A hybrid control framework for impulsive control of satellite rendezvous

Luca Zaccarian

Joint work with Mirko Brentari, Sofia Urbina, Denis Arzelier, Christophe Louembet

LAAS-CNRS and University of Trento

Seoul National University, October 17, 2017

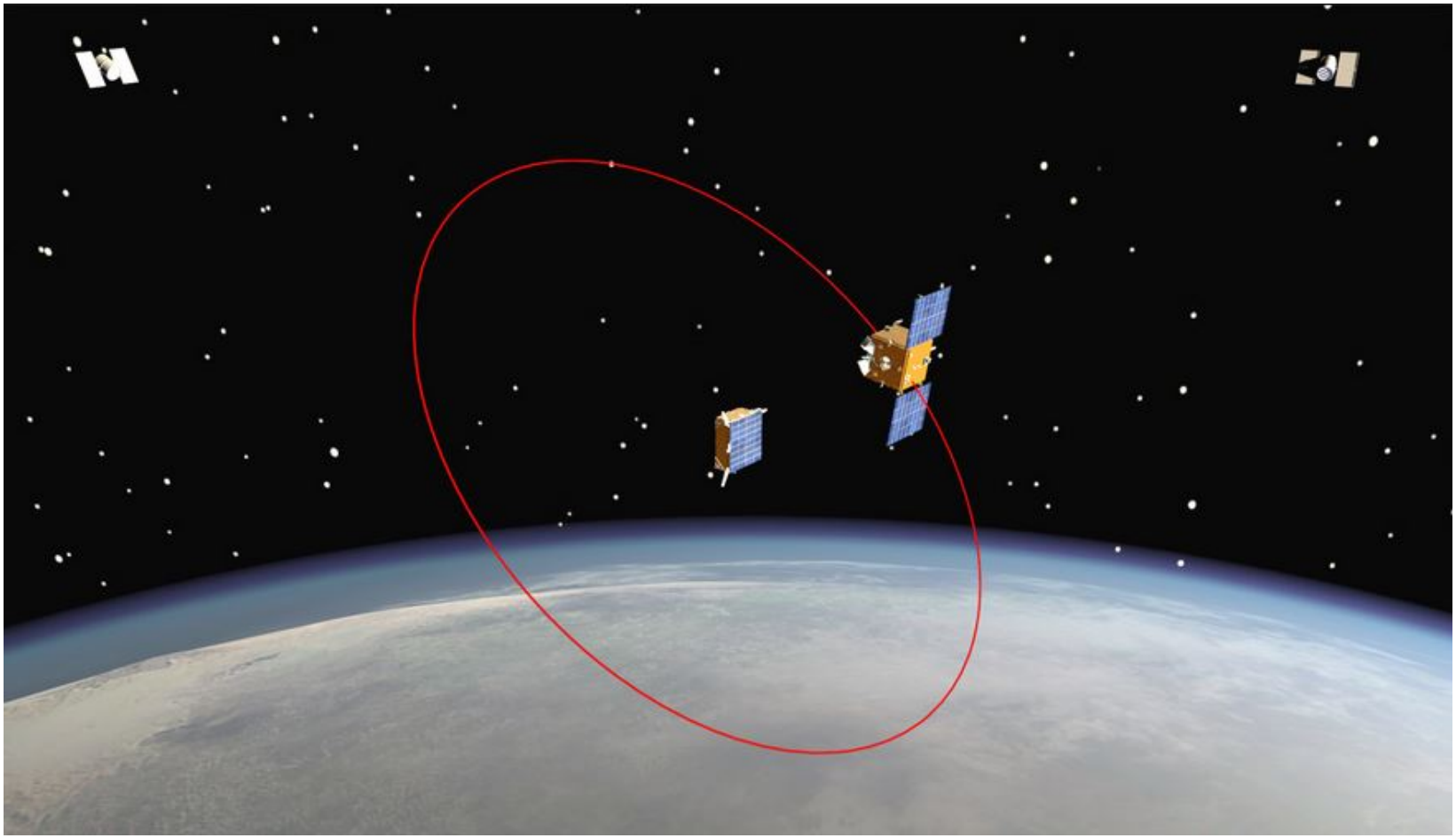
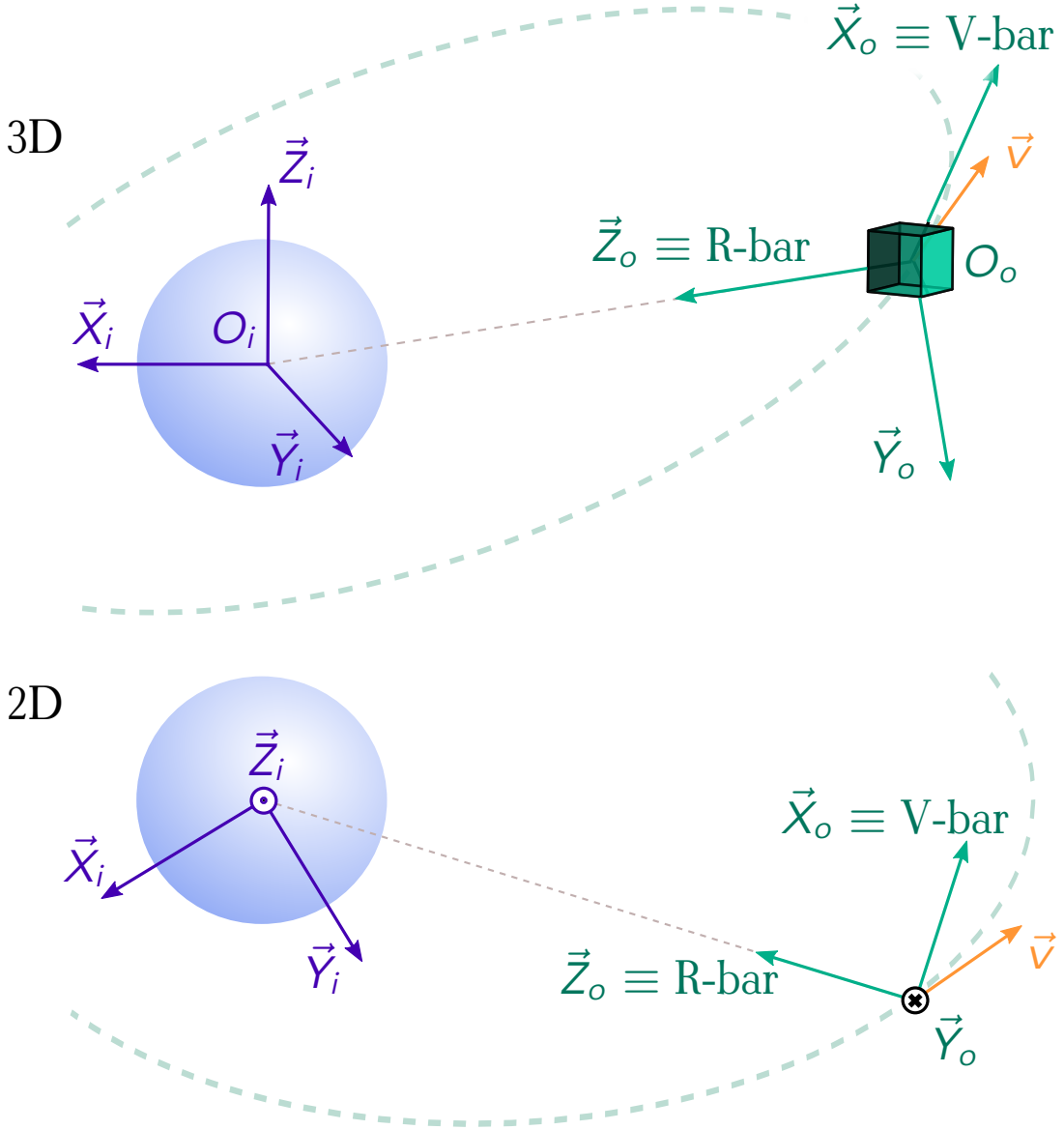


Figure 1 : The PRISMA mission. Photo courtesy of CNES.

- Proximity operations (active debris removal, reparation of a spacecraft, refueling, etc).
- Target/chaser.

