

# Input allocation for systems with redundant actuators

## Organizers:

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## 1 Motivation and outline of the tutorial session

The goal of this tutorial is to provide an overview of recent systematic constructions to deal with plants having redundant actuators. For these plants, the presence of multiple actuators with overlapping features presents an interesting allocation problem, typically encompassing a suitable performance goal that pertains the selected input pattern. Within the tutorial we plan to cover a number of topics, ranging from well established solutions to open problems and future trends.

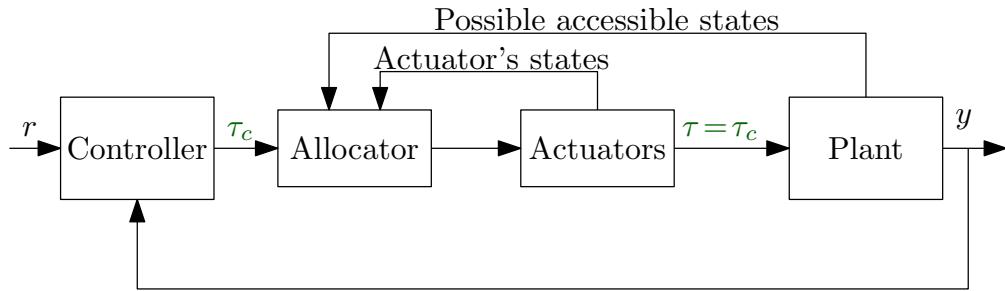


Figure 1: A control system with input allocation.

“Input allocation” denotes the design of a suitable portion of a control system to be placed between the plant to be controlled and an existing controller, in such a way that certain requested “efforts” on the plant (typically comprising physical quantities) are obtained by a combination of the actuators’ actions that correspond to an optimal distribution (or allocation) of the requested stroke. For example, in a cooperative manipulation system a request for a certain wrench at the center of mass of the manipulated object could be obtained by different wrenches at the end effectors of the manipulators performing the grasping action, but one may be interested in the “allocation” that minimizes the internal stress, to avoid damage to the manipulated object. Similarly, performance goals that one may want to optimize when distributing the input authority among the available actuators may be power consumption, or minimizing the use of an expensive actuator, or minimizing

alternative quantities that are dynamically related to the control input, such as the angular momenta in reaction wheels in satellite attitude stabilization, or state of charge of the batteries in hybrid electric vehicles ad so on.

The tutorial will be organized in four 30 minutes slots, each of them presented by one of the organizers. These four slots will provide a general overview of the available constructive techniques for input allocation, illustrating them on several applications, and finally presenting a number of future trends and open problems.

1. *Control allocation overview and application to marine vessel control* (Speaker: T.A. Johansen) The control algorithm hierarchy of motion control for over-actuated mechanical systems with a redundant set of effectors and actuators commonly includes three levels. First, a high-level motion control algorithm commands a vector of virtual control efforts (i.e. forces and moments) in order to meet the overall motion control objectives. Second, a control allocation algorithm coordinates the different effectors such that they together produce the desired virtual control efforts, if possible. Third, low-level control algorithms may be used to control each individual effector via its actuators. Control allocation offers the advantage of a modular design where the high-level motion control algorithm can be designed without detailed knowledge about the effectors and actuators. Important issues such as input saturation and rate constraints, actuator and effector fault tolerance, and meeting secondary objectives such as power efficiency and tear-and-wear minimization is handled within the control allocation algorithm. The objective of the presentation is survey control allocation algorithms. The survey classifies the different algorithms according to two main classes based on the use of linear or nonlinear models, respectively. The presence of physical constraints (e.g input saturation and rate constraints), operational constraints and secondary objectives makes optimization-based design a powerful approach. The simplest formulations allow explicit solutions to be computed using numerical linear algebra in combination with some logic and engineering solutions, while the more challenging formulations with nonlinear models or complex constraints and objectives call for iterative numerical optimization procedures and dynamic methods. Experiences using the different methods in marine vessel control application are discussed.
2. *Dynamic allocation and application to Tokamak plasmas control* (Speaker: L. Zaccarian) The concepts of weak and strong input redundancy are introduced and their use for the design of a dynamic allocator are explained. Suitable formal properties of the allocated closed loop are then established in terms of fast allocation (with strong redundancy) and slow allocation (with weak redundancy) allowing to obtain a fast or slow drift of the actuator signals towards the desired optimal value of a suitably specified cost function enjoying some regularity properties. The proposed approach will be illustrated by way of their technological application, notably to plasma elongation and plasma shape control in the JET and FTU Tokamaks. Further illustration of the proposed techniques may be briefly outlined with reference to additional applications such as control of hybrid electric vehicles, cooperative manipulation, satellite attitude

stabilization.

