Lyapunov-based Reset Control

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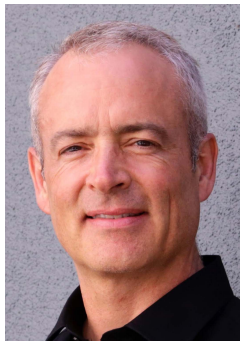
Outline

Clegg integrator and its use in a simple planar feedback loop

Collaborative work with:

D Nesic

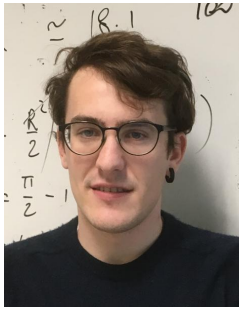


AR Teel

F Fichera

C Prieur

S Tarbouriech



M Della Rossa

R Goebel

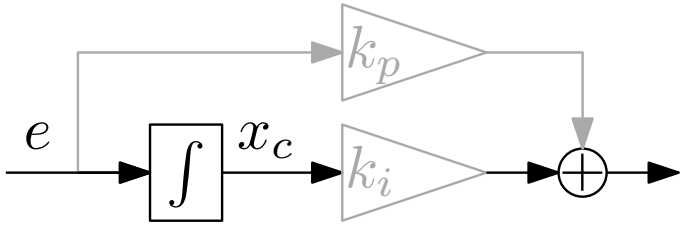
A Tanwani

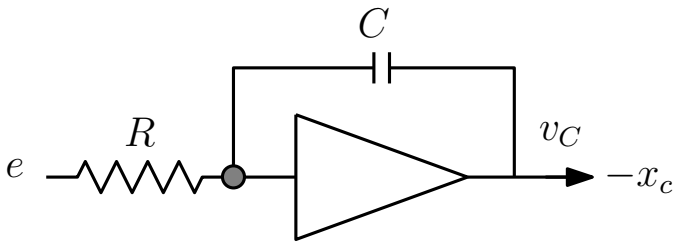
An analog integrator and its Clegg extension Clegg [1958]

Proportional Integral (PI) control comprise an integrator

Example: Error feedback

$$u(t) = k_p e(t) + k_i \underbrace{\int_0^t e(\tau)d\tau}_{=x_c}$$





$$\dot{x}_c = (RC)^{-1} e$$

- In an analog integrator, the state information is stored in a capacitor:

An analog integrator and its Clegg extension Clegg [1958]

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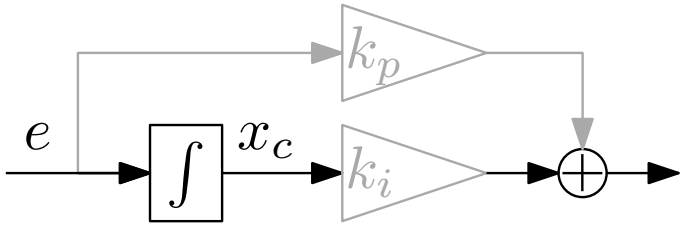
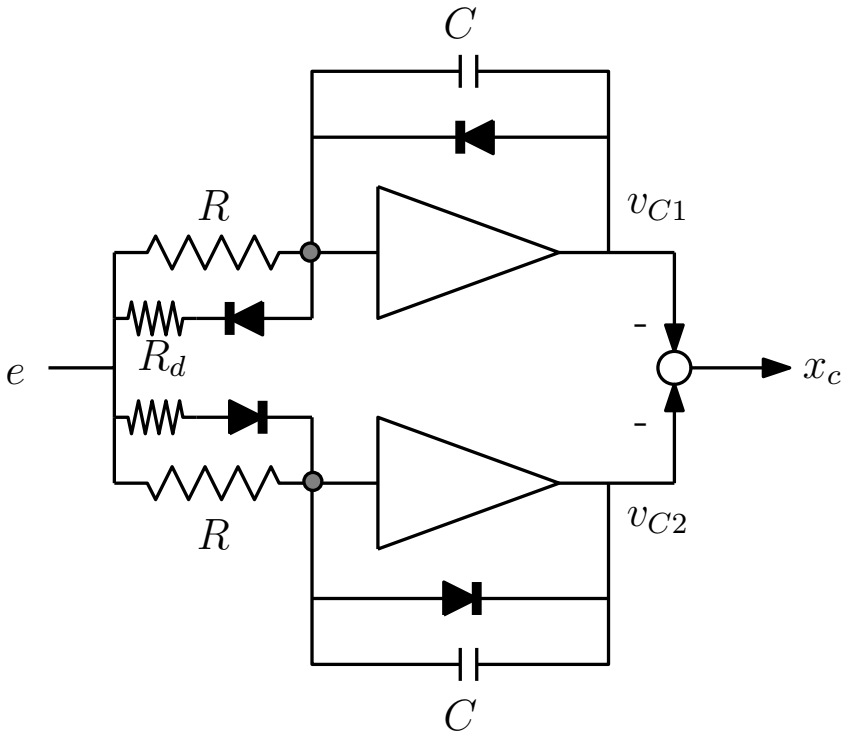




- Clegg's integrator Clegg [1958]:
  - *feedback diodes*: the **positive** part of x_c is all and only coming from the **upper** capacitor (and viceversa)
  - *input diodes*: when $e \leq 0$ the upper capacitor is reset and the lower one integrates (and viceversa) [$R_d \ll 1$]
- As a consequence $\Rightarrow$ e and x_c **never have opposite signs**

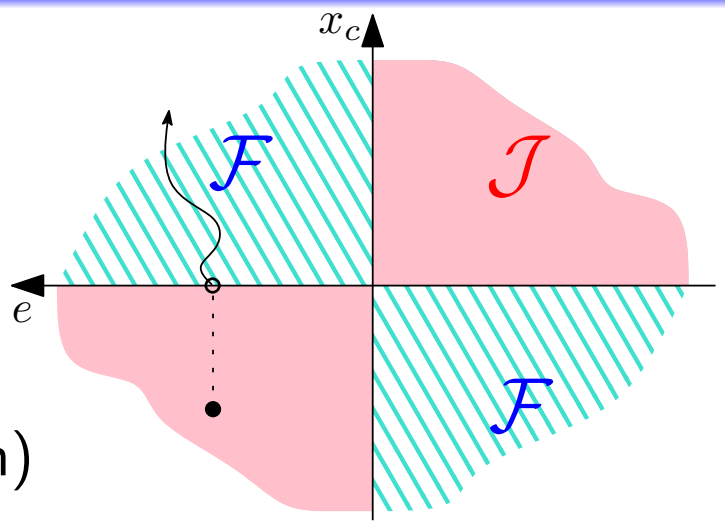
Hybrid dynamics rule flowing or jumping of solutions

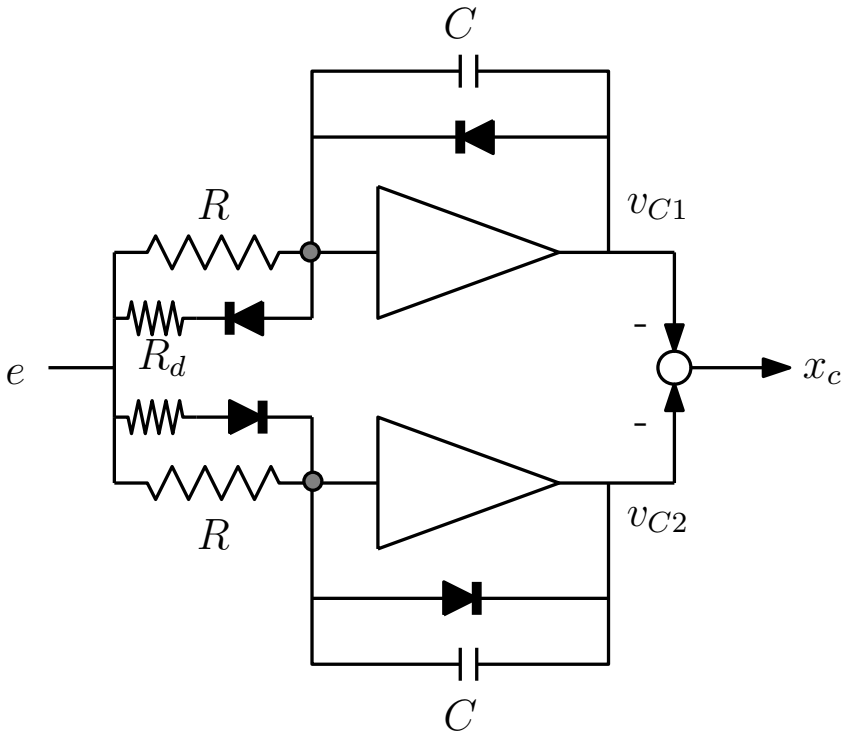
Hybrid Clegg integrator:

$\dot{x}_c = (RC)^{-1} e$, allowed when $x_c e \geq 0$,

$x_c^+ = 0$, allowed when $x_c e \leq 0$,

- Flow set $\mathcal{F}$: where x_c may flow (1st eq'n)
- Jump set $\mathcal{J}$: where x_c may jump (2nd eq'n)





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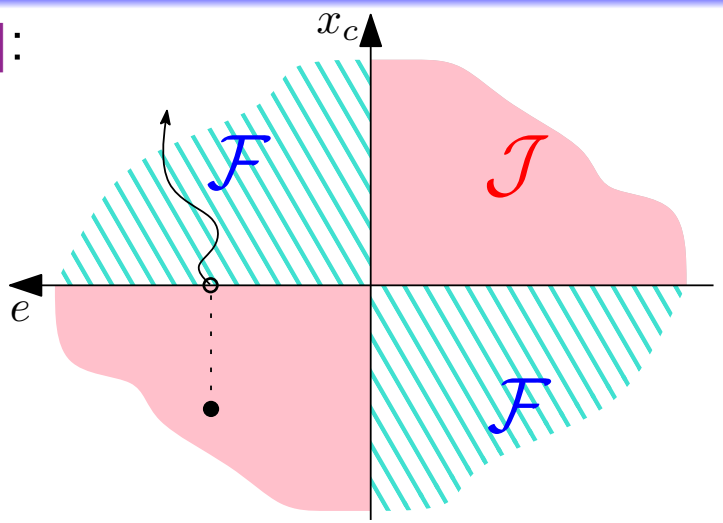
Hybrid solutions of Clegg flow with t and jump with j

Hybrid Clegg integrator Zaccarian et al. [2005]:

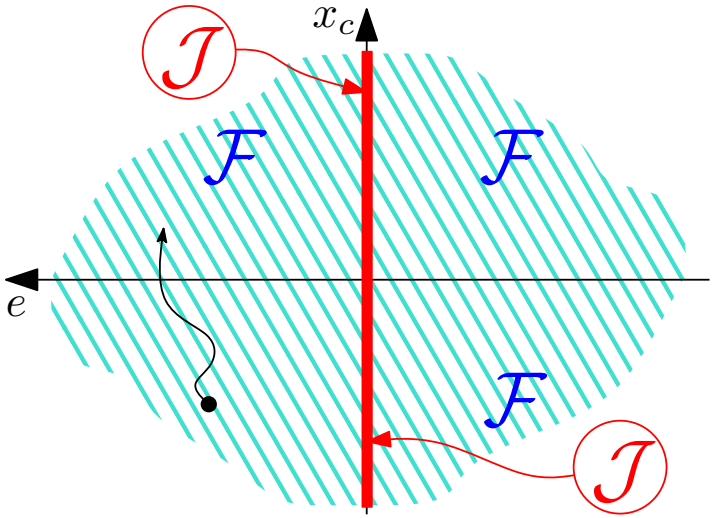
$$\frac{\mathrm{d}x_c(t,j)}{\mathrm{d}t} = (RC)^{-1}e(t,j), \quad x_c(t,j)e(t,j) \geq 0,$$
$$x_c(t,j+1) = 0, \quad x_c(t,j)e(t,j) \leq 0,$$



- Flow set $\mathcal{F} := \{(x_c, e) : x_c e \geq 0\}$ is closed
- Jump set $\mathcal{J} := \{(x_c, e) : x_c e \leq 0\}$ is closed
- Stability is robust! (Teel 2006–2012)

Previous models Clegg [1958], Krishnan and Horowitz [1974], Horowitz and Rosenbaum [1975], Beker et al. [2004]:

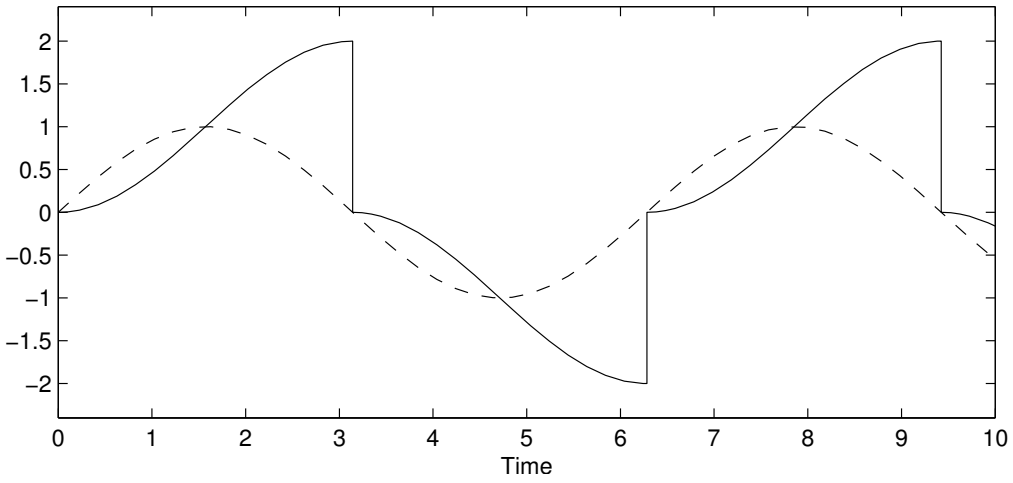
$$\dot{x}_c = (RC)^{-1}e, \quad \text{if } e \neq 0,$$
$$x_c^+ = 0, \quad \text{if } e = 0,$$



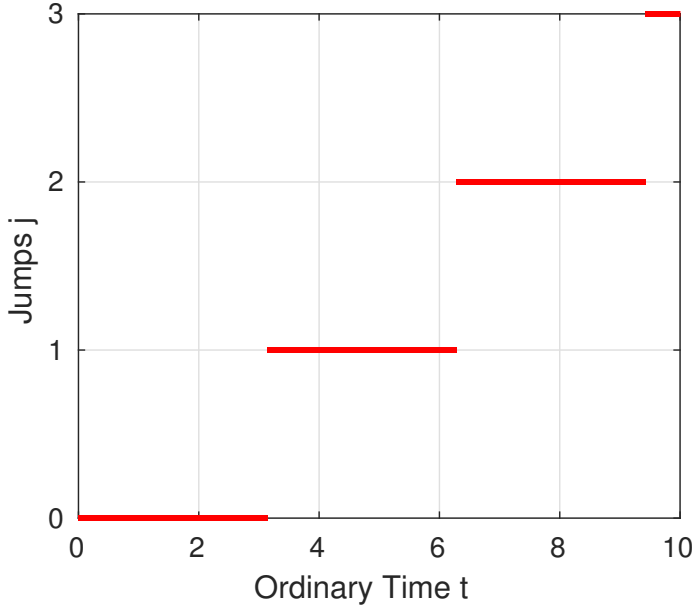
- <u>Inaccurate</u>: solutions $\exists$ s.t. $x_c e < 0$, but Clegg's x_c and e **always** have same sign!
- <u>Unrobust</u>: $\mathcal{F}$ is almost all of $\mathbb{R}^2$ (arbitrary small noise disastrous)
- <u>Conservative</u>: Adds extra solutions

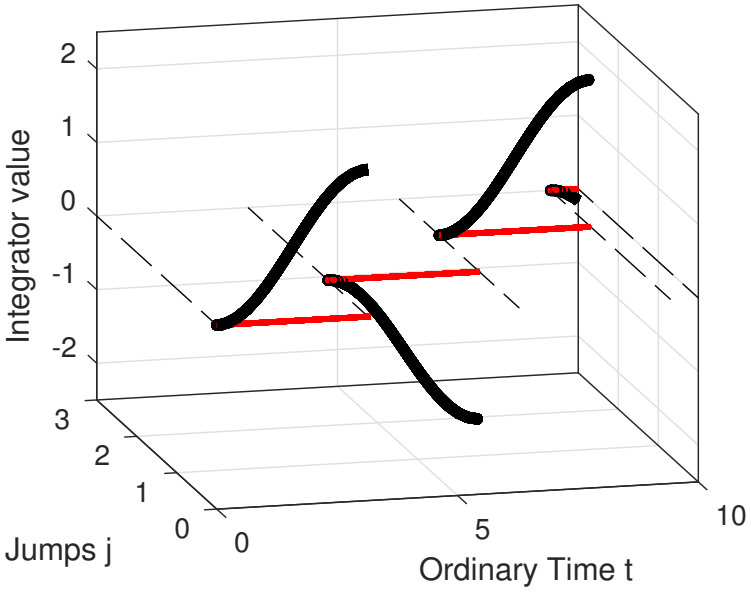
Example: Clegg response to a sine input $e(t,j) = \sin(t)$