Reference frames



Inertial reference frame:

$$\mathcal{I} = \left(O_i, \vec{X}_i, \vec{Y}_i, \vec{Z}_i\right)$$

LVLH reference frame:

$$\mathcal{O}_o = \left(O_o, \vec{X}_o, \vec{Y}_o, \vec{Z}_o\right)$$

Linear translational equations (Tschauner-Hempel)

Linearization $\dot{X}(t) = A(t)\,X(t) + B\,F(t)$ derived by Tschauner-Hempel

Denoting: $\dfrac{d\nu}{dt} = \dfrac{n}{(1-e^2)^{3/2}}\,(\underbrace{1+e\cos\nu}_{\rho(\nu)})^2 =: k^2\rho(\nu)^2$

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \\ \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{bmatrix}}_{\dot{X}(t)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{\nu}^2 + 2k^4\rho(\nu)^3 & 0 & \ddot{\nu} & 0 & 0 & 2\dot{\nu} \\ 0 & \dot{\nu}^2 - k^4\rho(\nu)^3 & 0 & 0 & 0 & 0 \\ -\ddot{\nu} & 0 & -k^4\rho(\nu)^3 & -2\dot{\nu} & 0 & 0 \end{bmatrix}}_{A(t)} \underbrace{\begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix}}_{X(t)} + \underbrace{\begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \frac{F^x_{prop_C}}{m_C} \\ \frac{F^y_{prop_C}}{m_C} \\ \frac{F^z_{prop_C}}{m_C} \end{bmatrix}}_{F(t)} \tag{1}$$

Linear translational equations (Tschauner-Hempel)

Linearization $\dot{X}(t) = A(t)\,X(t) + B\,F(t)$ derived by Tschauner-Hempel

Denoting: $\dfrac{d\nu}{dt} = \dfrac{n}{(1-e^2)^{3/2}}\,(\underbrace{1+e\cos\nu}_{\rho(\nu)})^2 =: k^2\rho(\nu)^2$

$$\underbrace{\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \\ \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{bmatrix}}_{\dot{X}(t)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{\nu}^2 + 2k^4\rho(\nu)^3 & 0 & \ddot{\nu} & 0 & 0 & 2\dot{\nu} \\ 0 & \dot{\nu}^2 - k^4\rho(\nu)^3 & 0 & 0 & 0 & 0 \\ -\ddot{\nu} & 0 & -k^4\rho(\nu)^3 & -2\dot{\nu} & 0 & 0 \end{bmatrix}}_{A(t)} \underbrace{\begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \dot{x}(t) \\ \dot{y}(t) \\ \dot{z}(t) \end{bmatrix}}_{X(t)} + \underbrace{\begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \frac{F^x_{prop_c}}{m_c} \\ \frac{F^y_{prop_c}}{m_c} \\ \frac{F^z_{prop_c}}{m_c} \end{bmatrix}}_{F(t)} \tag{1}$$

Chemical propulsion $\rightarrow$ impulsive problems

- Impulsive control input: $u(t_k) = \Delta V(t_k) = \Delta V_k = \int_{t_k^-}^{t_k^+} F(t)\,dt = \int_{t_k^-}^{t_k^+} \dfrac{\vec{F}_{prop_c}}{m_c}\,dt.$
- $\begin{cases} \dot{X} = A(t)X & \text{in free motion} \\ X^+ = X + \begin{bmatrix} 0_{3\times 3} \\ I_3 \end{bmatrix} u & \text{across impulsive thrusts} \end{cases}$

Linearized translational equations (Tschauner-Hempel)

Transformations of the free dynamics

$$X(t) \xrightarrow{H(\nu)} X(\nu) \xrightarrow{T(\nu)} \tilde{X}(\nu) \xrightarrow{C(\nu)} \bar{\xi}(\nu) \xrightarrow{S(\nu)} \widehat{\xi}(\nu)$$

1. $\dot{X}(t) = A(t)\, X(t) \xrightarrow{H(\nu)} X(\nu)' = A(\nu)\, X(\nu).$
   From the t-domain to the ν-domain (change of time scale).
2. $X(\nu)' = A(\nu)\, X(\nu) \xrightarrow{T(\nu)} \tilde{X}(\nu)' = \tilde{A}(\nu)\, \tilde{X}(\nu).$
   Provides simple periodic equations governed by true anomaly $\nu \in [0, 2\pi]$
3. $\tilde{X}(\nu)' = \tilde{A}(\nu)\, \tilde{X}(\nu) \xrightarrow{C(\nu)} \bar{\xi}(\nu)' = \bar{A}(\nu)\, \bar{\xi}(\nu).$
   Explicit solution of free motion exists (Yamanaka-Ankersen)
   It provides a basis for parametrization of constant solutions
   Representation of periodic trajectories $\Leftrightarrow \bar{\xi}_6 = 0$
4. $\bar{\xi}(\nu)' = \bar{A}(\nu)\, \bar{\xi}(\nu) \xrightarrow{S(\nu)} \widehat{\xi}(\nu)' = \widehat{A}\, \widehat{\xi}(\nu).$
   Application of Floquet-Lyapunov theory $\rightarrow$ transformation of LTV periodic system into an equivalent LTI system (with a constant dynamic matrix $\hat{A}$).

LTI dynamics derivation

Transformation $T(\nu)$

$$X(t) \xrightarrow{H(\nu)} X(\nu) \xrightarrow{T(\nu)} \tilde{X}(\nu) \xrightarrow{C(\nu)} \bar{\xi}(\nu) \xrightarrow{S(\nu)} \widehat{\xi}(\nu)$$

Second coordinate transformation $T(\nu)$:

$$\tilde{X}(\nu) = \underbrace{\begin{bmatrix} \rho(\nu)\, I_3 & 0_3 \\ \rho(\nu)^T\, I_3 & \rho(\nu)\, I_3 \end{bmatrix}}_{T(\nu)} X(\nu). \tag{2}$$

Simplified Tschauner-Hempel equations:

$$\begin{cases} \tilde{x}'' = 2\tilde{z}', \\ \tilde{y}'' = -\tilde{y}, \\ \tilde{z}'' = \dfrac{3}{1 + e\cos\nu}\tilde{z} - 2\tilde{x}'. \end{cases} \tag{3}$$

State vector: $\tilde{X}(\nu) = [\tilde{x} \quad \tilde{y} \quad \tilde{z} \quad \tilde{x}' \quad \tilde{y}' \quad \tilde{z}']^T$.

LTI dynamics derivation

Transformation $C(\nu)$

$$X(t) \xrightarrow{H(\nu)} X(\nu) \xrightarrow{T(\nu)} \tilde{X}(\nu) \xrightarrow{C(\nu)} \bar{\xi}(\nu) \xrightarrow{S(\nu)} \hat{\xi}(\nu)$$

Third coordinate transformation $C(\nu)$: it allows a periodic representation of the trajectories in the original coordinates.

$$\bar{\xi}(\nu) = \underbrace{\begin{bmatrix} 0 & c_\nu & 0 & 0 & -s_\nu & 0 \\ 0 & s_\nu & 0 & 0 & c_\nu & 0 \\ 1 & 0 & -\frac{3es_\nu(1+\rho)}{\rho(e^2-1)} & \frac{es_\nu(1+\rho)}{e^2-1} & 0 & \frac{\rho^2-ec_\nu-3}{e^2-1} \\ e & 0 & -3s_\nu & s_\nu(1+\rho) & 0 & c_\nu\rho \\ 0 & 0 & \frac{3(c_\nu+e)}{e^2-1} & -\frac{c_\nu(1+\rho)+e}{e^2-1} & 0 & s_\nu\rho \\ 0 & 0 & -\frac{3(3ec_\nu+e^2+2)}{e^2-1} & \frac{3\rho^2}{e^2-1} & 0 & -\frac{3es_\nu\rho}{e^2-1} \end{bmatrix}}_{C(\nu)} \tilde{X}(\nu).$$

State vector: $\bar{\xi}(\nu) = [\bar{\xi}_1 \quad \bar{\xi}_2 \quad \bar{\xi}_3 \quad \bar{\xi}_4 \quad \bar{\xi}_5 \quad \bar{\xi}_6]^T$.

LTI dynamics derivation

Transformation $S(\nu)$

$$X(t) \xrightarrow{H(\nu)} X(\nu) \xrightarrow{T(\nu)} \tilde{X}(\nu) \xrightarrow{C(\nu)} \bar{\xi}(\nu) \xrightarrow{S(\nu)} \widehat{\xi}(\nu)$$

Fourth coordinate transformation: based on Floquet-Lyapunov theory. It allows an LTI representation of the dynamics.

$$\widehat{\xi}(\nu) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \dfrac{\sigma(\nu)}{(1-e^2)^{3/2}} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{S(\nu)} \bar{\xi}(\nu),$$

where $\sigma(\nu) := \nu - \dfrac{2\pi}{T} t$.

State vector: $\widehat{\xi}(\nu) = [\widehat{\xi}_1 \quad \widehat{\xi}_2 \quad \widehat{\xi}_3 \quad \widehat{\xi}_4 \quad \widehat{\xi}_5 \quad \widehat{\xi}_6]^T$.

The final model is obtained after the four coordinate changes:

$$\widehat{\xi}(\nu) = R(\nu)\, X(\nu) := S(\nu)\, C(\nu)\, T(\nu)\, X(\nu)\ .$$

Linearized translational equations (Recap)

Transformations of the free dynamics

$$X(t) \xrightarrow{H(\nu)} X(\nu) \xrightarrow{T(\nu)} \tilde{X}(\nu) \xrightarrow{C(\nu)} \bar{\xi}(\nu) \xrightarrow{S(\nu)} \widehat{\xi}(\nu)$$

Equations of motion are transformed into:

$$\begin{cases} \hat{\xi}' = \hat{A}\hat{\xi} & \text{in free motion} \\ \hat{\xi}^+ = \hat{\xi} + \hat{B}(\nu)u & \text{across impulsive thrusts} \end{cases}$$

Proposition. $R(\nu)$ is a coordinate change (bounded and with bounded inverse) and motion of $X(t)$ is periodic if and only if $\hat{\xi}_6 = 0$.

