Passification-Based Adaptive Control of Linear Systems: Robustness Issues

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July 29, 2008

Abstract

Passivity is a widely used concept in control theory having lead to many significant results. The manuscript concentrates on one characteristic of passivity, namely passification-based adaptive control. This concept applies to MIMO systems for which exists a combination of outputs that renders the open-loop system hyper-minimum-phase. Under such assumptions, the system may be passified by both high-gain static output-feedback and by a particular adaptive control algorithm. This last control law is modified here to guarantee its coefficients to be bounded. The contribution of the paper is to investigate its robustness with respect to parametric uncertainty. Time response characteristics are illustrated on examples including realistic situations with noisy output and saturated input. Theoretical results are formulated as linear matrix inequalities and can hence be readily solved with semi-definite programming solvers.
Keywords: Passivity, Robustness, Adaptive Control, LMI

1 INTRODUCTION

One of a variety of adaptive control approaches is the so-called *passification based adaptive control* (PBAC). First introduced in 1974 for linear systems [9, 10, 1] and later extended to nonlinear systems [29, 30, 18, 13] it provides efficient design procedures and simple controller structures. Compared to adaptive schemes with combined parameter estimation and controller tuning [16, 4], PBAC needs no estimation and tuning is performed via a simple differential equation. For this reason it is also called simplified adaptive control in [19].

In PBAC framework, passification (sometimes called passivation) is understood as finding a state or output feedback control rendering the closed loop system passive [29, 20]. In the paper two passification control strategies are considered. More precisely, both strategies are required to be G-passifing [11]. That is, closed-loop passivity should be attested not for the entire measurement output vector but for a linear combination defined by a matrix G. As G may not be square, the results apply for non-square systems. An algorithm based on Bilinear Matrix Inequalities (BMIs) is provided for the design of G matrices.

Among applications areas of PBAC are process control [2], flight control [3, 12] and irrigation systems [32]. But many more applications would be possible if providing methods and proofs for assessing robustness of PBAC with respect to disturbances and parametric uncertainties. These long standing issues (see e.g. [8, 27]) are the main questions addressed in the paper.

Lack of robustness with respect to disturbances is noticed for many adaptive algorithms and is often characterized by diverging adaptation parameters. Many practical solutions have been proposed and one of the most popular ones is introducing a negative feedback into the adaptation differential equation.It was first employed in 1971 [24, 7] and rediscovered later [8, 15]. In the present paper, such negative feedback on the controller parameters is proposed based on a dead-zone type function. It is shown to have good behavior to disturbances and moreover it guarantees convergence to bounded equilibrium points. It is therefore called a *Bounded Passification-Based Adaptive Control* (BPBAC).

But the central result of the paper is to address robustness with respect to parametric uncertainty. While robustness is often complex to evaluate in a non-linear context, many results exist for linear systems [6, 17, 28]. The proposed approach therefore relies on a theoretical result relating linear static output-feedback (SOF) and BPBAC. Namely it is demonstrated that BPBAC has robust closed-loop passivity properties if one can prove the existence of a passifying parameter-dependent SOF with bounded gains. Limiting the study to state-space modeled linear time-invariant systems with uncertainties on the the matrix of dynamics A, a method is provided for such parameter-dependent SOF design. The result is derived from nowadays widely known Linear Matrix Inequality (LMI) techniques [6] which are adapted for the passification problem following the first results of [25, 26].

The outline of the manuscript is as follows. In the next section the robust G-passification problem is stated and the class of systems for which it is solved are presented. Section 3 is then devoted to LMI-based results for robust parameter-dependent SOF design. The obtained numerically testable conditions are proved in section 4 to attest robustness of the BPBAC. Section 5 gives the BMI algorithm for selecting appropriate G matrices. In section 6 all exposed results are applied to a numerical example and realistic closed-loop simulations are provided. All numerical calculations are performed in the MATLAB environment. YALMIP [23] is used to enter LMIs and BMIs. Semi-definite programming problems are solved with SeDuMi [31] and BMI problems are solved with PenBMI [21, 22].

Notations: $\mathsf{R}^{m\times n}$ and $\mathsf{C}^{m\times n}$ are the sets of m-by-n real and complex matrices respectively. For a matrix A, the notations $A_{(i,j)}$ indicates the element of row i and column j. A^T is the transpose of the matrix A and A^* is its transpose conjugate. $\mathrm{Tr}(A)$ is the trace of matrix A. $\mathsf{1}$ and $\mathsf{0}$ are respectively the identity and the zero matrices of appropriate dimensions depending on the context. For Hermitian matrices, $A > (\geq) B$ if and only if $A - B$ is positive (semi) definite.

2 PROBLEM FORMULATION

Control of uncertain LTI systems is considered. The systems are represented in state-space by

$$\begin{cases} \dot{x}(t) = A(\xi)x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathsf{C}^n$ is the state, $u(t) \in \mathsf{C}^m$ is the vector of control inputs and $y(t) \in \mathsf{C}^l$ is the vector of measured outputs. ξ is a constant parameter describing dependency with respect to operating conditions. This parameter is decomposed as a sum of two terms $\xi = \theta + \Delta$ where θ stands for an estimate of the operation point while Δ is an uncertainty on the conditions. The matrix $A(\theta + \Delta)$ is assumed rationally dependent with respect to Δ, which dependency can be generically written as:

$$A(\theta + \Delta) = A_0(\theta) + B_\Delta(\theta)\Delta(\mathbf{1} - D_\Delta(\theta)\Delta)^{-1} C_\Delta(\theta) \ .$$

The uncertainty matrix Δ is assumed constant full-block norm-bounded, $\Delta \in \mathbf{\Delta}_{\rho(\theta)}$ where

$$\mathbf{\Delta}_\rho = \{\Delta \in \mathsf{C}^{m_\Delta \times l_\Delta} \ : \ \Delta^* \Delta \leq \rho^2 \mathbf{1}\} \tag{2}$$

and $\rho(\theta)$ is the radius of the ball of uncertainties. It depends of the operating point. The larger is ρ, the bigger is the domain of admissible uncertainties. Many other types of uncertainties may be considered in the same framework as exposed in the following. Norm-bounded type is adopted to simplify presentation of results.

The B and C matrices are assumed to be exactly known and full rank. This assumption restricts the paper application to systems with uncertainties only on the system dynamics. Difficulty to extend the exposed results for systems with uncertainties on B and C matrices are discussed further in the paper. Such extensions are planned for future research.

The defined uncertain model is said to be a Linear Fractional Transform (LFT), build as the feedback connection of the uncertain matrix ($w_\Delta = \Delta z_\Delta$) with the linear system

$$\begin{cases} \dot{x} = A_0(\theta)x + B_\Delta(\theta)w_\Delta + Bu \\ z_\Delta = C_\Delta(\theta)x + D_\Delta(\theta)w_\Delta \\ y = Cx \end{cases} \tag{3}$$

The LFT is assumed to be well-posed, that is $(\mathbf{1} - D_\Delta(\theta)\Delta)$ is non-singular for all admissible uncertainties $\Delta \in \mathbf{\Delta}_{\rho(\theta)}$ and $A(\theta, \Delta)$ lies in a bounded set.

