Input allocation: hierarchical design paradigm with redundant actuators and its aerospace applications

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The fascinating experience of scientific exchange

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Nonlinear cascades as hierarchical behaviors

- Practical experience with several applications
- Abstraction reveals a pervasive pattern of control specs hierarchies

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- Stability/Hierarchy analysis stems from nonlinear cascades
Historical results on nonlinear cascades by E. Sontag

▷ Cascaded systems intrinsically represent hierarchical tasks
\[
\dot{x}_U = f_U(x_U) \quad x_U = u \quad \dot{x}_L = f_L(x_L, u) \quad x_L
\]

▷ Stability analysis results date back to the 1980’s

**Theorem ISS**
If \((U)\) is GAS and \((L)\) is 0-GAS and ISS, then cascade is GAS

**Theorem SON**
If \((U)\) is GAS and \((L)\) is 0-GAS, then cascade is LAS with basin of attraction \(\mathcal{B}_A = \{\text{largest set from where solutions don’t diverge}\}\).

**Corollary SON**
If \((U)\) is GAS and \((L)\) is 0-GAS, and all solutions are bounded, then cascade is GAS
Cascades generalize to reduction theorems useful next

- It is not always possible to write “cascaded-like” coordinates: in reduction theorems, the upper system \( (U) \) comprises convergence to a closed set \( \Gamma \)

- Reduction theorems for continuous-time discrete-time and hybrid dynamics

**Theorem RED**

If \( \Gamma \) is GAS and \( \mathcal{A} \) is GAS starting from \( \Gamma \), then \( \mathcal{A} \) is LAS with basin of attraction \( \mathcal{B}_\mathcal{A} = \{ \text{largest set from where solutions don’t diverge} \} \).

**Corollary RED**

if \( \Gamma \) is GAS and \( \mathcal{A} \) is GAS starting from \( \Gamma \) and all solutions are bounded, then \( \mathcal{A} \) is GAS
Attitude control performed by two actuators

Control law

Attitude target

Actuators

Sensors

Satellite dynamics

Current attitude

Disturbance torque

Reaction wheels

Magnetorquers

Initial attitude
desired attitude
Reaction wheels suffer from total momentum problems

\[ T_w = \dot{h}_w \]

\[ T_w (N.m) \]

\[ h_w (N.m.s) \]

\[ t (s) \]

\[ \times 10^{-3} \]

Cross:

- Total momentum can’t be modified (wheel turns CW, satellite turns CCW)
- Risk of saturation of \( h_w \)

\[ \Rightarrow h_w(t) = \int_0^t T_w(\tau)d\tau \] needs to be controlled

Nomenclature

- \( h_w \in \mathbb{R}^3 \): angular momentum
- \( T_w \in \mathbb{R}^3 \): control torque
Magnetorquers confined to exert torque in rotating plane

\[ T_m = -\tilde{b} \times (t, q) \tau_m = -(R(q)\tilde{b}_o(t)) \times \tau_m \]

\( \tilde{b} \): magnetic field
\( \tau_m \): magnetic momentum
\( q \): quaternion
\( R \): rotation matrix

\( z^\times := \begin{bmatrix} 0 & -z_z & z_y \\ z_z & 0 & -z_x \\ -z_y & z_x & 0 \end{bmatrix} \)

\( \times \): instantaneous controllability restricted to a plane (\( z^\times \) is singular)
\( \tilde{b}_o(t) \): almost periodic and uncertain
Stabilization problem requires coordination of the actuators

**Equations of the attitude motion**

\[
\begin{align*}
J\dot{\omega} &= -\omega \times (J\omega + h_w) - \tau_w - \tilde{b}(t, q)\tau_m \\
\dot{h}_w &= \tau_w \\
\begin{bmatrix}
\dot{\varepsilon} \\
\dot{\eta}
\end{bmatrix} &= \frac{1}{2} \begin{bmatrix}
-\omega \times & \omega \\
-\omega^T & 0
\end{bmatrix} \begin{bmatrix}
\varepsilon \\
\eta
\end{bmatrix}
\end{align*}
\]

