Chapter 3.

Inverse kinematics

Pseudo inverse and redundancy

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Rigid kinematics

- Group of rigid transformation: SE(3)

$$ {}^{A}M_b : \quad {}^{B}p \to {}^{A}p = {}^{A}M_b \; {}^{B}p $$

- Group of rigid velocity: se(3) = M^6
  - Tangent to SE(3)

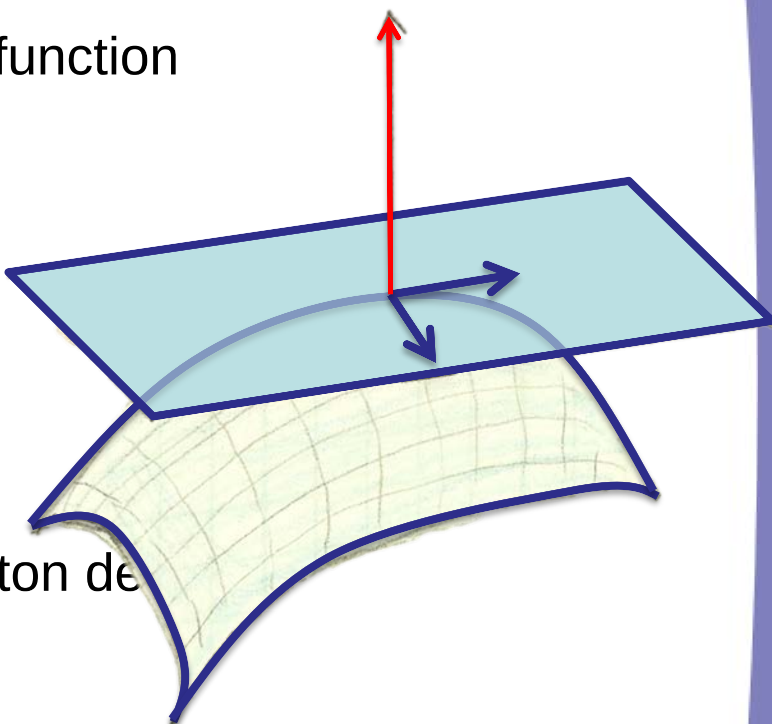
Robot kinematics

- Direct geometry

continuous function

- Direct kinematics

$\dot{q}$ $\dot{q}$ $\dot{q}$



- Inverse geometry
  - Ill defined, singular points
  - Numerical inversion by Newton de
- Integration of the descent
  - Robot trajectory
  - Quadratic problem at each step

A linear minimization solution

- If is not invertible, two cases:
  - There is no way to

    so that
  - There is many optimal solutions for

    { so that }

Pseudo inverse definition

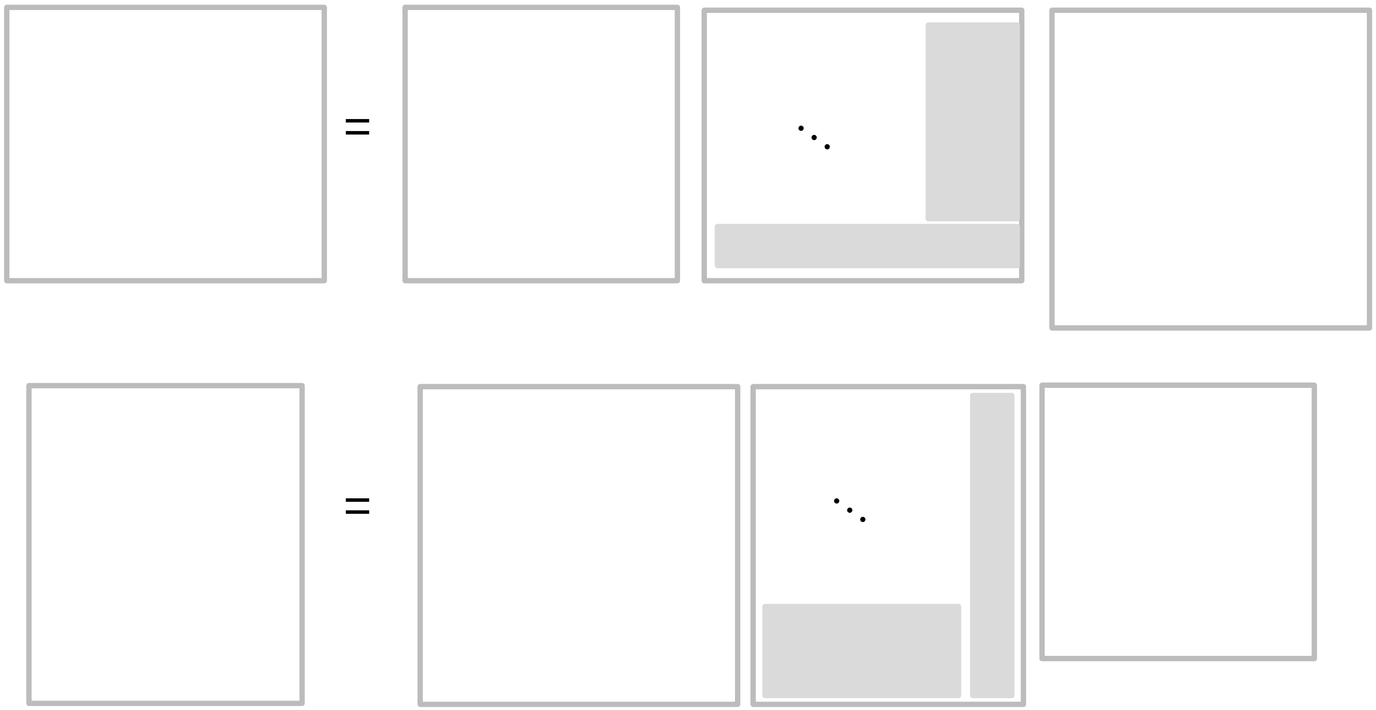
- The four conditions of Moore & Penrose

