Reset control systems:
theoretical and experimental results within hybrid control design and analysis

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Outline

1. Clegg integrators and First Order Reset Elements (FORE) and an overview of hybrid dynamical systems

2. Exponential stability of reset control systems

3. Set-point regulation of linear plants using adaptive FORE
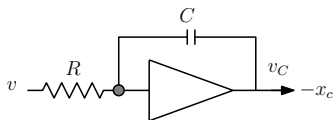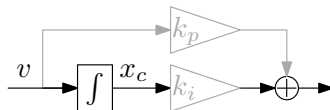
4. Some additional hybrid applications

An analog integrator and its Clegg extension Clegg [1958]

Integrators: core components of dynamical control systems
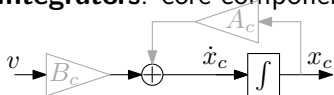
$$\dot{x}_c = A_c x_c + B_c v$$

Example: PI controller

$$\dot{x}_c = \frac{1}{RC} v$$

- In an analog integrator, the state information is stored in a capacitor:

An analog integrator and its Clegg extension Clegg [1958]

Integrators: core components of dynamical control systems

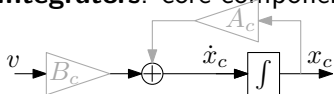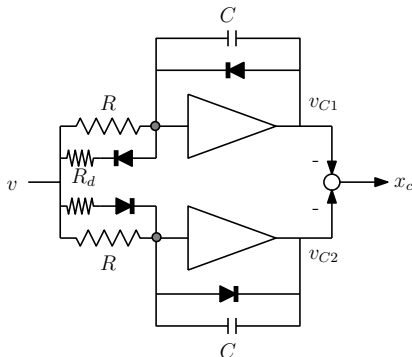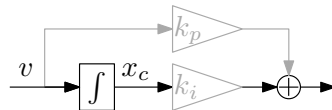$$\dot{x}_c = A_c x_c + B_c v$$

Example: PI controller

- Clegg's integrator Clegg [1958]:
  - *feedback diodes*: the **positive** part of x_c is all and only coming from the **upper** capacitor (and viceversa)
  - *input diodes*: when $v \leq 0$ the upper capacitor is reset and the lower one integrates (and viceversa) $[R_d \ll 1]$
- As a consequence $\Rightarrow$ v and x_c **never have opposite signs**
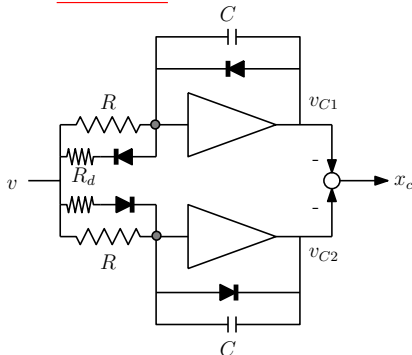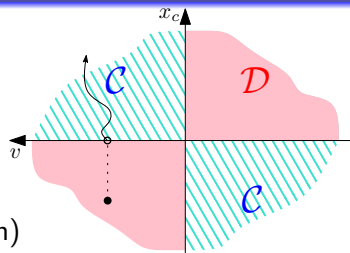
Hybrid dynamics may flow or jump

Hybrid Clegg integrator:

$$\dot{x}_c = \frac{1}{RC} v, \quad \text{allowed when } x_c v \geq 0,$$
$$x_c^+ = 0, \qquad \text{allowed when } x_c v \leq 0,$$

- <u>Flow set</u> $\mathcal{C}$: where x_c may flow (1st eq'n)
- <u>Jump set</u> $\mathcal{D}$: where x_c may jump (2nd eq'n)

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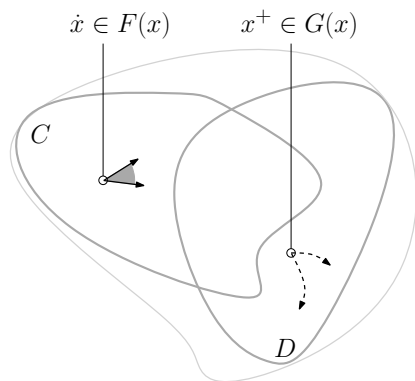
Hybrid dynamical systems review: **dynamics**

Key works: Goebel et al. [2009, 2012], Teel et al. [2013], Prieur et al. [2013]

$\mathcal{H} = (\mathcal{C}, \mathcal{D}, F, G)$

- $n \in \mathbb{N}$ (state dimension)
- $\mathcal{C} \subseteq \mathbb{R}^n$ (flow set)
- $\mathcal{D} \subseteq \mathbb{R}^n$ (jump set)
- $F : \mathcal{C} \rightrightarrows \mathbb{R}^n$ (flow map)
- $G : \mathcal{D} \rightrightarrows \mathbb{R}^n$ (jump map)

$$\mathcal{H} : \begin{cases} \dot{x} \in F(x), & x \in \mathcal{C} \\ x^+ \in G(x), & x \in \mathcal{D} \end{cases}$$

