LMI conditions for robust adaptive control of MIMO LTI systems

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CNRS-RAS cooperative research project

"Robust and adaptive control of complex systems: Theory and applications"

CNRS-RAS cooperation objectives

➙ Investigate robustness issues of adaptive algorithms for control

both theoretically and through experiments

➙ Adaptive Identification (CCA'07, ALCOSP'07)

➙ Direct adaptive control (ROCOND'06, ALCOSP'07, ACC'07, ACA'07)

➙ State-estimation in limited-band communication channel

Other cooperations

➙ Also part of ECO-NET project ”Polynomial optimization for complex systems”, funded by French Ministry of Foreign Affairs, and handled by Egide.

Concerned countries : Czech Republic, France, Russian Federation, Slovakia.

➙ Submitted a PICS project ”Robust and adaptive control of complex systems” (funded by CNRS and RFBR).

Control strategies to be compared

Output-feedback passification of LTI uncertain system

$$\dot{x}(t) = A(\Delta)x(t) + B(\Delta)u(t) \quad , \quad y(t) = C(\Delta)x(t)$$

where Δ a constant uncertainty in $\boldsymbol{\Delta}$ a compact set.

Parameter-Dependent SOF control $u(t) = v(t) + F(\Delta)y(t)$

✪ Possible if Δ is measured or estimated

Direct adaptive OF control $u(t) = v(t) + K(t)y(t)$

$$\dot{K}(t) = -Gy(t)y^T(t)\Gamma + \phi(K(t))\Gamma$$

✪ Nonlinear closed-loop with states $\eta = \begin{pmatrix} x^T & \text{vec}(K)^T \end{pmatrix}^T$.

✪ $\phi(K)$ to prevent $K(t)$ from growing to infinite values (burst).

Central result: If $\exists$ passifying SOF $\Rightarrow$ AOF is passifying

Control objectives

x-strict passification with respect to transfer $v \longrightarrow z$:

$$\exists V(\eta) > 0,\ \exists \rho(x) > 0 \ : \ V(\eta(t)) \leq V(\eta(0)) + \int_0^t [v^T(\theta) z(\theta) - \rho(x(\theta))] d\theta$$

➙ V: storage function

➙ $\rho = 0$: passivity

➙ $\rho(x) > 0\ ,\ \ \forall x \neq 0$: passivity and asymptotic stability to zero of x

Considered choices of output signals

✪ $z = y = Cx$, possible only for square systems

✪ $z = Gy = GCx$, extends passification, e.g. to non-square systems,

only for Hyper-Minimum Phase open-loop systems.

✪ $z = Gy + Dv = GCx + Dv$, further extension

the feed-through "shunt" D makes robustness issues possible.

SOF results for nominal case

Nominal system $\dot{x} = Ax + Bu \;,\;\; y = Cx$

SOF x-strict passivity w.r.t. $v \to Gy$ ($V(\eta) = x^T P x$, $\rho(x) = \frac{\epsilon}{2} x^T x$)

$$(A+BFC)^T P + P(A+BFC) \leq \epsilon \mathbf{1} \;,\;\; PB = C^T G^T$$

➙ LMI problem if G is given

$$A^T P + C^T F^T GC + PA + C^T G^T FC \leq \epsilon \mathbf{1} \;,\;\; PB = C^T G^T$$

➙ Robustness cannot be achieved if $B(\Delta)$ and $C(\Delta)$ uncertain

SOF x-strict passivity w.r.t. $v \to Gy + Dv$ May be robust, but BMI

$$\begin{bmatrix} (A+BFC)P + P(A+BFC) & PB \\ B^T P & 0 \end{bmatrix} \leq \begin{bmatrix} -\epsilon \mathbf{1} & C^T G^T \\ GC & D + D^T \end{bmatrix}$$

Linearization of BMI problem

"Shunt" D should be small

➙ Feed-through not appropriate for engineering problems

➙ Keep "close" to the linearizing $PB = C^T G^T$: exists R "small" s.t.