is symmetrical

is symmetrical

Computing the pseudo inverse

- Singular Value Decomposition (SVD)



- Proof using the 4 Moore-Penrose conditions

  (with and)

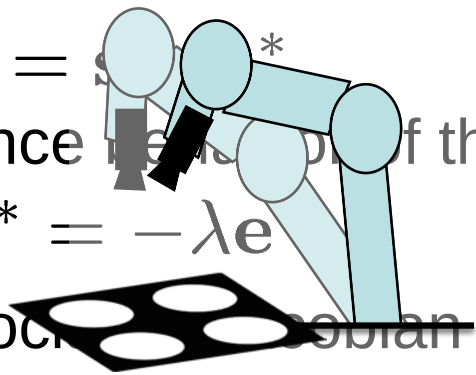
What is a task?

- Robot position: $\mathbf{q}$ Robot control input: $\dot{\mathbf{q}}$

- An error between and sensor values

[Samson91],[Espiau91]

$$\mathbf{e} = \mathbf{s} - \mathbf{s}^*$$



- A reference behavior of the error

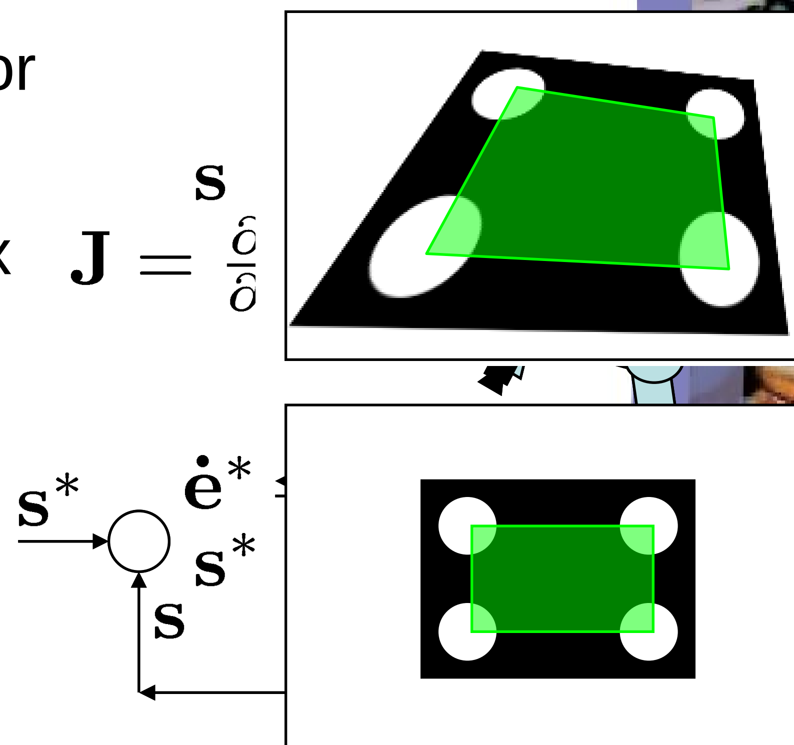
$$\dot{\mathbf{e}}^* = -\lambda \mathbf{e}$$

- The associated Jacobian matrix $\mathbf{J} = \frac{\partial \mathbf{s}}{\partial \mathbf{q}}$