Reference Periodic Motion can be specified in terms of **periodic relative trajectory**:

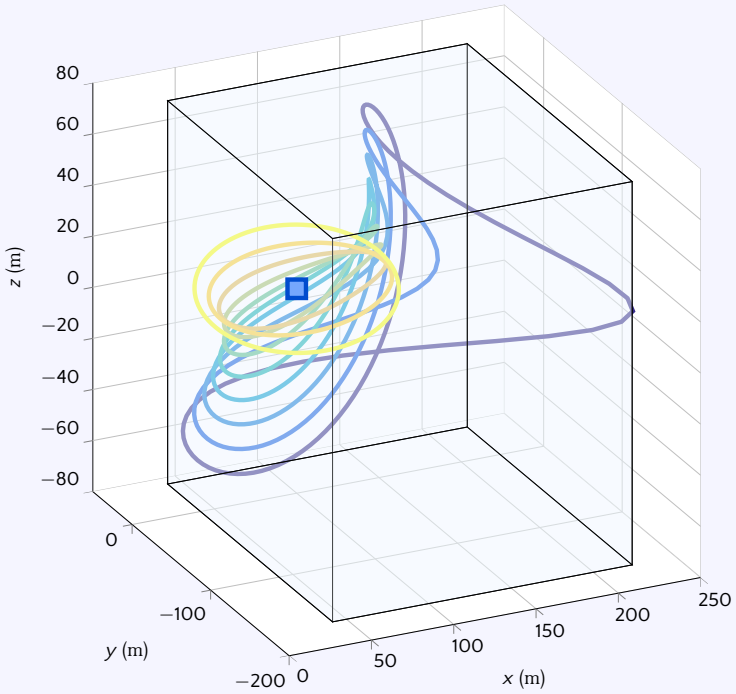
$$\xi^{ref} := [\xi_1^{ref} \ \xi_2^{ref} \ \xi_3^{ref} \ \xi_4^{ref} \ \xi_5^{ref} \ 0]^T$$

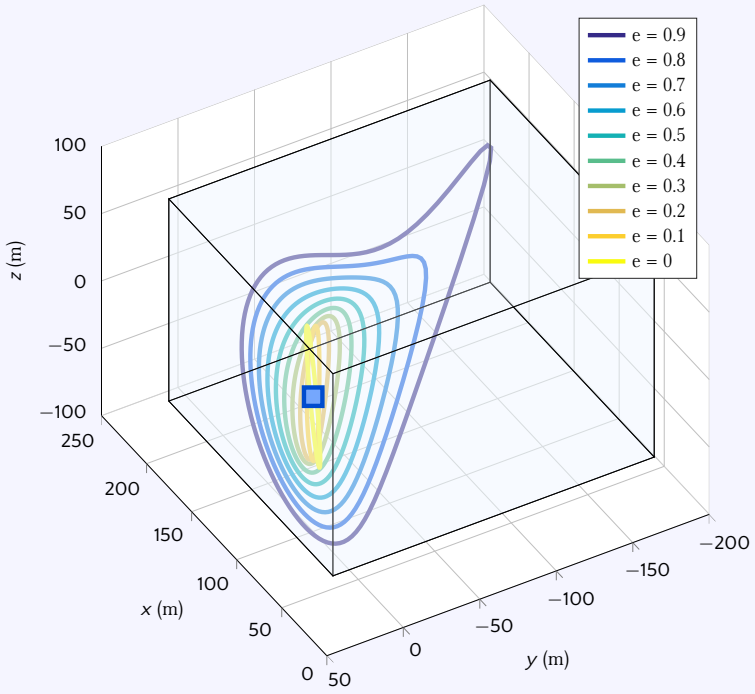
May also define the tracking error: $\widehat{\varepsilon} = \widehat{\xi} - \xi^{ref}$.

Reference periodic trajectories

Periodic trajectories

Periodic free motion $\Rightarrow \widehat{\xi}^{ref} = \begin{bmatrix} \widehat{\xi}_1^{ref} & \widehat{\xi}_2^{ref} & \widehat{\xi}_3^{ref} & \widehat{\xi}_4^{ref} & \widehat{\xi}_5^{ref} & 0 \end{bmatrix}^T.$





System representation: dynamics mismatch between current state and reference orbit:

$$\widehat{\varepsilon} = \widehat{\xi} - \widehat{\xi}^{ref}.$$

Summary of proposed hybrid context

Closed-loop dynamics

- Tracking of $\widehat{\xi}^{ref} \Rightarrow$ periodic trajectory $\Rightarrow$ chaser inside a tolerance box.
- System representation: error w.r.t. a desired reference orbit $\widehat{\varepsilon} = \widehat{\xi} - \widehat{\xi}^{ref}$.
- Linear translational model:

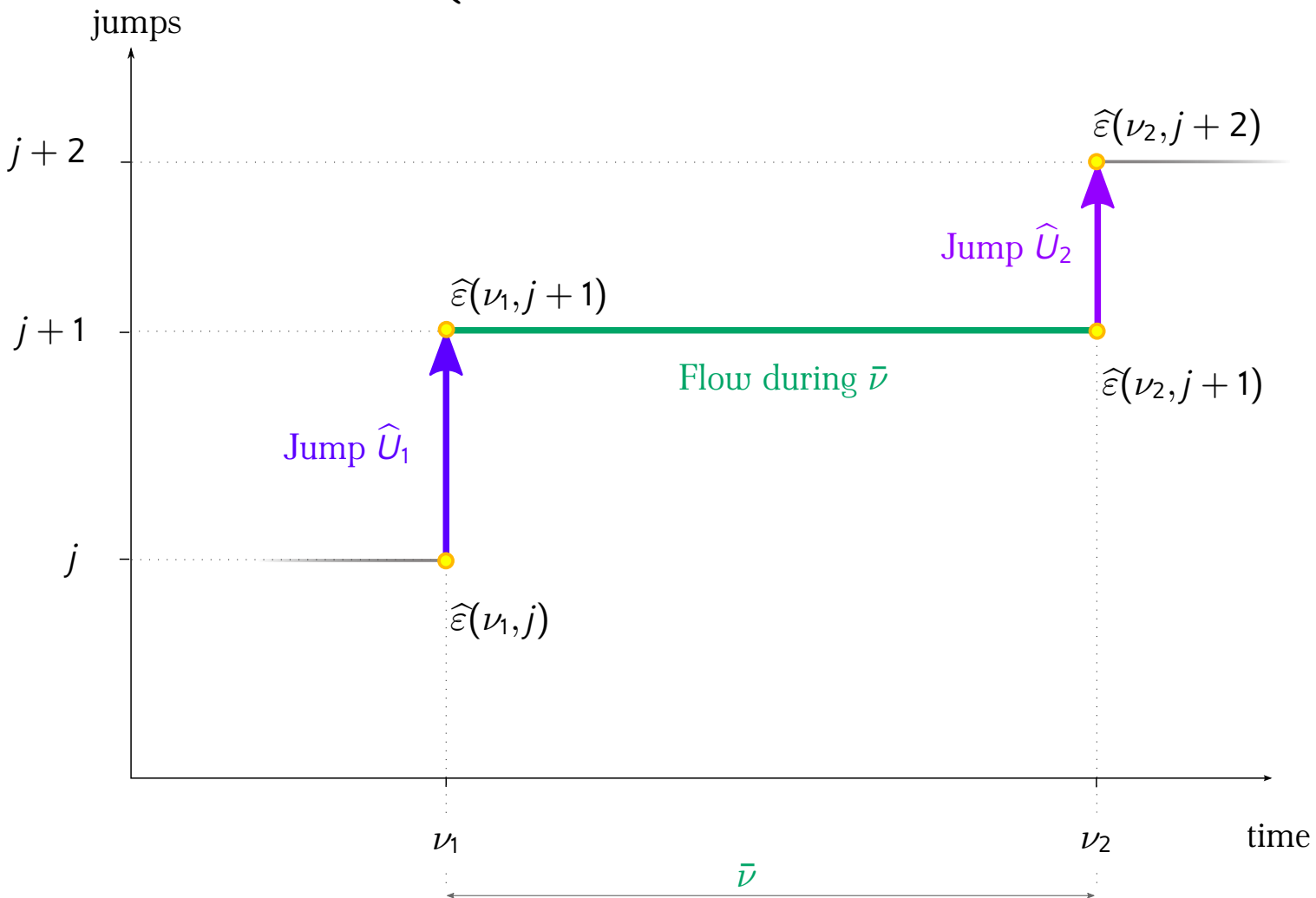
$$\begin{cases} \widehat{\varepsilon}' = \widehat{A}\,\widehat{\varepsilon}, & \widehat{\varepsilon} \in \mathcal{C} \text{ (free motion)}, \\ \widehat{\varepsilon}^{+} = \widehat{\varepsilon} + \widehat{B}(\nu)\,u, & \widehat{\varepsilon} \in \mathcal{D} \text{ (across impulsive thrusts)}. \end{cases}$$

Hybrid dynamical systems (Teel, Goebel, Sanfelice 2012)

- Continuous-time differential equations: free motion dynamics $\rightarrow$ flow set $\mathcal{C}$.
- Discrete-time difference equations: impulses execution $\rightarrow$ jump set $\mathcal{D}$.
- Control law amounts to selecting u and $\mathcal{D}$ ($\mathcal{C}$ = closed complement of $\mathcal{D}$):
- We propose a dynamical solution depending on timers (controller states).