$$\begin{bmatrix} R & C^T G^T - PB \\ GC - B^T P & 1 \end{bmatrix} \geq 0$$

which modifies the BMI problem into

$$C^T F^T B^T P + PBFC \leq C^T F^T GC + C^T G^T FC + R + C^T F^T FC$$

which may be guaranteed via LMIs if F is constrained to be bounded

$$\begin{bmatrix} T & F^T \\ F & 1 \end{bmatrix} \geq 0 \;, \quad \begin{array}{l} \mathrm{Trace}(T) \leq \gamma \\ \beta > 1 \end{array} \quad \Rightarrow \quad \begin{array}{l} F^T F \leq \beta\gamma 1 \\ \mathrm{Trace}(F^T F) \leq \gamma \end{array}$$

SOF result: LMI formulation for existence of <u>bounded</u> SOF gain F

that x-strictly passifies w.r.t. $v \longrightarrow z = Gy + Dv$.

Extension to uncertain systems (polytopic uncertainty case)

- If ”nominal” problem is LMI without equality constraints

  ➙ possible to give a robust LMI version

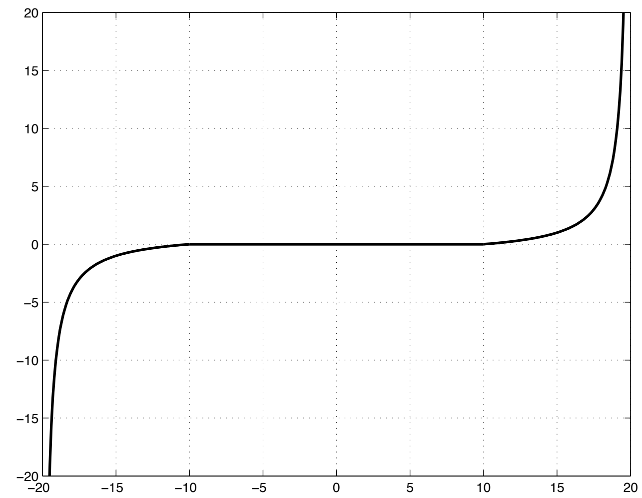
- Polytopic uncertain system

$$\begin{bmatrix} A(\Delta) & B(\Delta) \\ C(\Delta) & 0 \end{bmatrix} = \sum_{i=1}^{N} \zeta_i \begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix}$$

$$\zeta_i \geq 0 \ , \quad \sum_{i=1}^{N} \zeta_i = 1$$

➙ THM 1 if $\forall i = 1 \dots N$: $\mathcal{L}(H_1, H_2, P_i, T_i, R_i, F_i, D_i, \epsilon) \leq 0$
then define $P(\Delta) = \sum_{i=1}^{N} \zeta_i P_i$, $F(\Delta) = \sum_{i=1}^{N} \zeta_i F_i$, $D(\Delta) = \sum_{i=1}^{N} \zeta_i D_i$
$F(\Delta)$ is a bounded x-passifying SOF w.r.t. $v \longrightarrow z = Gy + D(\Delta)v$,
such that Trace$(F^T(\Delta)F(\Delta)) \leq \gamma$

Proof with storage function $V(\eta, \Delta) = x^T P(\Delta) x$



Choice of $\phi(K)$ to keep AOF admissible

$$\phi(K) = 0 \quad \text{if} \quad \text{Trace}(K^T K) \leq \gamma$$
$$\phi(K) = \frac{\text{Trace}(K^T K) - \gamma}{\beta\gamma - \text{Trace}(K^T K)} K \quad \text{otherwise}$$

- ➙ $\text{Trace}(K^T K) \leq \beta\gamma$ is guaranteed whatever bounded perturbations
- ➙ THM 2: Solution to THM 1 (LMI problem) guarantees that

$$u(t) = v(t) + K(t)y(t) \quad , \quad \dot{K}(t) = -Gy(t)y^T(t)\Gamma + \phi(K(t))\Gamma$$

x-strictly passifies the system for all uncertainties Δ in the polytopic set $\mathbf{\Delta}$.