Hybrid dynamical systems review: **continuous dynamics**

Key works: Goebel et al. [2009, 2012], Teel et al. [2013], Prieur et al. [2013]

$\mathcal{H} = (\mathcal{C}, \mathcal{D}, F, G)$

- $n \in \mathbb{N}$ (state dimension)
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$\mathcal{H} : \begin{cases} \dot{x} \in F(x), & x \in \mathcal{C} \\ x^+ \in G(x), & x \in \mathcal{D} \end{cases}$

$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + x_2(1 - x_1^2) \end{cases}$

Van der Pol

Hybrid dynamical systems review: **discrete dynamics**

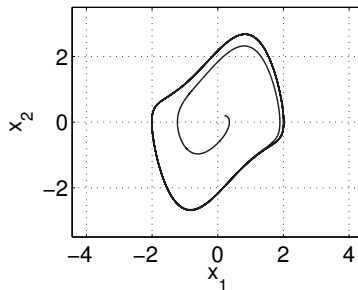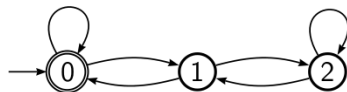Key works: Goebel et al. [2009, 2012], Teel et al. [2013], Prieur et al. [2013]

$\mathcal{H} = (\mathcal{C}, \mathcal{D}, F, G)$

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- $\mathcal{C} \subseteq \mathbb{R}^n$ (flow set)
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- $F : \mathcal{C} \rightrightarrows \mathbb{R}^n$ flow map)
- $G : \mathcal{D} \rightrightarrows \mathbb{R}^n$ (jump map)

$$x^+ \in \begin{cases} \{0,1\} & \text{if } x = 0 \\ \{0,2\} & \text{if } x = 1 \\ \{1,2\} & \text{if } x = 2 \end{cases}$$

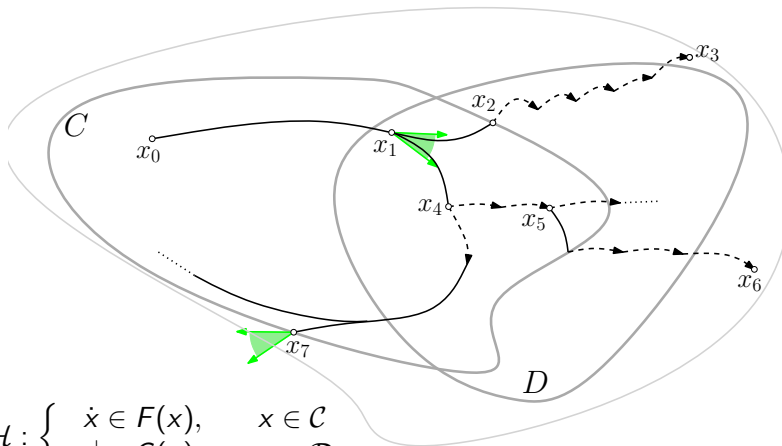
A possible sequence of states
from $x_0 = 0$ is:

$$(0 \cdot 1 \cdot 2 \cdot 1)^i \quad i \in N$$

$$\mathcal{H} : \begin{cases} \dot{x} \in F(x), & x \in \mathcal{C} \\ x^+ \in G(x), & x \in \mathcal{D} \end{cases}$$

Hybrid dynamical systems review: **trajectories**

Key works: Goebel et al. [2009, 2012], Teel et al. [2013], Prieur et al. [2013]

$$\mathcal{H} : \begin{cases} \dot{x} \in F(x), & x \in \mathcal{C} \\ x^+ \in G(x), & x \in \mathcal{D} \end{cases}$$

Hybrid dynamical systems review: **hybrid time**

The motion of the state is parameterized by two parameters:

- $t \in \mathbb{R}_{\geq 0}$, takes into account the elapse of time during the continuous motion of the state;
- $j \in \mathbb{Z}_{\geq 0}$, takes into account the number of jumps during the discrete motion of the state.

$$\forall \tau \in [0,5],\ \xi(\tau,0) \qquad \forall \tau \in [5,8],\ \xi(\tau,2) \qquad \forall \tau \geq 8,\ \xi(\tau,3)$$
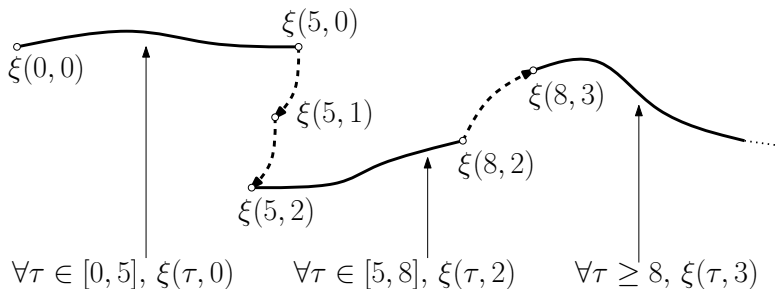
Hybrid dynamical systems review: **solution**

