Input allocation using dynamics: theory and applications

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Outline

1. Main Ideas and Static Actuators
2. Industrial Application: FTU elongation control
3. Considering Actuator Dynamics
4. Industrial application: JET Current Limit Avoidance
Weak and strong input redundancy in linear plants

A linear plant with weak or strong input redundancy

- **Strong**: some nonzero time-varying inputs produce zero transients
- **Weak**: some nonzero converging inputs produce zero steady-state

\[
\begin{align*}
\dot{x} &= Ax + Bu + B_d d \\
y &= Cx + Du + D_d d,
\end{align*}
\]

Def’n: A plant is input-redundant if one of the following two conditions is satisfied

- it is strongly input-redundant from \( u \) if it satisfies \( \text{Ker} \left( \begin{bmatrix} B \\ D \end{bmatrix} \right) \neq \emptyset \); denote

  \[
  B_\perp \text{ such that } \text{Im}(B_\perp) = \text{Ker} \left( \begin{bmatrix} B \\ D \end{bmatrix} \right) ;
  \]

- it is weakly input-redundant from \( u \) to \( y \) if

  \[
  P^* := \lim_{s \to 0} (C(sI - A)^{-1}B + D) \text{ is finite and satisfies } \text{Ker}(P^*) \neq \emptyset ;
  \]

  denote

  \[
  B_\perp \text{ such that } \text{Im}(B_\perp) = \text{Ker}(P^*) .
  \]
Allocator dynamics may only act in the $B_\perp$ directions

Assume that a controller has been designed disregarding input redundancy to obtain a desirable plant output response $y$

$$\dot{x}_c = A_c x_c + B_c y + B_r r$$
$$y_c = C_c x_c + D_c y + D_r r,$$

Design an input allocator which
- exploits strong redundancy to get fast reallocation during transients
- exploits weak redundancy to get slow reallocation at the steady-state

The allocator measures controller output $y_c$ and adds compensating signal
- Choose that signal as $B_\perp w$ for some $w$
- Pick $w$ as the output of a pool of integrators (dynamic solution)
Linear dynamic allocation minimizes $J = (u - u_O)^T \tilde{W} (u - u_O)$

- Linear solution only relying on the knowledge of the controller output $y_c$

\[
\dot{w} = -2\rho K B_\perp^T \tilde{W} (u - u_0) = -\rho K B_\perp^T \nabla J
\]

\[
u = y_c + B_\perp w,
\]

**Th’m**: With *strong redundancy*, if $K > 0$ and $B_\perp^T \tilde{W} B_\perp > 0$ then internal stability and output response $y$ unaffected by allocator

**Th’m**: With *weak redundancy*, if $K > 0$ and $B_\perp^T \tilde{W} B_\perp > 0$ then internal stability and steady-state output response $y$ unaffected by allocator for small enough $\rho$

- Role of $\tilde{W}$: assign the steady-state plant input, solution to:

\[
\min_w J(u) := (u - u_0)^T \tilde{W} (u - u_0), \quad \text{subject to: } u = y_c^* + B_\perp w,
\]

\[
\text{corresponding to } u^* = u_0 + \left( I - B_\perp (B_\perp^T \tilde{W} B_\perp)^{-1} B_\perp^T \tilde{W} \right) y_c^*.
\]

- $u_0$ is a useful drift term (e.g., center of saturation range)
Linear dynamic allocation minimizes $J = (u - u_O)^T \bar{W}(u - u_O)$

▶ Linear solution only relying on the knowledge of the controller output $y_c$

$$\dot{w} = -2\rho KB^T \bar{W}(u - u_0) = -\rho KB^T \nabla J$$

$$u = y_c + B_w,$$

**Th’m:** With *strong redundancy*, if $K > 0$ and $B^T \bar{W}B > 0$ then internal stability and output response $y$ unaffected by allocator

**Th’m:** With *weak redundancy*, if $K > 0$ and $B^T \bar{W}B > 0$ then internal stability and steady-state output response $y$ unaffected by allocator for small enough $\rho$