Proof with storage function

$$V(\eta, \Delta) = \frac{1}{2} x^T P(\Delta) x + \frac{1}{2} \text{Trace}\left((K - F(\Delta))\Gamma^{-1}(K - F(\Delta))^T \right) .$$

Example

$$\left[\begin{array}{cc} A(\Delta) & B(\Delta) \end{array}\right] = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 12-7.5\delta_1 & -0.6+0.7\delta_1 & 5-4.5\delta_1 & 0 \\ 0 & 0 & 0 & -20+\delta_2 & 20-\delta_2 \end{array}\right]$$

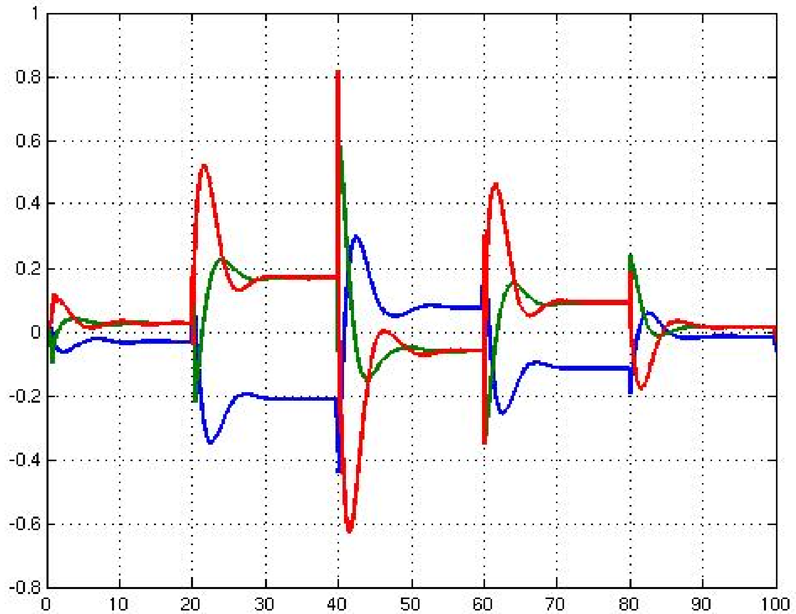
$$C(\Delta) = \left[\begin{array}{cccc} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1+0.1\delta_2 \end{array}\right], G = \left[\begin{array}{ccc} 400 & 300 & 200 \end{array}\right], \delta_2 \in [0, 2.5]$$

$\delta_1 \in$	LMIs
$[\,-1\;\;0.7\,]$	feasible
$[\,-1\;\;0.72\,]$	infeasible
$[\,0.7\;\;0.72\,]$	feasible
$[\,0.72\;\;0.722\,]$	feasible
0.723	infeasible

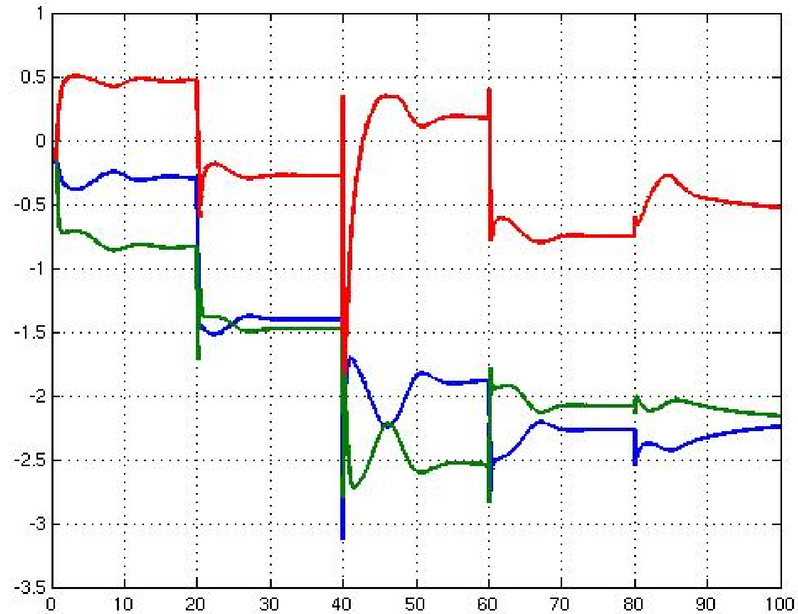
- ➙ AOF valid for all $\delta_1 \in [-1 \;\; 0.722]$
- ➙ $F(\Delta)$ would be switching if applied to $\delta_1 \in [-1 \;\; 0.722]$
- ➙ infeasibility for $\delta_1 \in [-1 \;\; 0.72]$ illustrates conservatism
- ➙ Computation time less than half a second