Two control strategies are adopted and compared. One is parameter-dependent static output-feedback (SOF)

$$u(t) = F(\theta)y(t) + v(t) \tag{4}$$

which supposes to have measurements of the operating point parameters (or to have access to a good estimate); the second, is adaptive control defined as

$$\begin{array}{l} u(t) = K(t)y(t) + v(t) \\ \dot{K}(t) = -Gy(t)y^*(t)\Gamma - \phi(K(t))\Gamma \end{array} \tag{5}$$

where ϕ is any dead-zone type function satisfying property $\mathbf{P}(\mathcal{B})$.

Definition 1 *Let $\mathcal{B} \subset \mathsf{C}^{m\times l}$ be a compact set of complex valued matrices. The function $\phi(K) : \mathsf{C}^{m\times l} \to \mathsf{C}^{m\times l}$, it is said to satisfy property $\mathbf{P}(\mathcal{B})$ if*

- *the function is zero valued inside $\mathcal{B}$:*

$$\phi(K) = 0 \quad , \quad \forall K \in \mathcal{B}$$

- *and, outside $\mathcal{B}$, makes strictly positive the following scalar product:*

$$Tr(\phi(K)(K-F)^*) > 0 \quad , \quad \forall K \in \mathsf{C}^{m\times l} \backslash \mathcal{B} \quad , \quad \forall F \in \mathcal{B} \ .$$

A sub-case of adaptive control (5) is when $\phi(K)$ is identically zero for all $K \in \mathsf{C}^{m\times l}$. It corresponds to an infinite dead-zone where $\mathcal{B} = \mathsf{C}^{m\times l}$. This infinite dead-zone case is exactly the control strategy adopted in [9, 19] and is known to diverge in the presence of disturbances on the measurements $y(t)$. A classical practical solution to prevent the gain $K(t)$ for diverging is to add a term such as $-\phi(K(t))$ (see for example [8, 15, 4]) which will force $K(t)$ to keep close to a region $\mathcal{B}$ of implementable gains. The idea of using dead-zone functions rather than other proportional gain regularizations is not new (see for example [14] for adaptive observer case). The present contribution is to consider dead-zone regularization in a control framework and to prove that properties holding for ϕ identically zero are kept true when the dead-zone type function is introduced.

The provided results prove robust closed-loop stability and passivity of (1) with adaptive control (5) for multiple operating points $\theta \in \{\theta_1 \dots \theta_N\}$ while maximizing the size $\rho(\theta_i)$ of admissible uncertainties Δ around each one of these values. Robustness of the adaptive control is then proved for ξ lying in the union of all "balls" centered at the operating points θ_i thus giving an inner approximation of the actual region of admissible parameters $\xi \in \Xi$ (see figure 1). Results are provided assuming the systems matrices of equations (3) are given and the region $\mathcal{B}$ is set *a priori* by practical implementation considerations.

Before getting into details, the definition of passivity on which are based all results is stated. It is done in a non-linear context because it corresponds to the situation when (1) is in closed-loop with the adaptive non-linear control (5).

Consider a closed-loop non-linear uncertain system

$$\left\{ \begin{array}{l} \dot{\eta} = f(\eta, \Delta) + g(\eta)v \\ y = h(\eta) \end{array} \right. \tag{6}$$

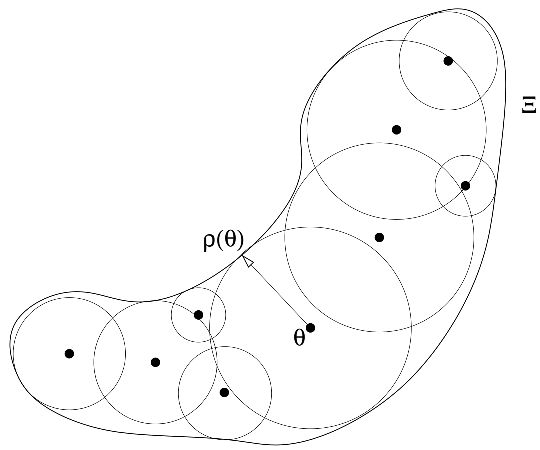


Figure 1: Inner approximation of the admissible domains Ξ by finite number of "balls"

where $\eta = \begin{pmatrix} x^* & K^* \end{pmatrix}^*$ is the state, v is the input, y is the measurements, Δ is an uncertainty, f, g and h are smooth functions. Let G be a prespecified $m \times l$-matrix. Extending the definitions of [11] to uncertain systems and insisting of the sub-part x of the state that corresponds to the original system to be controlled, the following definition is stated.

Definition 2 *The system (6) is called robustly globally x-strictly G-passive if for any $\Delta \in \mathbf{\Delta}$ there exists a nonnegative scalar function $V(\eta, \Delta)$ (storage function) and a scalar function $\gamma(x, \Delta)$ (strictly positive for all $x \neq 0$) such that*

$$V(\eta(t), \Delta) \leq V(\eta(0), \Delta) + \int_0^t \left[v(\tau)^* G y(\tau) - \gamma(x(\tau), \Delta) \right] d\tau \tag{7}$$

holds for all $t \geq 0$ and all solutions of the system (6).

The adjective "robustly" indicates passivity should hold for all admissible values of the uncertainty $\Delta \in \mathbf{\Delta}$, when omitted it indicate the property holds for only one considered value of Δ. The adjective "globally" indicates passivity does not depend on the initial conditions. As usual when dealing with control of linear systems, specifications are always global and the adjective is removed to alleviate. The adjective "strict" indicates that not only the system is passive, but for zero inputs the sub-state $x(t)$ converges to zero. As usually for adaptive control no convergence is required for the sub-state $K(t)$. On the contrary, except if the open-loop system is already asymptotically stable and passive, $K(t)$ has to be different from zero for stability to hold. "G-passivity" indicates that the usual passivity properties hold with respect to a linear combination of the closed-loop outputs characterized by the matrix G. This notion allows to extend results to many more systems than when taking $G = 1$, in particular systems may not be square. Note that design of G-passifying controllers K for a system Σ (*i.e.* find K such that the closed-loop with linearly combined outputs $G(\Sigma \star K)$ is passive) is not equivalent to passification of the "squared" system $G\Sigma$ (*i.e.* find K such that the closed-loop $(G\Sigma) \star K$ is passive). G-passification as the advantage to be more general and it preserves the number of adjustable parameters potentially giving better transient responses, see [1].

3 ROBUST PASSIFYING SOF DESIGN

Static-output feedback design is considered at first with an additional constraint on modulus-bounded entries. Let $\overline{F} \in \mathsf{R}^{m\times l}$ be a real valued matrix with strictly positives entries ($\overline{F}_{(i,j)} > 0$). The sub-set $\mathcal{B}_{\overline{F}} \subset \mathsf{C}^{m\times l}$ is defined as the set of complex valued matrices with entries bounded in modulus by the entries of $\overline{F}$:

$$F \in \mathcal{B}_{\overline{F}} \Leftrightarrow |F_{(i,j)}| \leq \overline{F}_{(i,j)} \ .$$

Based on results of [10, 11], an LMI test for robust G-passifiability was recently published in [25]. Slightly modified to take into account uncertain sets with $\rho \neq 1$, and including bounds on the controller gain, it writes as follows.

