Static input allocation for reaction wheels desaturation using magnetorquers

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Attitude control performed with two actuators

- Reaction wheels
- Magnetorquers

![Diagram of attitude control system with initial and desired attitudes, control law, actuators, sensors, and satellite dynamics](image)

- Control law
- Disturbance torque
- Current attitude
Reaction wheels suffer from total momentum problems

Reaction wheels

\[ T_w = \dot{h}_w \]

\[ T_i \]

\[ h \]

\[ h_w \in \mathbb{R}^3: \text{angular momentum} \]
\[ T_w \in \mathbb{R}^3: \text{control torque} \]

The total momentum cannot be modified (wheel turns CW, satellite turns CCW)

risk of saturation of \( h_w \)

\[ h_w(t) = \int_0^t T_w(\tau) d\tau \] needs to be controlled
Magnetorquers confined to exert 2D torque

\[ T_m = -\tilde{b} \times (t, q) \tau_m = -(R(q)\tilde{b}_o(t)) \times \tau_m \]

Notation

\[ z \times = \begin{bmatrix} 0 & -z_z & z_y \\ z_z & 0 & -z_x \\ -z_y & z_x & 0 \end{bmatrix} \]

Nomenclature

- \( T_m \in \mathbb{R}^3 \): control torque
- \( \tilde{b} \in \mathbb{R}^3 \): magnetic field
- \( \tau_m \in \mathbb{R}^3 \): magnetic momentum
- \( q \in \mathbb{R}^4 \): quaternion
- \( R \in \mathbb{R}^{3 \times 3} \): rotation matrix

\( \times \): instantaneous controllability restricted to a plane (\( \forall z \in \mathbb{R}^3 \), \( z \times \) is singular)

\( \tilde{b}_o(t) \): almost periodic and uncertain
Stabilization problem requires coordination of the actuators

Equations of the attitude motion

\[
\begin{align*}
J \dot{\omega} &= -\omega \times (J \omega + h_w) - \tau_w - \tilde{b} \times (t, q) \tau_m \\
h_w &= \tau_w \\
[\dot{\varepsilon} \dot{\eta}] &= \frac{1}{2} \begin{bmatrix} -\omega \times & \omega \\ -\omega^T & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}
\end{align*}
\]

Nomenclature

Satellite:

- $\omega$: angular velocity
- $q = (\varepsilon, \eta)$: quaternion
- $J$: inertia matrix

Reaction wheels:

- $h_w$: angular momentum
- $\tau_w = T_w$: control torque

Magnetorquers:

- $\tilde{b}(t, q)$: geomagnetic field
- $\tau_m$: magnetic momentum

Stabilizing state-feedback problem: find $\tau_w(x)$ and $\tau_m(x)$ such that $x = \begin{bmatrix} \omega \\ q \\ h_w \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ q \circ \\ h_{\text{ref}} \end{bmatrix}$

Actuators may badly interact
Global attitude properties via hybrid feedback laws

Ideal attitude feedback $u_{\text{att}}$ must be selected as a hybrid control law

$$J\dot{\omega} = -\omega \times J\omega + u_{\text{att}} + d$$

$$\begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega \times & \omega \\ -\omega^T & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix}$$

- Even if $d = 0$, no time-invariant continuous selection $u_{\text{att}}(x)$ stabilizes the compact attractor $\mathcal{A} := \{\omega = \varepsilon = 0, \eta = \pm 1\}$  [Bhat et al, 2000]

- hybrid solution available in the literature [Mayhew et al, 2009]:

For any scalars $c > 0$, $\delta \in (0, 1)$ and any matrix $K_\omega \succ 0$, the attractor $\mathcal{A}$ is globally asymptotically and locally exponentially stabilized by the control law:

$$u_{\text{att}} := -cx_c\varepsilon - K_\omega\omega,$$

$$\dot{x}_c = 0, \quad (q, \omega, x_c) \in C,$n$$

$$x_c^+ = -x_c, \quad (q, \omega, x_c) \in D,$n$$

where the flow set $C$ and the jump set $D$ are defined as

$$C := \{(q, \omega, x_c) \in S^3 \times \mathbb{R}^3 \times \{-1, 1\} : x_c\eta \geq -\delta\}$$

$$D := \{(q, \omega, x_c) \in S^3 \times \mathbb{R}^3 \times \{-1, 1\} : x_c\eta \leq -\delta\},$$

does not take into account limitations of the actuators
I. The industrial solution: “cross product control law”

Ignore the interaction of the two inputs

\[ J \dot{\omega} = -\omega \times J \omega - \tau_w - \omega \times h_w + \tau_m, \]

\[ \begin{bmatrix} \dot{\varepsilon} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\omega \times & \omega \\ -\omega^T & 0 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} \]

- loop 1: Attitude control performed by the reaction wheels
- loop 2: Regulation of \( h_w \) by the magnetorquers
- the two loops are treated separately

The cross-product control law

\[ \tau_w = -\omega \times h_w - u_{att}, \quad \tau_m = -\frac{\tilde{b} \times (t)}{|\tilde{b}(t)|^2} k_p (h_w - h_{ref}) \]