- Formally, a solution satisfies the red flow dynamics when flowing and satisfies the jump dynamics when jumping
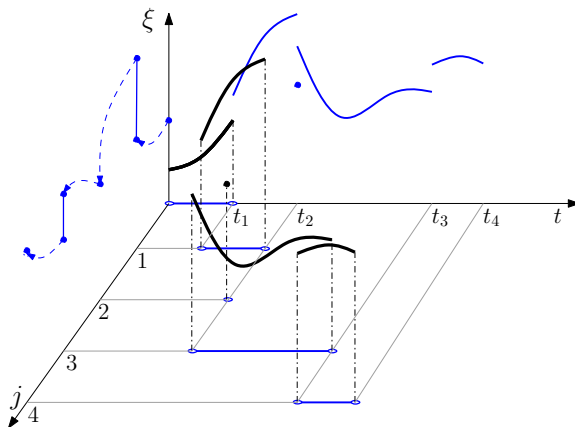
Hybrid dynamical systems review: **Lyapunov theorem**

Th'm Teel et al. [2013] Given Euclidean norm $|x| = \sqrt{x^T x}$ and system

$$\mathcal{H} : \begin{cases} \dot{x} = f(x), & x \in \mathcal{C} \\ x^+ = g(x), & x \in \mathcal{D}, \end{cases}$$

aassume that function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ satisfies for some scalars c_1, c_2 positive and c_3 positive:

$$c_1 |x|^2 \leq V(x) \leq c_2 |x|^2, \qquad \forall x \in \mathcal{C} \cup \mathcal{D} \cup G(\mathcal{D})$$

$$\langle \nabla V(x), f(x) \rangle \leq -c_3 |x|^2, \qquad \forall x \in \mathcal{C},$$

$$V(g(x)) - V(x) \leq -c_3 |x|^2, \qquad \forall x \in \mathcal{D},$$

then the origin is uniformly globally exponentially stable (UGES) for $\mathcal{H}$, namely there exist $K, \lambda > 0$ such that all solutions satisfy

$$|\xi(t,j)| \leq K e^{\lambda(t+j)} |\xi(0,0)|, \quad \forall (t,j) \in \operatorname{dom} \xi$$

<u>Note</u>: Lyapunov conditions comprise **flow** and **jump** conditions.
<u>Note</u>: UGES is characterized in terms of hybrid time (t, j).

Hybrid dynamical systems review: **Lyapunov theorem**

Th'm Goebel et al. [2012] Given a closed set $\mathcal{A} \subset \mathbb{R}^n$ and system

$$\mathcal{H} : \begin{cases} \dot{x} \in F(x), & x \in \mathcal{C} \\ x^+ \in G(x), & x \in \mathcal{D}, \end{cases}$$

aassume that function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ satisfies for some $\alpha_1, \, \alpha_2 \in \mathcal{K}_\infty$ and ρ positive definite:

$$\alpha_1(|x|_\mathcal{A}) \leq V(x) \leq \alpha_2(|x|_\mathcal{A}), \qquad \forall x \in \mathcal{C} \cup \mathcal{D} \cup G(\mathcal{D})$$

$$\langle \nabla V(x), f \rangle \leq -\rho(|x|_\mathcal{A}), \qquad \forall x \in \mathcal{C}, f \in F(x),$$

$$V(g) - V(x) \leq -\rho(|x|_\mathcal{A}), \qquad \forall x \in \mathcal{D}, g \in G(x)$$

then $\mathcal{A}$ is uniformly globally asymptotically stable (UGAS) for $\mathcal{H}$, namely there exists $\beta \in \mathcal{KL}$ such that all solutions satisfy

$$|\xi(t,j)|_\mathcal{A} \leq \beta(|\xi(0,0)|_\mathcal{A}, t+j), \quad \forall (t,j) \in \operatorname{dom} \xi$$

<u>Note</u>: Lyapunov conditions comprise **flow** and **jump** conditions.
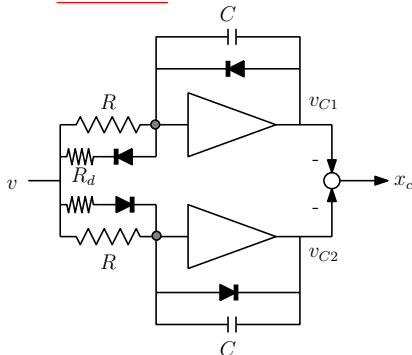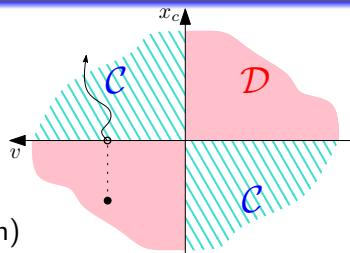<u>Note</u>: UGAS is characterized in terms of hybrid time (t, j)