Theorem 1 *Let $G \in \mathsf{C}^{m\times l}$ a given matrix, if there exists a solution $H(\theta) \in \mathsf{C}^{n\times n}$, $F(\theta) \in \mathsf{C}^{m\times l}$ to the LMI constraints*

$$H(\theta) > 0 \ \ , \ \ H(\theta)B = C^*G^* \ \ , \tag{8}$$

$$\begin{array}{r} \left[\begin{array}{cc} H(\theta)A_0(\theta) + A_0^*(\theta)H(\theta) + C^*(G^*F(\theta) + F^*(\theta)G)C & H(\theta)B_\Delta(\theta) \\ B_\Delta^*(\theta)H(\theta) & 0 \end{array} \right] \\ + \left[\begin{array}{cc} C_\Delta^*(\theta) & 0 \\ D_\Delta^*(\theta) & 1 \end{array} \right] \left[\begin{array}{cc} \rho^2(\theta)1 & 0 \\ 0 & -1 \end{array} \right] \left[\begin{array}{cc} C_\Delta(\theta) & D_\Delta(\theta) \\ 0 & 1 \end{array} \right] < 0 \end{array} \tag{9}$$

and for $(i,j) \in \{1 \dots m\} \times \{1 \dots l\}$

$$\left[\begin{array}{cc} \overline{F}^2_{(i,j)} & F_{(i,j)}(\theta) \\ F^*_{(i,j)}(\theta) & 1 \end{array} \right] \geq 0 \ \ , \tag{10}$$

then $F(\theta) \in \mathcal{B}_{\overline{F}}$ and the uncertain system (1) at operating point θ is robustly x-strictly G-passified via SOF $u(t) = F(\theta)y(t) + v(t)$ for all uncertainties $\Delta \in \mathbf{\Delta}_{\rho(\theta)}$.

Proof: The fact that $F(\theta) \in \mathcal{B}_{\overline{F}}$ is immediate applying a Schur complement argument to the inequalities (10). (9) being a strict inequality, there exists a positive scalar ϵ such that

$$\mathcal{L}(9) \leq \left[\begin{array}{cc} -\epsilon 1 & 0 \\ 0 & 0 \end{array} \right]$$

where $\mathcal{L}(9)$ stands for the left-hand side of inequality (9). Pre and post multiply the obtained inequality by the vector $\begin{pmatrix} x^* & w_\Delta^* \end{pmatrix}$ and its transpose respectively to get

$$2x^*H(\theta)(A_0(\theta)x + B_\Delta(\theta)w_\Delta) + 2x^*C^*G^*F(\theta)y + \epsilon x^*x \leq w_\Delta^* w_\Delta - \rho^2(\theta) z_\Delta^* z_\Delta \ . \tag{11}$$

By definition of the uncertainty, the right-hand side of this inequality is negative. Due to (8) and feedback control law (4) one gets $x^*C^*G^*F(\theta)y = x^*H(\theta)Bu - y^*G^*v$. Hence (11) implies

$$2x^*H(\theta)\dot{x} \leq 2v^*Gy - \epsilon x^*x \ .$$

Taking its integral over time gives (7) where $V(x) = x^*H(\theta)x$ and $\gamma(x) = 1/2\epsilon x^*x$. ■

Although the current paper considers only full-block norm-bounded uncertainties to limit the presentation complexity, note that the result of Theorem 1 extends easily to many more types of uncertainties. Following methodology of papers such as [17, 28] it simply needs to modify the $\begin{bmatrix} \rho^2(\theta)\mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{bmatrix}$ matrix accordingly to the uncertainty description.

Some characteristics of Theorem 1:

- The equality constraint of (8) is a strong limiting constraint that, except maybe in very special cases, cannot hold for uncertainty dependent matrices B and C. The assumption that B and C are exactly known is at this stage necessary. Future work will be devoted to removing this limitation.

- The constraints (8), (9) and (10) are linear with respect to the decision variables $H(\theta)$ and $F(\theta)$ hence finding such a solution can be done efficiently using semi-definite programming solvers such as those interfaced in YALMIP [23].

- The LMI conditions are also linear with respect to $\rho(\theta)$. Therefore, the solvers can as well maximize $\rho(\theta)$ for increasing the uncertainty domains on which robustness is guaranteed.

Solving separately optimization problems for several operating points $\theta \in \overline{\theta} = \{\theta_1 \dots \theta_N\}$ gives a sub-set of operating points $\hat{\theta} \subset \overline{\theta}$ for which LMIs of Therorem 1 are feasible. Associated to this sub-set are sequences of bounded feedback gains $\{F(\theta) \in \mathcal{B}_{\overline{F}}\}_{\theta\in\hat{\theta}}$ and associated bounds on uncertainties $\{\rho(\theta)\}_{\theta\in\hat{\theta}}$. Define the overall set of obtained operating points:

$$\hat{\Xi}_{\overline{F}} = \left\{ \ \xi = \theta + \Delta(\theta) \quad : \quad \theta \in \hat{\theta} \ \ , \ \ \Delta(\theta) \in \mathbf{\Delta}_{\rho(\theta)} \ \right\} \ .$$

Denote $\Xi_{\overline{F}}$ the set of all operating conditions ξ such that system (1) is G-passifiable via a parameter-dependent SOF (4) with gains bounded in $\mathcal{B}_{\overline{F}}$. The set $\hat{\Xi}_{\overline{F}}$ obtained by solving a finite number of LMI conditions, is trivially an inner approximation of $\Xi_{\overline{F}}$. Moreover, taking a sufficiently thin discretization

$\bar{\theta}$ of the parameter space, the inner approximation can be made as exact as wanted. Without entering into details on building and optimizing an algorithm for picking appropriately the elements of $\bar{\theta}$, note that such algorithm would need only to solve at each tested point LMIs of relatively low dimensions with few variables. All the calculations being done off-line, the overall computational burden is relatively limited.

4 ROBUST PASSIFYING ADAPTIVE CONTROL

The previous section concludes with the possibility to compute a set $\hat{\Xi}_{\overline{F}}$ such that for all operating conditions $\xi \in \hat{\Xi}_{\overline{F}}$ there exists a SOF control gain $F(\xi) \in \mathcal{B}_{\overline{F}}$ that G-passifes system (1), moreover the such gains are known and belong to the finite set $\{F(\theta)\}_{\theta\in\hat{\theta}}$. But the underlying parameter-dependent control strategy is not recommended. In case ξ is indeed constant, implementation would mean to be able to measure it (or to build a sufficiently fast estimator to get an appropriate value before the system diverges) and couple that with a look-up table for $F(\xi) \in \{F(\theta)\}_{\theta\in\hat{\theta}}$ selection. In case ξ is not constant but slowly varying, the control strategy may totally fail because of switching between controllers. Rather than entering such complex, computation and memory demanding strategies, direct adaptive control is adopted in the following.