Lack of proof of stability

- formally proving desirable stabilization properties of the overall scheme seems hard
- frequency separation between the two loops (\( = \) very aggressive action of the attitude stabilizer) gives an engineering solution [Camillo, 1980; Carrington 1981; Chen 1999]
II. New revisited version of “cross product control law” highlights cascade

New point of view on the classical approach

- quasi cascaded structure where $h_T^I$ refers to the total angular momentum (satellite + wheels)

A revisited version of the cross-product control law

$$\tau_w = -\omega \times h_w - u_{att}, \quad \tau_m = -\frac{\tilde{b} \times (t)}{|\tilde{b}(t)|^2} k_p (h_w + J\omega - R(q) h_{ref})$$

- the feedback branch (the dashed line) can be avoided by redefining $\tau_m$
- GAS is achieved for any stabilizer $u_{att}$ (under ISS and reasonable assumptions on $\tilde{b}_{\circ}(t)$)
- attitude dynamics is affected by the secondary task of momentum damping
III. New static-allocation-based controller induces desirable attitude

Allocation-based controller equations

\[ \tau_w = - \omega \times h_w - (R(q) \tilde{b}_o(t)) \times \tau_m - u_{\text{att}}, \quad \tau_m = - \frac{(R(q) \tilde{b}_o(t)) \times}{|\tilde{b}_o(t)|^2} k_p (h_w - h_{\text{ref}}) \]

Reversing the cascaded structure

- giving priority to the attitude control goal
- equivalent to a new different partition of the dynamics equation:

\[ J \dot{\omega} + \omega \times J \omega = - \tau_w - \omega \times h_w + T_m + u_{\text{att}} \]

✓ GAS is achieved for any stabilizer \( u_{\text{att}} \) (No ISS needed but same mild assumptions on \( \tilde{b}_o(t) \))
III. New static-allocation-based controller induces desirable attitude

Allocation-based controller equations

\[
\tau_w = -\omega \times h_w - (R(q)\tilde{b}_o(t)) \times \tau_m - u_{\text{att}}, \\
\tau_m = -\left(\frac{(R(q)\tilde{b}_o(t))}{|\tilde{b}_o(t)|^2}\right) k_p (h_w - h_{\text{ref}})
\]

Proof of stability uses reduction theorem for hybrid systems

- if attractor \( \mathcal{A} \) is GAS (and LES) for the upper system
- if the origin is GAS for the lower system with zero input
- if all solutions are bounded (proved with exponential convergence of \( u \) + Gronwall)

Then the attractor \( \mathcal{A} \times \{h = h_{\text{ref}}\} \) is GAS for the overall system.
Simulation results reveal advantages of the proposed controller

Context of the simulations
- mission of the micro-satellite Demeter designed by CNES, the French space agency
- $\tilde{b}_o(t)$ evaluated by means of the IGRF (high fidelity model of the geomagnetic field)
- rest-to-rest maneuvers with non-nominal $h_w$

Controllers used
- Classical “cross product control” controller
- Revisited version of the classical controller
- Allocation-based controller

Simulation tests
- Nominal: Shows that the classical solution diverges
- Perturbed $J$: Allocation outperforms Revisited
- Periodic disturbances: Allocation outperforms Revisited
Aggressive attitude controller $u_{\text{att}}$

Similar results

![Graph showing aggressive attitude controller $u_{\text{att}}$](attachment:image.png)
Allocation for attitude control

Position on orbit (%)

\( h_w (N.m.s) \)

\( \tau_w (N.m) \)

\( \tau_m (A.m^2) \)

\( x 10^{-3} \)

classical
revisited
allocation

\( \times \) saturation

(University of Padova)
Non-aggressive attitude controller $u_{att}$

![Graph showing comparison between classical, revisited, and allocation controllers.]

- **revisited and allocation controllers preserve stability**
- **Attitude transient is more regular for the allocation-based strategy**
\textbf{✓ Actuators do not saturate}
Monte-Carlo study with uncertainties on \( J \) reveals improved transients

- Clear advantages emerge from swapping the cascaded structure

- Improved attitude transients with allocation-based controller
Monte-Carlo study with uncertainties on $J$ reveals smaller inputs

- Reduced spread and usage of the actuators efforts

✓ Improved attitude transients with allocation-based controller
Periodic disturbances are best handled by allocator

- No formal analysis has been performed for this case

✔ Improved attitude response with allocation-based controller
Conclusions

Summary of the advantages of the new allocation-based controller

- ✓ actuators are less inclined to saturate (non-aggressive attitude stabilizers can be handled)
- ✓ attitude dynamics independent of the momentum damping
- ✓ rigorous proof of stability
- ✓ good properties of robustness w.r.t. uncertainties on $\tilde{b}_c(t)$ (according to simulation results)

Perspectives

✗ mean value of attitude perturbations induces a drift of the momenta of the reaction wheels [Lovera, 2001]

⇒ How this new allocation framework can prevent these phenomena to occur?