Simulations for extremal values of $\delta_1 \in [-1 \;\; 0.722]$, $\delta_2 \in [0 \;\; 2.5]$

- Random step disturbance on the measurements every 20 seconds
- Parameters values $\delta_1 = -1$, $\delta_2 = 2.5$



outputs $y(t)$



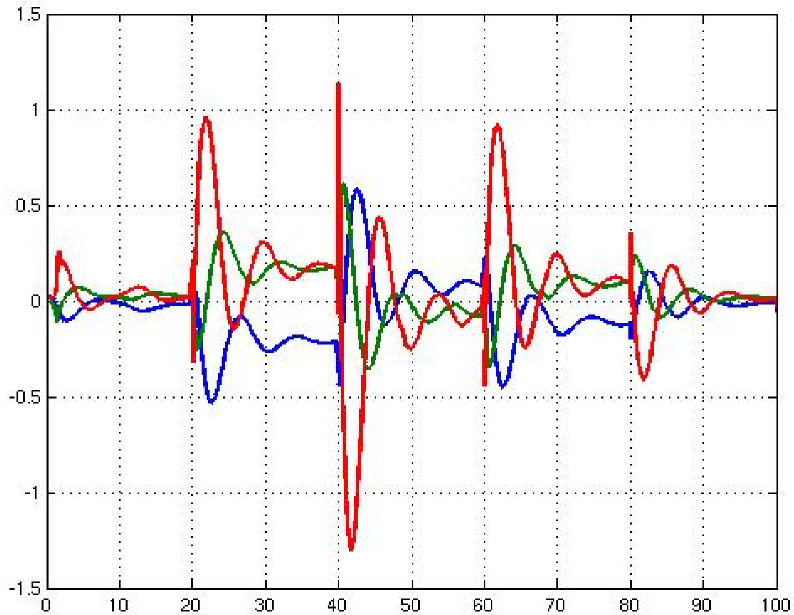
control gains $K(t)$

↗ Stability and bounded signals

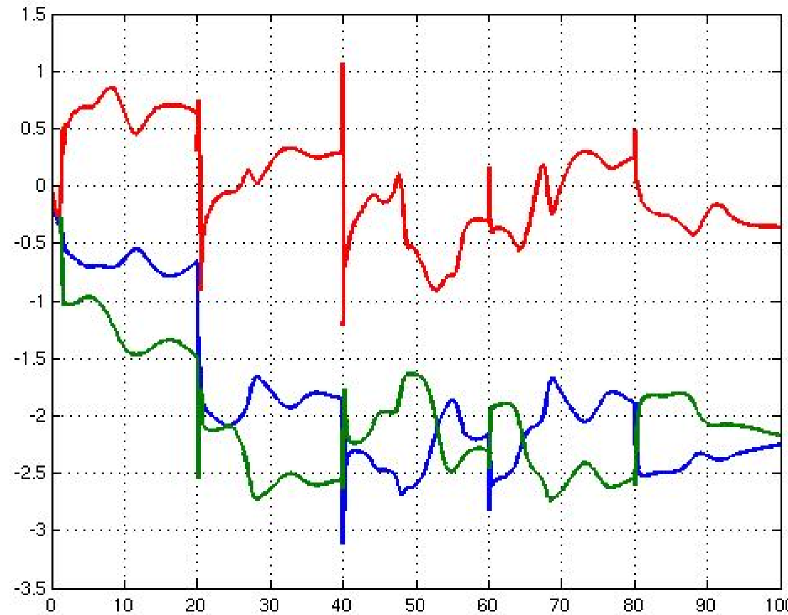
Simulations for extremal values of $\delta_1 \in [-1 \ \ 0.722]$, $\delta_2 \in [0 \ \ 2.5]$

✪ Same experimental conditions (same disturbance signal)