- Solid: projection of $x_c(t,j)$ on continuous time axis t
- Dash: projection of $e(t,j)$ on continuous time axis t



- Domain $\mathrm{dom}\, x_c$ (in red) and **graph (bold black)** of the solution x_c





Hybrid Lyapunov theory to study exponential stability

Th'm Teel et al. [2013] Given Euclidean norm $|x| = \sqrt{x^T x}$ and system

$$\mathcal{H} : \begin{cases} \dot{x} = f(x), & x \in \mathcal{F} \\ x^+ = g(x), & x \in \mathcal{J}, \end{cases}$$

assume that function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ satisfies for some scalars c_1, c_2 positive and c_3 positive:

$$c_1|x|^2 \leq V(x) \leq c_2|x|^2, \qquad \forall x \in \mathcal{F} \cup \mathcal{J} \cup g(\mathcal{J})$$

$$\dot{V}(x) := \langle \nabla V(x), f(x) \rangle \leq -c_3|x|^2, \qquad \forall x \in \mathcal{F}, \text{(Flow cond'n)}$$

$$\Delta V(x) := V(g(x)) - V(x) \leq -c_3|x|^2, \qquad \forall x \in \mathcal{J}, \text{(Jump cond'n)}$$

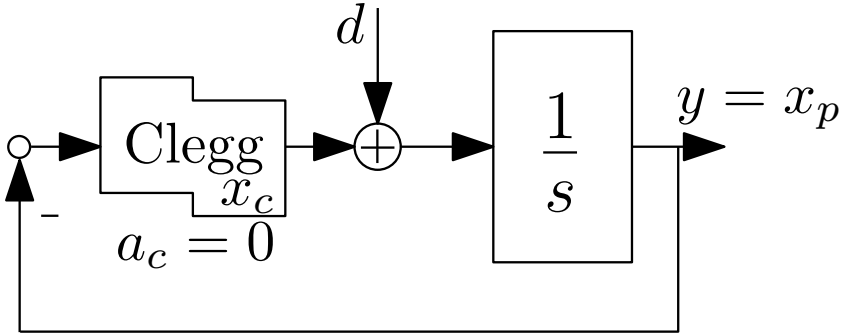
then the origin is uniformly globally exponentially stable (UGES) for $\mathcal{H}$, namely there exist $K, \lambda > 0$ such that all solutions x satisfy

$$|x(t,j)| \leq K e^{\lambda(t+j)} |x(0,0)|, \quad \forall (t,j) \in \operatorname{dom} x$$

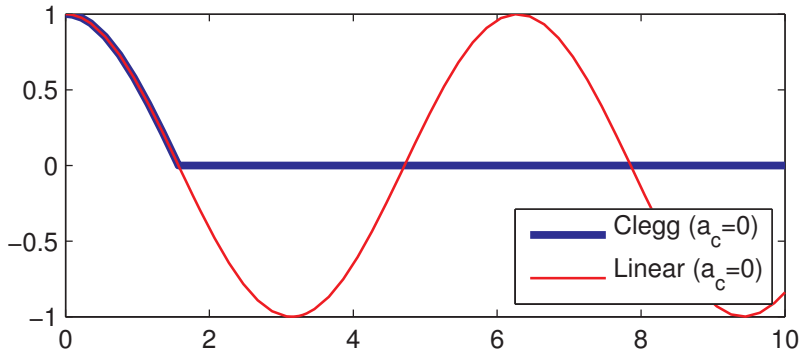
Note: Lyapunov conditions comprise **flow** and **jump** conditions.
Note: UGES is characterized in terms of hybrid time (t,j)

Example 1: Clegg connected to an integrator plant

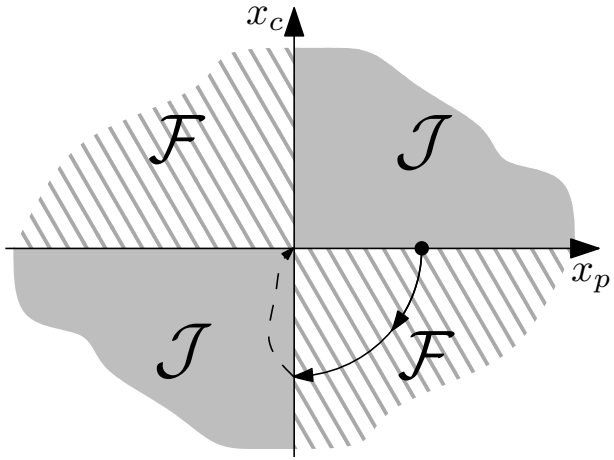
- Block diagram:

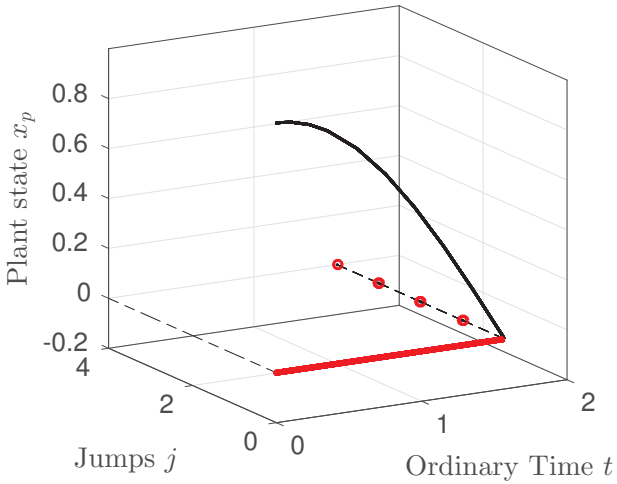


- Output response overcomes linear control limitations Beker et al. [2001]



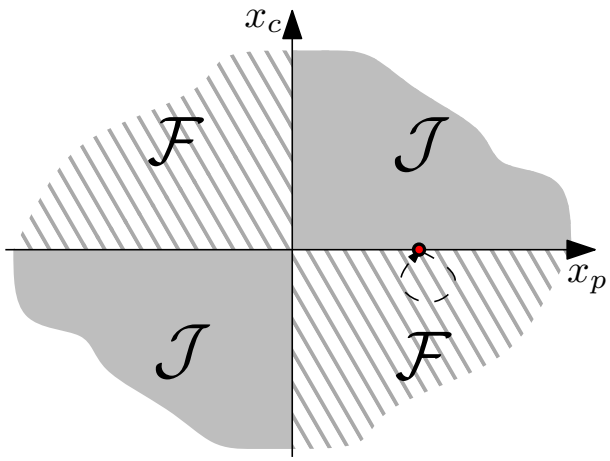
- Hybrid solution on the phase-plane and plant state response

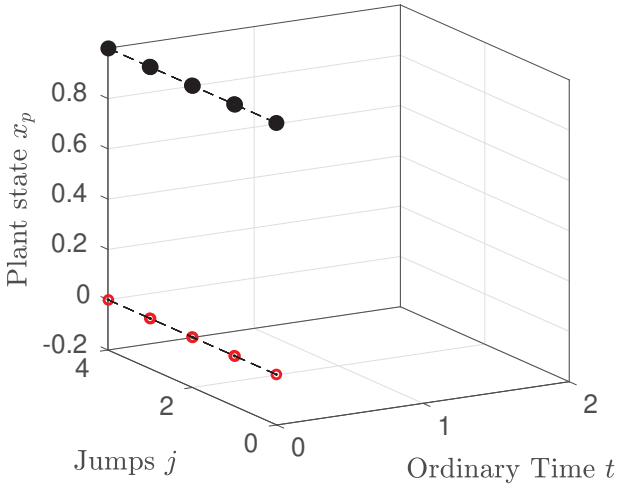




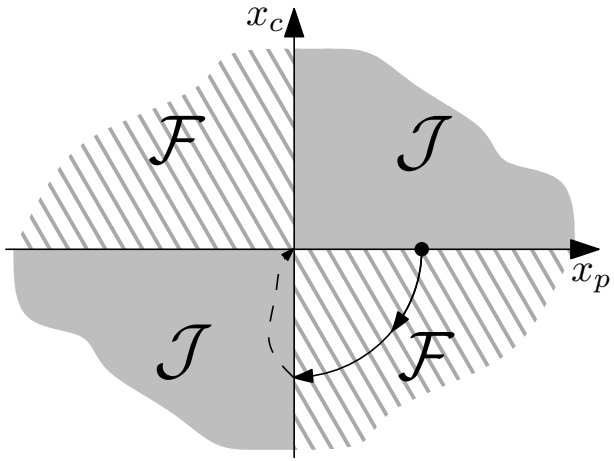
Example 1: there exists another **bad** solution!

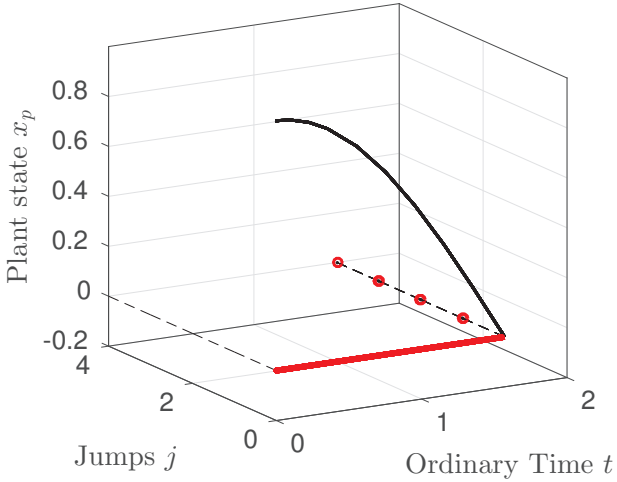
- A **bad** nonconverging "discrete" solution





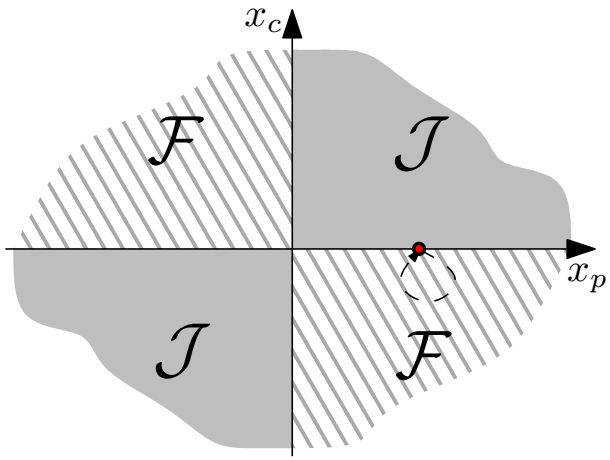
- Hybrid solution on the phase-plane and controller response

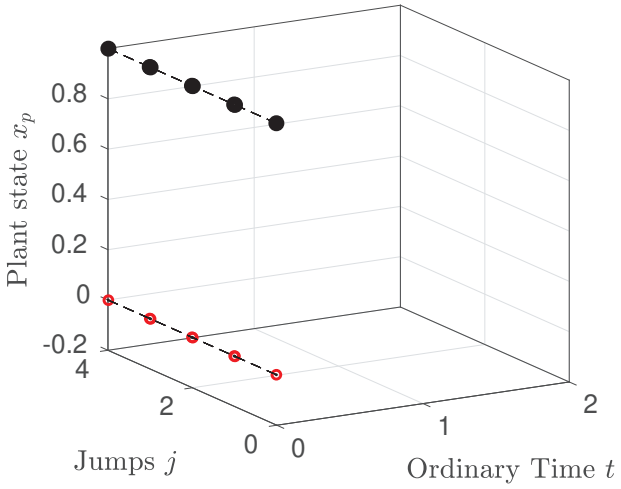




Space or time regularization to eliminate **bad** solutions

- A **bad** nonconverging "discrete" solution





- Two options to avoid multiple instantaneous resets (jumps)

Inhibit jumps for ρ continuous time after each jump

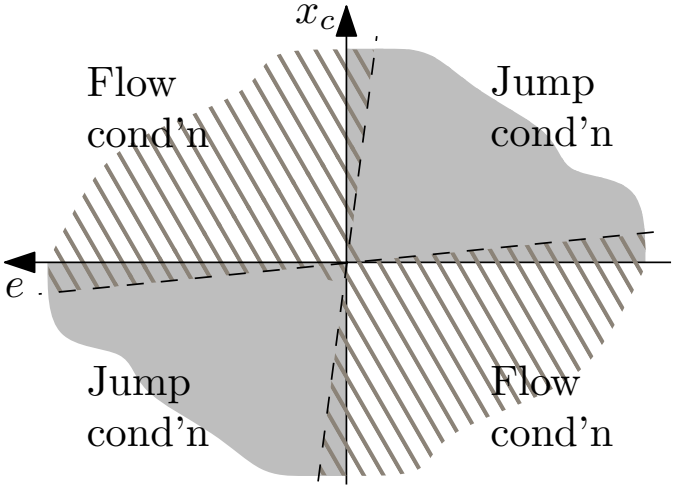
Time regularization

Ensure that jumps map to the interior of $\mathcal{F}$, away from $\mathcal{J}$

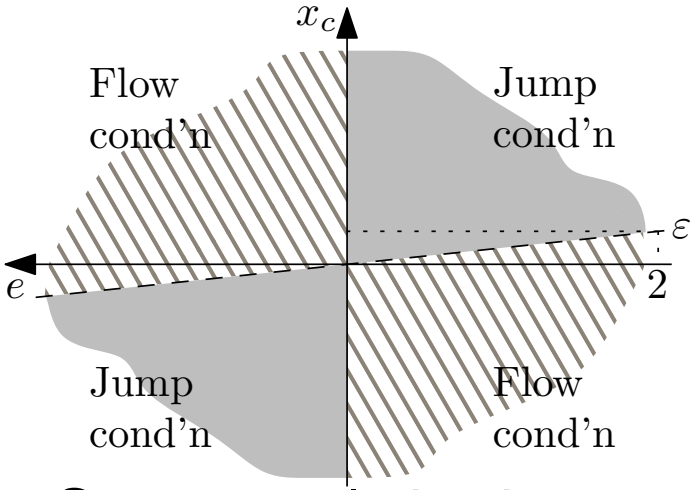
Space regularization

Space or time regularization to eliminate **bad** solutions

- **Lyapunov conditions** must be enforced on suitable sets



Time regularization



Space regularization

- Two options to avoid multiple instantaneous resets (jumps)

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Time regularization

Ensure that jumps map to the interior of $\mathcal{F}$, away from $\mathcal{J}$

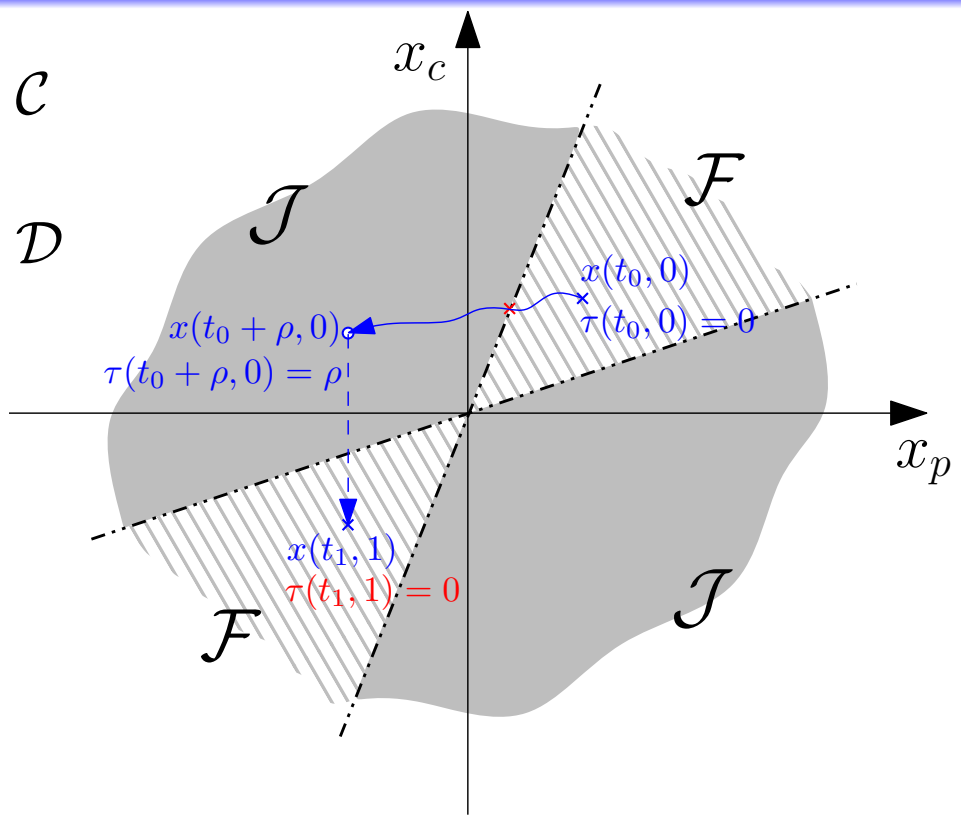
Space regularization

Time Regularization can induce overflowing in the set $\mathcal{J}$

$$\mathcal{H}\begin{cases} \dot{x} &= Ax + Bd \\ \dot{\tau} &= 1 \\ x^+ &= Gx \\ \tau^+ &= 0 \\ z &= C_z x + D_{zd} d \end{cases}$$