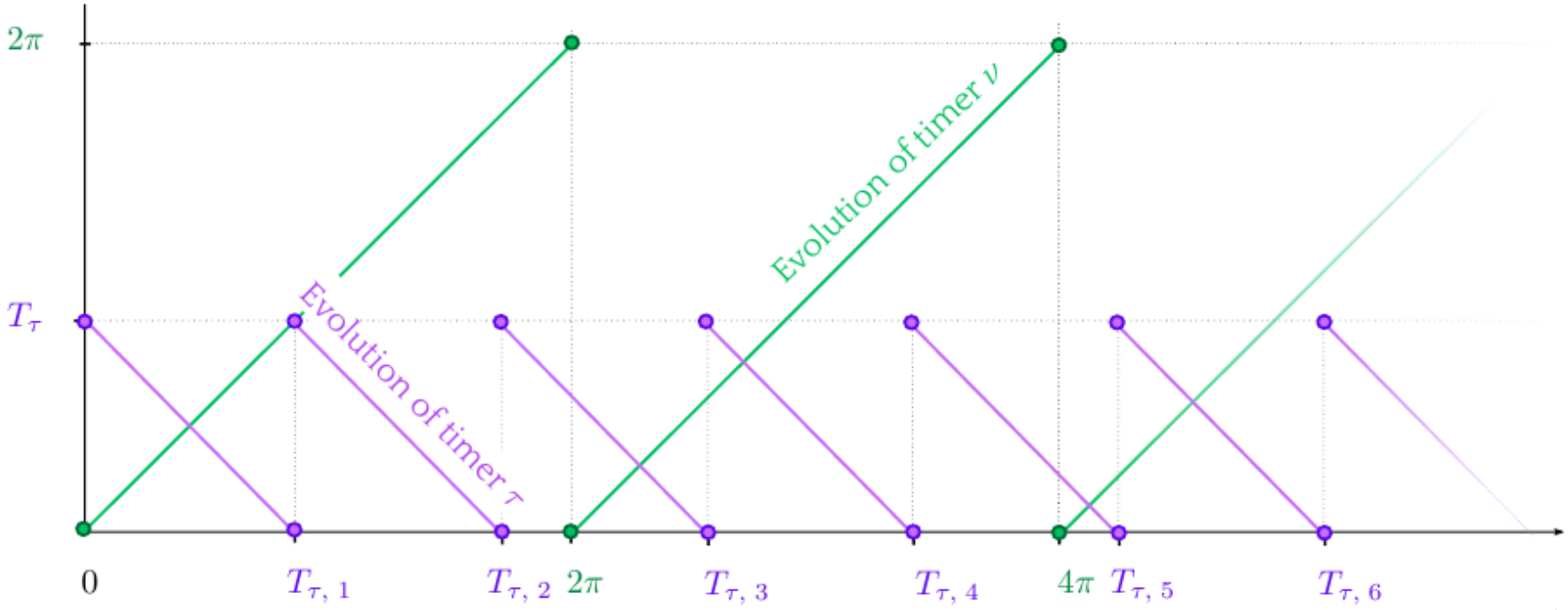
A typical evolution of a hybrid solution

$$\begin{cases} \widehat{\varepsilon}\,' = \widehat{A}\,\widehat{\xi}, & \widehat{\varepsilon} \in \mathcal{C}, \\ \widehat{\varepsilon}^{\,+} = \widehat{\varepsilon} + \widehat{B}(\nu)\,u, & \widehat{\varepsilon} \in \mathcal{D}. \end{cases}$$



Timers are the states of the dynamic controller

- Satellite dynamics $\rightarrow$ periodic of period 2π. **Timer ν** used to track the periodic time-varying nature of the satellite dynamics. $\nu^+ = 0$ each time it reaches 2π.
- Thrusters firings instants. **Timer τ** used to capture the information about how long we need to wait until the next impulsive control action. When $\tau = 0 \rightarrow$ thrusters are fired by $u = \gamma_u(\widehat{\varepsilon}, \nu)$ and next firing instant is selected $\tau^+ = \gamma_\tau(\widehat{\varepsilon}, \nu)$
- $\mathcal{D}$ is now selected. Design problem reduced to: choose γ_u and γ_τ



Hybrid closed-loop with controller states

FLOW DYNAMICS

$$(\widehat{\varepsilon}, \nu, \tau) \in \mathcal{C}$$

$$\begin{cases} \widehat{\varepsilon}' = \widehat{A}\,\widehat{\varepsilon}, \\ \nu' = 1, \\ \tau' = -1, \end{cases}$$

$$\mathcal{C} := \overline{(\mathbb{R}^6 \times [0, 2\pi] \times [0, 2\pi]) \setminus \mathcal{D}}$$

JUMP DYNAMICS

$$(\widehat{\varepsilon}, \nu, \tau) \in \mathcal{D}_\nu$$

$$\begin{cases} \widehat{\varepsilon}^+ = \widehat{\varepsilon}, \\ \nu^+ = 0, \\ \tau^+ = \tau, \end{cases}$$

$$\mathcal{D}_\nu := \mathbb{R}^6 \times \{2\pi\} \times [0, 2\pi]$$

$$(\widehat{\varepsilon}, \nu, \tau) \in \mathcal{D}_\tau$$

$$\begin{cases} \widehat{\varepsilon}^+ = \widehat{\varepsilon} + \widehat{B}(\nu)\,\gamma_u(\widehat{\varepsilon}, \nu), \\ \nu^+ = \nu, \\ \tau^+ = \gamma_\tau(\widehat{\varepsilon}, \nu). \end{cases}$$

$$\mathcal{D}_\tau := \mathbb{R}^6 \times [0, 2\pi] \times \{0\}$$

Timer	Action	Set
ν	Captures periodicity of motion. Reset to 0 when it reaches 2π	$[0, 2\pi]$
τ	Impulsive control actions sequencing. How long need to wait until the next impulse	$[0, 2\pi]$

Hybrid problem statement

Objective

We wish to (partially) meet the following specifications:

A. The chaser departs from an initial state and converges to a final state $\left\{\widehat{\xi} = \widehat{\xi}^{\,ref}\right\}$ (reference periodic trajectory) which is uniformly globally asymptotically stable.

B. The chaser tracks the reference periodic trajectory during the inspection.

C. The cost J in (??) is minimized over the full maneuver:

$$J = \sum_{k=1}^{N} ||\Delta V_k||_1 = \sum_{k=1}^{N} \left(|\Delta v_{x\ k}| + |\Delta v_{y\ k}| + |\Delta v_{z\ k}|\right).$$

Goal: Design state feedback impulsive guidance laws by selecting:

- the firing instants ν_k, $k \in \mathbb{N}$ given by the trigger law $\gamma_\tau(\widehat{\varepsilon}, \nu)$;
- the corresponding input law $\gamma_u(\widehat{\varepsilon}, \nu)$.

Proposed guidance laws

Controller	Trigger law $\gamma_\tau(\widehat{\varepsilon}, \nu)$	Input law $u = \gamma_u(\widehat{\varepsilon}, \nu)$
#1: uniform sampling norm-minimizing	$\gamma_\tau(\widehat{\varepsilon}, \nu) = \bar{\nu}$	Arrival at $\widehat{\xi}^{ref}$ by means of periodic trajectories. Minimize the Euclidean norm of the error $\widehat{\varepsilon}$.
#2: u.s. bi-impulsive	$\gamma_\tau(\widehat{\varepsilon}, \nu) = \bar{\nu}$	Tracking of $\widehat{\xi}^{ref}$ after 2 control actions.
#3: non-u.s. bi-impulsive	$\gamma_\tau(\widehat{\varepsilon}, \nu) = \arg\min_{\bar{\nu} \in [0, 2\pi]} J$	Tracking of $\widehat{\xi}^{ref}$ after 2 control actions. Minimize the cost J.
#4: u.s. tri-impulsive	$\gamma_\tau(\widehat{\varepsilon}, \nu) = \bar{\nu}$	Arrival at $\widehat{\xi}^{ref}$ by means of periodic trajectories. Tracking of $\widehat{\xi}^{ref}$ after 3 control actions.

A closer look at the dynamic matrices

Hybrid equations:

$$\begin{cases} \widehat{\varepsilon}\,' = \widehat{A}\,\widehat{\varepsilon}, & \widehat{\varepsilon} \in \mathcal{C}, \\ \widehat{\varepsilon}^{\,+} = \widehat{\varepsilon} + \widehat{B}(\nu)\,u, & \widehat{\varepsilon} \in \mathcal{D}. \end{cases}$$

Matrices $\hat{A}$ and $\hat{B}$ have useful structure:

$$\widehat{A} = (1\text{-}e^2)^{-3/2} \left[\begin{array}{cc|cccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad \widehat{B}(\nu) = \frac{1}{k^2\rho(1-e^2)} \begin{bmatrix} 0 & -(1\text{-}e^2)s_\nu & 0 \\ 0 & (1\text{-}e^2)c_\nu & 0 \\ -e(1+\rho)s_\nu - \dfrac{3\sigma\rho^2}{(1\text{-}e^2)^{3/2}} & 0 & \dfrac{3\sigma e\rho s_\nu}{(1\text{-}e^2)^{3/2}} - \rho^2 + \rho - 2 \\ (1\text{-}e^2)(1+\rho)s_\nu & 0 & (1\text{-}e^2)\rho c_\nu \\ (1+\rho)c_\nu + e & 0 & -\rho s_\nu \\ -3\rho^2 & 0 & 3e\rho s_\nu \end{bmatrix}$$

where $c_\nu := \cos(\nu) \quad s_\nu := \sin(\nu)$

Uniform sampling and norm-minimizing

Control law 1

Theorem. The reference periodic orbit $\widehat{\xi}^{ref}$ is stable (attractivity not guaranteed).

The chaser moves through periodic trajectories $\Rightarrow \widehat{\varepsilon}_6^{\,+} = 0$, while the rest of the states are minimized.