✪ Parameters values $\delta_1 = 0.722$, $\delta_2 = 0$



outputs $y(t)$



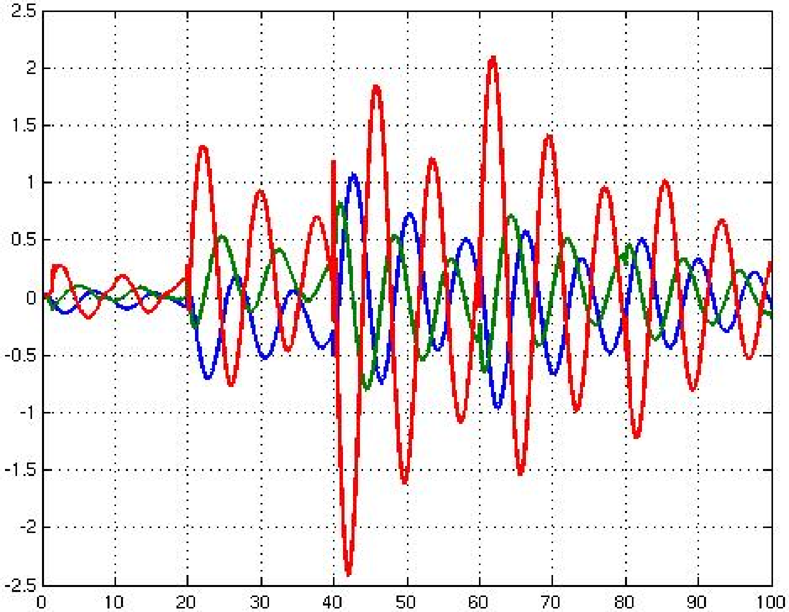
control gains $K(t)$

↗ Stability and bounded signals

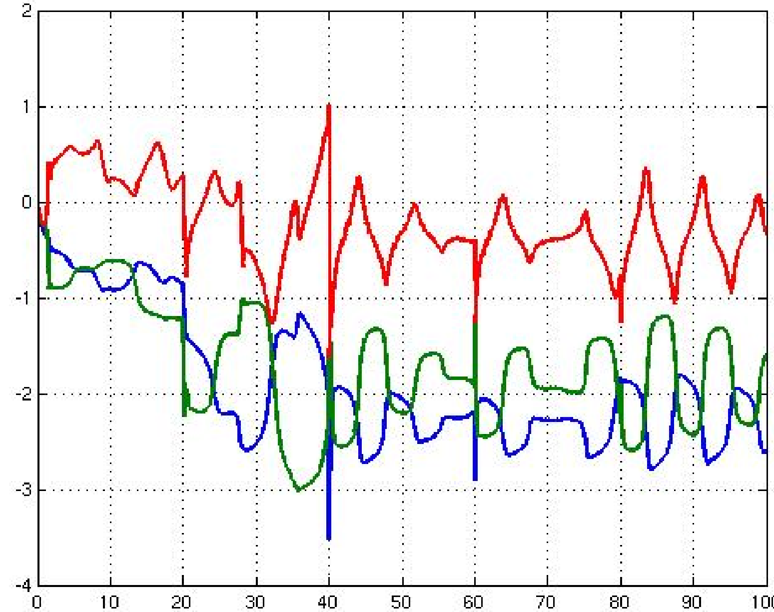
↘ More oscillations and longer convergence time

Simulations for extremal values of $\delta_1 \in [-1 \;\; 0.722]$, $\delta_2 \in [0 \;\; 2.5]$

- ✪ Same experimental conditions (same disturbance signal)
- ✪ Parameters values $\delta_1 = 0.722$, $\delta_2 = 2.5$



outputs $y(t)$



control gains $K(t)$

- ↗ Stability and bounded signals
- ↘ More oscillations and longer convergence time: close to instability
- ↗ Instability if δ_i are further increased: result not conservative

Conclusions

Proof of robust stability with bounded AOF gains

↗ LMI based results: efficient test (for low system dimension)

↗ No need for identification, nor gain scheduling

↘ Results assume G given

↘ No proof for the case of varying parameters

↘ Need for performance guarantees:

convergence-time, oscillations, consumption...

Promising results

↗ AOF always performs better L_2-gain attenuation than SOF

↗ Stability preserved for varying parameters $\Delta(t)$

that temporarily exit the stability region

See invited session "Simple Adaptive Control" this afternoon

[I. Barkana] at 16:50 and [R. Ben Yamin, I. Yaesh, U. Shaked] at 18:30