Hybrid dynamics and the Clegg integrator (recall)

Hybrid Clegg integrator:

$$\dot{x}_c = \frac{1}{RC}v, \quad \text{allowed when } x_c v \geq 0,$$
$$x_c^+ = 0, \qquad \text{allowed when } x_c v \leq 0,$$

- <u>Flow set</u> $\mathcal{C}$: where x_c may flow (1st eq'n)
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- Clegg's integrator Clegg [1958]:
  - *feedback diodes*: the **positive** part of x_c is all and only coming from the **upper** capacitor (and viceversa)
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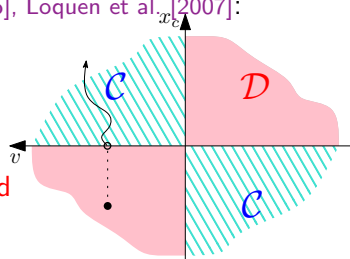
Hybrid dynamics of the Clegg integrator (revisited)

Hybrid Clegg integrator Zaccarian et al. [2005], Loquen et al. [2007]:

$$\dot{x}_c(t,j) = (RC)^{-1} v(t,j), \quad x_c(t,j) v(t,j) \geq 0,$$
$$x_c(t,j+1) = 0, \qquad\qquad x_c(t,j) v(t,j) \leq 0,$$

- Flow set $\mathcal{C} := \{(x_c, v) : x_c v \geq 0\}$ is closed
- Jump set $\mathcal{D} := \{(x_c, v) : x_c v \leq 0\}$ is closed
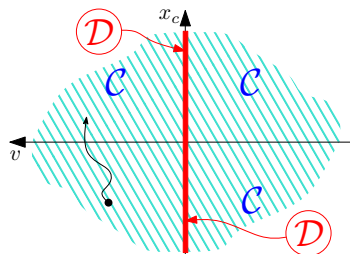- Stability is robust! (Teel 2006–2012)

Previous models Clegg [1958], Krishnan and Horowitz [1974], Horowitz and Rosenbaum [1975], Beker et al. [2004]:

$$\dot{x}_c = (RC)^{-1} v, \quad \text{if } v \neq 0,$$
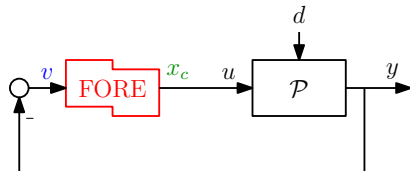$$x_c^+ = 0, \qquad\qquad \text{if } v = 0,$$

- <u>Imprecise</u>: solutions $\exists$ s.t. $x_c v < 0$, but
  Clegg's x_c and v **always** have same sign!
- <u>Unrobust</u>: $\mathcal{C}$ is almost all $\mathbb{R}^2$
  (arbitrary small noise disastrous)
- <u>Unsuitable</u>: Adds extra solutions

Stabilization using hybrid jumps to zero

First Order Reset Element Nešić et al. [2011], Loquen et al. [2007]:

$$\dot{x}_c = a_c x_c + b_c v, \qquad x_c v \geq 0,$$
$$x_c^+ = 0, \qquad x_c v \leq 0,$$

Theorem If $\mathcal{P}$ is linear, minimum phase and relative degree one, **then**
a_c, b_c or (a_c, b_c) large enough $\Rightarrow$ global exponential stability
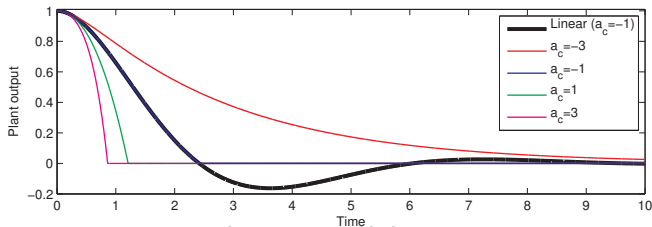
Theorem In the planar case, γ_{dy} shrinks to zero as parameters grow