$(x, \tau) \in \mathcal{C}$

$(x, \tau) \in \mathcal{D}$



$$\mathcal{C} = \{(x, \tau) : x \in \mathcal{F} \text{ or } \tau \in [0, \rho]\}$$
$$\mathcal{D} = \{(x, \tau) : x \in \mathcal{J} \text{ and } \tau \geq \rho\}$$

$$\mathcal{F} = \left\{x \in \mathbb{R}^n : x^\top M x \leq 0\right\}$$
$$\mathcal{J} = \left\{x \in \mathbb{R}^n : x^\top M x \geq 0\right\}$$

☺ Persistent flowing of solutions

☹ Overflow in the set $\mathcal{J}$ $(t_1 \geq t_0 + \rho)$

Theorem: Nešić et al. [2011] From partial homogeneity it follows that

$$\text{LAS} \Leftrightarrow \text{GES} \Leftrightarrow \mathcal{L}_p \text{ stable from } d \Leftrightarrow \text{ISS from } d$$

From dwell time ⇒ performance wrt ordinary time t

Nešić et al. [2008], Fichera et al. [2016a]

- Dwell-time allows using *classical* continuous-time performance indexes

Definition (t-decay rate)

Hybrid system $\mathcal{H}$ has t-decay rate $\alpha > 0$ if there exists $K > 0$ such that all solutions satisfies

$$|x(t,j)| \leq K \exp(-\alpha t)|x(0,0)|, \quad \text{for all } (t,j) \in \mathrm{dom}(x).$$

Definition (t-$\mathcal{L}_2$ gain)

System $\mathcal{H}$ is finite t-$\mathcal{L}_2$ gain stable from d to z with gain (upper bounded by) $\gamma > 0$ if any solution to $\mathcal{H}$ starting with $x(0,0) = 0$ satisfies

$$\|x\|_{2t} \leq \gamma \|d\|_{2t} := \gamma \left(\sum_{j \in \mathbb{Z}} \int_{t_j}^{t_{j+1}} |d(\tau,j)|^2 d\tau \right)^{\frac{1}{2}}, \quad \text{for all } d \in t\text{-}\mathcal{L}_2.$$

Performance analysis result: $V(x) = x^\top P x$ quadratic

Nešić et al. [2008], Fichera et al. [2016a]

Theorem: Consider system $\mathcal{H}$. If there exist $P = P^\top > 0$, non-negative τ_F, $\tau_R \in \mathbb{R}_{\geq 0}$ and positive $\bar{\gamma}$, s.t.

$$(Flow) \qquad \begin{pmatrix} A^\top P + PA - \tau_F M & PB & C_z^\top \\ B^\top P & -\bar{\gamma} I & D_{zd}^\top \\ C_z & D_{zd} & -\bar{\gamma} I \end{pmatrix} < 0,$$

$$(Jump) \qquad G^\top P G - P + \tau_R M \leq 0,$$

Then, by virtue of $V(x) = x^T P x$, for any γ satisfying

$$\gamma \geq \bar{\gamma}, \quad \gamma > \sqrt{2}|D_{zd}|,$$

there exists $\overline{\rho} > 0$ such that for any $\rho \in (0, \overline{\rho})$:

1. the set $\mathcal{A} = \{(x, \tau) : x = 0\}$ is globally exponentially stable for the hybrid system $\mathcal{H}$ with $d = 0$;
2. the t-$\mathcal{L}_2$ gain from d to z is $\leq \gamma$, for all $d \in t$-$\mathcal{L}_2$.

Piecewise quadratic Lyapunov function construction

- Proposed in Zaccarian et al. [2011], Loquen [2010], Aangenent et al. [2010]
  - Given $N \geq 2$ (number of sectors)
  - Patching angles:

$$-\theta_\epsilon = \theta_0 < \theta_1 < \cdots < \theta_N = \frac{\pi}{2} + \theta_\epsilon$$

  - Patching hyperplanes

$$\Theta_i = \begin{bmatrix} 0_{1\times(n-2)} & \sin(\theta_i) & \cos(\theta_i) \end{bmatrix}^T$$

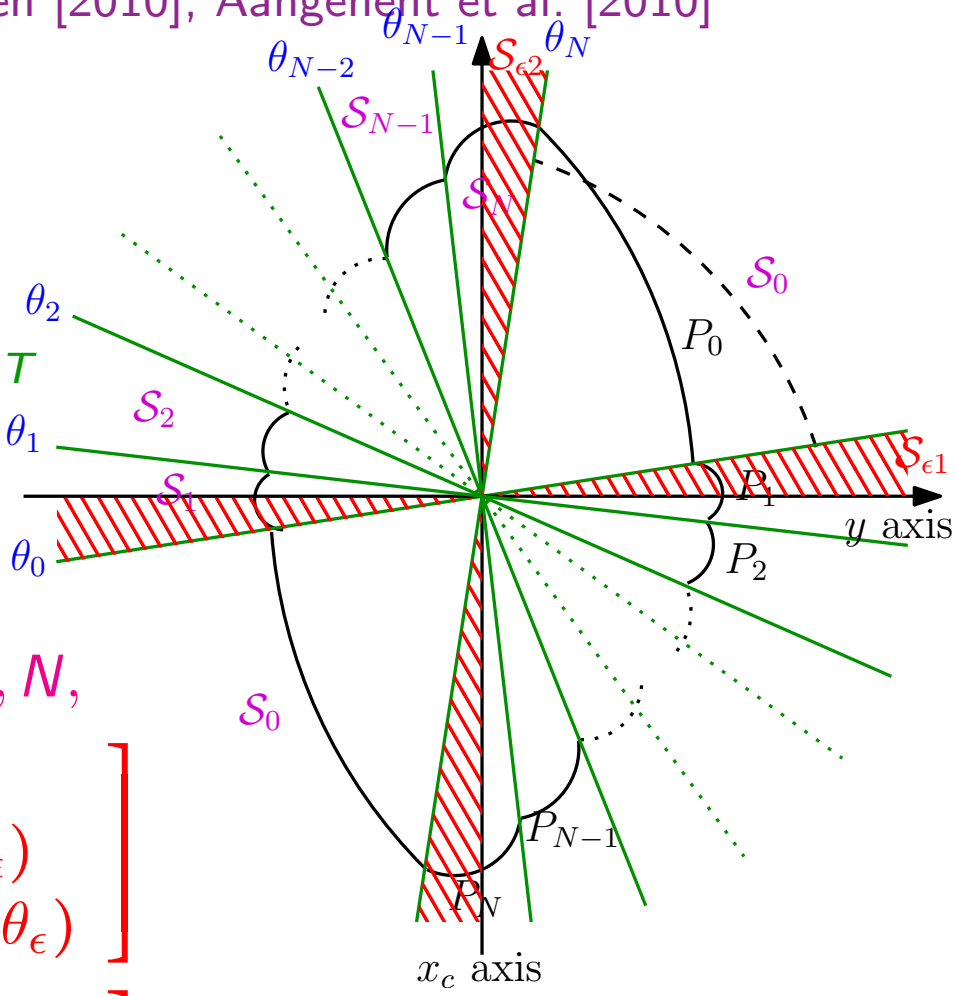
  - Sector matrices:

$$S_0 := \Theta_0\Theta_N^T + \Theta_N\Theta_0^T$$

$$S_i := -(\Theta_i\Theta_{i-1}^T + \Theta_{i-1}\Theta_i^T), \quad i = 1, \ldots, N,$$

$$S_{\epsilon 1} := \begin{bmatrix} 0_{(n-2)\times(n-2)} & 0 & 0 \\ 0 & 0 & \sin(\theta_\epsilon) \\ 0 & \sin(\theta_\epsilon) & -2\cos(\theta_\epsilon) \end{bmatrix}$$

$$S_{\epsilon 2} := \begin{bmatrix} 0_{(n-2)\times(n-2)} & 0 & 0 \\ 0 & -2\cos(\theta_\epsilon) & \sin(\theta_\epsilon) \\ 0 & \sin(\theta_\epsilon) & 0 \end{bmatrix}$$



Plant **outputs**:

$$y = C_p x = [0 \cdots 0\ 1]x$$
$$z = C_z x \text{ (that is, } D_{zd} = 0)$$

Piecewise quadratic Lyapunov theorem

Theorem Zaccarian et al. [2011], Loquen [2010]: If the following LMIs in the green unknowns (where $Z = [I_{n-2}\ 0_{(n-2)\times 2}]$) are feasible:

$$(Flow)\begin{bmatrix} A^T P_i + P_i A + \tau_{Fi} S_i & P_i B_d & C_z^T \\ \star & -\gamma I & 0 \\ \star & \star & -\gamma I \end{bmatrix} < 0, i = 1, \dots, N,$$

$$(Jump) \quad G^T P_1 G - P_0 + \tau_J S_0 \leq 0$$

$$(Cont'ty) \quad \Theta_{i\perp}^T (P_i - P_{i+1}) \Theta_{i\perp} = 0, \quad i = 0, \dots, N-1,$$

$$(Cont'ty) \quad \Theta_{N\perp}^T (P_N - P_0) \Theta_{N\perp} = 0$$

$$(Overlap) \quad G^T P_1 G - P_1 + \tau_{\epsilon 1} S_{\epsilon 1} \leq 0$$

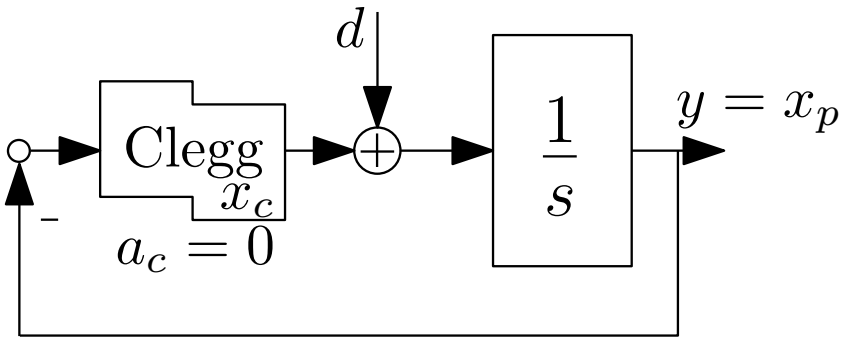
$$(Overlap) \quad G^T P_1 G - P_N + \tau_{\epsilon 2} S_{\epsilon 2} \leq 0$$

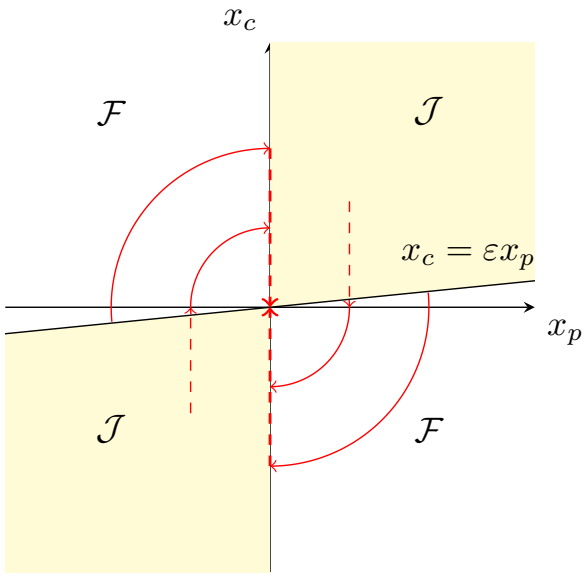
$$(Origin)\begin{bmatrix} Z(A^T P_0 + P_0 A) Z^T & Z P_0 B_d & Z C_z^T \\ \star & -\gamma I & 0 \\ \star & \star & -\gamma I \end{bmatrix} < 0,$$

then global exponential stability + finite t-$\mathcal{L}_2$ gain γ from d to z

Example 1: Clegg connected to an integrator

- Block diagram:





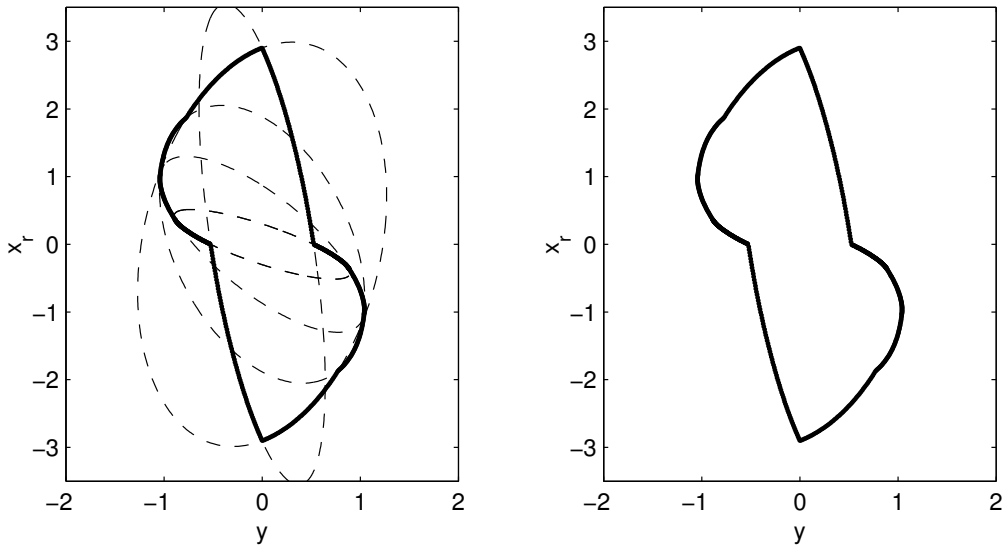
- Quadratic Lyapunov functions are unsuitable Zaccarian et al. [2011]

- Gain γ_{dy} estimates (N = # of sectors)

N	2	4	8	50
gain γ_{dy}	2.834	1.377	0.914	0.87

- A lower bound: $\sqrt{\frac{\pi}{8}} \approx 0.626$

- Lyapunov func'n level sets for $N = 4$

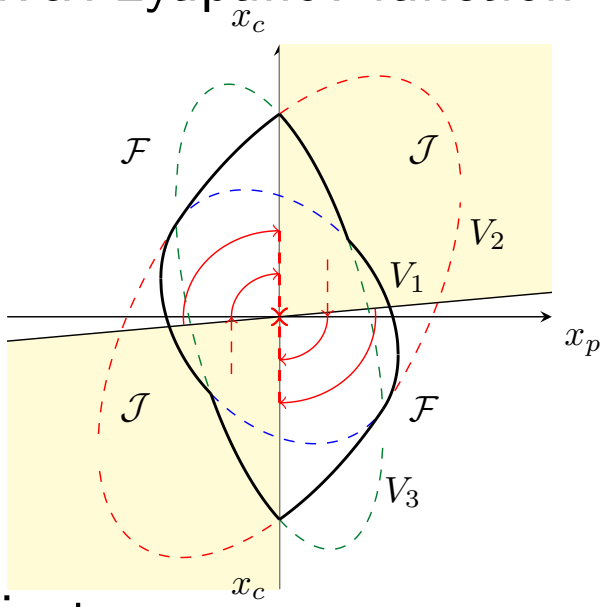


- $P_1, \ldots, P_4$ cover 2nd/4th quadrants
- P_0 covers 1st/3rd quadrants

Convex Lyapunov function for Example 1 Della Rossa et al. [2019]