**Nomenclature**

**Satellite:**
- \(\omega\): angular velocity
- \(q = (\varepsilon, \eta)\): quaternion
- \(J\): inertia matrix

**Reaction wheels:**
- \(h_w\): angular momentum
- \(\tau_w = T_w\): control torque

**Magnetorquers:**
- \(\tilde{b}(t, q)\): geomagnetic field
- \(\tau_m\): magnetic momentum

⇒ Design goal: find \(\tau_w(x)\) and \(\tau_m(x)\) such that \(x := \begin{bmatrix}
\omega \\
q \\
h_w
\end{bmatrix} \rightarrow \begin{bmatrix}
0 \\
q_o \\
h_{ref}
\end{bmatrix}\)

✗ actuators may badly interact
Global attitude stabilization via hybrid feedback

▷ Ideal attitude feedback $u_{\text{att}}$ may be selected as a hybrid control law

$$J\dot{\omega} = -\omega \times J\omega + u_{\text{att}}$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega \times & \omega \\ -\omega^T & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

• No time-invariant continuous selection $u_{\text{att}}(x)$ stabilizes the compact attractor $\mathcal{A} := \{\omega = \varepsilon = 0, \eta = \pm 1\}$ [Bhat et al, 2000]

Hybrid solution available in the literature [Mayhew et al, 2009]

For any scalars $k_p > 0, k_d > 0, \delta \in (0, 1)$, the attractor $\mathcal{A}$ is globally asymptotically and locally exponentially stabilized by the hybrid PD-like dynamic controller:

$$u_{\text{att}}(x_c, \varepsilon, \omega) := -k_p x_c \varepsilon - k_d \omega$$

$$\begin{align*}
\dot{x}_c &= 0, \quad \text{when } (q, \omega, x_c) \in C := \{(q, \omega, x_c) : x_c \eta \geq -\delta\} \\
\dot{x}_c^+ &= -x_c, \quad \text{when } (q, \omega, x_c) \in D := \{(q, \omega, x_c) : x_c \eta \leq -\delta\},
\end{align*}$$

where the $C$ is the flow set and $D$ is the jump set.
I. The industrial solution: “cross product control law”

The cross-product control law

\[ \tau_w = -\omega \times h_w - u_{att}, \quad \tau_m = -\frac{\tilde{b}(t)}{|b(t)|^2} k_p (h_w - h_{ref}) \]

Ignore the interaction of the two inputs

\[ \dot{\omega} = -\omega \times J\omega - \tau_w - \omega \times h_w + \frac{d}{T_m} \]

\[ \begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega \times & \omega \\ -\omega^T & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} + u_{att}(x_c, \varepsilon, \omega) \]

- loop 1: Attitude control performed by the reaction wheels
- loop 2: Regulation of \( h_w \) by the magnetorquers
- the two loops are treated separately

▷ frequency separation between the two loops (\( = \) very aggressive attitude stabilizer) gives engineering solution [Camillo, 1980; Carrington 1981]

✗ formally proving stability properties of the overall scheme seems hard
II. Revisited “cross product control law” highlights cascade

A revisited version of the cross-product control law

\[
\tau_w = -\omega \times h_w - u_{\text{att}}, \quad \tau_m = -\frac{\tilde{b} \times (t)}{|\tilde{b}(t)|^2} k_p (h_w + J\omega - R(q)h_{\text{ref}})
\]

-Classical approach reveals quasi cascaded structure where \(h_T^{[I]}\) refers to the total angular momentum (satellite + wheels)

✓ the feedback branch (the dashed line) can be avoided by redefining \(\tau_m\)

✗ attitude dynamics is affected by the momentum dumping action

GAS can be established using Theorem ISS

GAS is proven for any \(u_{\text{att}}\) under ISS of attitude closed loop if \(\tilde{b}_o(t)\) is persistently exciting
III. Allocation-based controller prioritizes attitude