Before stating the main result of the paper, equilibrium points of the closed-loop with adaptive control are investigated. The closed-loop system is non-linear with $n+ml$ states corresponding to the n original states $x(t)$ of (1) and the ml elements of the matrix $K(t)$ of system (5). Define the set

$$\mathcal{E} = \{\ (x_e, K_e)\ :\ x_e = 0\ \ ,\ \ K_e = F \in \mathcal{B}\ \}\ .$$

As $\phi(F) = \mathsf{0}$ for all $F \in \mathcal{B}$, assuming zero input $v(t) = 0$, the states included in $\mathcal{E}$ are such that

$$\dot{x} = 0\ \ ,\ \ \dot{K} = \mathsf{0}\ .$$

$\mathcal{E}$ is a set of equilibrium points for system (1) with adaptive control (5). For a given operating condition ξ, around the equilibrium point $(x_e = 0, K_e = F(\xi))$, the linearized model is such that

$$\dot{x}(t) = A(\xi)x(t) + BK_eCx(t) + BK(t)Cx_e = (A(\xi) + BF(\xi)C)x(t)\ .$$

First Lyapunov's theorem indicates that the equilibrium point is locally unstable if $A(\xi) + BF(\xi)C$ has a positive eigenvalue. This result indicates that $\mathcal{E}$ cannot be an asymptotically stable set if $\mathcal{B}$ does not contain a stabilizing SOF gain. Moreover, if $\mathcal{E}$ is proved to be a globally asymptotically stable set and assuming the initial conditions are not at an unstable equilibrium, the system states can only converge, if it does converge to a fixed point (which is the case in practice), to a value $K(+\infty) = F(\xi)$ which defines a stabilizing SOF gain for system (1).

The central result is now formulated.

Theorem 2 *Let $G \in \mathsf{C}^{m\times l}$ a given matrix, the following condition (i) implies (ii):*

(i) There exists a parameter-dependent $m \times l$ matrix $F(\xi) \in \mathcal{B}$ such that system (1) with the SOF (4) is robustly x-strictly G-passive for all $\xi \in \Xi$.

(ii) For any Hermitian positive definite matrix $\Gamma \in \mathsf{C}^{l\times l}$ and any function ϕ satisfying property $\mathbf{P}(\mathcal{B})$, the choice (5) is a time-varying output-feedback that renders system (1) robustly x-strictly G-passive for all $\xi \in \Xi$ and, in case of zero input $v(t) = 0$, the set $\mathcal{E}$ is globally asymptotically stable.

Proof : Take any $\xi \in \Xi$, let $F(\xi) \in \mathcal{B}$ be an asymptotically stabilizing gain and assume it x-strictly G-passifies the system. According to [11], this is equivalent to the existence of a quadratic storage function $V(x,\xi) = x^*H(\xi)x$ and a positive scalar $\gamma(\xi) > 0$ fulfilling the matrix inequalities

$$\begin{array}{l} H(\xi) = H^*(\xi) > \mathbf{0} \quad : \quad H(\xi)B = C^*G^* \\ H(\xi)A(\xi,F) + A^*(\xi,F)H(\xi) < -2\gamma(\xi)\mathbf{1} \end{array} \tag{12}$$

where $A(\xi,F) = A(\xi) + BF(\xi)C$ is the closed-loop dynamics matrix. Let any Hermitian positive definite matrix $\Gamma > 0$ and let the output-feedback law (5). Consider the following storage function

$$V(x,K,\xi) = \frac{1}{2}x^*H(\xi)x + \frac{1}{2}\mathrm{Tr}\left((K - F(\xi))\Gamma^{-1}(K - F(\xi))^*\right) \ . \tag{13}$$

Along the trajectories of (1) with the control law (5) the derivatives of $V(x,K,\xi)$ write

$$\dot{V}(x,K,\xi) = x^*H(\xi)(A(\xi)x + BKy + Bv) + \mathrm{Tr}\left((K - F(\xi))\Gamma^{-1}\dot{K}^*\right) \ .$$

Add and subtract $x^*H(\xi)BF(\xi)y$ in the equation to get

$$\begin{array}{r} \dot{V}(x,K,\xi) = x^*H(\xi)(A(\xi)x + BF(\xi)y + Bv) + x^*H(\xi)B(K - F(\xi))y \\ +\mathrm{Tr}\left((K - F(\xi))\Gamma^{-1}\dot{K}^*\right) \end{array}$$

which, taking $y = Cx$, $H(\xi)B = C^*G^*$ and $\dot{K}$ as in (5), reads

$$\begin{array}{l} \dot{V}(x,K,\xi) = x^*H(\xi)A(\xi,F)x + y^*Gv + y^*G^*(K - F(\xi))y \\ \qquad -\mathrm{Tr}\left((K - F(\xi))yy^*G^*\right) - \mathrm{Tr}\left((K - F(\xi))\phi(K)^*\right) \ . \end{array}$$

As $\mathrm{Tr}(M_1M_2) = \mathrm{Tr}(M_2M_1)$ one gets that

$$\begin{array}{l} y^*G^*(K - F(\xi))y - \mathrm{Tr}\left((K - F(\xi))yy^*G^*\right) \\ \qquad = y^*G^*(K - F(\xi))y - \mathrm{Tr}\left(y^*G^*(K - F(\xi))y\right) \\ \qquad = 0 \end{array}$$

therefore, due to (12), the derivative of the Lyapunov function is over-bounded as

$$\dot{V}(x,K,\xi) \leq -\gamma(\xi)||x||^2 + y^*Gv - \mathrm{Tr}\left(\phi(K)(K - F(\xi))^*\right) \ . \tag{14}$$

Since $F(\xi) \in \mathcal{B}$ and due to property $\mathbf{P}(\mathcal{B})$, one gets that for zero input $v = 0$ and for any vector (x, K) not belonging to $\mathcal{E}$ the time derivative of the Lyapunov function V is strictly negative:

$$\dot{V}(x, K, \xi) < 0 \quad , \quad v = 0 \quad , \quad \forall (x, K) \notin \mathcal{E}$$

which proves global asymptotically stability of $\mathcal{E}$. Taking the integral over time of (14), one gets

$$V(x(t), K(t), \xi) \leq V(x(0), K(0), \xi) + \int_0^t \left[v(\tau)^* G y(\tau) - \gamma(\xi) ||x(\tau)||^2 \right] d\tau$$

which proves x-strict G-passivity for the closed-loop. ∎

Some characteristics of Theorem 2:

- The time-varying control (5) is called the Bounded Passification-Based Adaptive Controller (BPBAC). The values of the gain $K(t)$ are automatically tuned given the measures $y(t)$, it adapts whatever the values of the uncertain parameters ξ and thanks to the dead-zone function ϕ, the controller gains are constrained to converge in a bounded set. Moreover, it is shown that the BPBAC converges asymptotically to a stabilizing SOF gain.

- The BPBAC is not claimed to ensure robustness for any parameter ξ (hardly believable in practice). But applying results of section 3, in case $\mathcal{B}$ is of the type $\mathcal{B}_{\overline{F}}$, admissible sets $\hat{\Xi}_{\overline{F}}$ fulfilling property *(i)* of Theorem 2 may be computed with LMI tools.

5 BMI DESIGN OF G MATRICES

All results exposed up to this point assume a given $G \in \mathsf{C}^{m \times l}$ matrix for which the system is desired to be closed-loop G-passive. Trivially, not all matrices are suitable. For that purpose (except if physical considerations indicate a good choice or if techniques of [5] apply) there is a need for a guide to choose the matrix G. Ideally, one may formulate the problem as finding G such that the set $\Xi_{\overline{F}}$ is maximal. Not only such problem would be difficult to state mathematically but as seen in the following it would be much complex from a numerical point of view. Therefore the paper adopts a sub-optimal strategy consisting of designing G such that $\Xi_{\overline{F}}$ is non-empty and contains at least one value ξ_0 to be chosen *a priori*.