3. *A geometric viewpoint on dynamic allocation* (Speaker: A. Serrani) The concepts of weak and strong input redundancy, introduced in the previous talk, are further characterized for linear time-invariant state-space representations using geometric techniques. In particular, it is shown that weak redundancy entails the existence of multiple and independently controllable state trajectories that are compatible with a given reference output, whereas strong input redundancy essentially confines redundancy to the null-space of the input operator, which can be factored out by projection. This interpretation opens the door to the possibility of defining a unifying paradigm where both weak and strong input redundancy are exploited in the system inverse rather than in the plant model itself. The geometric characterization of input redundancy leads naturally to the synthesis of a dynamic allocator, whose structure comprises a dynamic optimization module and an “annihilator” or “extended reference model,” whose role is to generate reference trajectories in the input and state spaces that do not affect the generation of the desired output reference in the system inverse. This leads to a scheme where the steady-state behavior of the closed-loop system is shaped through dynamic optimization of selected performance criteria penalizing both the control input and the state trajectory, while maintaining invariance of the error-zeroing subspace. An illustrative example will be presented and discussed to highlight the main features of the approach. The organization of the talk proceeds as follows:
  - 3.1) A state-space approach to weak and strong redundancy
  - 3.2) Redundancy in the inverse plant model
  - 3.3) Geometric characterization and design of the annihilator
  - 3.4) Simple gradient-based allocation: examples
4. *Hybrid and nonlinear dynamic allocation and output regulation* (Speaker: S. Galeani) Based on the geometric framework described in the previous talk, the rich interplay between control allocation and output regulation will be explored and exemplified in this last talk. When the output regulation problem is addressed for systems having more inputs than outputs, an infinite number of different steady-state solutions become available, so that using a suitable extension of dynamic allocation algorithms it becomes possible to perform a transfer from one solution to another in order to choose an optimal steady-state solution in real time; as an example, a specific approach will be discussed, based on the use of a hybrid allocator. Looking at the steady-state motions achieved after the allocation transients are exhausted, it becomes apparent that dynamic allocation actually provides nonlinear output regulators for linear time invariant systems, where nonlinearity is essential in order to provide optimal performance or constraint satisfaction. The study of such nonlinear regulators is the subject of the last part of the talk. To sum up, the talk will address the following sequence of topics:
  - 4.1) Parametrizing the solutions of the Francis equations in input-redundant systems
  - 4.2) Hybrid input allocation for output regulation

- 4.3) Nonlinear solutions of the Francis equations
- 4.4) Nonlinear regulators for linear fat plants

## 2 About the Organizers

**Tor Arne Johansen** (MSc, PhD) worked at SINTEF Electronics and Cybernetics as a researcher before he was appointed Associated Professor at the Norwegian University of Science and Technology in Trondheim in 1997 and was promoted as Professor in 2001. He has published more than 100 articles in international journals as well as numerous conference articles and book chapters in the areas of control, estimation and optimization with applications in the marine, automotive, biomedical and process industries. In 2002 Johansen co-founded the company Marine Cybernetics AS where he was Vice President until 2008. Prof. Johansen is currently a principal researcher within the Center of Excellence on Autonomous Marine Operations and Systems (AMOS) and director of the UAV Laboratory at NTNU.

**Sergio Galeani** received the Laurea degree in 1998 and the Ph.D. in Computer Science and Control Engineering in 2002 from the University of Roma Tor Vergata, where he currently works as a researcher. His research interests include periodic and multirate control systems, control systems with constraints, robust/adaptive anti-windup techniques, and hybrid, linear and nonlinear output regulation. He is an Associate Editor in the Conference Editorial Board of the IEEE Control Systems Society and vice chair of the IFAC Technical Committee 2.1 (Control Design).

**Andrea Serrani** received the Laurea (B.Eng.) degree and the Ph.D. degree from the University of Ancona, Italy, in 1993 and 1997, respectively. From 1994 to 1999, he was a Fulbright Fellow at Washington University, St. Louis, MO, where he obtained the M.S. and D.Sc. degrees in Systems Science and Mathematics in 1996 and 2000, respectively. Since 2002, he has been with the Department of Electrical and Computer Engineering at The Ohio State University, where he is a Professor and Chair of Graduate Studies. He has held visiting positions at the University of Bologna and at the University of Padua, Italy, and multiple summer faculty positions at AFRL. The research interests of Prof. Serrani lie at the intersection of nonlinear, adaptive and geometric control theory with applications in aerospace, marine and automotive systems, fluidic systems, and robotics. His work has been supported by AFRL, NSF, NASA and Ford Motor Co., among others. Prof. Serrani is a Distinguished Lecturer of the IEEE CSS and an Associate Editor for the IEEE Transactions on Control Systems Technology, the IEEE CSS Conference Editorial Board, and the EUCA Conference Editorial Board. He has been an Associate Editor for Automatica and the International Journal of Robust and Nonlinear Control, and has served on operating and program committees of several IEEE and IFAC conferences.

**Luca Zaccarian** received the Laurea and the Ph.D. degrees from the University of Roma Tor Vergata (Italy) in 1995 and 2000, respectively. He has been Assistant Professor in control engineering at the University of Roma, Tor Vergata (Italy), from 2000 to 2006 and then Associate Professor. Since 2012 he is Directeur de Recherche at the LAAS-CNRS,

Toulouse (France) and since 2013 he holds a part-time associate professor position at the University of Trento, Italy. Luca Zaccarian's main research interests include analysis and design of nonlinear and hybrid control systems, modeling and control of mechatronic systems. He has served in the organizing committee and TPC of several IEEE and IFAC conferences. He has been a member of the IEEE-CSS Conference Editorial Board and an associate editor for Systems and Control Letters. He is currently a member of the EUCA-CEB, an associate editor for the IEEE Transactions on Automatic Control and the IFAC journal Automatca. He was a member of the Board of Governors of the IEEE-CSS in 2014 and he is a senior member of the IEEE since 2009. He is currently Student Activities Chair and Associate Editor of Electronic Publications (Conference Information) for the IEEE-CSS. He was a recipient of the 2001 O. Hugo Schuck Best Paper Award given by the American Automatic Control Council.

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