Jump equation:

$$\left[\begin{array}{c} \widehat{\varepsilon}_{1\ldots5}^{\,+} \\ \hline 0 \end{array}\right] = \left[\begin{array}{c} \widehat{\varepsilon}_{1\ldots5} \\ \hline \widehat{\varepsilon}_6 \end{array}\right] + \widehat{B}(\nu)\, u \longrightarrow u = u^{\|_6} + u^{\perp_6}$$

- $u^{\|_6}$ in charge of $\widehat{\varepsilon}_6^{\,+} = 0$.
- $u^{\perp_6}$ must *min* $\widehat{\varepsilon}_{1\ldots5}$ while having no influence over $u^{\|_6}$.

Computation of the input law $u = \gamma_u(\widehat{\varepsilon}, \nu)$:

$$\gamma_u\left(\widehat{\varepsilon}, \nu\right) = -\frac{\widehat{b}_6(\nu)^T}{\|\widehat{b}_6(\nu)\|^2}\,\widehat{\varepsilon}_6 - \widehat{B}_6^{\perp}(\nu)\,\left(\widehat{B}(\nu)\,\widehat{B}_6^{\perp}(\nu)\right)^{-L}\left(\widehat{\varepsilon} - \widehat{B}(\nu)\,\frac{\widehat{b}_6(\nu)^T}{\|\widehat{b}_6(\nu)\|^2}\,\widehat{\varepsilon}_6\right).$$

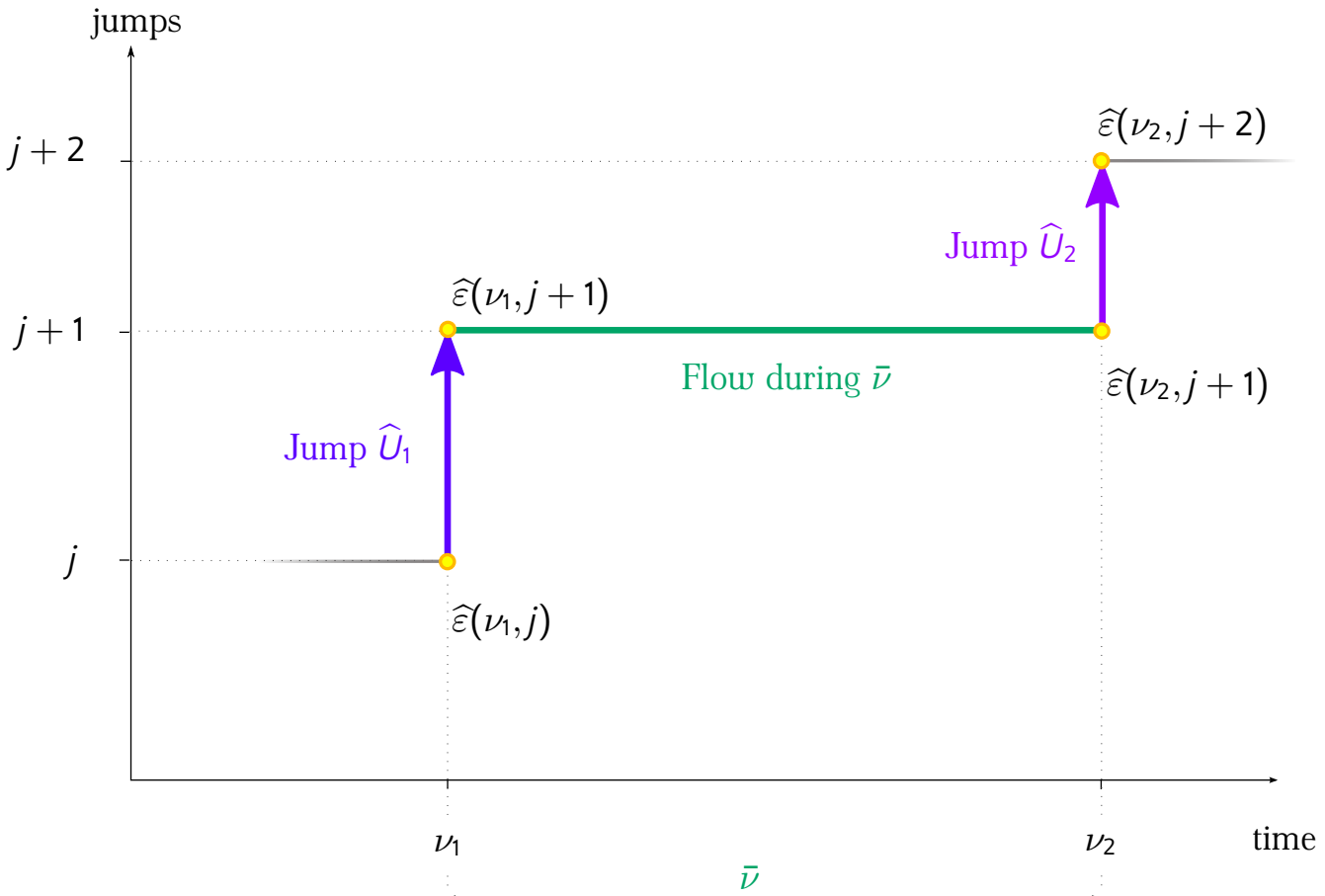
Uniform sampling bi-impulsive

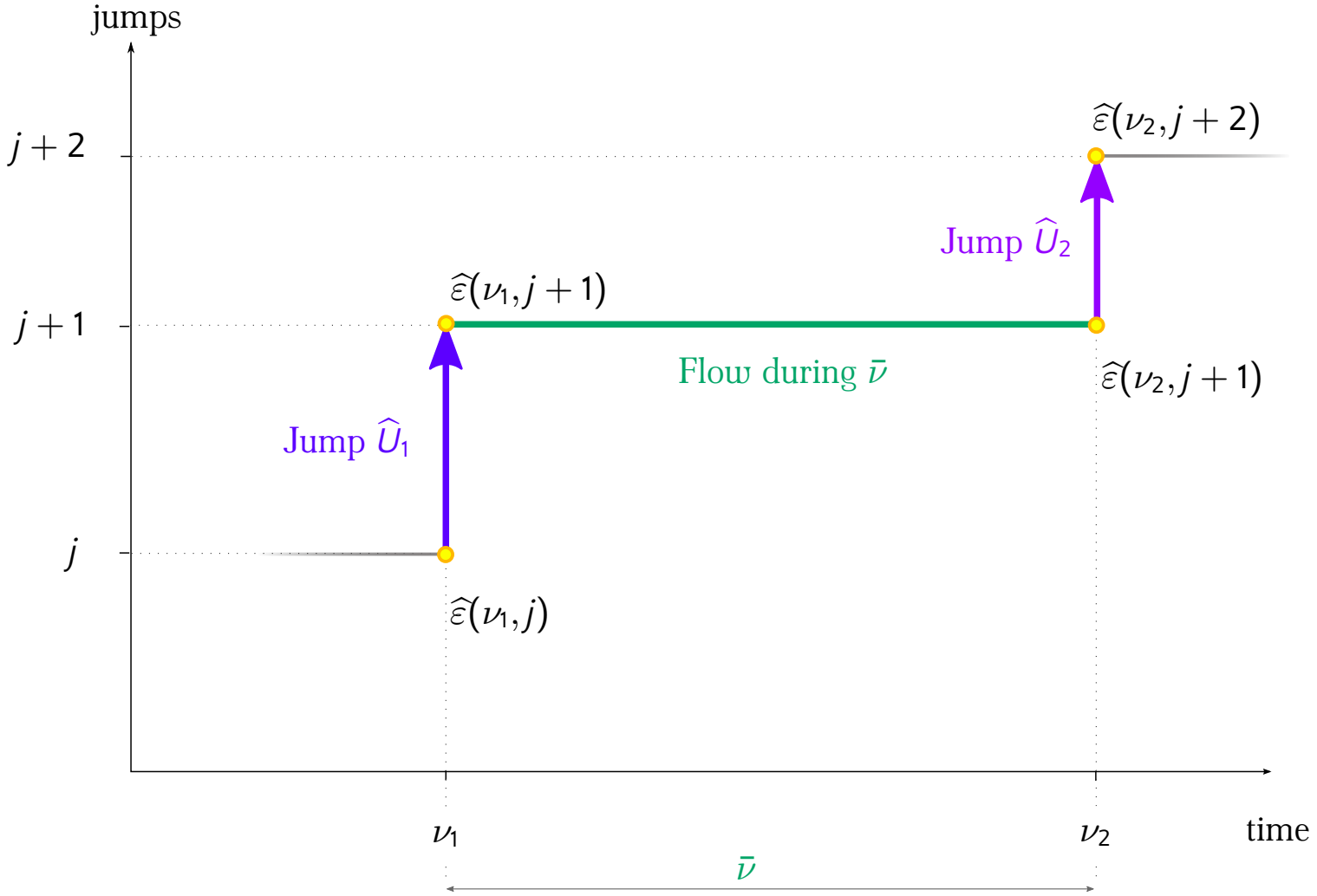
Control law 2

Theorem. The reference periodic orbit $\widehat{\xi}^{ref}$ is stable and attractive.