Simulation uses:
$$\mathcal{P} = \frac{1}{s}$$
$$b_c = 1$$

Interpretation: Resets remove overshoots, instability improves transient

Piecewise quadratic Lyapunov function construction

- Proposed in Zaccarian et al. [2011], Loquen [2010]
- Given $N \geq 2$ (number of sectors)
- Patching angles:
$$-\theta_\epsilon = \theta_0 < \theta_1 < \cdots < \theta_N = \frac{\pi}{2} + \theta_\epsilon$$
- Patching hyperplanes ($C_p = [0 \ \cdots \ 0 \ 1]$)
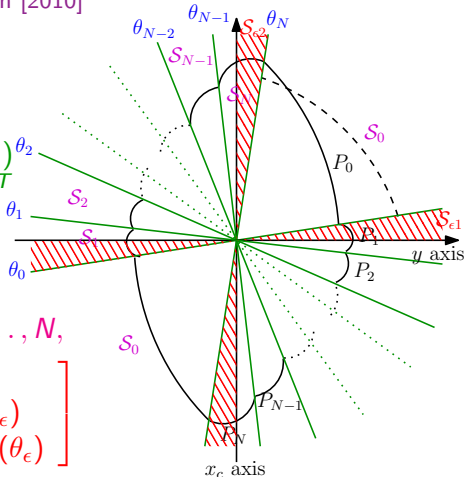$$\Theta_i = \begin{bmatrix} 0_{1\times(n-2)} & \sin(\theta_i) & \cos(\theta_i) \end{bmatrix}^T$$
- Sector matrices:

$S_0 := \Theta_0 \Theta_N^T + \Theta_N \Theta_0^T$

$S_i := -(\Theta_i \Theta_{i-1}^T + \Theta_{i-1} \Theta_i^T), \quad i = 1, \ldots, N,$

$$S_{\epsilon 1} := \begin{bmatrix} 0_{(n-2)\times(n-2)} & 0 & 0 \\ 0 & 0 & \sin(\theta_\epsilon) \\ 0 & \sin(\theta_\epsilon) & -2\cos(\theta_\epsilon) \end{bmatrix}$$

$$S_{\epsilon 2} := \begin{bmatrix} 0_{(n-2)\times(n-2)} & 0 & 0 \\ 0 & -2\cos(\theta_\epsilon) & \sin(\theta_\epsilon) \\ 0 & \sin(\theta_\epsilon) & 0 \end{bmatrix}$$

Hybrid closed-loop:

$\dot{x} = A_F x + B_d d, \quad x \in \mathcal{C}$

$x^+ = A_J x, \quad x \in \mathcal{D}$

Piecewise quadratic Lyapunov theorem

Theorem Zaccarian et al. [2011], Loquen [2010]: If the following LMIs in the green unknowns (where $Z = [I_{n-2} \; 0_{(n-2)\times 2}]$) are feasible:

$$(Flow) \begin{bmatrix} A_F^T P_i + P_i A_F + \tau_{Fi} S_i & P_i B_d & C^T \\ \star & -\gamma I & 0 \\ \star & \star & -\gamma I \end{bmatrix} < 0, i = 1, \ldots, N,$$

$$(Jump) \quad A_J^T P_1 A_J - P_0 + \tau_J S_0 \leq 0$$

$$(Cont'ty) \quad \Theta_{i\perp}^T (P_i - P_{i+1}) \Theta_{i\perp} = 0, \quad i = 0, \ldots, N-1,$$

$$(Cont'ty) \quad \Theta_{N\perp}^T (P_N - P_0) \Theta_{N\perp} = 0$$

$$(Overlap) \quad A_J^T P_1 A_J - P_1 + \tau_{\epsilon 1} S_{\epsilon 1} \leq 0$$
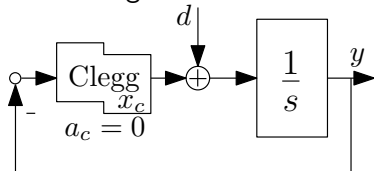
$$(Overlap) \quad A_J^T P_1 A_J - P_N + \tau_{\epsilon 2} S_{\epsilon 2} \leq 0$$

$$(Origin) \begin{bmatrix} Z(A_F^T P_0 + P_0 A_F)Z^T & Z P_0 B_d & Z C^T \\ \star & -\gamma I & 0 \\ \star & \star & -\gamma I \end{bmatrix} < 0,$$
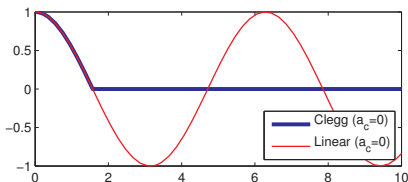
then global exponential stability + finite $\mathcal{L}_2$ gain γ_{dy} from d to y

Example 1: Clegg ($a_c = 0$) connected to an integrator

- Block diagram:

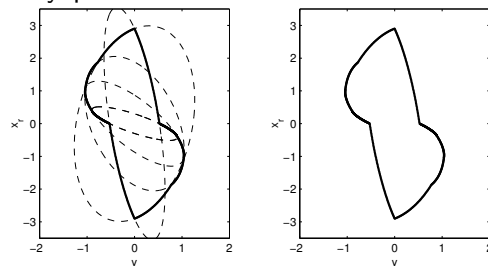- Output response (overcomes linear systems limitations)

- Quadratic Lyapunov functions are unsuitable Zaccarian et al. [2011]