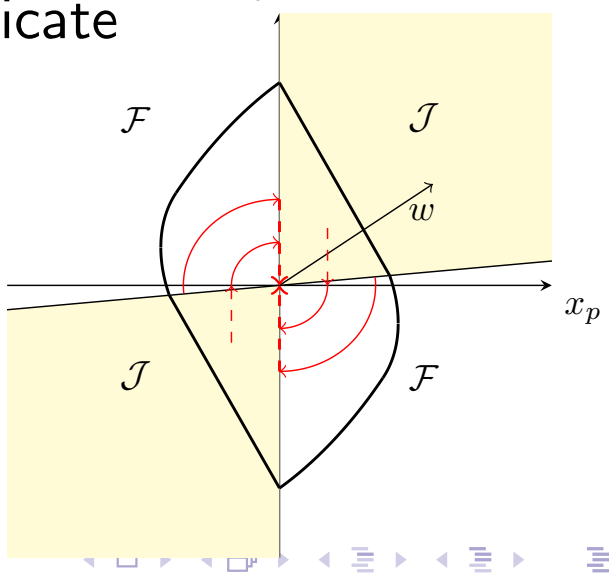
- Mid of quadratics provides another nonconvex Lyapunov function certifying GES for the previous example

$$
\begin{aligned}
V_{\mathrm{mid}}(x) &= \mathrm{mid}\{V_1, V_2, V_3\} \\
&:= \max\{\min\{V_1, V_2\}, \\
&\quad \min\{V_2, V_3\}, \min\{V_1, V_3\}\}
\end{aligned}
$$



- May twist it into a convex Lyapunov certificate

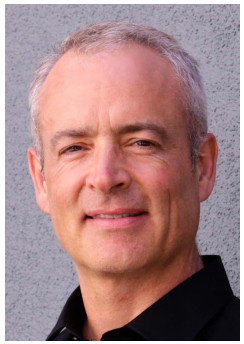
$$
V_{\mathrm{conv}}(x) = \begin{cases} V_{\mathrm{mid}}(x), & \text{if } x \in \mathcal{F} \\ \langle w, x \rangle^2 & \text{if } x \in \mathcal{J} \end{cases}
$$



Stabilization, regulation and tracking using adaptive First Order Reset Elements (FORE) with automotive applications

Collaborative work with:

D Nesic



AR Teel

D Alberer

FS Panni

H Waschl

M Cordioli

F Palazzetti

F Panizzolo

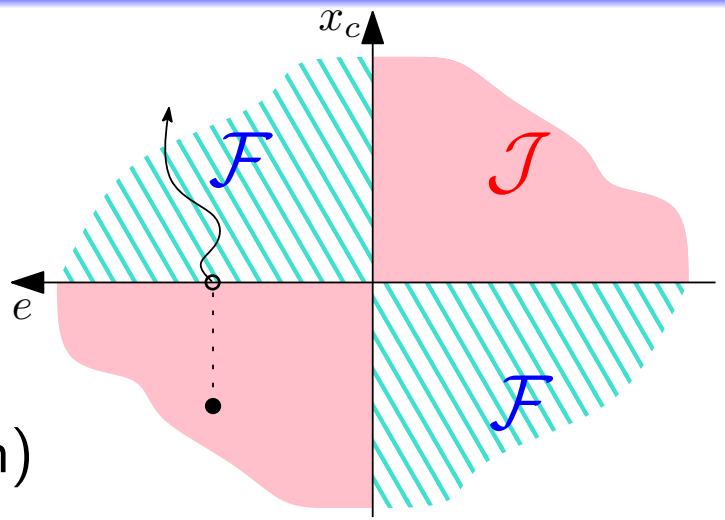
Hybrid dynamics of the Clegg integrator (recall)

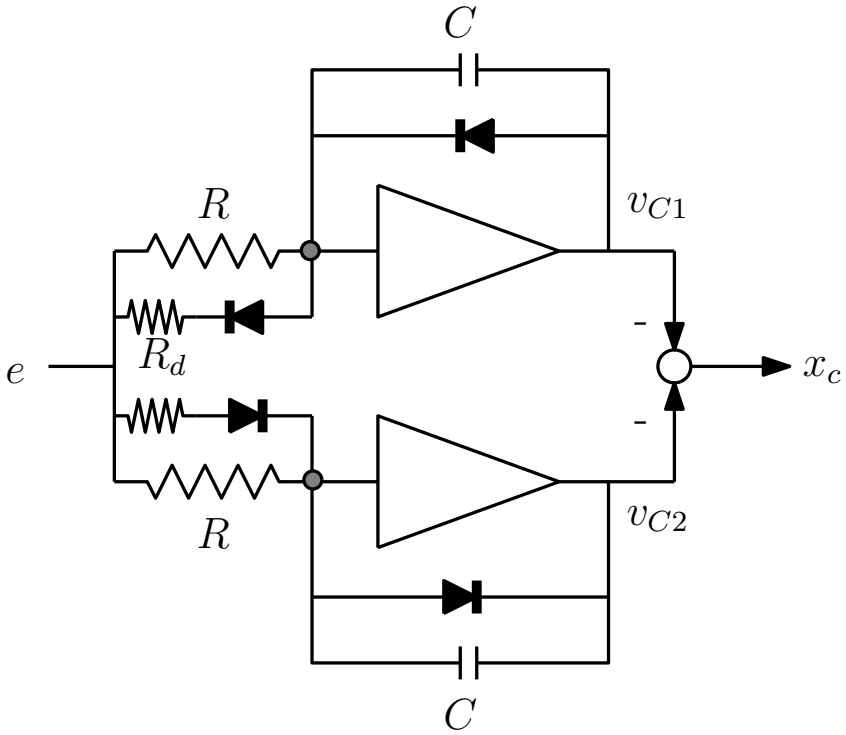
Hybrid Clegg integrator:

$$\dot{x}_c = (RC)^{-1} e, \quad \text{allowed when } x_c e \geq 0,$$

$$x_c^+ = 0, \quad \text{allowed when } x_c e \leq 0,$$

- Flow set $\mathcal{F}$: where x_c may flow (1st eq'n)
- Jump set $\mathcal{J}$: where x_c may jump (2nd eq'n)



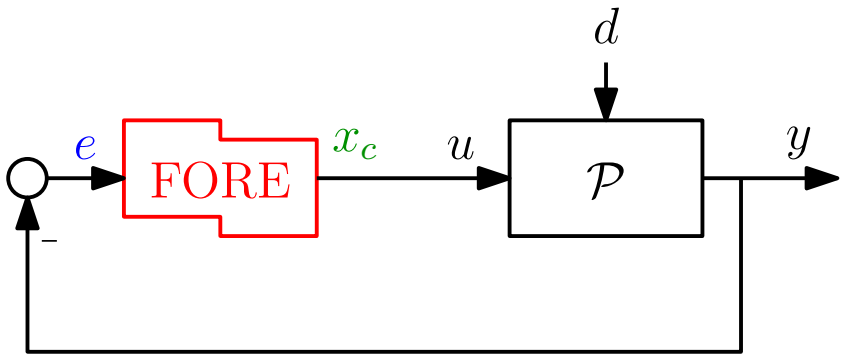


- Clegg's integrator Clegg [1958]:
  - *feedback diodes*: the **positive** part of x_c is all and only coming from the **upper** capacitor (and viceversa)
  - *input diodes*: when $e \leq 0$ the upper capacitor is reset and the lower one integrates (and viceversa) [$R_d \ll 1$]
- As a consequence $\Rightarrow$ e and x_c **never have opposite signs**

Stabilization using hybrid jumps to zero

First Order Reset Element Nešić et al. [2011], Loquen et al. [2007]:

$$\dot{x}_c = a_c x_c + b_c e, \qquad x_c e \geq 0,$$
$$x_c^+ = 0, \qquad x_c e \leq 0,$$



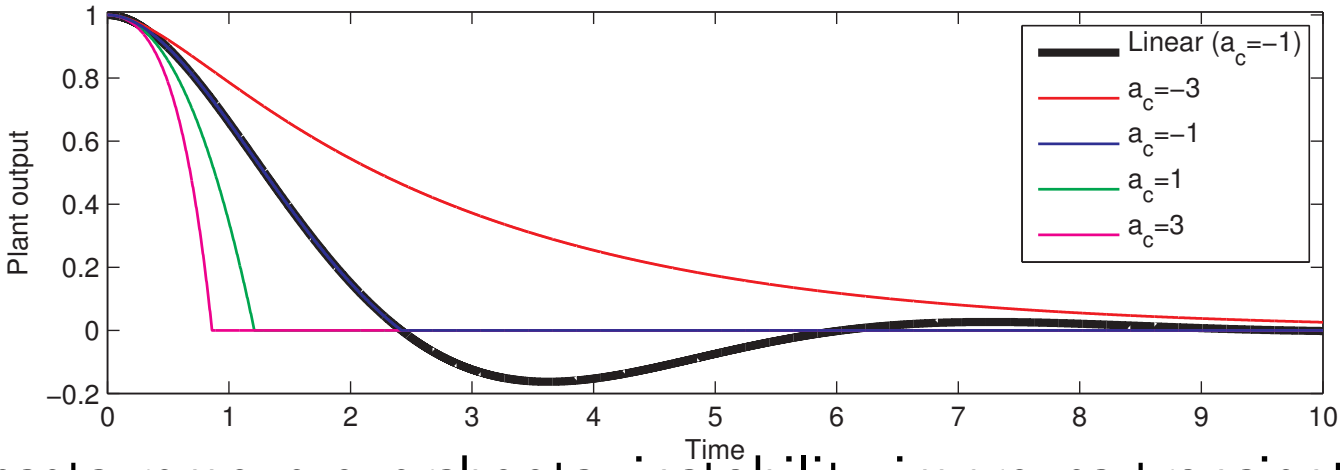
Theorem If $\mathcal{P}$ is linear, minimum phase and relative degree one, **then** a_c, b_c or (a_c, b_c) large enough $\Rightarrow$ global exponential stability

Theorem In the planar case, γ_{dy} shrinks to zero as parameters grow

Simulation with:

$$\mathcal{P} = \frac{1}{s}$$

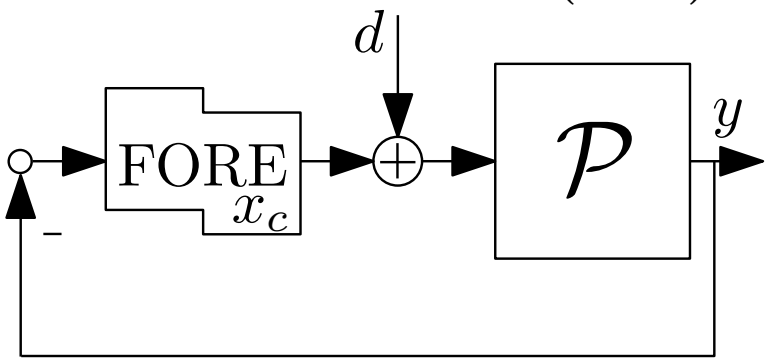
$$b_c = 1$$



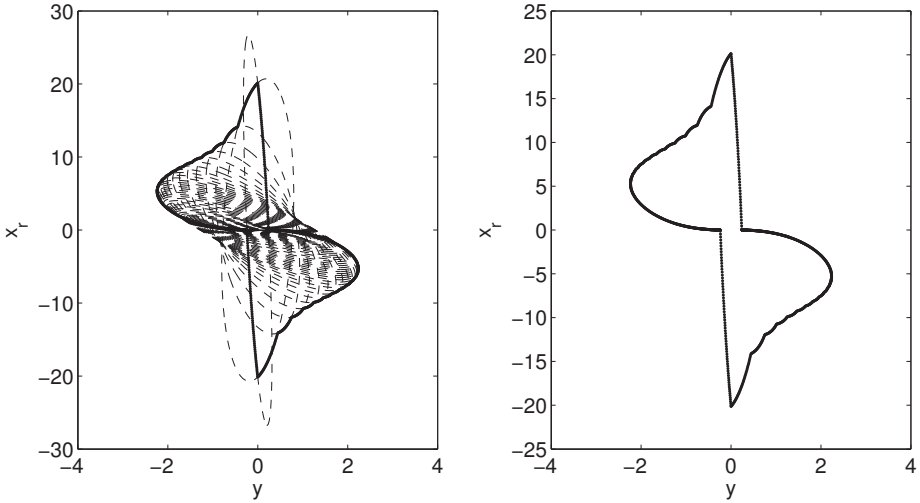
Interpretation: Resets remove overshoots, instability improves transient

Example 2 (Hollot et al.): FORE (any a_c) and linear plant

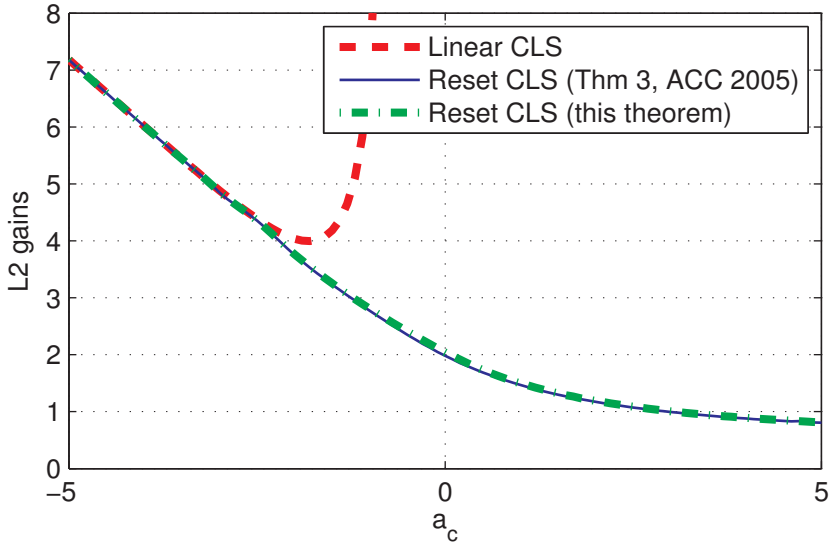
- Block diagram ($\mathcal{P} = \frac{s+1}{s(s+0.2)}$)



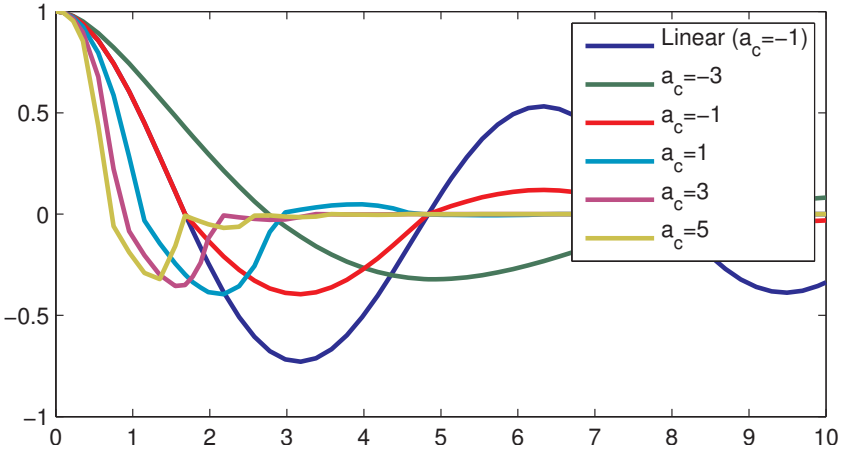
- $a_c = 1$: level set with $N = 50$



- Gain γ_{dy} estimates



- Time responses

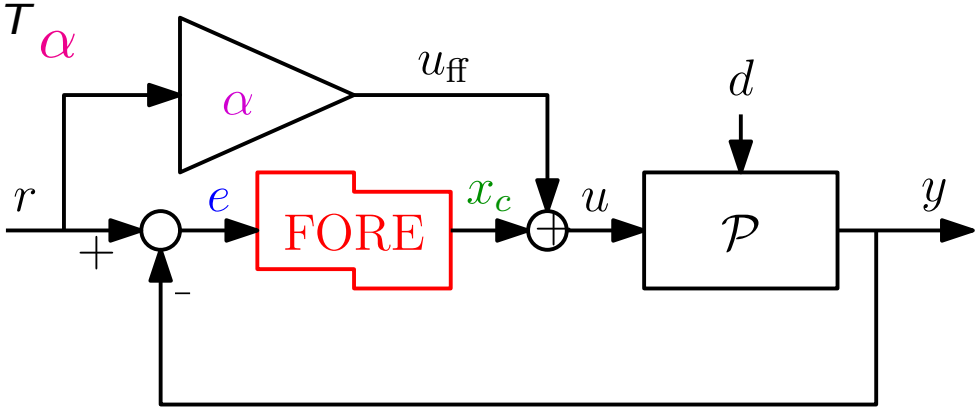


Set point adaptive **regulation** using hybrid jumps to zero

- Relevant works Panni et al. [2014], Loquen et al. [2008]
- Parametric feedforward $u_{\text{ff}} = \Psi(r)^T \alpha$

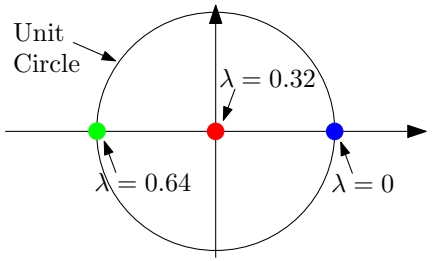
$$\begin{cases} \dot{x}_c = a_c x_c + b_c e, \\ \dot{\alpha} = 0, \end{cases} \qquad x_c e \geq 0,$$