▶ Role of $K$ diagonal: promote/penalize different redundant directions while not affecting the steady-state input:

$$u^* = u_0 + (I - B_w(B^T \bar{W}B)^{-1}B^T \bar{W}) y_c^*$$

▶ Role of $\rho \in \mathbb{R}_{>0}$ is to assign any (arbitrily fast or slow) allocation speed
Plant is strongly input redundant (one direction), controller is LQG

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
-0.157 & -0.094 & 0.87 & 0.253 & 0.743 \\
-0.416 & -0.45 & 0.39 & 0.354 & 0.65 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}.
\]


\[K = 10I\] and \[\bar{W} = I\]

\[
\bar{W} = \begin{bmatrix}
100 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad \bar{W} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 100 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad \text{and} \quad \bar{W} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]
Randomly generated academic example (strong)

- Plant is strongly input redundant (one direction), controller is LQG

\[
\begin{align*}
K &= 10I \quad \text{and} \quad \bar{W} = I \\
\bar{W} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad \bar{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix}
\end{align*}
\]
Randomly generated academic example (strong)

- Plant is strongly input redundant (one direction), controller is LQG

\[ K = 10I \text{ and } \bar{W} = I \]

\[ K = 10 \text{ (solid) and } K = 0.01 \text{ (dash-dotted)} \]
Randomly generated academic example (weak)

- Plant is weakly input redundant (two directions), controller is LQG

\[ r \rightarrow \text{Controller} \rightarrow y_c \rightarrow \text{Plant} \rightarrow y \]

\[ B_\perp \rightarrow \text{Input Optimizer} \rightarrow u_0 \]

\( \rho K = 0.1I \) and \( \bar{W} = I \): OK!

\( \rho K = 1I \) and \( \bar{W} = I \): unstable!
Randomly generated academic example (weak)

- Plant is weakly input redundant (two directions), controller is LQG

\[ \rho K = 0.1I \text{ and } \bar{W} = I: \text{ OK!} \]

\[ \rho K = \begin{bmatrix} 100 & 0 \\ 0 & 0.1 \end{bmatrix} \text{ and } \bar{W} = I: \text{ Better!} \]
Nonlinear allocation with magnitude saturation

- Select nonlinear $W(\cdot)$ to increasingly penalize each actuator as it approaches its magnitude saturation limit $M$

$$W(u) = \left(\text{diag}((1 + \epsilon)M - \text{abs}({\text{sat}_M(u)}))\right)^{-1}$$

**Interpretation:** *anti-windup deals with saturation during transients; dynamic allocation avoids saturation at the steady-state*
Example 1 (revisited with magnitude saturation)

▷ Input usage after allocation [9.5 3.37 7]% (note $u_2^* \approx 0.5 \gg m_2 = 0.01$)
Solution readily applies to cooperative manipulation

▷ Multiple robots grasp an object by applying a desired wrench $h_0^d$

▷ Given $h_0^d$, strong redundancy in selection of effectors wrenches $h^d$

▷ Dynamic allocation allows to gracefully steer operating point away from joint torque saturation levels:

▷ Alternative allocation goal: nonlinearly adjust the internal forces (breaking limits, maximum beding, etc)
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Application: plasma position and elongation control

- Frascati Tokamak Upgrade (FTU): a nuclear fusion experiment

**Coils and toroidal plasma**

**Cross section**

- Poloidal field coils regulate plasma position and elongation
Previous FTU horizontal position regulation scheme

- Frascati Tokamak Upgrade:
  \[ \Delta \psi = \text{plasma horizontal position}, \; i_P = \text{plasma current} \]

- Tools:
  - \( V \) coil: very slow and powerful;
  - \( F \) coil: fast and squeezes the plasma

- Goal:
  - Want to use the \( F \) coil to perform two actions:
    - high bandwidth disturbance rejection on \( \Delta \psi = y \)
    - low bandwidth elongation, equivalently, \( i_F = u_2 \) regulation
Solution with allocator uses weak redundancy

- Transfer (slowly) control authority from $F$ to $V$ using dynamic allocation

- Zoom of the allocator block (note the drift term $u_0 = u_r$ which is now a reference signal for $i_F$)

**Th’m:** With weak redundancy, if $K > 0$ then internal stability and steady-state output response $y = \Delta \Psi$ unaffected by allocator for small enough $\rho$
Experiments: F current regulation