The chaser arrives at the reference $\widehat{\xi}^{ref}$ in 2 control impulses separated by an angle $\bar{\nu}$:

$$\widehat{\varepsilon}(\nu_1 + \bar{\nu}, j+2) = 0. \tag{4}$$





Thrust u_1 at time ν_1: $\widehat{\varepsilon}(\nu_1, j+1) = \widehat{\varepsilon}(\nu_1, j) + \widehat{B}(\nu_1)\, u_1.$

Flow during $\bar{\nu}$: $\widehat{\varepsilon}(\nu_1 + \bar{\nu}, j+1) = \widehat{\Phi}(\bar{\nu}) \left[\widehat{\varepsilon}(\nu_1, j) + \widehat{B}(\nu_1)\, u_1\right].$

Thrust u_2 at time $\nu_1 + \bar{\nu}$: $\widehat{\varepsilon}(\nu_1 + \bar{\nu}, j+2) = \widehat{\Phi}(\bar{\nu}) \left[\widehat{\varepsilon}(\nu_1, j) + \widehat{B}(\nu_1)\, u_1 + \widehat{\Phi}(-\bar{\nu})\widehat{B}(\nu_1 + \bar{\nu})\, u_2\right].$

Uniform sampling bi-impulsive

Control law 2

Theorem. The reference periodic orbit $\widehat{\xi}^{ref}$ is stable and attractive. This results from forward invariance and global uniform attractivity.

The chaser arrives at the reference $\widehat{\xi}^{ref}$ in 2 control impulses separated by an angle $\bar{\nu}$:

$$\widehat{\varepsilon}(\nu_1 + \bar{\nu}, j+2) = 0. \tag{5}$$

Computation of the input law $\gamma_u(\widehat{\varepsilon}, \nu) = u$:

$$\gamma_u(\widehat{\varepsilon}, \nu) = [-I \;\; 0] \underbrace{\begin{bmatrix} \widehat{B}(\nu) & \widehat{\Phi}(-\bar{\nu})\,\widehat{B}(\nu+\bar{\nu}) \end{bmatrix}^{-1}}_{M(\nu,\bar{\nu})^{-1}} \widehat{\varepsilon}. \tag{6}$$

Conjecture: (no proof)

Matrix M is invertible $\Leftrightarrow \bar{\nu} \neq k\pi, k \in \mathbb{Z}$.

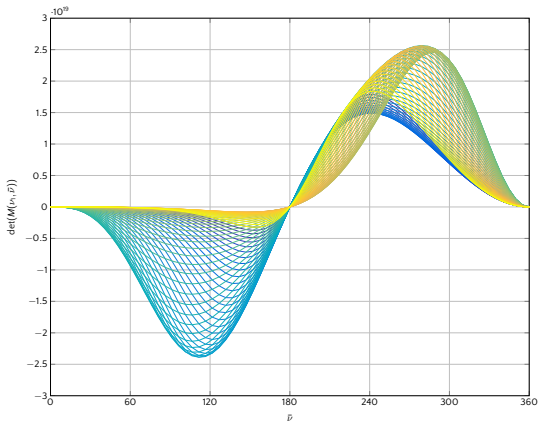


Figure 3 : Determinant of M.

Non-uniform sampling bi-impulsive

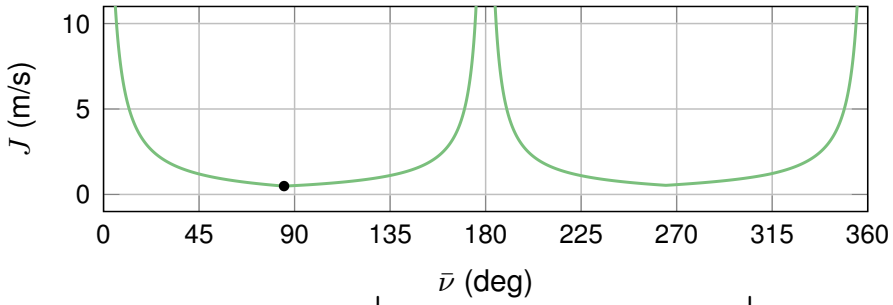
Control law 3

Theorem. The reference periodic orbit $\widehat{\xi}^{ref}$ is stable and attractive. This results from forward invariance and global uniform attractivity.

The chaser arrives at the reference $\widehat{\xi}^{ref}$ in 2 control impulses separated by an angle $\bar{\nu}$: $\widehat{\varepsilon}(\nu_1 + \bar{\nu}, j+2) = 0$.
The input law $u = \gamma_u(\widehat{\varepsilon}, \nu)$ is the same as for the previous controller:

$$\gamma_u(\widehat{\varepsilon}, \nu) = [-I \;\; 0]\, M(\nu, \nu^*)^{-1}\, \widehat{\varepsilon}.$$

The trigger function is evaluated at $\nu^* \neq k\pi, k \in \mathbb{Z}$, which minimizes the cost:



$$\nu^* = \gamma_\tau(\widehat{\varepsilon}, \nu) = \underset{\bar{\nu}\in[0,2\pi]}{\arg\min} \left| [I \;\; 0]\, M(\nu, \bar{\nu})^{-1}\, \widehat{\varepsilon} \right|_1 = \underset{\bar{\nu}\in[0,2\pi]}{\arg\min}\; J.$$

The corresponding minimization is not (not even locally) convex: hard to implement.

Uniform sampling tri-impulsive

Control law 4

Theorem. The reference periodic orbit $\widehat{\xi}^{ref}$ is stable and attractive. This results from forward invariance and global uniform attractivity.

Take advantage of the decoupling in matrices $\widehat{A}$ and $\widehat{B}(\nu)$ such that the control design is separated in two completely decoupled problems corresponding to the following partitions:

$$\begin{cases} \begin{bmatrix} \widehat{\varepsilon}_{1,2}{}' \\ \hline \widehat{\varepsilon}_{3\cdots5}{}' \\ \widehat{\varepsilon}_{6}{}' \end{bmatrix} = \underbrace{\left[\begin{array}{c|cc} 0_{2\times2} & 0_{2\times3} & 0_{2\times1} \\ \hline 0_{3\times2} & 0_{3\times3} & [f(e) \;\; 0_{2\times1}]^T \\ 0_{1\times2} & 0_{1\times3} & 0 \end{array}\right]}_{\widehat{A}} \begin{bmatrix} \widehat{\varepsilon}_{1,2} \\ \hline \widehat{\varepsilon}_{3\cdots5} \\ \widehat{\varepsilon}_{6} \end{bmatrix}, \\ \\ \begin{bmatrix} \widehat{\varepsilon}_{1,2}^{+} \\ \hline \widehat{\varepsilon}_{3\cdots5}^{+} \\ \widehat{\varepsilon}_{6}^{+} \end{bmatrix} = \begin{bmatrix} \widehat{\varepsilon}_{1,2} \\ \hline \widehat{\varepsilon}_{3\cdots5} \\ \widehat{\varepsilon}_{6} \end{bmatrix} + \underbrace{\left[\begin{array}{c|c} \widehat{B}_{\alpha}(\nu)_{2\times1} & 0_{2\times2} \\ \hline 0_{4\times1} & \widehat{B}_{\beta}(\nu)_{4\times2} \end{array}\right]}_{\widehat{B}(\nu)} \begin{bmatrix} \Delta\widehat{v}_b \\ \hline \Delta\widehat{V}_{ac} \end{bmatrix}. \end{cases} \tag{7}$$

Guidance law #4: tri-impulsive

$$\widehat{A} = \left[\begin{array}{cc|ccc c} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & \dfrac{1}{(1-e^2)^{3/2}} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

$$\widehat{B}(\nu) = \underbrace{\frac{1}{k^2\rho^2(1-e^2)}}_{K} \left[\begin{array}{c|c|c} 0 & -(1-e^2)\,\rho\sin(\nu) & 0 \\ 0 & (1-e^2)\,\rho\cos(\nu) & 0 \\ \hline -e(1+\rho)\rho\sin(\nu) - \dfrac{3\sigma\rho^3}{(1-e^2)^{3/2}} & 0 & \dfrac{3\sigma e\rho^2\sin(\nu)}{(1-e^2)^{3/2}} - \rho^3 + \rho^2 + 2\rho \\ (1-e^2)(1+\rho)\rho\sin(\nu) & 0 & (1-e^2)\rho^2\cos(\nu) \\ (1+\rho)\rho\cos(\nu) + e\rho & 0 & -\rho^2\sin(\nu) \\ -3\rho^3 & 0 & 3e\rho^2\sin(\nu) \end{array}\right]$$

Periodic tri-impulsive

Control law 4

The chaser moves through periodic trajectories $\Rightarrow \widehat{\varepsilon}_6^{\,+} = 0$.

$$\begin{cases} \widehat{\varepsilon}_{1,2}^{\,+} = \widehat{\varepsilon}_{1,2} + \widehat{B}_\alpha(\nu)\, u_b, \\ \begin{bmatrix} \widehat{\varepsilon}_{3\ldots5}^{\,+} \\ 0 \end{bmatrix} = \begin{bmatrix} \widehat{\varepsilon}_{3\ldots5} \\ \widehat{\varepsilon}_6 \end{bmatrix} + \widehat{B}_\beta(\nu)\, u_{ac} \longrightarrow u_{ac} = u_{ac}^{\parallel 6} + u_{ac}^{\perp 6}. \end{cases} \tag{8}$$

- u_b can be computed as in the bi-impulsive method. In 2 impulses $\widehat{\varepsilon}_{1,2} = 0$.
- $u_{ac}^{\parallel 6}$ in charge of $\widehat{\varepsilon}_6^{\,+} = 0$. In 1 impulse $\widehat{\varepsilon}_6 = 0$.
- $u_{ac}^{\perp 6}$ must have no influence over $u_{ac}^{\parallel 6}$. In 3 impulses $\widehat{\varepsilon}_{3\ldots5} = 0$.