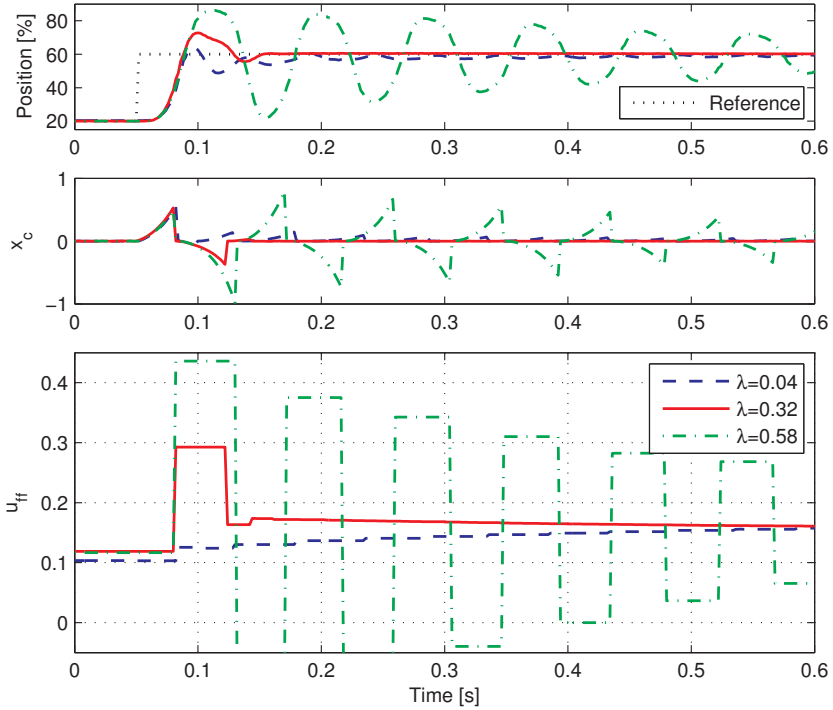
$$\begin{cases} x_c^+ = 0, \\ \alpha^+ = \alpha + \lambda \frac{\Psi(r)}{|\Psi(r)|^2} x_c, \end{cases} \qquad x_c e \leq 0,$$



Theorem: If FORE stabilizes with $r = 0$, then for constant r, $y \to r$

Lemma: Tuning of λ using discrete-time rules (Ziegler-Nichols)

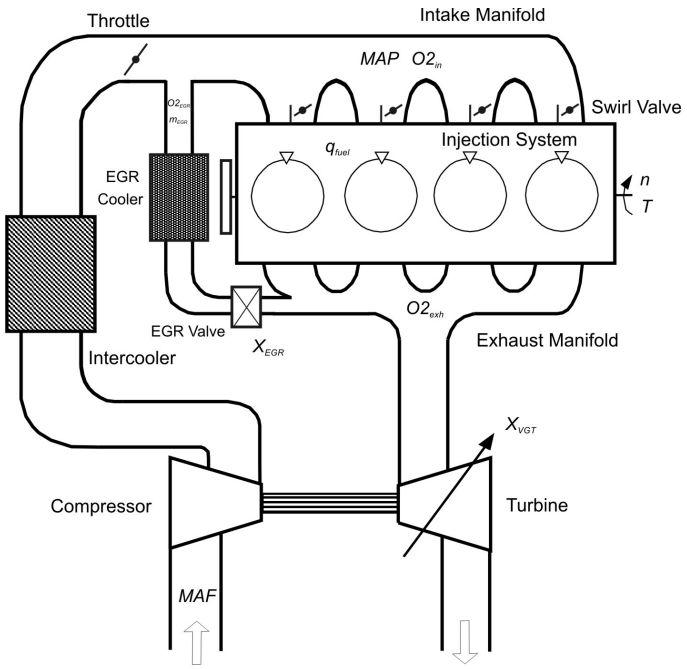


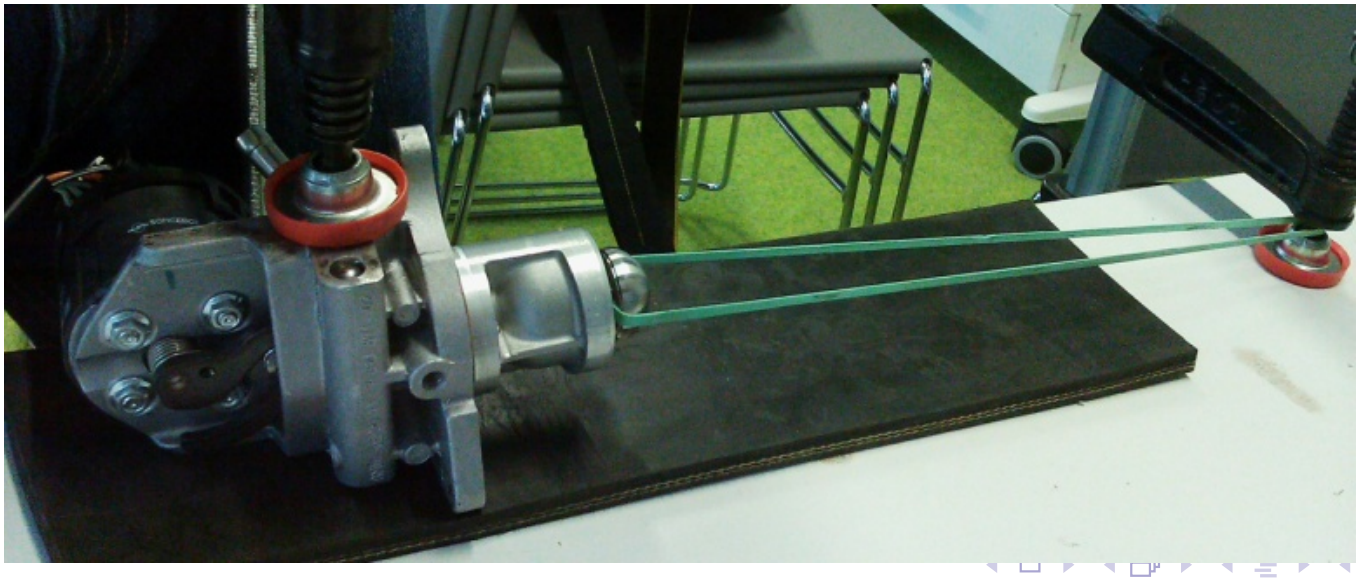


Example: EGR Experiment (next slide)

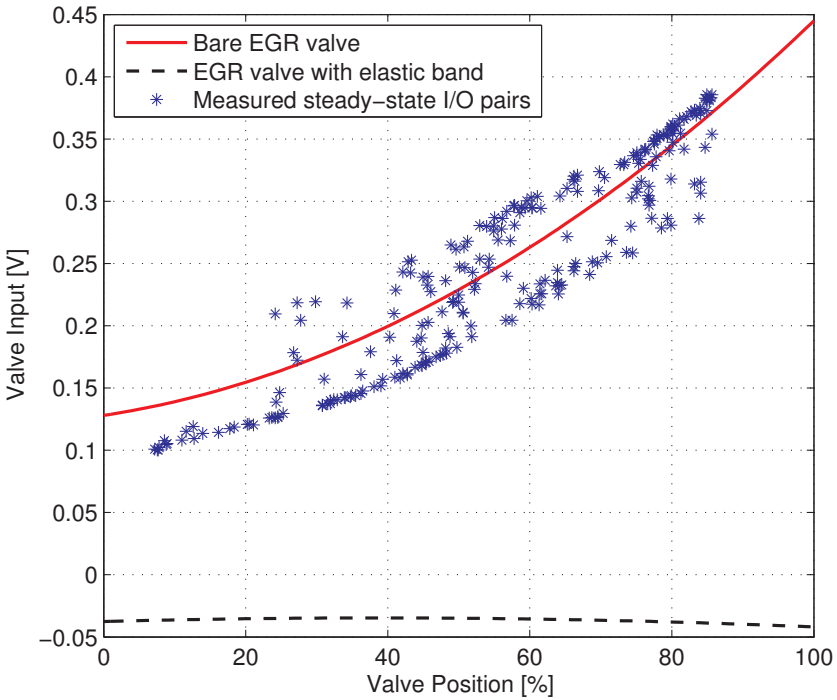
Fast regulation of EGR valve position in Diesel engines

- Reported in Panni et al. [2014]
- EGR: Recirculates Exhaust Gas in Diesel engines
- Subject to strong disturbances
  ⇒ need aggressive controllers
  (recall exp. unstable transients)

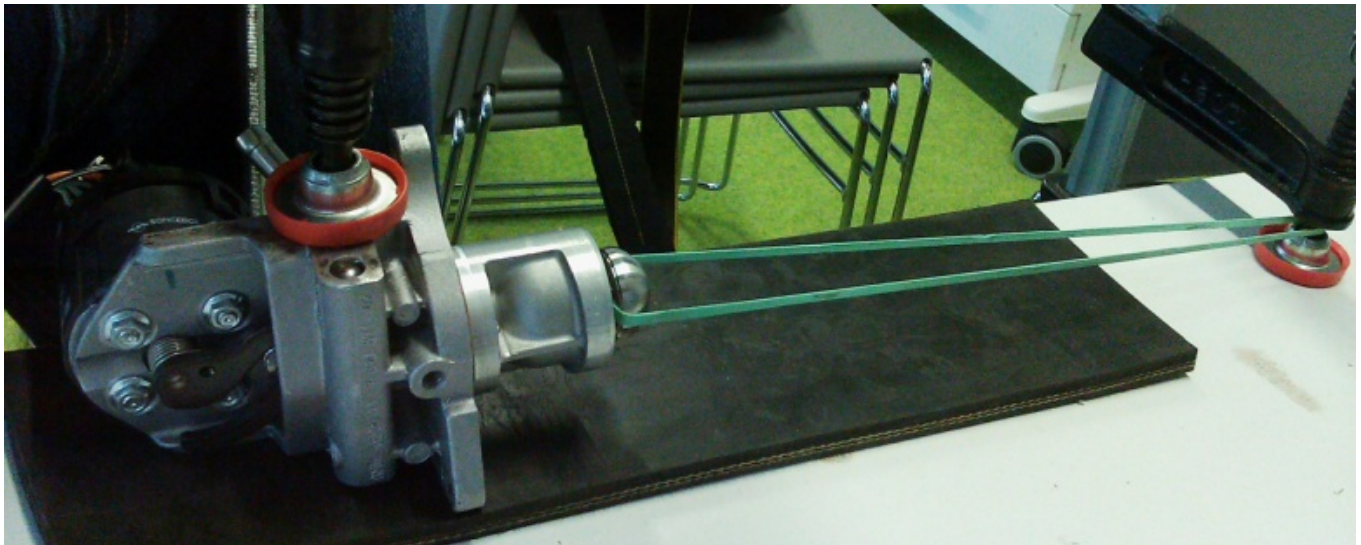




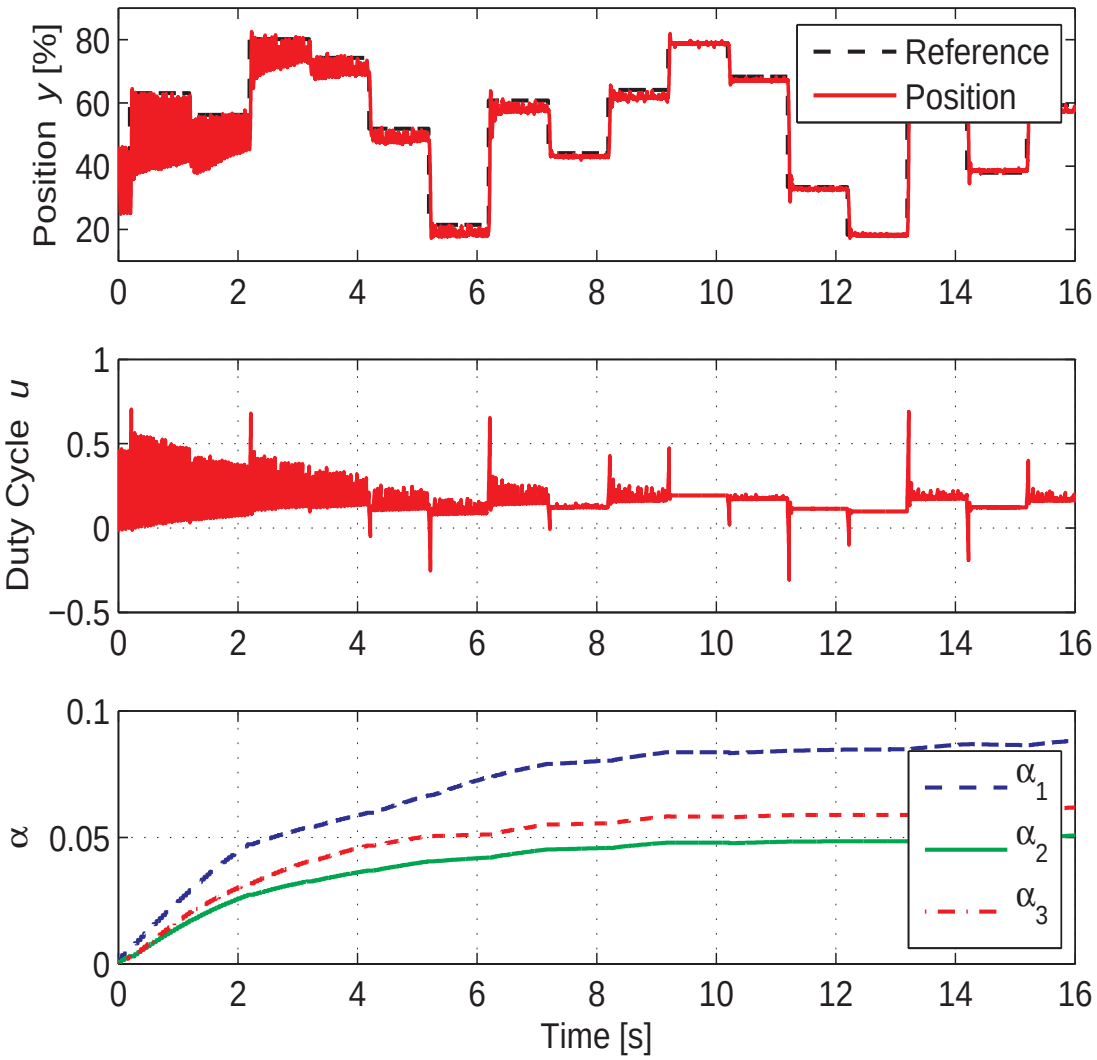
Feedforward: α converges to suitable parametrization



- $\star$: steady-state input/output pairs (stiction!!)
- Red Solid: $u_{ff} = \Psi^T(r)\alpha^*$, with α^* steady-state for α
- Black dashed: $u_{ff} = \Psi^T(r)\bar{\alpha}^*$ when pulling the valve with an elastic band