In [11, Corollary 3] it is proved that if there exists a SOF gain that G-passifies the system (1) at operation point ξ_0 then the choice $F(\xi_0) = -k(\xi_0)G$, for a sufficently large value of $k(\xi_0)$, is also a G-passifying gain. This result is quite common in passivity context and is known as high-gain control. For our purpose, it allows to simplify the search of G. Indeed G-passifiability conditions

(12) write for $F(\xi_0) = -k(\xi_0)G$:

$$\begin{array}{c} H(\xi_0) = H^*(\xi_0) > 0 \quad : \quad H(\xi_0)B = C^*G^* \\ H(\xi_0)A(\xi_0) + A^*(\xi_0)H(\xi_0) < 2k(\xi_0)C^*G^*GC \end{array} \tag{15}$$

This problem is non-convex and, if $k(\xi_0)$ is fixed *a priori*, it is a problem constrained by Bilinear Matrix Inequalities (BMIs). It may nevertheless be solved using the PenBMI solver [21, 22]. Of course, since BMI problems are not convex, there is no guarantee for finding G even when it exists. However, as tested on several examples, PenBMI does succeed efficiently.

The design procedure we have adopted for G-design is

- Choose a large value of $k(\xi_0)$;
- Choose an upper bound on $H(\xi_0)$ (we took $\bar{h} = 1$) for scaling the solutions;
- Declare in YALMIP the following BMI problem where $H(\xi_0)$, G and t are the variables

$$\bar{h}\mathbf{1} > H(\xi_0) > 0 \quad , \quad H(\xi_0)B = C^*G^* \quad , \quad t > -1 \tag{16}$$

$$H(\xi_0)A + A^*H(\xi_0) - k(\xi_0)C^*G^*GC < t\mathbf{1} \tag{17}$$

$$\begin{bmatrix} \overline{F}^2_{(i,j)} & -k(\xi_0)G \\ -k(\xi_0)G^* & 1 \end{bmatrix} \geq 0 \quad , \tag{18}$$

- Minimize t using PenBMI, if it returns $t < 0$, the procedure succeeded.

Some characteristics of the procedure:

- The only bilinear term in the inequalities is G^*G. Every other term in linear, which may explain the relatively good behavior of PenBMI for these BMIs.
- The last inequality (18) is added to the optimization problem to guarantee the existence of at least one gain F in the set $\mathcal{B}_{\overline{F}}$.
- Once a matrix G has been found, any matrix proportional to that is also satisfactory for the considered problem. We recommend for numerical reasons to choose $k(\xi_0)G$.

6 NUMERICAL EXAMPLE

Consider the linearized fourth-order model of lateral dynamics for an autonomous aircraft including model of actuator dynamics, presented in [12]. The nominal model is defined for a medium value of the flight altitude $\xi_0 = 5$km. The measured plant output $y(t)$ is a vector of the yaw angle $\psi(t)$, yaw angular rate $r(t)$ and the rudder deflection angle $\delta_r(t)$: $y(t) = [\psi, r, \delta_r]^*$. The control input of the plant is the rudder servo command signal, i.e. $n = 4$, $m = 1$, $l = 3$.

Parameters of the nominal state-space model (1) in this case are as follows:

$$A(\xi_0) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 12 & -0.6 & 5.0 \\ 0 & 0 & 0 & -20 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 20 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and dependency with respect to fight altitude is given by

$$\begin{array}{c} B_\Delta = \begin{bmatrix} 0 & 0 & 0.2 & 0 \end{bmatrix}^T \quad , \quad D_\Delta = 0 \\ C_\Delta = \begin{bmatrix} 0 & -7.5 & 0.7 & -4.5 \end{bmatrix} \; . \end{array}$$

Note that for this data some coefficients of the matrix $A(\xi)$ vary in an order of magnitude when ξ varies from 0 to 10.

6.1 Design of G matrices

This design procedure is applied with two choices of k:

$$k_1 = 10^2 \quad , \quad k_2 = 10^4$$

and with $\overline{F} = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}$, *i.e.* all coefficients of the control gain should be norm bounded by 10. It gives the following two admissible values of G

$$\begin{array}{l} G_{1\text{PenBMI}} = 10^{-2} \begin{bmatrix} 10 & 6.14 & 3.55 \end{bmatrix} \\ G_{2\text{PenBMI}} = 10^{-4} \begin{bmatrix} 10 & 10 & 4.87 \end{bmatrix} \; . \end{array}$$

The computation time of PenBMI is less than half a second for the example (Sunblade 150 computer). The scaled values

$$G_1 = \begin{bmatrix} 10 & 6.14 & 3.55 \end{bmatrix} \quad , \quad G_2 = \begin{bmatrix} 10 & 10 & 4.87 \end{bmatrix}$$

are admissible as well. These are used next.

6.2 Computation of the admissible set of flight altitudes

LMIs of Theorem 1 are solved with $\overline{F} = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}$, for each matrix G_1 and G_2 and for sequences of values θ_i. The latest version of SeDuMi [31] (SeDuMi 1.1 available at `http://sedumi.mcmaster.ca/`) is used to solve the LMIs along with the parser YALMIP [23]. The computation time of each individual LMI problem is less than half a second (Sunblade 150 computer). No numerical problems are encountered.

The results are given in Tables 1 and 2. The obtained admissible uncertainty sets $\hat{\Xi}_{\overline{F}}$ are the unions of the discs centered at θ_i with radius $\rho(\theta_i)$ plotted in Figure 2.

One can notice that depending on the flight conditions, the SOF gains are quite different. Although there may be a single SOF gain G-passifying the system for the entire sets $\hat{\Xi}_{\overline{F}}$, the results tend to indicate that such gain would

Table 1: Results of Theorem 1 for G_1

θ_i	$\rho(\theta_i)$	$F(\theta_i)$		
0	1.811	−9.5880	−6.2087	−10
3	1.8557	−9.8017	−9.3363	−10
6	1.6791	−10	−10	−7.7524
8	1.2443	−10	−10	−3.8156
9.2	0.4667	−6.4047	−6.2846	0.1371
9.6	0.0753	−1.0534	−1.1990	0.8271

Table 2: Results of Theorem 1 for G_2

θ_i	$\rho(\theta_i)$	$F(\theta_i)$		
0	2.0850	−9.2301	−7.2487	−10
4	2.1685	−9.9957	−9.9653	−10
7	1.7141	−10	−10	−5.7759
9	0.5217	−4.7837	−6.0888	−0.0732
9.5	0.0248	−0.5089	−0.7243	0.7936

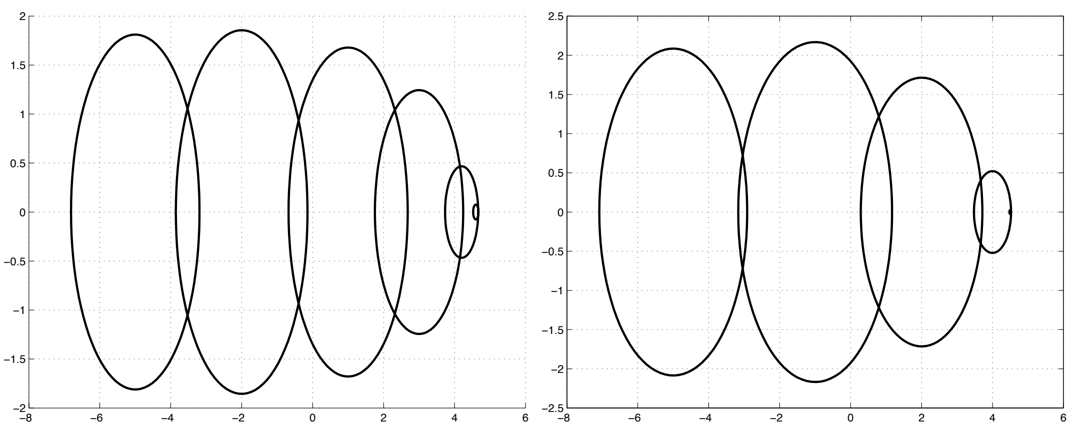


Figure 2: Unions of admissible uncertain sets for G_1 and G_2 respectively

have coefficients with norm larger than 10. When limiting the control gain it is therefore needed to have an auto-tuning rule to adapt the gains depending on the flight altitudes. BPBAC is applied for this purpose in the following.