$$\dot{\mathbf{e}} = \mathbf{J}\dot{\mathbf{q}}$$

- Classical control law

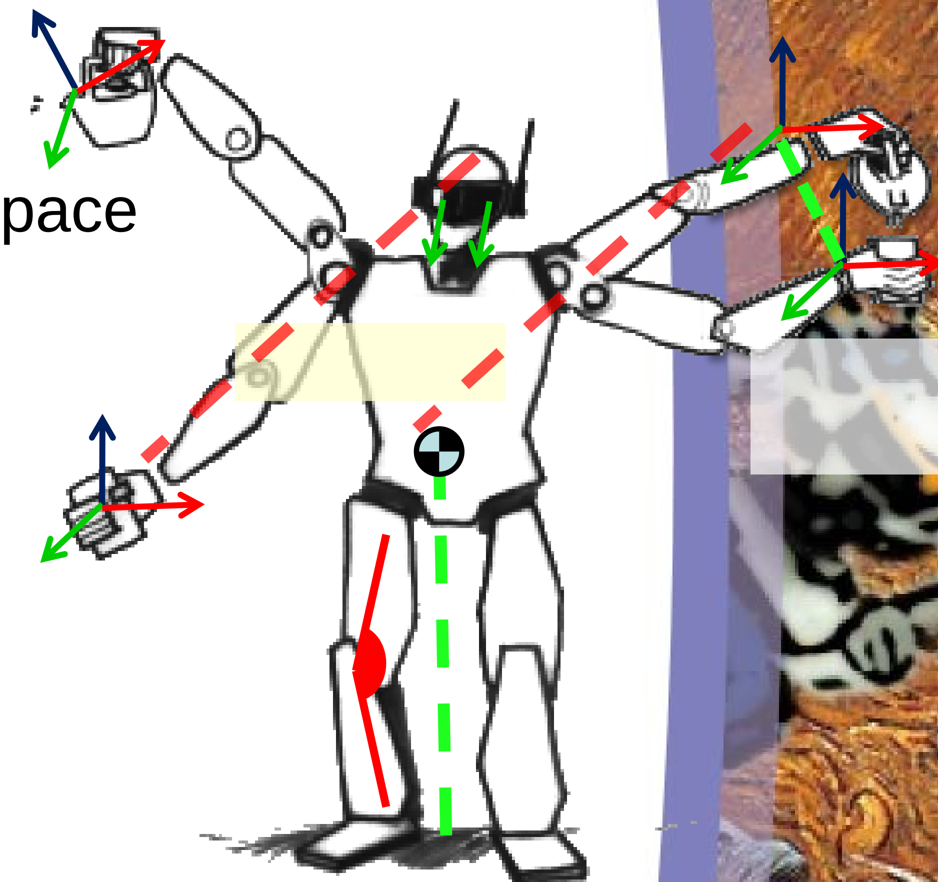
$$\dot{\mathbf{q}} = \mathbf{J}^+ \dot{\mathbf{e}}^*$$



Control in the task space

- Configuration space VS task space

- Easy motion specification
- Reusability - versatility
- Deformation of the motion
- Sensor feedback

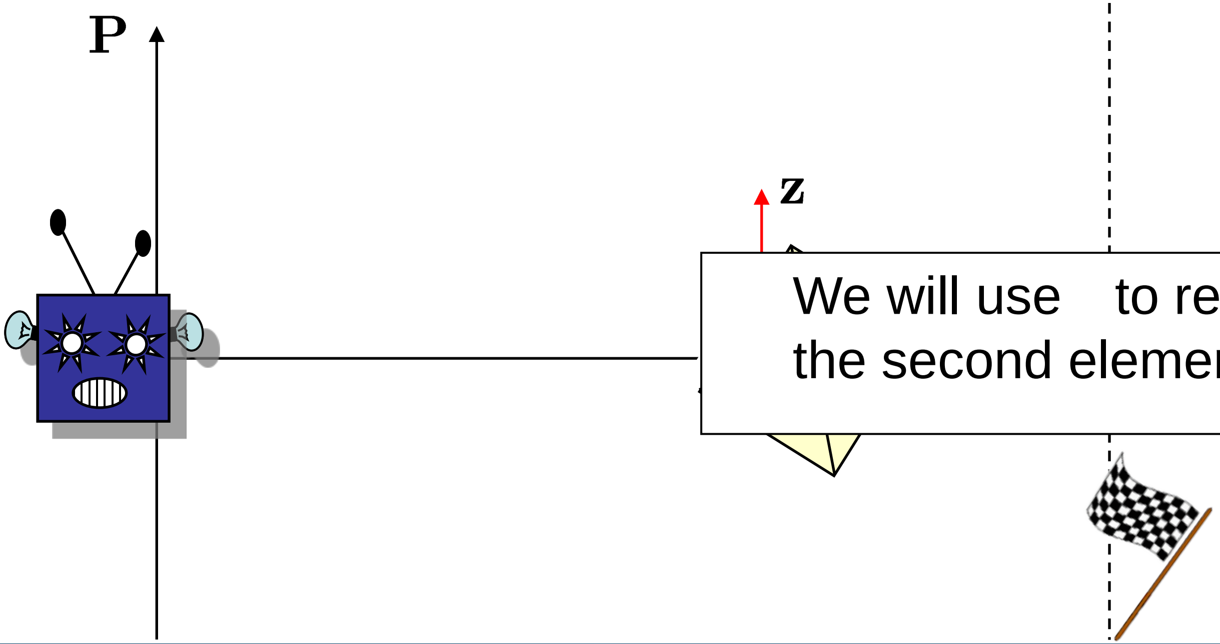


Task redundancy

- $\dot{\mathbf{e}} = \mathbf{J}\dot{\mathbf{q}}$
- $\dot{\mathbf{q}} = \mathbf{J}^{+}\dot{\mathbf{e}}^{*} + \underbrace{(\mathbf{I} - \mathbf{J}^{+}\mathbf{J})}\mathbf{z}$

can be used to vary the trajectory (obstacle avoidance)

[Rosen60],[Liegeois77]



We will use to realize at best the second elementary task