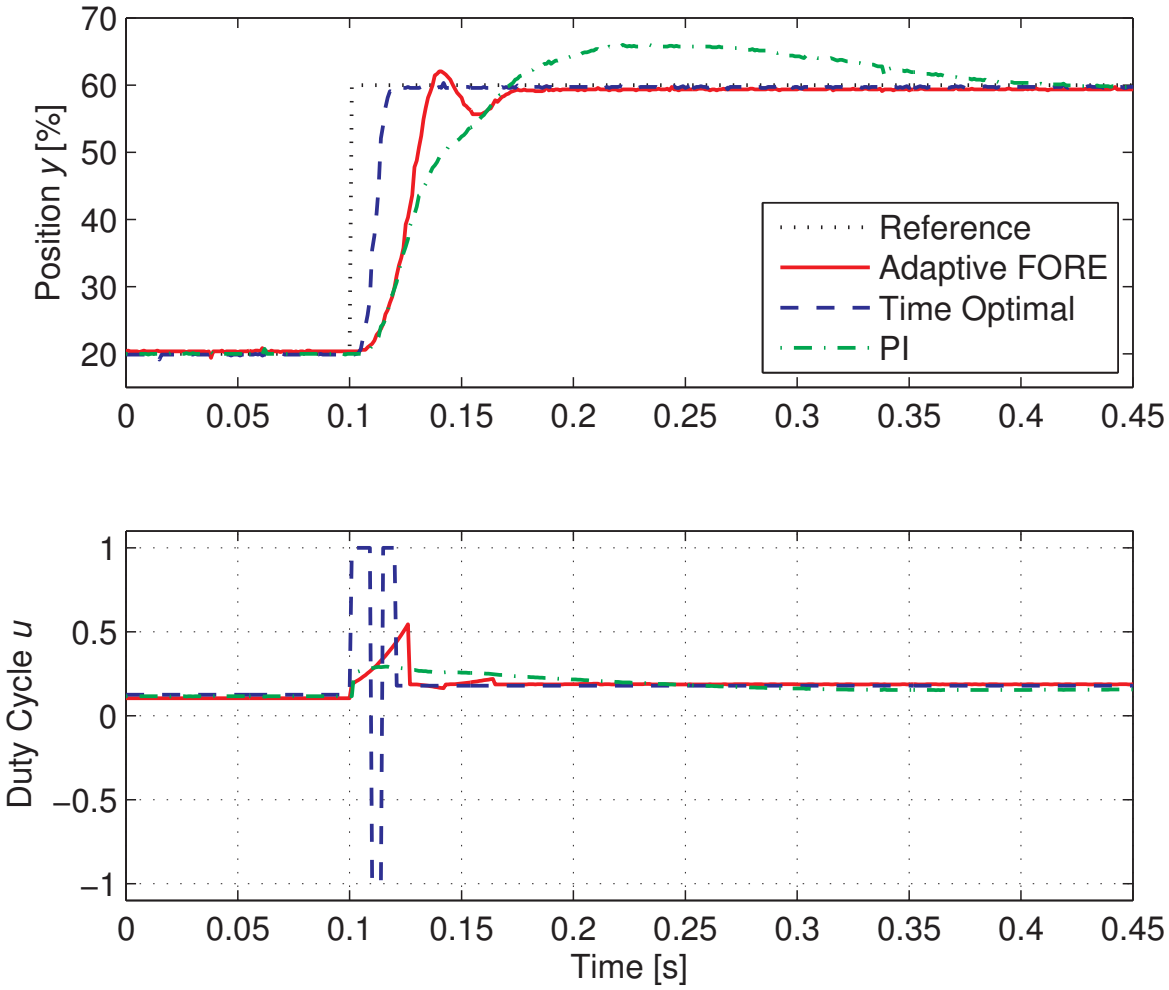


Experimental adaptation of feedforward in lab setup



- **Random sequence** of position reference steps
- Adaptation gain λ intentionally selected small and α initialized at zero to **appreciate transient**
- Initial transient shows **typical oscillations** arising with inaccurate feedforward
- As $\alpha \to \alpha^*$, the step responses become **increasingly desirable**

Laboratory experiments close to time-optimal

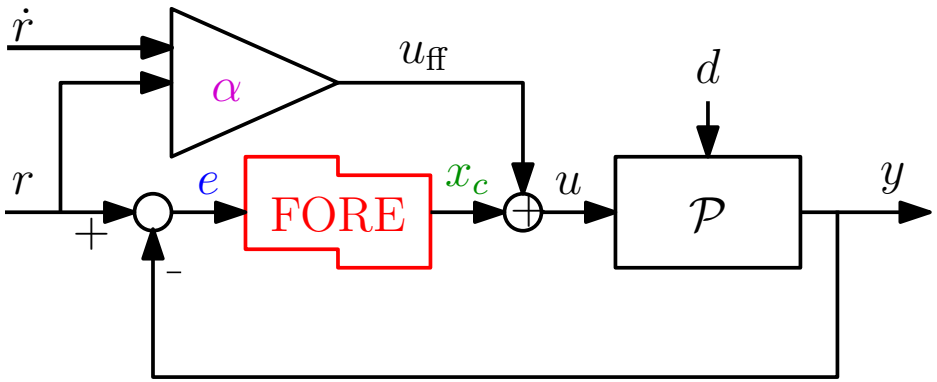


- Time-optimal: unrobust, obtained via trial and error
- PI: Tuned using standard MATLAB tools
- Adaptive FORE: Response after $\alpha \to \alpha^* = (0.128,\ 0.087,\ 0.115)$

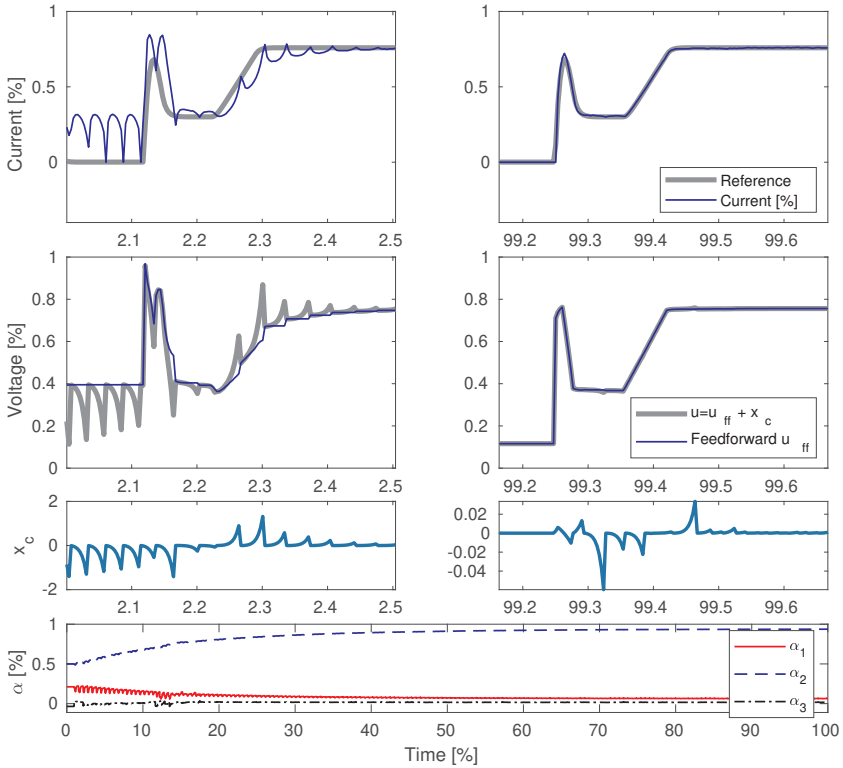
- Note the exponentially diverging voltage: aggressive action for disturbance rejection on the real engine

Extension to **reference tracking** is ongoing work

- NEW Parametric feedforward:
  $u_{\text{ff}} = \Psi(r)^T \alpha \Rightarrow \Psi(r, \dot{r})^T \alpha$
- Proposed in Cordioli et al. [2015]
- Revisited in Cocetti et al. [2019]



- Electromechanical valve current tracking in power-split transmissions application



High-order reset elements and LMI-based reset plant-order designs

Collaborative work with:

F Fichera

C Prieur

S Tarbouriech

F Ferrante

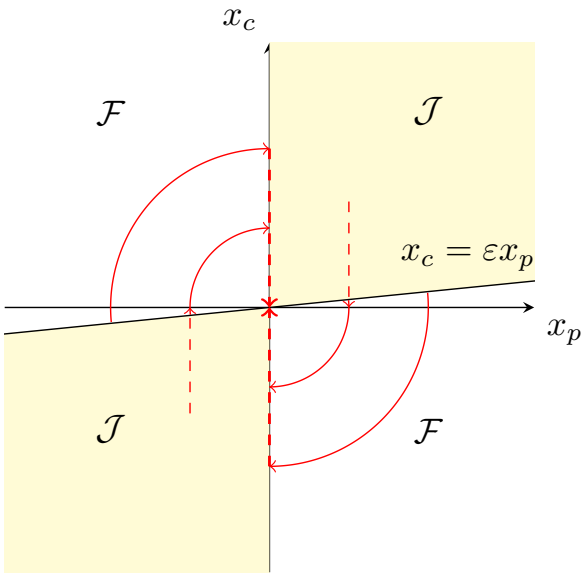
A Lyapunov interpretation of the Clegg integrator logic

Prieur et al. [2013]

Reset to the minimizer of the hybrid Lyapunov function $V(x) = \left\| \begin{bmatrix} x_p \\ x_c \end{bmatrix} \right\|^2$:

$$x_c^+ = \phi(x_p) := \underset{x_c}{\operatorname{argmin}}\, V(x_p, x_c) = 0$$

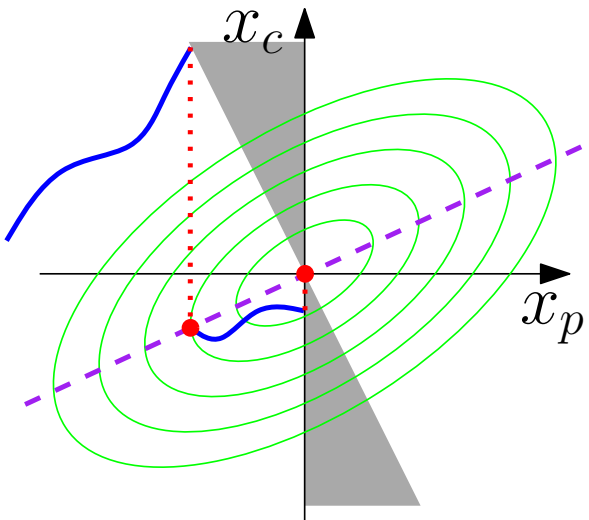
- Reset whenever function $V_p(x_p) := V(x_p, \phi(x_p)) = |x_p|^2$ starts increasing



- Jump set selected as

$$\mathcal{J} = \left\{ \left[\begin{smallmatrix} x_p \\ x_c \end{smallmatrix}\right] \in \mathbb{R}^n : \left[\begin{smallmatrix} x_p \\ x_c \end{smallmatrix}\right]^\top M \left[\begin{smallmatrix} x_p \\ x_c \end{smallmatrix}\right] \geq 0 \right\},$$

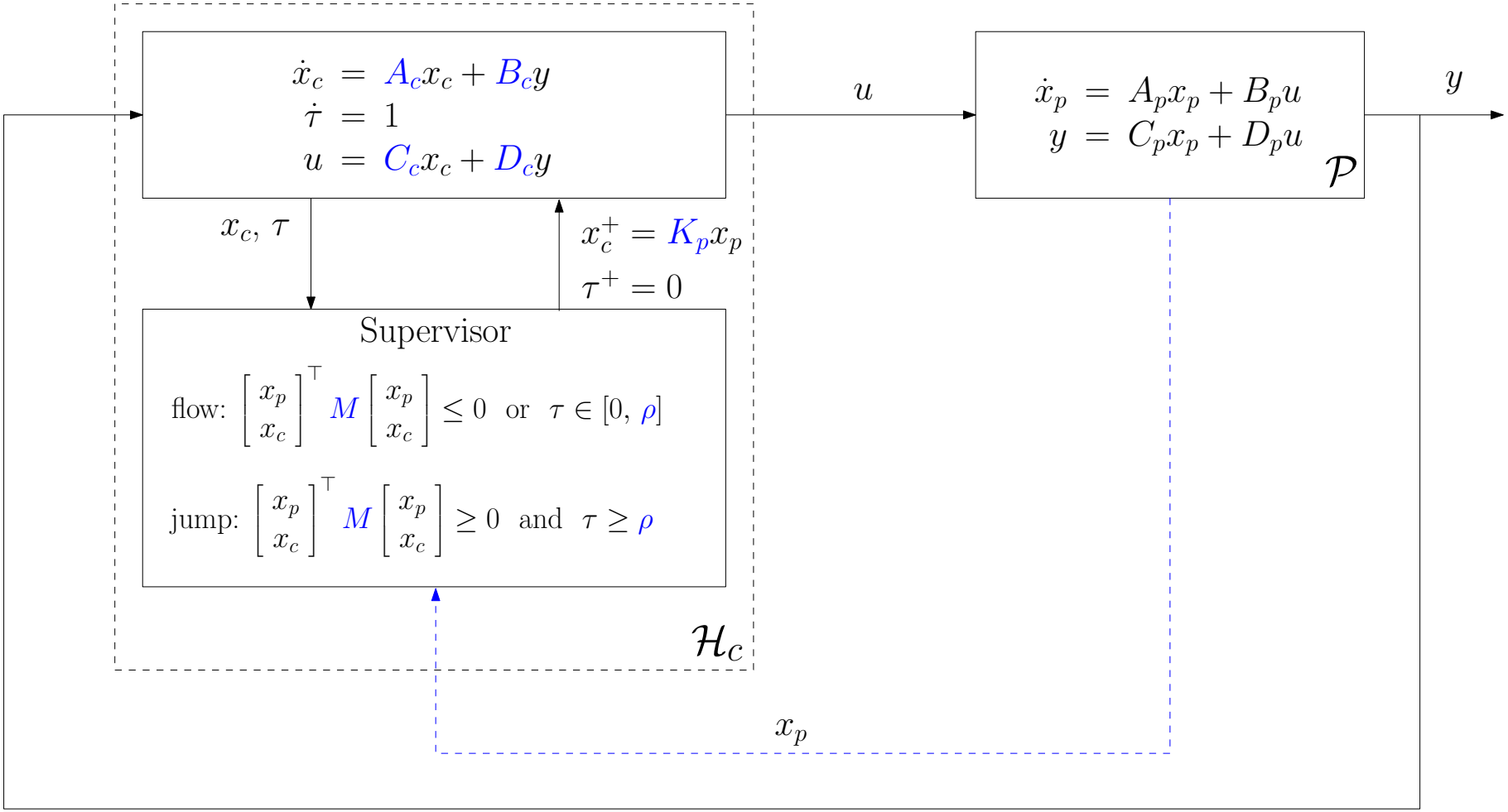
$$\left[\begin{smallmatrix} x_p \\ x_c \end{smallmatrix}\right]^\top M \left[\begin{smallmatrix} x_p \\ x_c \end{smallmatrix}\right] = \langle \nabla V_p(x_p), A_p x_p + B_p x_c \rangle = \dot{V}_p(x_p, x_c)$$



- Nonlinear extensions in Prieur et al. [2013]

An architecture for homogeneous reset controller design

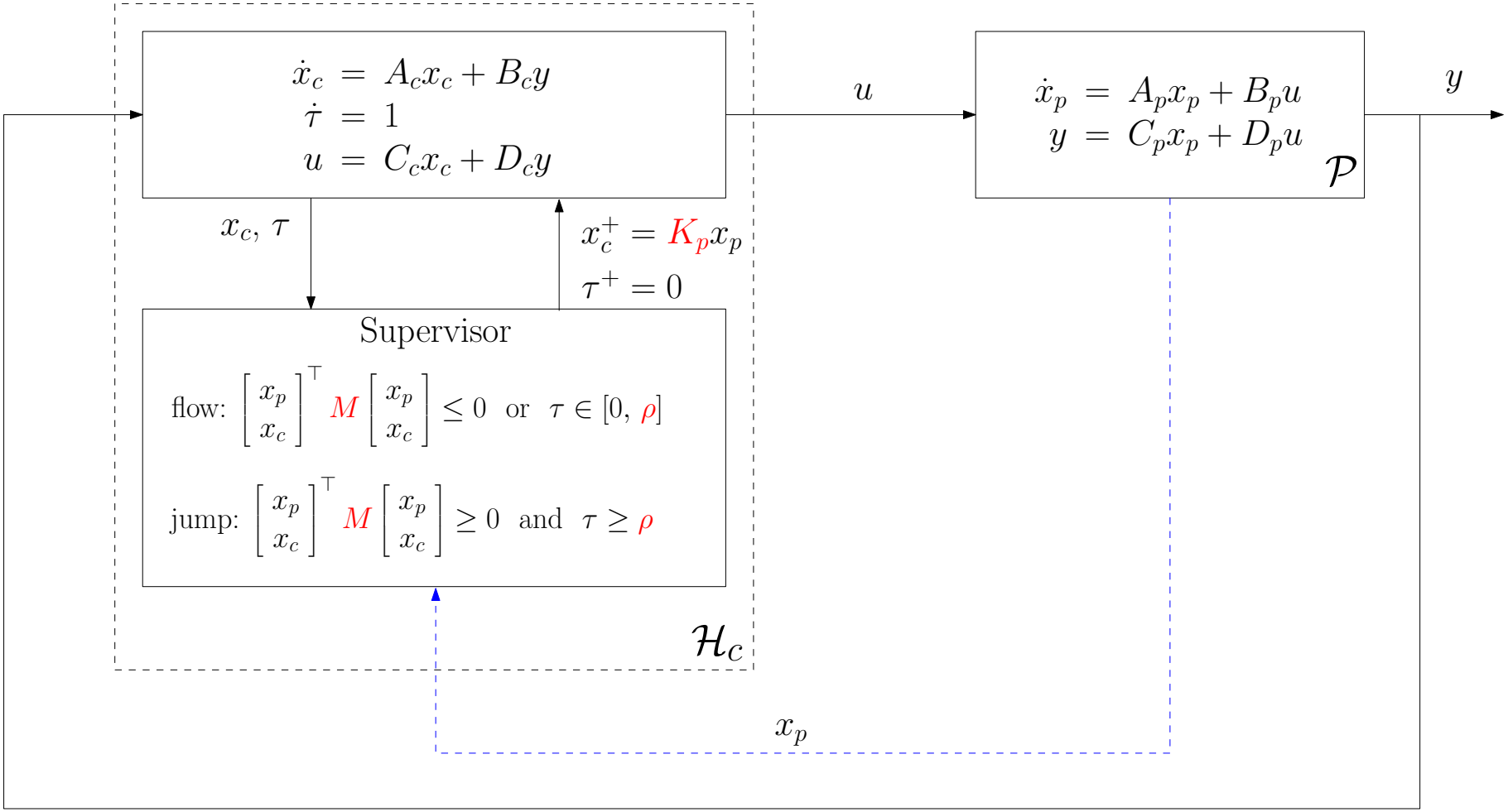
May design the reset rules K_p, M, ρ only or the whole dynamics