Note as well that uncertainties are assumed complex in Theorem 1. This is why the regions are discs of the complex plane. For practical purpose only the real part of these sets are of interest. The result is that there exists a bounded parameter-dependent SOF such that the system is robustly G_1-passsifiable for all real valued flight altitude $\xi \in [0\ 9.6753]$ and G_2-passifiable for all $\xi \in [0\ 9.5248]$. Theorem 2 guarantees that for these sets the BPBAC (5) asymptotically stabilizes the system and makes it G-passive. In the following all tests are performed for $G = G_1$ that gives the largest admissible set of flight altitudes.

6.3 Implementing the BPBAC

To implement BPBAC one needs to chose an positive definite matrix Γ and a function ϕ satisfying property $\mathbf{P}(\mathcal{B}_{\overline{F}})$. Two such functions are now exhibited. One is based on the dead-zone function $\psi^d_{\overline{f}} : \mathsf{C} \to \mathsf{C}$ parametrized by $\overline{f} > 0$, such that:

$$\psi^d_{\overline{f}}(k) = \begin{cases} 0 & (0 \leq |k| \leq \overline{f}) \\ (1 - \frac{\overline{f}}{|k|})k & (\overline{f} \leq |k|) \end{cases}$$

that is applied element-wize to define $\phi^d_{\overline{F}} : \mathsf{C}^{m\times l} \to \mathsf{C}^{m\times l}$ such that:

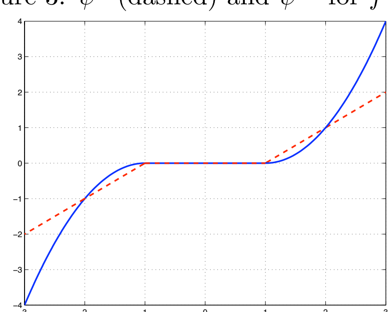
$$\hat{K} = \phi^d_{\overline{F}}(K) \Leftrightarrow \hat{K}_{(i,j)} = \psi^d_{\overline{F}_{(i,j)}}(K_{(i,j)}) \ .$$

The second function is a smoothed version of the dead-zone $\psi^{sd}_{\overline{f}} : \mathsf{C} \to \mathsf{C}$ parametrized by $\overline{f} > 0$, such that:

$$\psi^{sd}_{\overline{f}}(k) = (|k| - \overline{f})\psi^d_{\overline{f}}(k)$$

that is applied element-wize as well to define $\phi^{sd}_{\overline{F}} : \mathsf{C}^{m\times l} \to \mathsf{C}^{m\times l}$. For illustration the two functions ψ^d and ψ^{sd} are plotted on Figure 3 for the real scalar case.

Figure 3: ψ^d (dashed) and ψ^{sd} for $\overline{f} = 1$



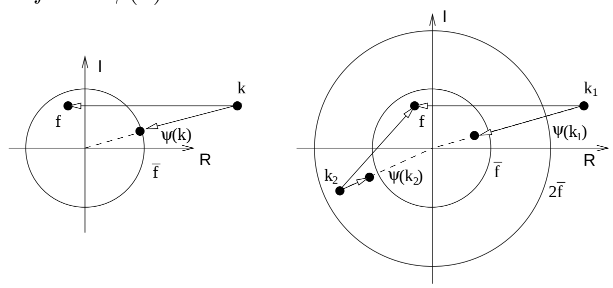
Lemma 1 *The functions $\phi^d_{\overline{F}}$ and $\phi^{sd}_{\overline{F}}$ fulfill property $\mathbf{P}(\mathcal{B}_{\overline{F}})$.*

Proof : First, note that $\phi(K) = 0$ for K inside the set $\mathcal{B}_{\overline{F}}$ is trivial. Second, recall that the scalar product is such that

$$\mathrm{Tr}(\phi(K)(K-F)^*) = \sum_{i=1}^{m}\sum_{j=1}^{l} \psi_{\overline{F}_{(i,j)}}(K_{(i,j)})(K_{(i,j)} - F_{(i,j)})^*$$

which is the sum over all terms of elements like $\psi_{\overline{f}}(k)(k-f)^*$. These terms are scalar products of two vectors in the complex plane as illustrated on Figure 4. They are necessarily nonnegative. Moreover, the sum is strictly positive as soon as one element of K is outside the set $\mathcal{B}_{\overline{F}}$. ∎

Figure 4: $k-f$ and $\psi(k)$ for the dead-zone and the second order dead-zone



6.4 Simulating the BPBAC

A first series of simulations is made taking initial conditions

$$x(0) = \begin{pmatrix} 10 & 0 & 0 & 0 \end{pmatrix}^T \quad , \quad K(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} .$$

taking $\Gamma = 1$, $\overline{F} = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}$ and $\phi = \phi^d_{\overline{F}}$. Robustness with respect to ξ is illustrated for $\xi = 0$, $\xi = 5$ and $\xi = 9.655$. Time histories of both the output $y(t)$ and the control gain $K(t)$ are plotted on Figures 5, 6 and 7 respectively.

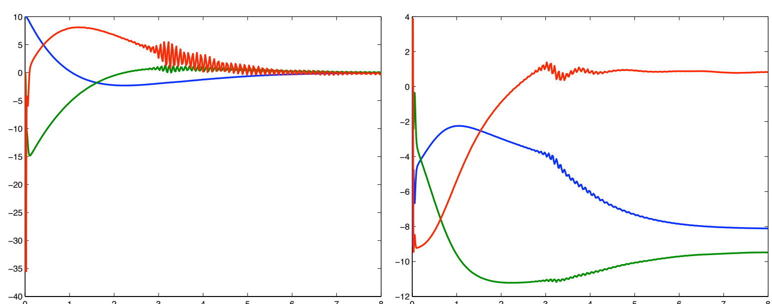


Figure 5: $y(t)$ and $K(t)$ histories for $\xi = 0$

As expected the stabilization of the system becomes more critical as the flight altitude is increased. Other simulations for $\xi = 9.67$ show that convergence to zero takes about 1000 seconds. In all cases the control gain $K(t)$ converge to the specified set $\mathcal{B}_{\overline{F}}$ as expected. This stationary value is necessarily such that the closed-loop is G-passive. But over time, not all values of $K(t)$ are possible SOF stabilizing gains. Indeed for $\xi = 0$ and $\xi = 5$ the time histories show temporary unstable oscillating behaviors. These situations occur as the control gains seem