- Gain γ_{dy} estimates ($N = \#$ of sectors)

N	2	4	8	50
gain γ_{dy}	2.834	1.377	0.914	0.87

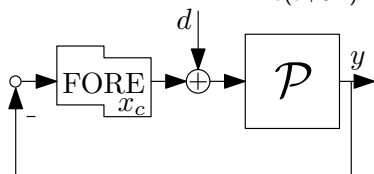- A lower bound: $\sqrt{\frac{\pi}{8}} \approx 0.626$
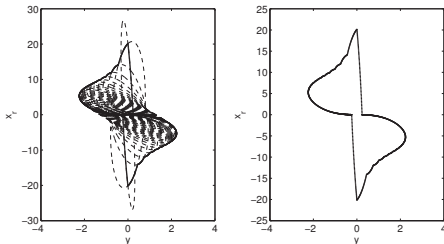
- Lyapunov func'n level sets for $N = 4$

- $P_1, \ldots, P_4$ cover 2nd/4th quadrants
- P_0 covers 1st/3rd quadrants

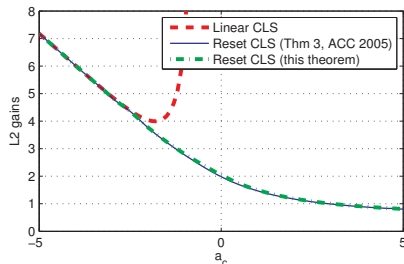# Example 2: FORE (any a_c) and linear plant (Hollot et al.)

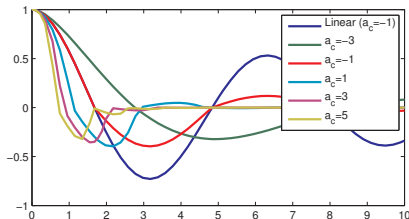- Block diagram ($\mathcal{P} = \frac{s+1}{s(s+0.2)}$)

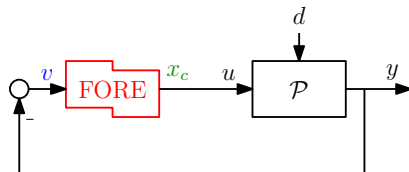- Gain γ_{dy} estimates

- $a_c = 1$: level set with $N = 50$

- Time responses

Stabilization using hybrid jumps to zero (recall)

First Order Reset Element Nešić et al. [2011], Loquen et al. [2007]:

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Theorem If $\mathcal{P}$ is linear, minimum phase
and relative degree one, **then**
a_c, b_c or (a_c, b_c) large enough $\Rightarrow$ global exponential stability
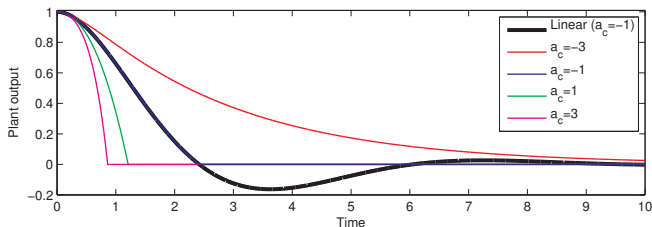
Theorem In the planar case, γ_{dy} shrinks to zero as parameters grow

Simulation
uses:
$$\mathcal{P} = \frac{1}{s}$$

$b_c = 1$

Interpretation: Resets remove overshoots, instability improves transient
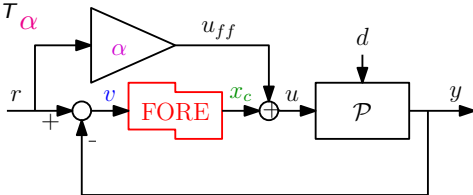
Set point adaptive **regulation** using hybrid jumps to zero

Relevant works Panni et al. [2014], Loquen et al. [2008]

- Parametric feedforward $u_{ff} = \Psi(r)^T \alpha$

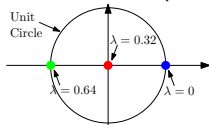$$\begin{cases} \dot{x}_c = a_c x_c + b_c v, \\ \dot{\alpha} = 0, \end{cases} \quad x_c v \geq 0,$$

$$\begin{cases} x_c^+ = 0, \\ \alpha^+ = \alpha + \lambda \frac{\Psi(r)}{|\Psi(r)|^2} x_c, \end{cases} \quad x_c v \leq 0,$$
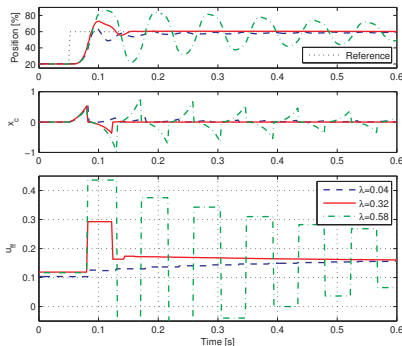