Flow dynamics is given, design Jump sets and rules

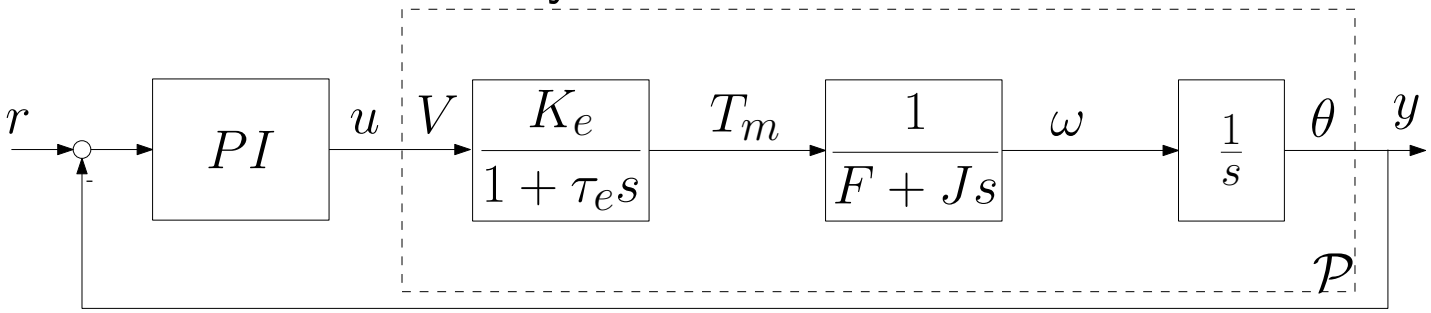
Fichera et al. [2012]

Matrices A_c, B_c, C_c, D_c are given. Design K_p, M and ρ



Positioning system: overshoot reduction Fichera et al. [2012]

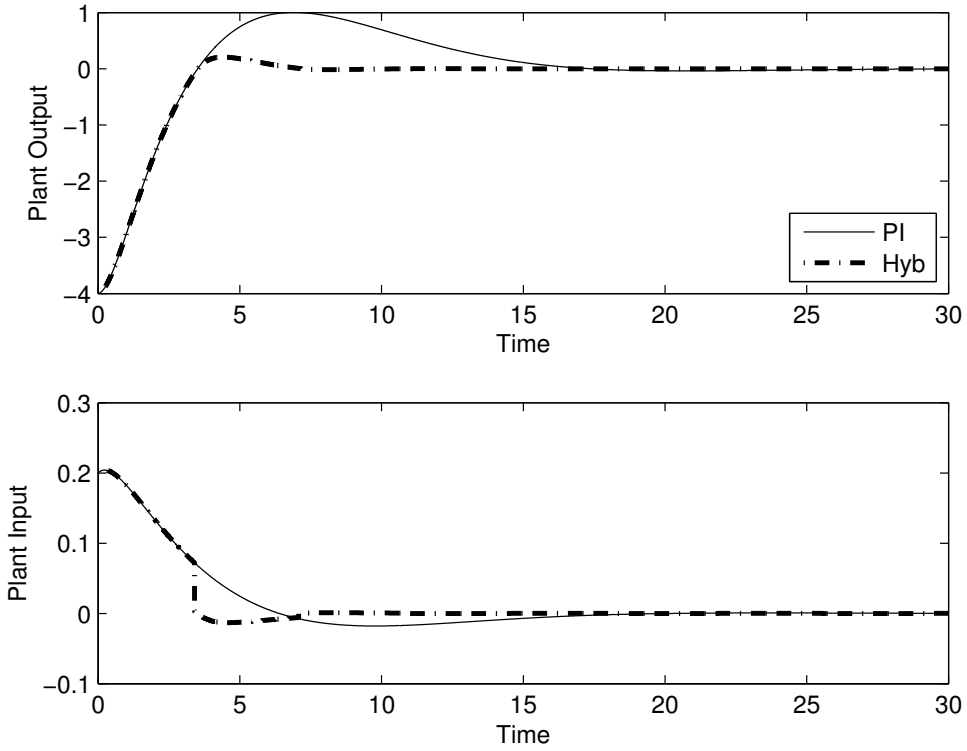
- A DC motor controlled by a PI controller



- LMI-based synthesis for **overshoot reduction**:

$$V_p(x_p) = x_p^\top P_p x_p \approx |y|^2$$

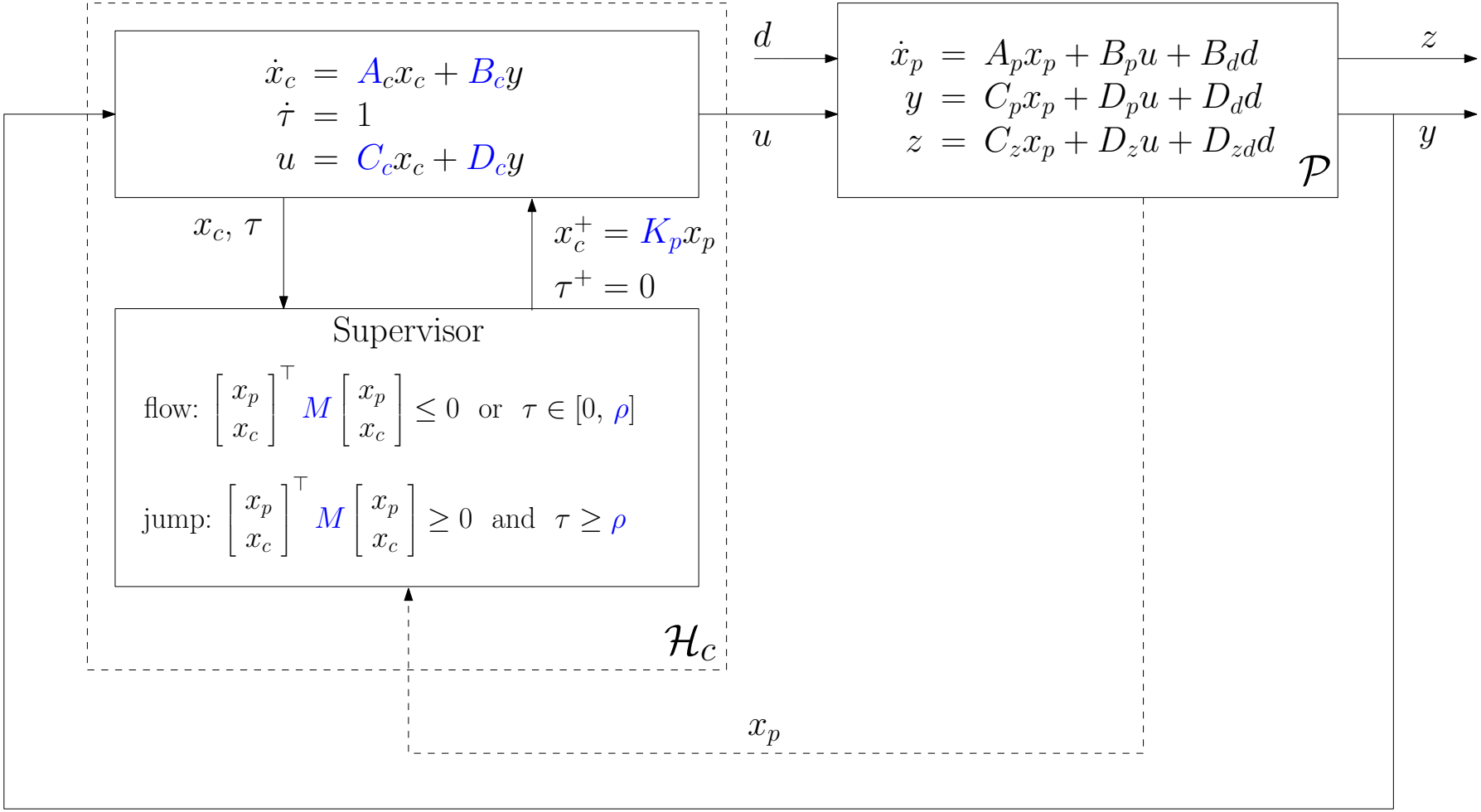
- may also maximize the **decay rate** α



Multi-objective hybrid $\mathcal{H}_\infty$ controller synthesis

Fichera et al. [2016b]

- Design all of $\mathcal{H}_c$, A_c, B_c, C_c, D_c, K_p, M and ρ, to minimize γ_{dz}



- LMI-based design using Scherer's like approach $P^{-1} = \begin{bmatrix} Y & Z \\ Z & Z \end{bmatrix}$, $P_p = Y^{-1}$

"Freeze and play" output feedback plant-order synthesis

Ferrante and Zaccarian [2019]

- Previous design still uses **state feedback**
- LMI-based **fully output feedback** design in Ferrante and Zaccarian [2019]

$$\begin{cases} \dot{x}_c = (1-q)(A_c x_c + B_c y) \\ \dot{v} = -(1-q)\lambda v \\ \dot{\tau} = 1 \end{cases} \qquad \begin{bmatrix} y \\ x_c \end{bmatrix}^\top M \begin{bmatrix} y \\ x_c \end{bmatrix} \leq 0 \text{ or } \tau \in [0, \rho]$$

$$\begin{cases} x_c^+ = K_c x_c + G_c y \\ v^+ = K_v x_c + G_v y \\ \tau^+ = 0 \end{cases} \qquad \begin{bmatrix} y \\ x_c \end{bmatrix}^\top M \begin{bmatrix} y \\ x_c \end{bmatrix} \geq 0 \text{ and } \tau \geq \rho$$

$$u = (C_c x_c + D_c y)(1-q) + vq$$

$$q = \begin{cases} 1, & \text{if } \tau \leq \rho, \ (\textbf{freeze} \text{ controller state } x_c \text{ and plant input } u = v) \\ 0, & \text{if } \tau \geq \rho, \ (\textbf{release} \text{ controller state } x_c \text{ and plant input } u) \end{cases}$$

- Lyapunov function based on dwell-time $\tau \in [0, \rho]$ and $x := (x_p, x_c, v)$,

$$W(x, \tau) := e^{-2\alpha\tau} (e^{A_1 \max\{0, \rho-\tau\}} x)^\top P e^{A_1 \max\{0, \rho-\tau\}} x$$

Resets improve PID controlled positioning systems with Coulomb and Stribeck effects

Collaborative work with:

R Beerens

A Bisoffi

M Heemels

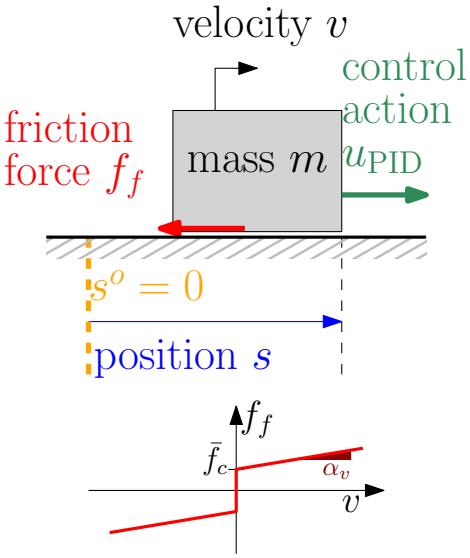
H Nijmeijer

N van de Wouw

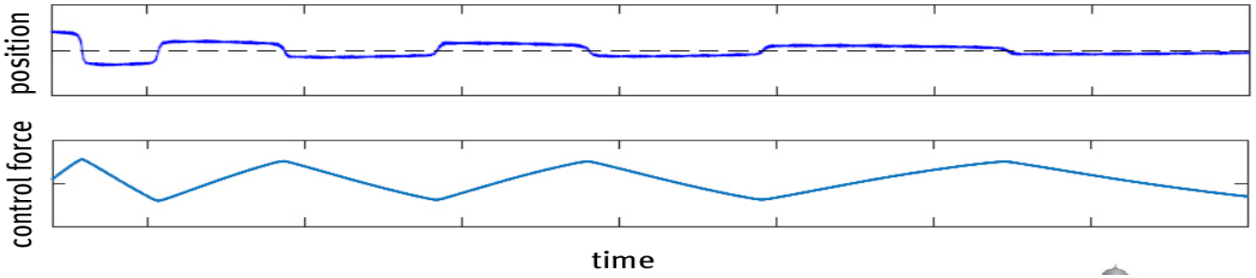
Reset PID control to compensate Coulomb friction

Bisoffi et al. [2018], Beerens et al. [2019a]

- Coulomb friction causes slow transients with PID feedback



- Manipulation stage of an electron microscope



Discontinuous Lyapunov function:

$$V(x) := \begin{bmatrix} \sigma \\ v \end{bmatrix}^T \begin{bmatrix} \frac{k_v}{k_i} & -1 \\ -1 & k_p \end{bmatrix} \begin{bmatrix} \sigma \\ v \end{bmatrix} + \min_{f \in f_c \,\mathrm{SGN}(v)} |\phi - f|^2$$

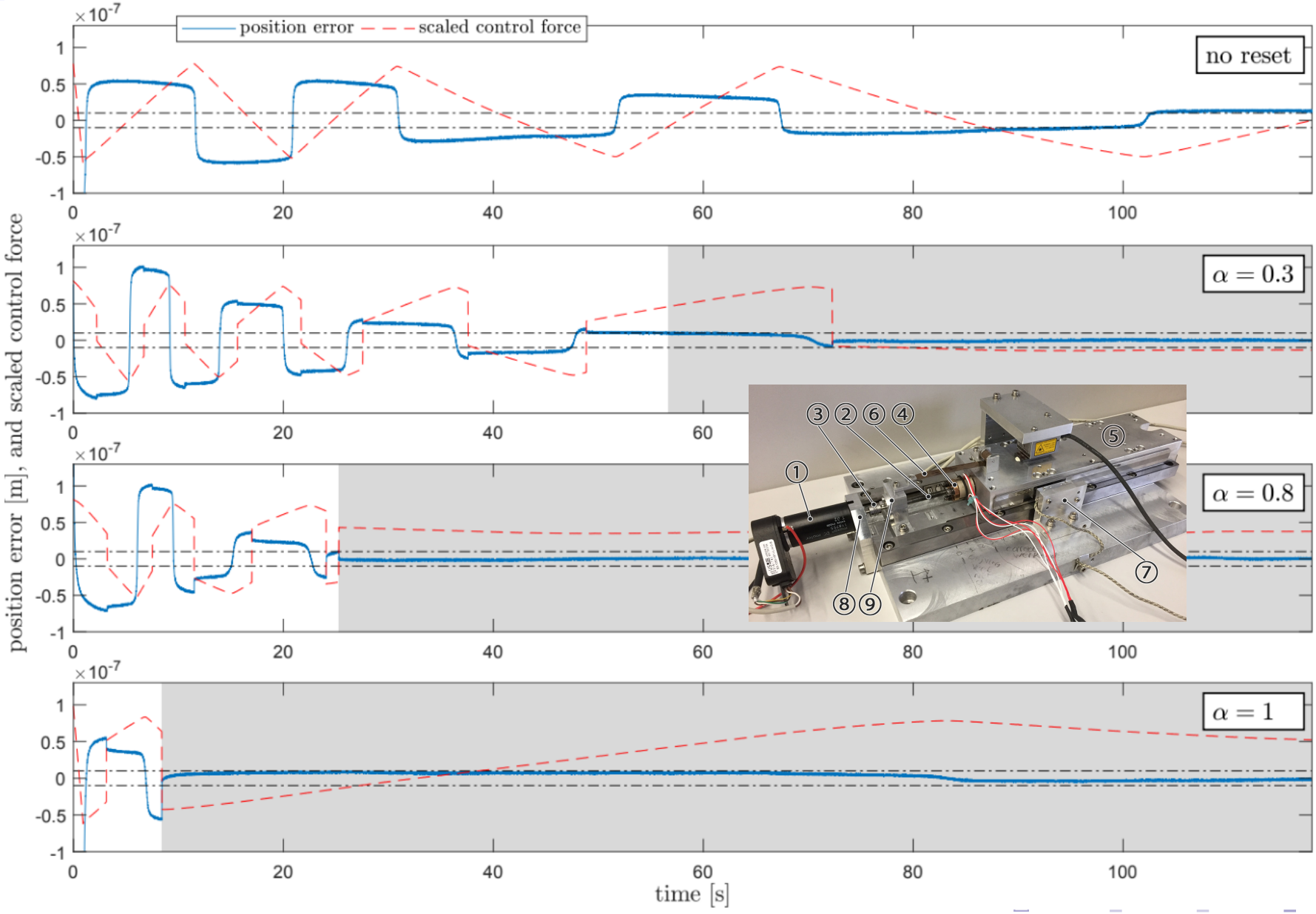
- Hybrid closed loop with reset PID (no knowledge of f_c required)

$$\dot{x} \in \begin{bmatrix} 0 & 0 & -k_i \\ 1 & 0 & -k_p \\ 0 & 1 & -k_v \end{bmatrix} \begin{bmatrix} \sigma \\ \phi \\ v \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ f_c \end{bmatrix} \mathrm{SGN}(v), \qquad x \in \mathcal{C} := \overline{\mathbb{R}^3 \setminus \mathcal{D}},$$

$$x^+ = \begin{bmatrix} \sigma & -\alpha\phi & v \end{bmatrix}^\top, \qquad x \in \mathcal{D} := \left\{ x \in \mathbb{R}^3 \mid \phi\sigma \le 0,\ \phi v \le 0,\ |\phi\sigma| \ge \varepsilon \right\},$$

