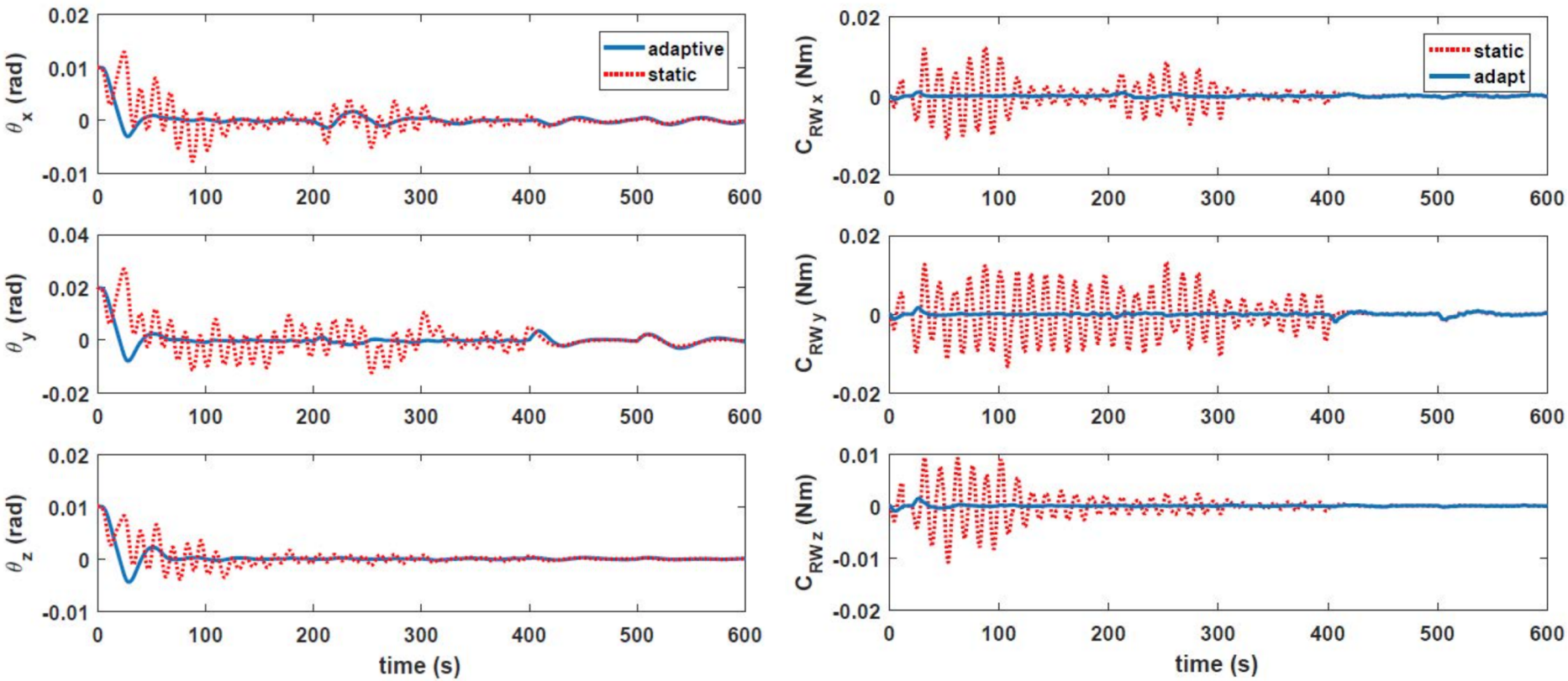
[Leduc 2017]

Uncertainties on the inertia (large variations during deployment of appendices)

Tested on the full size simulation model at CNES

Baseline LTI control designed for the fully deployed configuration

Adaptive control stable for all configurations



Adaptive & Robust !

To be continued...

Thank you !

and happy α^{70}!