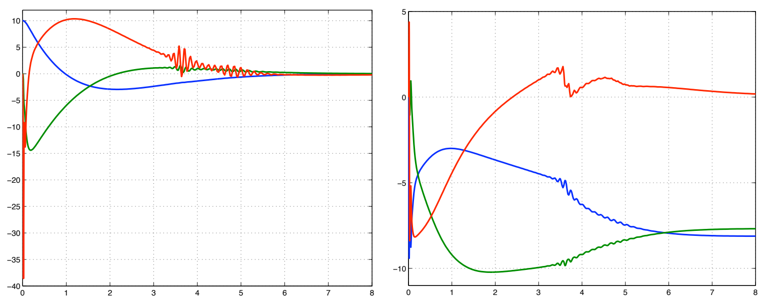


Figure 6: $y(t)$ and $K(t)$ histories for $\xi = 5$

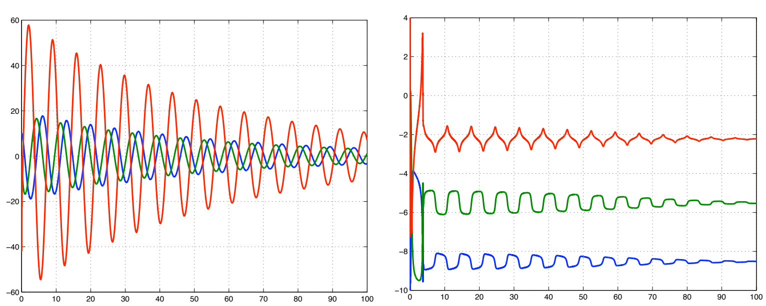


Figure 7: $y(t)$ and $K(t)$ histories for $\xi = 9.655$

to stop their convergence and their value at that time does not stabilize the linear system. The resulting instability "pushes" the control gain away from that value.

A second series of simulations is made to see the influence of the Γ matrix in particular with respect to these oscillating phenomena. The experiments are done for $\xi = 0$ and for the following values of Γ:

$$\Gamma_1 = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \Gamma_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \Gamma_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\Gamma_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \Gamma_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}, \ \Gamma_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

The time histories of $K(t)$ are represented on Figure 8. The influence of Γ is illustrated to be significant but as proved upper, any positive definite Γ makes the system asymptotically stable.

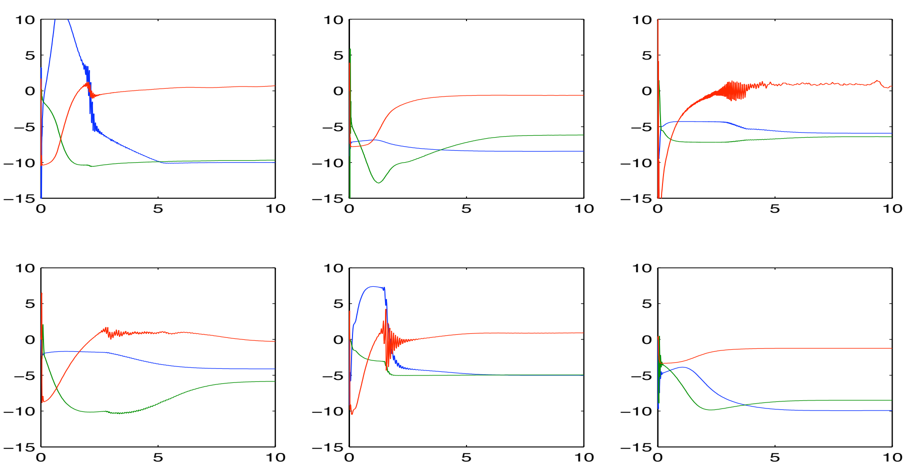


Figure 8: $K(t)$ histories for various values of Γ

Last series of simulations is performed for a more realistic situation with noise on the measurements ($y(t) = Cx(t) + n(t)$), saturation on the inputs ($u(t)$ is saturated between -20 and +20) and with slowly varying parameters (ξ grows linearly from 0 to 9.6 during the time interval [0 5] and then decreases linearly to 8 at $t = 10$). Γ is chosen to be equal to Γ_6 because it gave the best results in the last experiments. The time histories of $y(t)$, $K(t)$ and $u(t)$ for the two choices of dead-zone functions ϕ^d_F and ϕ^{sd}_F are plotted respectively in Figures 9 and 10.

The simulations illustrate that the BPBAC has good behavior in real situations. Comparisons of these two simulations show that whatever the dead-zone function the closed-loop system behaves globally the same. The differences are

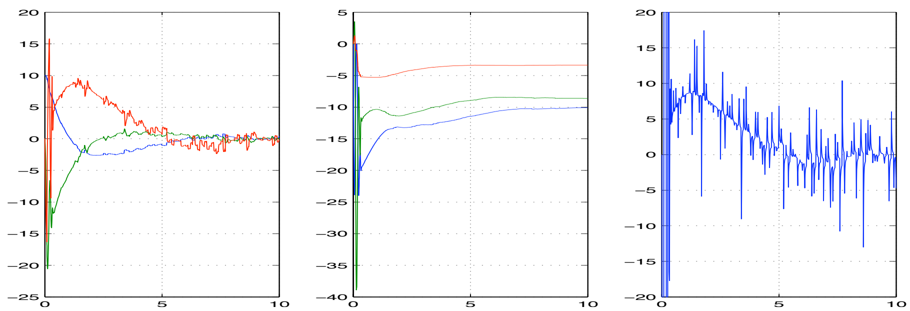


Figure 9: $y(t)$, $K(t)$ and $u(t)$ histories in real noisy saturated situation with $\phi = \phi^d_F$

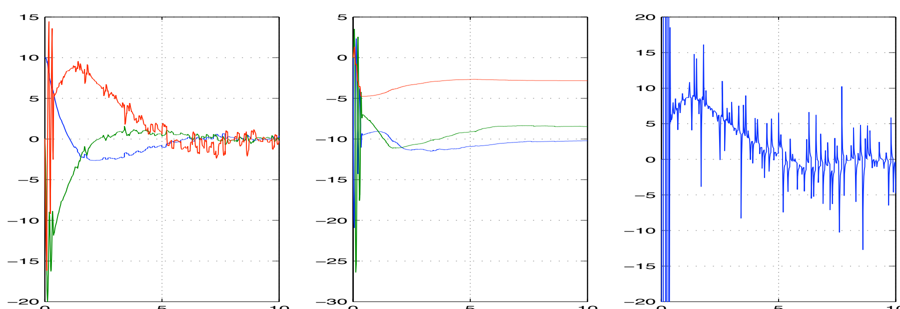


Figure 10: $y(t)$, $K(t)$ and $u(t)$ histories in real noisy saturated situation with $\phi = \phi^{sd}_F$

essentially in terms of convergence speed to the set $\mathcal{B}_{\overline{F}}$. The second order dead-zone function $\phi^{sd}_{\overline{F}}$ acting as a severe penalty function as the gains are far from the set $\mathcal{B}_{\overline{F}}$, it keeps the control gains closer to the set.