Experimental response shows transient improvement

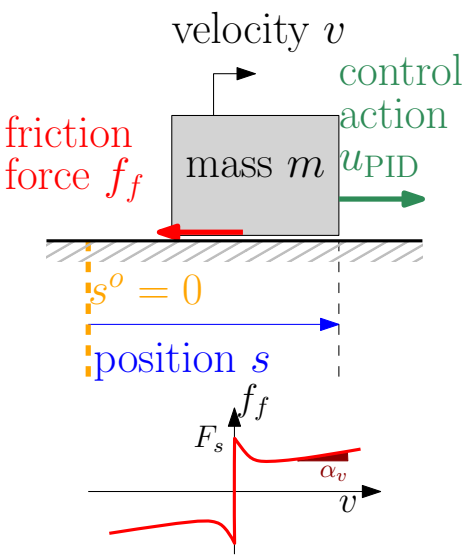
Bisoffi et al. [2018], Beerens et al. [2019a]

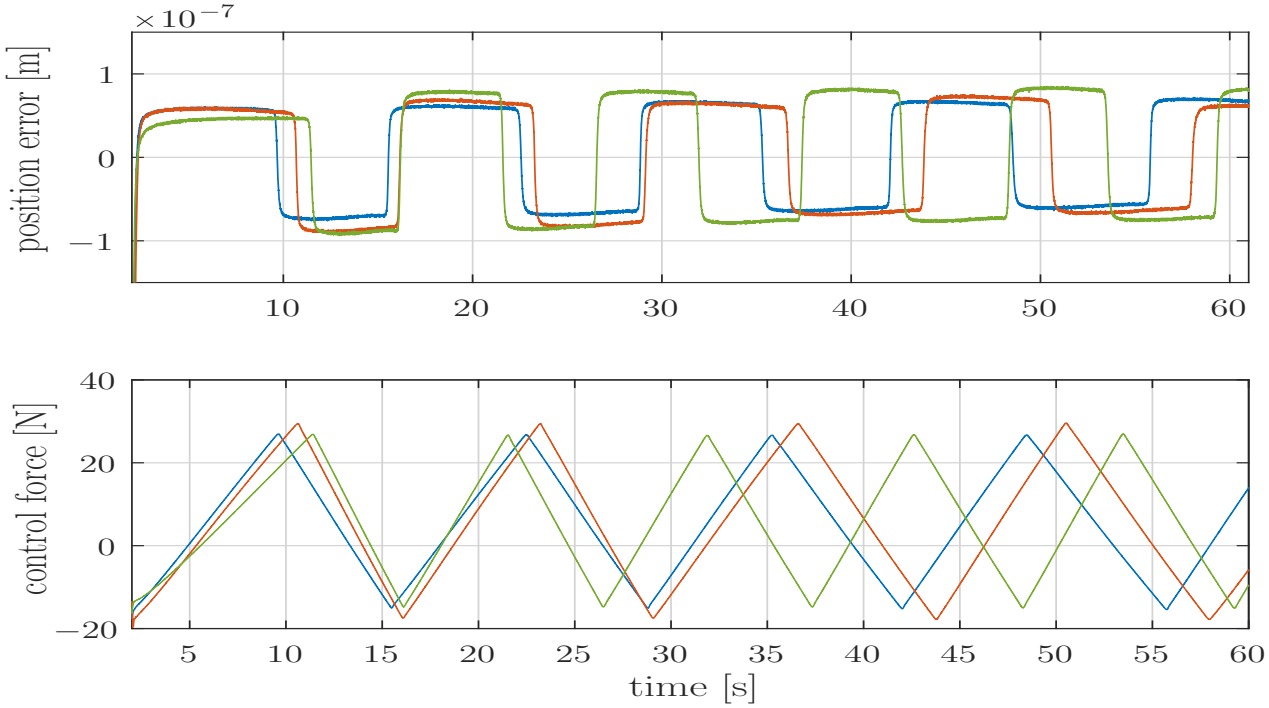


Reset PID control to compensate destabilizing Stribeck

Bisoffi et al. [2019], Beerens et al. [2019b]

- Stribeck effect causes "hunting" instability with PID feedback





- Reset controller with extra logical state $h \in \{-1, 1\}$, jumps from

$$\mathcal{D}_\sigma := \{x : \ \sigma = 0, h = 1\}, \quad \mathcal{D}_v := \{x : \ v = 0, \ \sigma\phi \geq \frac{k_p}{k_i}\sigma^2, \ h = -1\}$$

- Flow is constrained within $\mathcal{C} := \{x : \ hv\sigma \geq 0, \ \sigma\phi \geq \frac{k_p}{k_i}\sigma^2\}$

- Stability proof uses semiglobal dwell time and bisimulation of the dynamics with a rather convoluted hybrid Lyapunov function

Reset PID control to compensate destabilizing Stribeck

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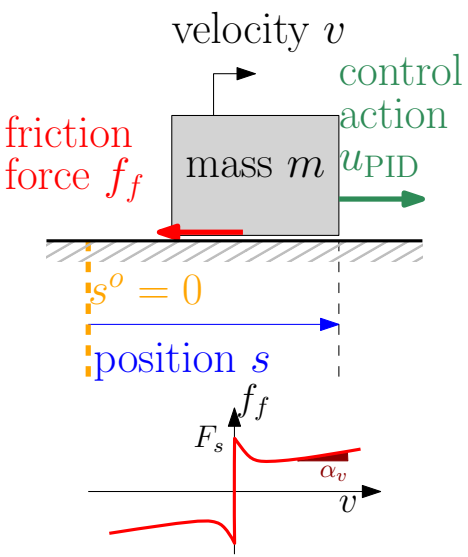

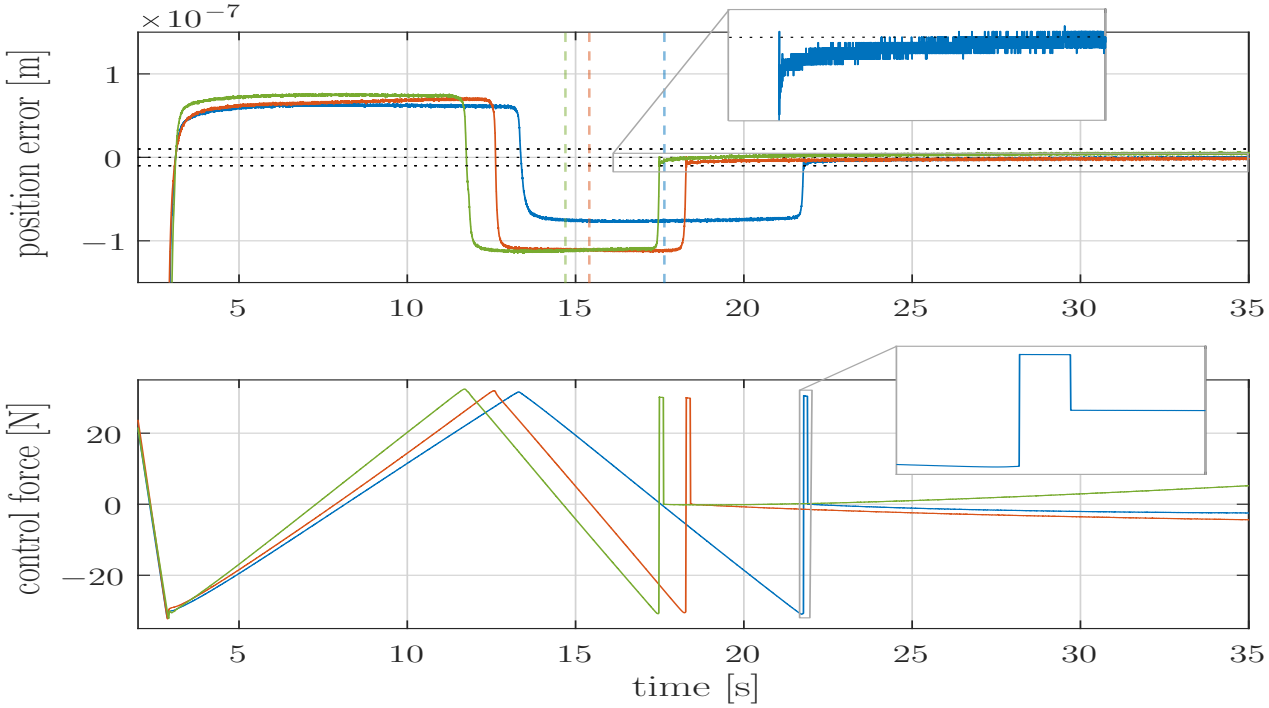

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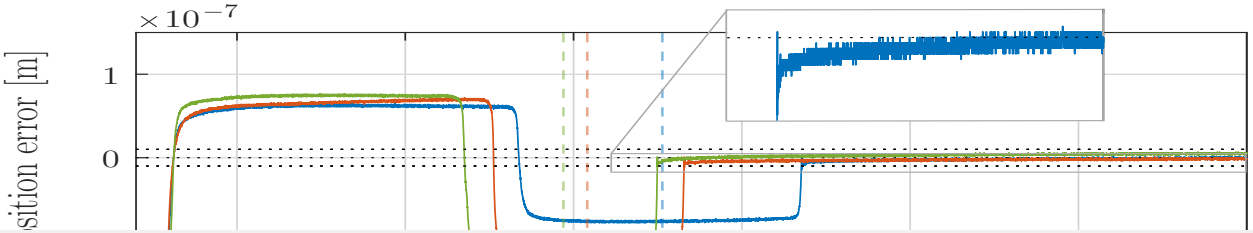
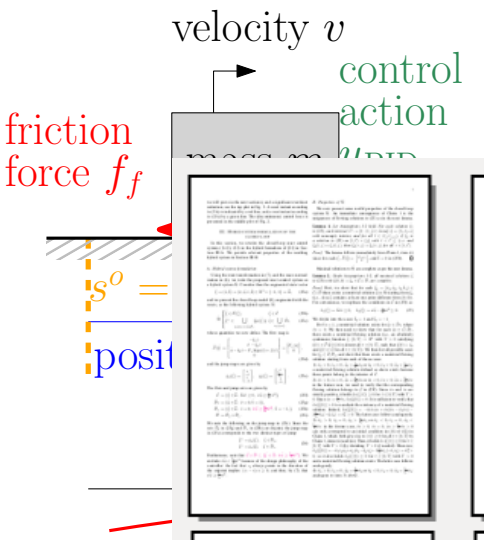
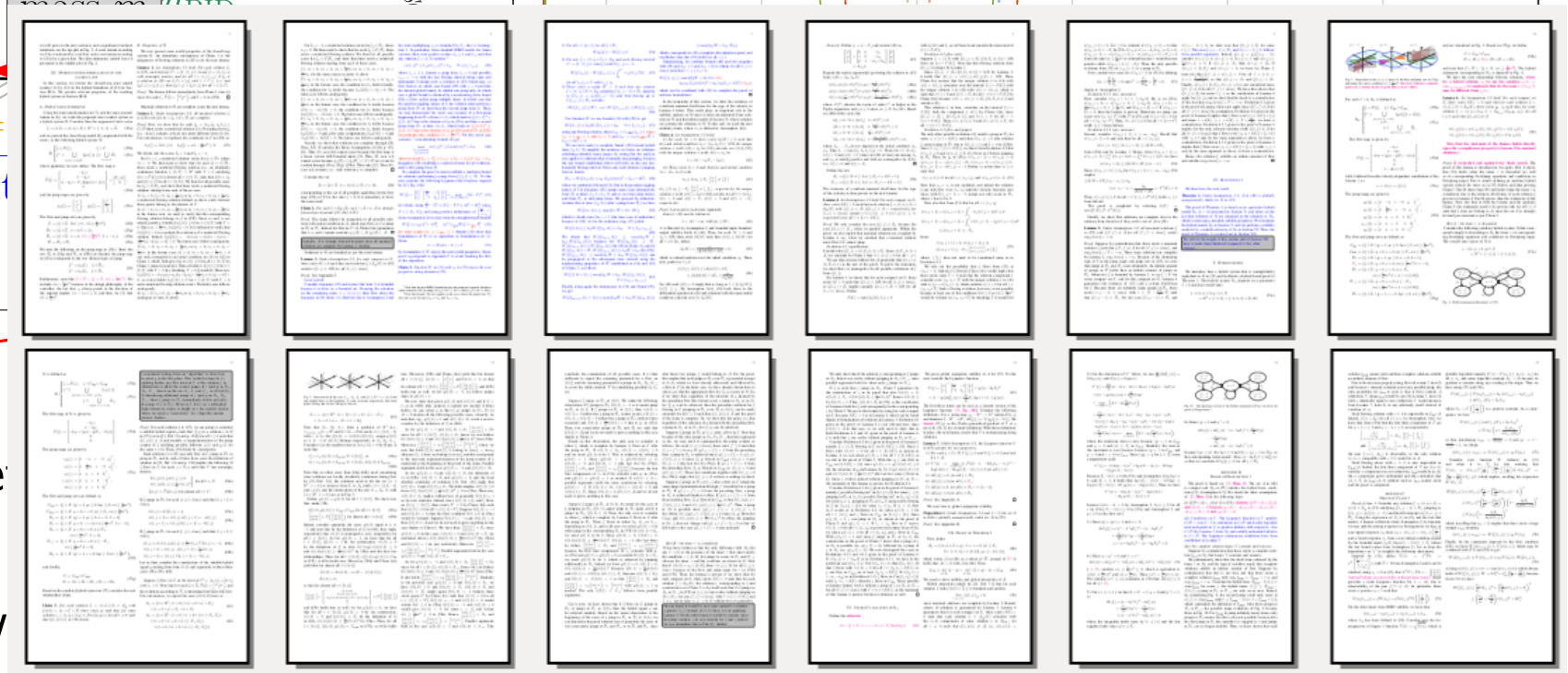
Proof of stability is, so far, a nightmare 12 pages blurb...

Bisoffi et al. [2019], Beerens et al. [2019b]

- Stribeck effect causes "hunting" instability with PID feedback







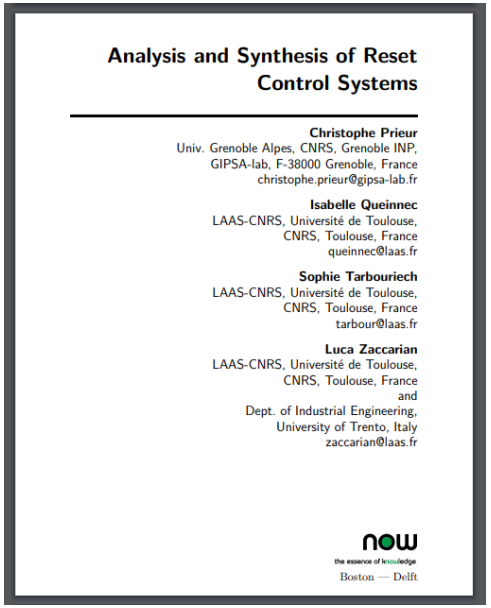
- Rese
$\mathcal{D}_\sigma$
- Flow

- Stability proof uses semiglobal dwell time and bisimulation of the dynamics with a rather convoluted hybrid Lyapunov function

The fascinating experience of scientific exchange

Conclusions:

- Resets promise aggressive stabilization with exponentially diverging transients
- We need to be careful about the internal model properties of reset controllers
- Most of the presented results in a **recently published survey paper** (NOW Publishers) **freely available until Sep 11, 2019** at `https://www.nowpublishers.com/article/Download/SYS-017`


Analysis and Synthesis of Reset Control Systems

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