Actions u_b and u_{ac} computed separately

Computation of input law $\gamma_{u,\ (1,2)}(\widehat{\varepsilon},\nu) = \Delta\widehat{v}_b$:

- State $\widehat{\varepsilon}_{1,2}$ becomes 0 after 2 impulses: $\widehat{\varepsilon}_{1,2}(\nu+\bar{\nu}, j+2) = 0$.
- Propagation of the hybrid dynamics along the 2 impulses, executed at ν and $\nu+\bar{\nu}$:

$$\gamma_{u,\ (1,2)}(\widehat{\varepsilon},\nu) = -\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\widehat{B}_{1,2}(\nu) & \widehat{B}_{1,2}(\nu+\bar{\nu})\end{bmatrix}^{-1}\widehat{\varepsilon}_{1,2}. \tag{9}$$

Computation of input law $\gamma_{u,\ (3\cdots6)}(\widehat{\varepsilon},\nu) = \Delta\widehat{V}_{ac}$:

- State $\widehat{\varepsilon}_6$ becomes and remains 0 after each impulse: $\widehat{\varepsilon}_6^{\,+} = 0$.
- Design of a control component $u_{ac}^{\|_6}$ that ensures periodicity $\widehat{\varepsilon}_6^{\,+} = 0$ while $u_{ac}^{\perp_6}$ has no influence over $\widehat{\varepsilon}_6$.
- Propagation of the hybrid dynamics along 3 impulses, executed at $\nu_1 = \nu$, $\nu_2 = \nu+\bar{\nu}$ and $\nu_3 = \nu+2\bar{\nu}$:

$$\gamma_{u,\ (3\cdots6)}(\widehat{\varepsilon},\nu) = -\frac{\hat{b}_6(\nu)}{|\hat{b}_6(\nu)|}\hat{\varepsilon}_6 - B_6^{\perp}(\nu)[1\ 0\ 0]\begin{bmatrix}\widehat{B}_{xz}(\nu) & \widehat{B}_{xz}(\nu+\bar{\nu}) & \widehat{B}_{xz}(\nu+2\bar{\nu})\end{bmatrix}^{-1}\widehat{\varepsilon}_{3\cdots6}$$

Periodic bi-impulsive

Control law 4

Theorem. The reference periodic orbit $\widehat{\xi}^{ref}$ is stable and attractive. The proof uses forward invariance and global attractivity.

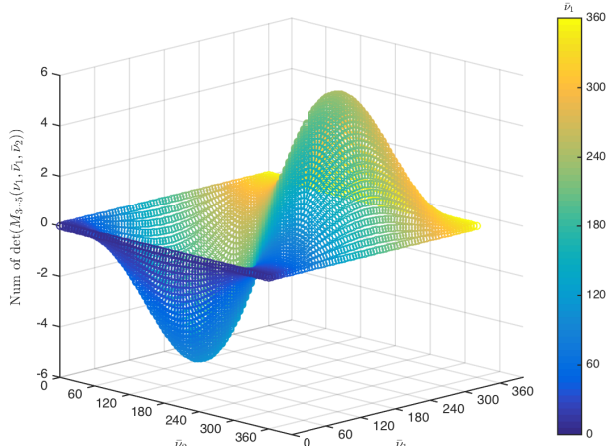
The chaser arrives at the reference $\widehat{\xi}^{ref}$ in 3 control impulses separated two angles $\bar{\nu}_1$ and $\bar{\nu}_2$:

$$\widehat{\varepsilon}(\nu_3, j+3) = 0. \tag{10}$$

Computation of the input law $u = \gamma_u(\widehat{\varepsilon}, \nu)$ when $\bar{\nu}_1 = \bar{\nu}_2 = \bar{\nu}$:

$$\gamma_u\left(\widehat{\varepsilon}, \nu\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \gamma_{u,\ (1\cdots 2)}(\widehat{\varepsilon}, \nu) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \gamma_{u,\ (3\cdots 6)}(\widehat{\varepsilon}, \nu) \tag{11}$$

Proposition. Matrix M is invertible **if and only if** $\bar{\nu}_1 + \bar{\nu}_2 \neq 2k\pi, k \in \mathbb{Z}$.