△ $i_F$ current is slowly regulated without affecting plant output $y = \Delta \Psi$
From current regulation to elongation regulation

- An approximately known nonlinear static map $f$ relates $I_F$ to the elongation $\kappa$

$$\Delta \Psi \leftarrow i_F \leftarrow i_{PPID} \leftarrow i_{FF} \leftarrow \Delta i_F \leftarrow \Delta i_V \leftarrow \text{PID} \leftarrow i_{PPID} \leftarrow \text{Allocator} \leftarrow \hat{i}_F \leftarrow i_F \leftarrow f^{-1} \leftarrow \kappa_0 \leftarrow \text{Plant} \leftarrow i_F \leftarrow \Delta \Psi$$

- Invert the map $f$ to perform feedback elongation regulation via allocation
- Experiments confirm that the scheme works only if $\rho$ is sufficiently slow

**Th’m:** With *weak redundancy*, if $K > 0$ and map $f$ is invertible, then internal stability and steady-state output response $y = \Delta \Psi$ unaffected by allocator for small enough $\rho +$ elongation regulation $\kappa \to \kappa_0$. 

Th’m: With *weak redundancy*, if $K > 0$ and map $f$ is invertible, then internal stability and steady-state output response $y = \Delta \Psi$ unaffected by allocator for small enough $\rho +$ elongation regulation $\kappa \to \kappa_0$. 

[Diagram of system with PID, Allocator, and Plant blocks and their connections]
Experiments: Elongation regulation

Shots 31970 (without allocator) and 31971 (with allocator)

- \( \kappa \) [m/m]
- \( i_F \) [A]
- \( i_V \) [A]
- \( \Delta \Psi \) [Wb]

Reference, without allocator, and with allocator comparison.
Experiments: loss of stability if parameter $\rho$ too large

Experiments with different values of $\rho$
(Shots 31937, 31971, 31975)
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Hybrid Electric Vehicle has ICE and EM actuators

▷ A prototype built at the “University of Rome, Tor Vergata”

▷ Extension of framework: redundancy after dynamic actuators
Hybrid Electric Vehicle has ICE and EM actuators

Redundancy: net torque = ICE torque + EM torque

Parallel HEV control strategy

Dynamic allocator inputs:
- $y_c$ represents the transient torque request (non-optimized),
- $u_0$ represents the steady-state torque allocation (energy efficient)
Dynamic allocation uses LCM of actuators dynamics

- Allocator dynamics $\tilde{G}(s)$, $W(s)$ designed following a systematic procedure

- Slow variation of the injected signals ensured by the presence of saturation

- Main result proven using saturated systems techniques

**Th’m:** If the actuator parameters are designed following the procedure, the transient response given by the controller is not modified by the allocator, and the **steady-state torque allocation** $u_0$ is asymptotically obtained.
Experimental response on the prototype car

Torque regulation. Steady-state reference $u_0$ changes at $t = 20$ s

Human driver in the loop. Reference $u_0$ changes at $t = 10$ s
Attitude control with reaction wheels and magnetorquers

- **Plant dynamics:**
  \[
  J\dot{\omega} + \omega \times J\omega = \omega \times h_w - \tau_w - \tilde{b} \times (t, q) \tau_m
  \]
  \[
  \dot{q} = F(\omega) q
  \]

- **Actuator dynamics (Reaction Wheels):**
  \[
  \dot{h}_w = \tau_w
  \]

- **Reaction wheels:** if \( \tau_w = k \) then \( h_w = kt \) → risk of saturation of \( h_w \)

- **Magnetorquers:** Controllability issues:
  \[
  T_m = -\tilde{b} \times (t, q) \tau_m = -(R(q)\tilde{b}_o(t)) \times \tau_m
  \]
Attitude control with reaction wheels and magnetorquers

- Classical solution: “Cross-product law” uses separate loops and high-gain
- Proposed-solution: use static allocation in feedback from actuator state

**Dynamics:**

\[
\begin{align*}
J \dot{\omega} + \omega \times J \omega &= -\tau_w - \omega \times h_w + T_m \\
\dot{q} &= F(\omega)q \\
\dot{h}_w &= -\omega \times h_w - (R(q)\tilde{b}_o(t)) \times \tau_m - \tau
\end{align*}
\]