Theorem: If FORE stabilizes with $r = 0$, then for constant r, $y \to r$

Lemma: Tuning of λ using discrete-time rules (Ziegler-Nichols)
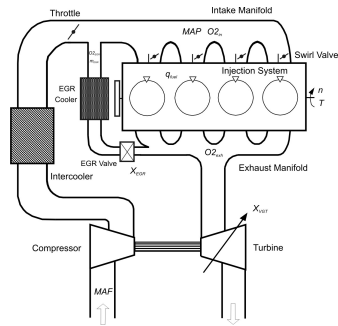

Example: EGR Experiment (next slide)

Fast regulation of EGR valve position in Diesel engines

- Reported in Panni et al. [2014]
- EGR: Recirculates Exhaust Gas
  in Diesel engines

- Subject to strong disturbances
  ⇒ need aggressive controllers
  (recall exp. unstable transients)
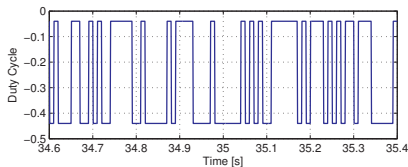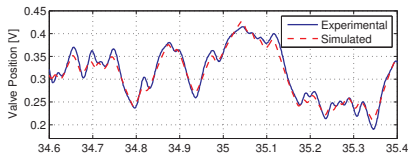
- Identified valve transfer function:

$$P(s) = \frac{2200}{(s + 164.4)(s + 10.69)}.$$

Feedforward: α converges to suitable parametrization
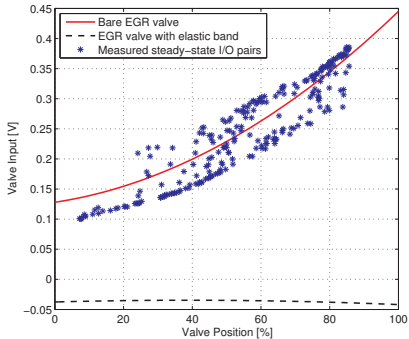
- $\star$: steady-state input/output pairs (stiction!!)

- Red Solid: $u_{ff} = \Psi^T(r)\alpha^*$, with α^* steady-state for α

- Black dashed: $u_{ff} = \Psi^T(r)\bar{\alpha}^*$ when pulling the valve with an elastic band
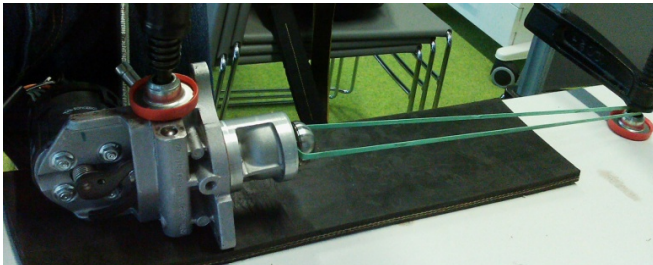
Laboratory experiments close to time-optimal
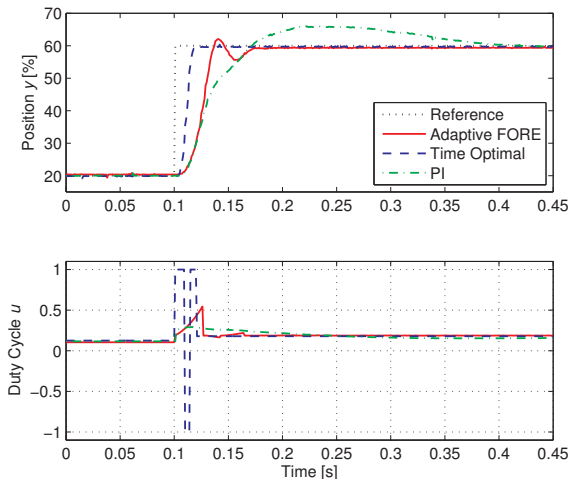
- Time-optimal:
  unrobust, obtained via trial and error

- PI:
  Tuned using standard MATLAB tools

- Adaptive FORE:
  Response after
  $\alpha \to \alpha^* =$
  $(0.128, \ 0.087, \ 0.115)$

- Note the exponentially diverging voltage:
  aggressive action for disturbance rejection on the real engine

Experiments on Diesel engine testbench (JKU)

Experimental testbench at the Johannes Kepler Universitet (Linz, Austria)
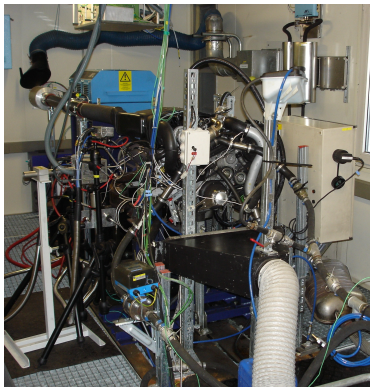
- **Specs**: 2 liter, 4 cylinder passenger car turbocharged Diesel engine