Allocation-based controller equations

\[ \tau_w = -\omega \times h_w - (R(q)\tilde{b}_\circ(t))^\times \tau_m - u_{att}, \quad \tau_m = -\frac{(R(q)\tilde{b}_\circ(t))^\times}{|\tilde{b}_\circ(t)|^2} k_p(h_w - h_{ref}) \]

▷ Reversing the cascaded structure giving priority to the attitude stabilization
▷ stemming from a different partition of the effected input: \( u_{att}(x_c, \varepsilon, \omega) \)

\[ J\dot{\omega} + \omega \times J\omega = -\tau_w - \omega \times h_w + T_m. \]

GAS can be established using Corollary SON

GAS is proven for any \( u_{att} \) (No ISS needed) if \( \tilde{b}_\circ(t) \) is persistently exciting. Boundedness from LES of \( (U) \) and Gronwall Lemma.
Simulations reveal advantages of the proposed controller

**Context of the simulations**
- Mission: micro-satellite Demeter by CNES, the French space agency
- $\tilde{b}_\circ(t)$ evaluated by the IGRF model of the geomagnetic field
- rest-to-rest maneuvers with non-nominal $h_w$

**Controllers used**
- **Classical** “cross product control law” controller
- **Revisited** version of the classical controller
- **Allocation**-based controller

**Simulation tests**
- **Nominal**: Shows that the classical solution diverges
- **Perturbed $J$**: Allocation outperforms Revisited
- **Periodic disturbances**: Allocation outperforms Revisited
Stabilization transients with aggressive controller

Similar results √ saturation of $h_w$
Stabilization transients with non aggressive controller

- revisited and allocation controllers preserve stability
Monte-Carlo with uncertainties on $J$: improved transients

- Clear advantages emerge from swapping the cascaded structure

- Improved attitude transients with allocation-based controller (right)

- Robustness rigorously established by intrinsic results of well-posed (hybrid) feedbacks
Periodic disturbances are best handled by allocator

- No rigorous analysis has been performed for this case
  - Interesting direction of future development (regulation theory, contraction theory/convergent dynamics)

☑ Improved attitude response with allocation-based controller (right)
ROSPO represents key challenges in UAV allocation

- ROtor graSPing Omnidirectional (ROSPO) ground platform developed at the LAAS-CNRS in Toulouse (France)

- **3 DoF** Task in SE(2): 2 DoFs position + 1 DoF orientation
  - each turret 2 actuators:
    - propeller: thrust magnitude
    - servo: thrust orientation
  - \(n\) turrets: \(n = 3, 4, \ldots\)
  - overactuated for \(n > 1\)

\[
\begin{align*}
\dot{p} &= v \\
mv\ddot{v} &= \sum_{i=1}^{n} R(\psi) f_i^B \\
\dot{\psi} &= \omega \\
J\dot{\omega} &= \sum_{i=1}^{n} (\nabla r_i)^T f_i^B
\end{align*}
\]

Platform equations

\[
\begin{align*}
\dot{\theta}_i &= u_{\theta,i} \\
\dot{w}_i &= u_{w,i}
\end{align*}
\]

Actuator dynamics

\[
\begin{align*}
f_i^B &= k_w w_i^2 \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}
\end{align*}
\]

Constraints

\[
\begin{align*}
\theta_i &\leq \theta_i \leq \bar{\theta}_i \\
w_i &\leq w_i \leq \overline{w}_i
\end{align*}
\]
Control Objective and Allocator Hierarchy

- **Design Goal:** Trajectory tracking in position and attitude SE(2)

- **High level control**
  - Ensures *trajectory tracking* by generating a suitable “commanded virtual input” $u_{v,c}$ for the allocator