**Control law:**

\[
\begin{align*}
\tau_w &= -\omega \times h_w - (R(q)\tilde{b}_o(t)) \times \tau_m - \tau \\
\tau_m &= -\frac{(R(q)\tilde{b}_o(t)) \times \tilde{b}_o(t)}{|\tilde{b}_o(t)|^2} k_p(h_w - h_{ref}) \\
\tau &= \text{Hybrid attitude controller command}
\end{align*}
\]

**Th’m:** If \( \tau \) ensures GAS of the origin for \((q, \omega)\) dynamics, then allocation scheme preserve the same exact \((q, \omega)\) response and ensures GAS of \(h_w = h_{ref}\).
Allocation scheme enables inverting the cascade
Stabilization transients with aggressive controller

- Classical
- Revisited
- Allocation

- Similar results
- Saturation of $h_w$
Stabilization transients with non aggressive controller

✓ revisited and allocation controllers preserve stability
Attitude transient decoupled from the $h_w$ transient

✓ allocation-based strategy gives more regular attitude transient
Nonlinear allocation with partial actuator measurements

- In some applications may be able to only access virtual input $\tau$
- If $g(x_a)$ is invertible and $f(x_a)$ is incrementally stable, may use scheme

\[
\dot{\tau} = -\gamma_p (\tau - \tau_c) \quad \text{and (slow) convergence of } x_a \text{ to the minimum of } J(x_a).
\]

**Th’m**: Under stated assumptions, we have $\dot{\tau} = -\gamma_p (\tau - \tau_c)$ and (slow) convergence of $x_a$ to the minimum of $J(x_a)$.

- Hydrodynamic dynamometer uses two valves with nonlinear output map $h$
An output regulation approach to weak redundancy

- Generalized Francis equations provide the “Annihilator” (dynamical $B^\perp$)

- Optimizer acts on Annihilator to obtain “Asymptotic Optimality” for

$$J_1(z) = |z|_1, \quad J_2(z) = |z|_2, \quad J_\infty(z) = \max_i |z_i|^2 + \epsilon |z|^2$$

- **Theorem**: The proposed scheme provides:
  - Robust UGAS without the exosystem
  - Convergence to a steady-state (contractive) if $S$ is Poisson stable
  - Robust Asymptotic Optimality
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Joint European Torus (JET) plasma shape control

- We want to control the plasma shape on a poloidal cross section.
- Shape is described by a finite number of geometrical parameters called **gaps**.

- Gaps are defined as the distances between the plasma boundary and the first wall along certain segments.
- Gaps values are evaluated from magnetic sensor measurements by estimation algorithms.
- We want to control: **32 outputs** $y$. 
JET shape control has no input redundancy

- JET has 8 poloidal field (PF) coils available as actuators for plasma shape control.
- JET PF coils are connected to form 9 circuits.
- Control inputs represented by currents flowing in the circuits.
- Inputs available:
  - 9 control inputs $u$.

▶ No redundant inputs: still need achieve saturation avoidance!
Recall allocator features for the redundant case

- Essential features of the dynamic allocator seen before
  
  \[
  \dot{w} = -\rho K B_{\perp}^T \nabla J \\
  u = y_c + B_{\perp} w
  \]

- The columns of \( B_{\perp} \) correspond to the redundant directions
- \( K \) diagonal allows to promote/penalize different redundant directions
- \( \tilde{W} \) imposes the optimality criterion: \( u \) converges to
  
  \[
  u^* = \arg\min_w (u - u_0)^T \tilde{W} (u - u_0), \text{ subject to: } u = y_{c^*} + B_{\perp} w,
  \]

  namely minizes cost \( J = (u - u_0)^T \tilde{W} (u - u_0) \).