- **Compared**: to factory EGR valve controller coded in ECU
  (gain scheduled PI with feedforward)

- **Test cycle**: Urban part of
  New European Driving Cycle

- **Relevance**: Faster EGR positioning
  $\Rightarrow$ Reduced NO_x emissions

Adaptive FORE provides substantial performance increase
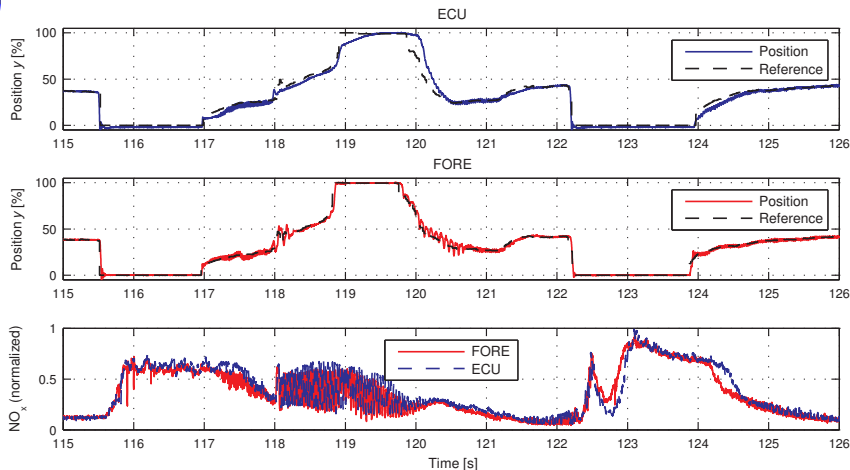
- Mean squared error: ECU = 6.68 (100%), FORE = 1.53 (23 %)
- Improvement most important with EGR almost closed ($t \approx 117,\ 124$)
- Recent results promise time-varying reference tracking

From discrete + continuous to hybrid dynamical systems

Continuous dynamical system

$$\frac{dx(t)}{dt} = f(x(t)), \ \forall t \in \mathbb{R}_{\geq 0}$$

Discrete dynamical system

$$x(k+1) = g(x(k)), \ \forall k \in \mathbb{Z}_{\geq 0}$$

(possible discrete variables)

$$\Downarrow$$

Hybrid dynamical system

$$\begin{cases} \dfrac{dx(t, k)}{dt} &=& f(x(t, k)), & x(t, k) \in \mathcal{C} \subset \mathbb{R}^n \\ x(t, k+1) &=& g(x(t, k)), & x(t, k) \in \mathcal{D} \subset \mathbb{R}^n \end{cases}$$

- **Continuous** time domain $t \in \mathbb{R}_{\geq 0}$ and **Discrete** time domain $k \in \mathbb{Z}_{\geq 0}$ merged into **Hybrid** time domain $(t, k) \in \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$
- Solution x can "flow" if $\in \mathcal{C}$, can "jump" if $\in \mathcal{D}$
- Fundamental stability results now available (Converse Lyapunov theorems, ISS, invariance principle, $\mathcal{L}_p$ stability) Goebel et al. [2012]

Hybrid systems tools utilized in diverse scenarios
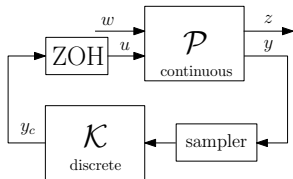
Sampled-data control design with saturation

Dai et al. [2007], Gomes Da Silva et al. [2014]

- uniform sampling time assumed
- jumps correspond to sampling actions

Lazy sensors for reduced transmission rate

Seuret et al. [2013], Forni et al. [2014]

- reduce sample transmission over networks
- a peculiar event-triggered sampling
- jumps occur at samples transmissions
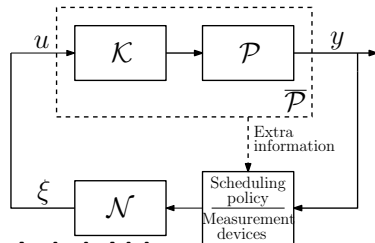
Nonlinear stabilization via hybrid loops

Fichera et al. [2013], Prieur et al. [2013], Fichera et al. [2012]

- enforces jumps of the controller state in some sets
- so as to guarantee decrease of suitable functions
- useful, e.g., for overshoot reduction with SISO plants
- enables design of hybrid $\mathcal{H}_\infty$ controllers

Clegg and FORE are hybrid | Exponential Stability | Set-point Regulation | More on hybrid | **References**
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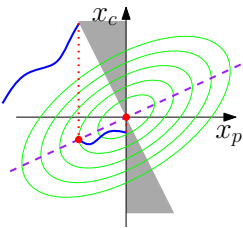
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