- **Allocator tasks with their priorities**
  - **HIGH** ensures that the commanded virtual input is *dynamically* exerted on the plant with time constant $\gamma_p$
    - $$\dot{u}_v = \gamma_p(-u_v + u_{v,c}), \text{ where } u_v := \left( \sum_{i=1}^{n} f_i^B, \sum_{i=1}^{n} (\Pi r_i)^T f_i^B \right)$$
  - **LOW** ensures optimal allocation w.r.t a cost function $J(w, \theta)$ penalizing constraints violation
Allocator Dynamics is based on combined effect of $u_J$ and $u_y$

- Feedback linearization transforms actuators in $\dot{x}_a = u_y + u_J$, $u_v = h(x_a)$
- $u_y$ takes care of assigning first order dynamics
  \[ \dot{u}_v = \gamma_P (-u_v + u_{v,c}) \]
  where $u_v := \left( \sum_{i=1}^{n} f_i^B, \sum_{i=1}^{n} (\Pi r_i)^T f_i^B \right)$
- $u_J$ takes care of the cost function $J$ via projection operator $\nabla_h(x_a)$

- Cost Function $J(w, \theta)$ penalizes:
  - approaching actuator saturation
  - energy consumptions of propellers
The block **Allocator + Actuators** externally appears as a first-order filter.

Design goal is to track a reference motion $t \mapsto p_R(t), \psi_R(t)$.

Design task is then simplified by allocator:
- Simple Feedforward + Feedback scheme ensures PD-like behavior.
- The selected gains ensure desirable damping and bandwidth.

**GAS can be established using Theorem RED**

- **EXP** $p(t), \psi(t)$ converge globally and exponentially to $p_R(t), \psi_R(t)$.
- **ACT** $u_v(t), u_{v,c}(t)$ asymptotically satisfy $\dot{u}_v = \gamma P(-u_v + u_{v,c})$.
- **OPT** if $p_R(t), \psi_R(t)$ is constant, then $x_a(t)$ converges to a stationary point of $J(x_a)$ subject to $h(x_a) = u_{v,c}$. 
Experiments show allocator-induced “external linearity”

▷ Step refs in directions $\mathbf{u}_v = (f_x, f_y, \tau)$ confirm linear $\mathbf{u}_v' = \gamma_P (-\mathbf{u}_v + \mathbf{u}_{v,c})$

▷ If $\gamma_P$ is too large, input saturation becomes relevant
Experiments following an $\infty$-shaped motion

- $\infty$-shaped motion with $n = 3$ turrets and $n = 4$ turrets configurations
- Allocator parameters do not change between $n = 3$ and $n = 4$

- Lower precision in the case $n = 3$ as compared to $n = 4$
- Cost Function $J$ highly improved by the allocator action
Beyond ISS cascades in hierarchical UAV control

- In underactuated UAVs, the cascaded sequence enforced by the dynamics

\[
\begin{align*}
\dot{x} &= v  \\
m\dot{v} &= -mge_3 + Rf_c \\
\dot{R} &= R\hat{\omega} \\
J\dot{\omega} &= (J\omega) \times \omega + \tau_c \\
\end{align*}
\]

- Cascade interconnection shows undesirable position feedback perturbation

Attitude dynamics controlled to guarantee \( \Delta \gamma \to 0 \)

- New iISS quasi-time-optimal stabilizer outperforms historical ISS approach
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Main references related to the presented works

Key works for this presentation:
- Reduction Theorems for Hybrid Dynamical Systems Maggiore et al. [2019]
- Applications of static allocation:
  - Satellite attitude stabilization Trégouët et al. [2015]
  - ROSPO experimental platform Nainer et al. [2017]
- Hierarchical paradigms for UAV control Invernizzi et al. [2018, 2019]

Additional related references
- Dynamic allocation paradigms for linear systems Zaccarian [2009], Cocetti et al. [2018], Galeani et al. [2015]
- Applications of dynamic allocation:
  - Internal wrenches control in interacting robots Zambelli Bais et al. [2015]
  - Tokamak plasma shape control Boncagni et al. [2012], De Tommasi et al. [2011, 2012]
  - Hybrid Electric Vehicle control Cordiner et al. [2014]
  - Hydrodynamic dynamometer application Passenbrunner et al. [2016]