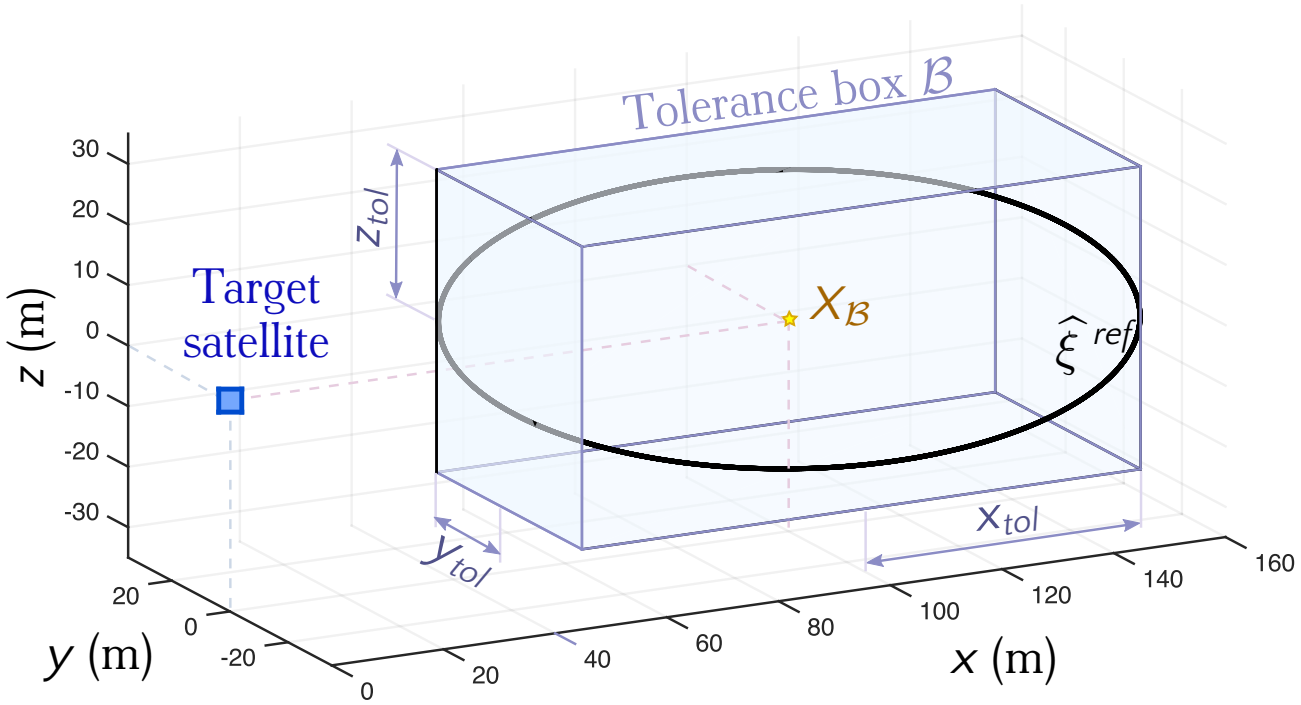


Simulations

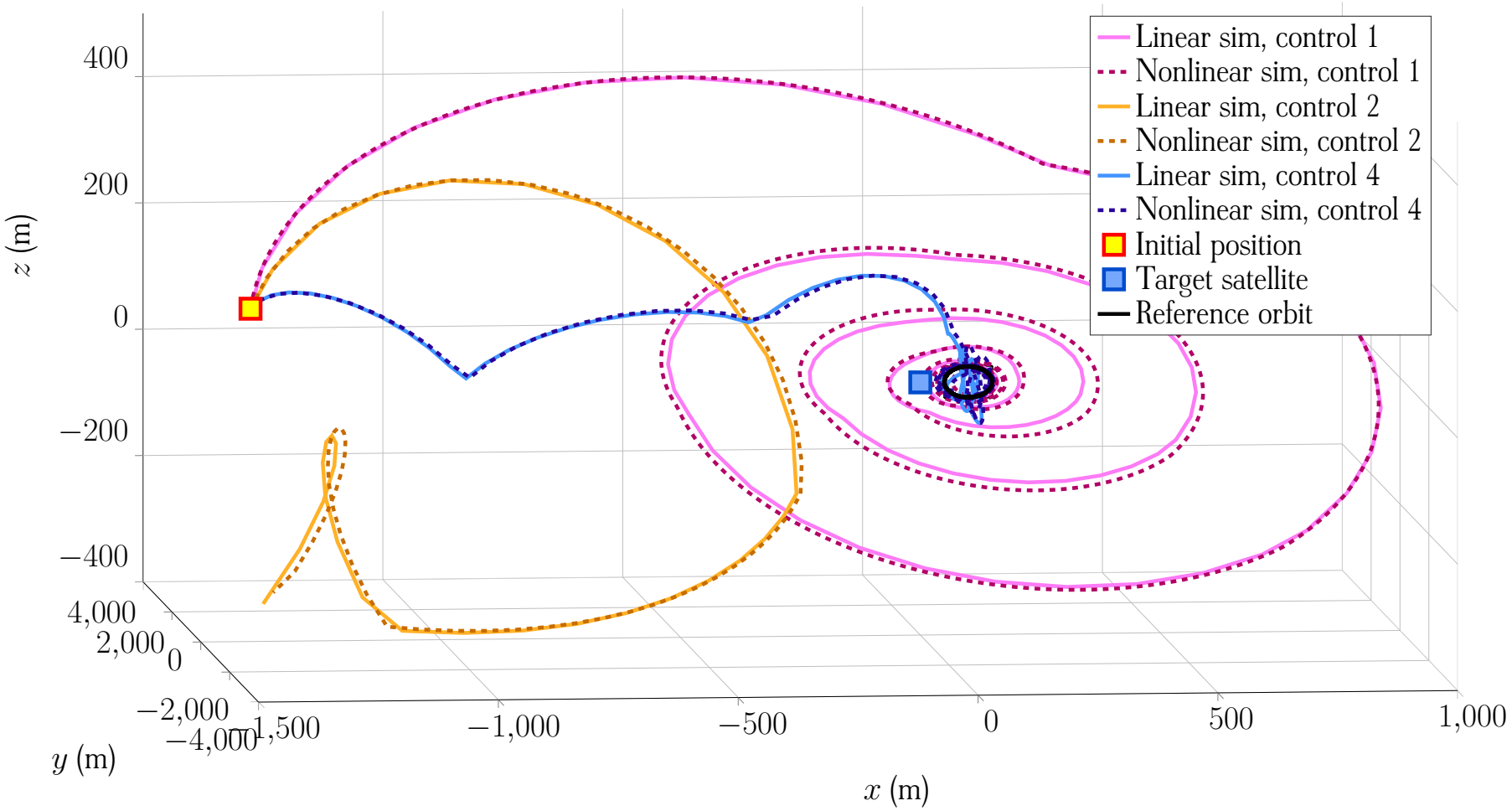
Matlab-Simulink

Simulators

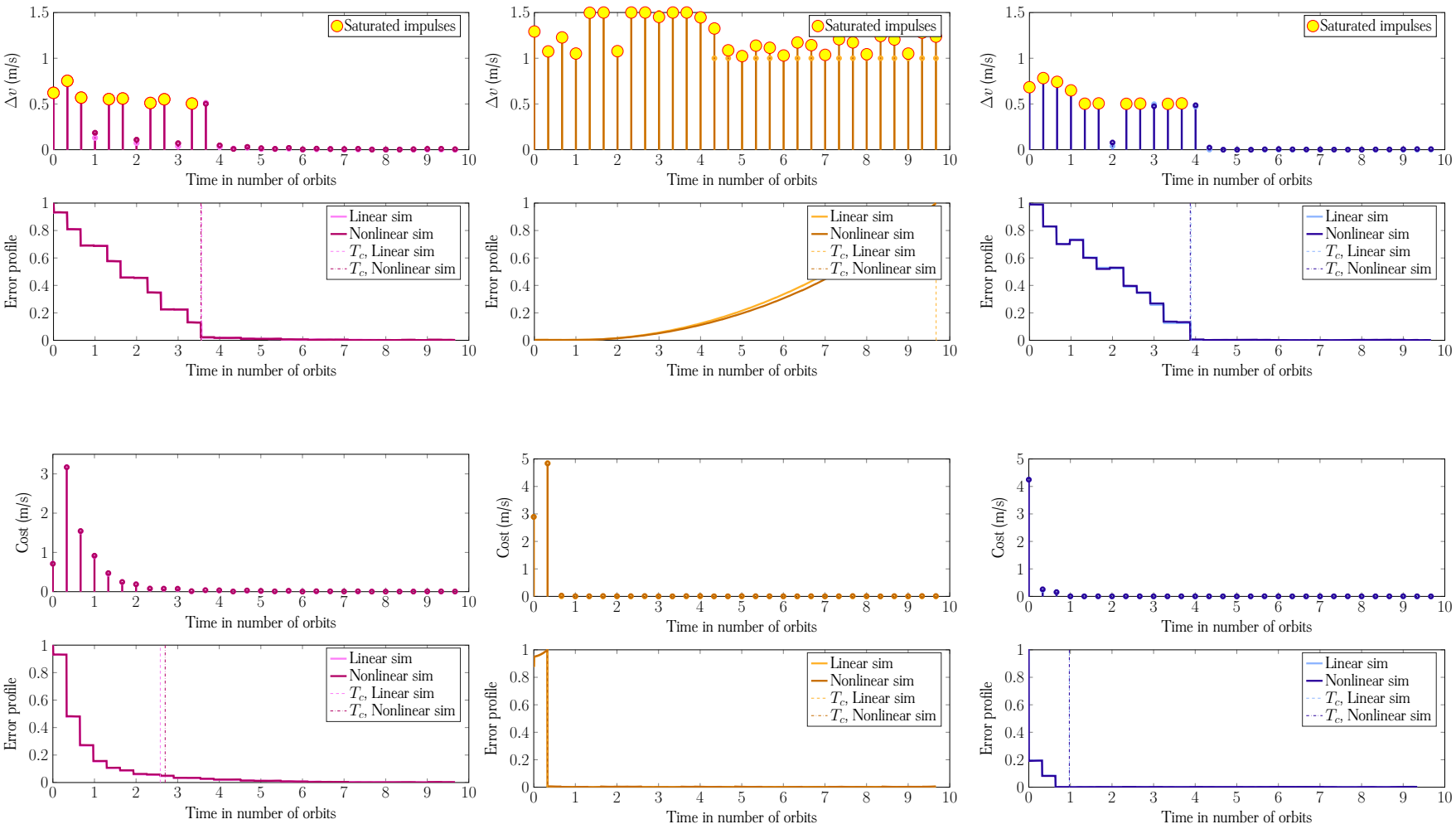
- Approach and inspection phases for the PRISMA mission.
- Linear vs nonlinear simulations (J_2, atmospheric drag, saturations, dead zones).
- Performance indices: fuel consumption & convergence time.

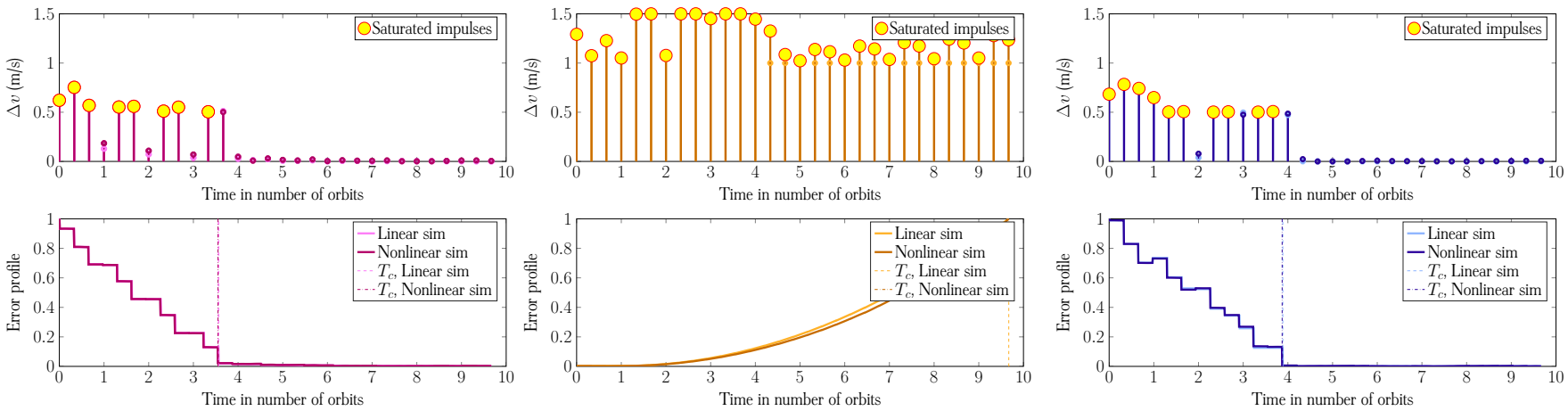


Initial state: $X_{chaser} = [-1300 \quad 3200 \quad 50 \quad 0 \quad 0 \quad 0]^T$.
Target position: $X_{target} = [0 \quad 0 \quad 0]^T$.
Tolerance box centered at $X_{tol} = [100 \quad 50 \quad 50]^T$ of widths $[50 \quad 25 \quad 25]^T$.



Norm-minimizing & tri-impulsive: periodic trajectories ⇒ safe.


Saturated impulses
Δv (m/s)
Time in number of orbits
Error profile
Linear sim
Nonlinear sim
T_c, Linear sim
T_c, Nonlinear sim
Cost (m/s)



Consumption (m/s) for the different uniform sampling guidance laws:

#1: norm-min		#2: 2-imp		#4: 3-imp	
LIN	NL	LIN	NL	LIN	NL
5.49	5.69	*	*	6.85	6.99

Convergence time (nb. orbits) for the different uniform sampling guidance laws:

#1: norm-min		#2: 2-imp		#4: 3-imp	
LIN	NL	LIN	NL	LIN	NL
3.56	3.56	*	*	3.87	3.87

Conclusions & perspectives

Conclusions

– Floquet-Lyapunov theory successfully applied to rendez-vous dynamics

- Suitable coordinate transformations provide simplified dynamics.
- Suggestive hybrid framework enables the use of intuitive laws (timers, periodicity).
- Fundamental theorems with compact attractors and data regularity allow to conclude stability from (uniform) convergence and invariance.
- Robustness of asymptotic stability is confirmed by perturbed simulations.

– Satellite rendez-vous control is an example where hybrid dynamics is useful:

Perspectives

- Physical meaning of the parameters after the coordinate transformations: all constant except one $\rightarrow$ link to relative orbital parameters?
- Address saturation more directly
- Design sub-optimal laws based on simple hybrid mechanisms (estimates of J?).