- \( \rho \), positive scalar allows to adjust convergence speed
We introduce a more general **cost function** [before] 

\[ J_e(u, \delta y) \quad [J = (u - u_0)^T \bar{W} (u - u_0)] \]

Minimum of \( J_e \) is a **trade-off between** (\( \star \) denotes steady state values).
- the modified steady state value of the **plant input** \( u^\star \) and
- the associated **output modification** \( \delta y^\star \) with respect to the original \( y^\star \)

The new allocator is described by the equations [before] :

\[ \dot{w} = -\rho KB_0^T \left[ \frac{I}{P^\star} \right]^T \nabla J_e \]
\[ u = y_c + B_0 w \]

\[ \begin{bmatrix} \dot{w} \\ u \end{bmatrix} = -\rho KB_{\perp}^T \nabla J \]
\[ \begin{bmatrix} u \\ y_c + B_{\perp} w \end{bmatrix} \]

\( B_0 \) is a suitable full column rank matrix, generalizing the matrix \( B_{\perp} \) (all input directions are potentially “redundant” now).
Allocator now also injects signals at plant output

- New allocator injects extra signal $\delta y = P^*y_a$ so as to not “fight” against the controller at the steady-state:

\[
    u_c = y - P^*B_0w = y - P^*y_a \\
    u = y_c + B_0w = y_c + y_a
\]

**Th’m** Under some convexity assumptions on nonlinear cost $J_e$, for sufficiently small $\rho$ the allocator is such that, under constant inputs, $(u(t), \delta y(t))$ converge to the minimizer of $J_e$. 
Example of a cost function: penalize $u$ and $\delta y$

A possible selection of the cost function is

$$J_e(u, \delta y) = \sum_{i=1}^{n_u} a_i dz(u_i)^2 + \sum_{i=1}^{n_y} b_i (\delta y_i)^2$$

where $dz(u_i) = \text{sign}(u_i) \max\{0, |u_i| - 1\}$, $a_i \geq 0$, $i = 1, \ldots, n_u$ and $b_i > 0$ $i = 1, \ldots, n_y$.

Alternative non symmetric choices are possible.
Steady-state allocation: penalize input $u$

- Allocated shape (red balloon) greatly modified wrt the nominal shape (blue balloon)
- ID1 is moved away from saturation by allocator

Input ranges (gray), controller output $y_c$ (blue), allocated input $u$ (red)
Steady-state allocation: penalize output $y$

- Allocated shape (red balloon) slightly modified wrt the nominal shape (blue balloon)
- Increasing output penalty, shape modification $\delta y^*$ is reduced
- ID1 comes back very close to saturation level

Input ranges (gray), controller output $y_c$ (blue), allocated input $u$ (red)
Steady-state allocation: restrict $B_0$ to nail down outputs

- Allocated shape (red balloon)
- Nominal shape (blue balloon)
- Penalize input $u$ as in first test
- Remove columns from $B_0$ to fix 5 outputs (CV-RX, CV-ZX, ZSOGB, RSIGB and RSOGB, i.e. X-point and strike points) and one input (IP4T current)
- ID1 again far from saturation level
Experiment during current ramp-down *without* allocator

▷ X-point and strike points severely compromised at $t = 19 \, \text{s}$
▷ Radial Inner Gap (RIG) also becomes very small

\begin{align*}
& t = 15 \text{s} \\
& t = 17 \text{s} \\
& t = 19 \text{s}
\end{align*}
Experiment during current ramp-down with allocator

- X-point, strike points and RIG better behaved in the same conditions
- Shape is sacrificed in the upper part of the vessel where space is available

$t = 15s$

$t = 16s$

$t = 18s$
Summary of presented works with references

- A recent survey about input allocation in Johansen and Fossen [2013]

- Main works on the presented dynamic allocation paradigm:
  - First ideas on dynamic allocation vs saturation Zaccarian [2007, 2009]
  - Input-output trade-offs with JET simulations De Tommasi et al. [2011]
  - Weak allocation within a regulation framework Galeani et al. [2015], Cocetti et al. [2016]

- The presented applications are reported in:
  - Internal wrenches control in interacting robots Zambelli Bais et al. [2015]
  - FTU elongation control Boncagni et al. [2012]
  - Hybrid Electric Vehicle control Cordiner et al. [2014]
  - Satellite attitude stabilization Trégouët et al. [2015]
  - Hydrodynamic dynamometer application Passenbrunner et al. [2012]

- JET current limit avoidance system:
  - Software implementation commissioning De Tommasi et al. [2012]
  - Closed-loop experimental results De Tommasi et al. [2013a,b]

- Additional references: Valmórbida and Galeani [2013], Serrani [2012], Cristofaro and Galeani [2014], Galeani and Pettinari [2014]