Note that without the dead-zone functions, the controller gains would diverge under the influence of the noise on measurements. Indeed, as seen in the theoretical part of the paper, the control can have infinite gain ($K = -kG$ with any k sufficiently large) and any perturbation may tend to "push" the gains in that direction. This phenomenon is illustrated in Figure 11.

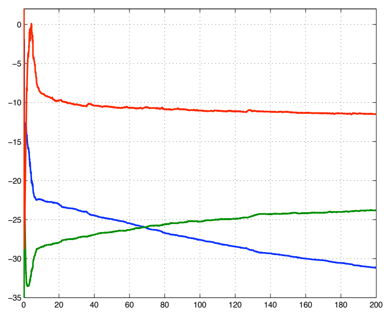


Figure 11: $K(t)$ histories in real noisy saturated situation with $\phi = 0$

7 CONCLUSIONS

Robustness problem of passification-based adaptive control is solved for the case of linear systems with uncertainties on the A matrix. Not only robustness is proved to hold but a constructive strategy is provided to evaluate the admissible uncertainty domains. Still several questions remain open and are left for future research. The major question is whether such techniques may apply in case the B and C matrices are uncertain as well. Results in that direction are expected provided a relaxation of the strong $HB = G^*C^*$ equality constraint. Other, more practical questions are relative to the choices of the Γ matrix and the dead-zone ϕ function used to implement BPBAC. Several matrices Γ have been tested on the example showing very diverse convergence of the control parameters. Providing theoretical tools for choosing Γ is clearly needed. Concerning the dead-zone type functions, we have tested two such functions without noticing much difference. A deeper study with comparisons to other existing disturbance rejections strategies would be of interest.

References

[1] B.R. Andrievsky, A.N. Churilov, and A.L. Fradkov. Feedback Kalman-Yakubovich lemma and its applications to adaptive control. In *IEEE Conference on Decision and Control*, pages 4537–4542, June 1996.

[2] B.R. Andrievsky and A.L. Fradkov. Adaptive controllers with implicit reference models based on Kalman-Yakubovich lemma. In *IEEE Conference on Control Application*, pages 1171–1174, Glasgow, September 1994.

[3] B.R. Andrievsky and A.L. Fradkov. Combined adaptive flight control system. In *5th Intern. ESA Conf. Spacecraft Guidance, Navigation and Control Systems*, pages 299–302, Frascati, Italy, October 2002. (ESA-516, Feb. 2003).

[4] K.J. Åström and B. Wittenmark. *Adaptive Control*. Addison-Wesley, 1989.

[5] I. Barkana, M.C.M. Teixeira, and L. Hsu. Mitigation of symmetry condition in positive realness for adaptive control. *Automatica*, 42(9):1611–1616, September 2006.

[6] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM Studies in Applied Mathematics, Philadelphia, 1994.

[7] R. Carroll and D. Lindorff. An adaptive observer and identifier for a linear system. *IEEE Trans. on Automat. Control*, 18:428–435, 1973.

[8] V. Fomin, A. Fradkov, and V. Yakubovich. *Adaptive control of dynamic plants*. Nauka, Moscow, 1981. In Russian.

[9] A.L. Fradkov. Adaptive stabilization of an linear dynamic plant. *Autom. Remote Contr.*, 35(12):1960–1966, 1974.

[10] A.L. Fradkov. Quadratic Lyapunov functions in the adaptive stabilization problem of a linear dynamic plant. *Siberian Math. J.*, 2:341–348, 1976.

[11] A.L. Fradkov. Passification of non-square linear systems and feedback Yakubovich-Kalman-Popov lemma. *European J. of Control*, 6:573–582, 2003.

[12] A.L. Fradkov and B.R. Andrievsky. Shunting method for control of homing missiles with uncertain parameters. In *IFAC Symposium on Automatic Control in Aerospace*, volume 2, pages 33–38, St. Petersburg, Russia, June 2004.

[13] A.L. Fradkov, I.V. Miroshnik, and V.O. Nikiforov. *Nonlinear and Adaptive Control of Complex Systems*. Kluwer Academic Publishers, Dordrekht, 1999.

[14] A.L. Fradkov, V.O. Nikiforov, and B.R. Andrievsky. Adaptive observers for nonlinear nonpassifiable systems with application to signal transmission. In *IEEE Conference on Decision and Control*, Las Vegas, December 2002.

[15] P. Ioannou and P. Kokotović. *Adaptive Systems with Reduced Models*. Springer-Verlag, Berlin, 1983.

[16] P. Ioannou and J. Sun. *Robust Adaptive Control*. Prentice Hall, Inc, 1996.

[17] T. Iwasaki and S. Hara. Well-posedness of feedback systems: Insights into exact robustness analysis and approximate computations. *IEEE Trans. on Automat. Control*, 43(5):619–630, 1998.

[18] Z. Jiang, A. Fradkov, and D. Hill. A passification approach to adaptive nonlinear stabilization. *Systems & Control Letters*, 28:73–84, 1996.

[19] H. Kaufman, I. Barkana, and K. Sobel. *Direct adaptive control algorithms*. Springer, New York, 1994.

[20] P. Kokotovic and M. Arcak. Constructive nonlinear control: a historical perspective. *Automatica*, 37(5):637–662, 2001.

[21] M. Kočvara and M. Stingl. PENNON - a code for convex nonlinear and semidefinite programming. *Opt. Methods and Software*, 18(3):317–333, 2003.

[22] M. Kočvara and M. Stingl. PENBMI, version 2.0, 2004. see `www.penopt.com` for a free developer version.

[23] J. Löfberg. *YALMIP : A Toolbox for Modeling and Optimization in MATLAB*, 2004.

[24] K. Narendra, S. Tripathi, G. Luders, and P Kudva. Technical Report N-CT-43, Yale Univ., New Haven, 1971.

[25] D. Peaucelle, A.L. Fradkov, and B.R. Andrievsky. Robust passification via static output feedback - LMI results. In *IFAC World Congress*, Prague, Czech Republic, July 2005.

[26] D. Peaucelle, A.L. Fradkov, and B.R. Andrievsky. Passification-based adaptive control : Robustness issues. In *IFAC Symposium on Robust Control Design*, Toulouse, July 2006. Paper in an invited session.

[27] C.E. Rohrs, L. Valavani, M. Athans, and G. Stein. Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics. *IEEE Trans. on Automat. Control*, 30:881–889, 1985.

[28] C.W. Scherer. *Advances in Linear Matrix Inequality Methods in Control*, chapter 10 Robust Mixed Control and Linear Parameter-Varying Control with Full Block Scallings, pages 187–207. Advances in Design and Control. SIAM, 2000. edited by L. El Ghaoui and S.-I. Niculescu.

[29] M.M. Seron, D.J. Hill, and A.L. Fradkov. Adaptive passification of nonlinear systems. In *IEEE Conference on Decision and Control*, pages 190–195, December 1994.

[30] M.M. Seron, D.J. Hill, and A.L. Fradkov. Nonlinear adaptive control of feedback passive systems. *Automatica*, 31:1053–1060, 1995.

[31] J.F. Sturm. Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones. *Optimization Methods and Software*, 11-12:625–653, 1999. URL: `http://sedumi.mcmaster.ca/`.

[32] A.M. Tsykunov. *Adaptive Control of Retarded Systems*. Nauka, Moscow (In Russian), 1984.