Nonlinear anti-windup design for fully actuated Euler-Lagrange systems

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“Windup” and “Anti-windup”

Unconstrained closed-loop behavior
- desirable performance
  (for all signals)

Saturated closed-loop behavior
- Desirable performance for small signals
- “Windup” effect for large signals:
  - stability and/or performance loss

“Anti-windup” augmentation goal
- Unconstrained performance for small signals
- “Anti-windup” for large signals
  - stability recovery
  - (partial) performance recovery
An illustrative example

- SCARA robot with limited torque/force inputs

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<tr>
<td>$m_i$</td>
<td>55 Nm</td>
<td>45 Nm</td>
<td>70 N</td>
<td>25 Nm</td>
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Feedback linearizing controller + PID action ($computed torque$) induces decoupled linear performance (for small signals)
An illustrative example (cont’d)

Saturation effects on the closed-loop system ($r = [6 \text{ deg}, -4 \text{ deg}, 4 \text{ cm}, 8 \text{ deg}]$)

- How can we retain the local linear performance and avoid the stability & performance loss?
The unconstrained scheme

- The E-L dynamics for \( L(q, \dot{q}) = T(q, \dot{q}) - V(q); T(q, \dot{q}) = \frac{1}{2} \dot{q}^T I(q) \dot{q} \)

\[
\frac{d}{dt} \left( \frac{\partial L(q, \dot{q})}{\partial \dot{q}} \right) - \frac{\partial L(q, \dot{q})}{\partial q} + \frac{\partial R(q, \dot{q})}{\partial \dot{q}} = \text{sat}(u), \tag{♠}
\]

\[
\dot{x} = f(x, \text{sat}(u)), \quad x := (q, \dot{q})
\]

- The "original" controller dynamics

\[
\dot{x}_c = g(x_c, u_c, r) \quad \text{(♡)}
\]

\[
y_c = k(x_c, u_c, r)
\]

- The unconstrained closed-loop

\[
\begin{align*}
\begin{cases}
    u_c &= x \\
    \text{"sat}(u) &= y_c
\end{cases} \quad \Rightarrow \quad \dot{x} = f(x, y_c)
\end{align*}
\]

- The unconstrained solution is \( u_u(\cdot), x_u(\cdot) \)
The anti-windup scheme

▷ Add extra dynamics
\[ \dot{x}_e = f(x, \text{sat}(u)) - f(x - x_e, y_c) \]  

▷ Condition the controller input
\[ u_c = x - x_e \]

▷ Condition the plant input
\[ u = \text{sat}(y_c + \gamma(x, x - x_e)) \]

▷ Select \( \gamma(\cdot, \cdot) \) to achieve

**Property GOAL:** If \( u_u(\cdot), x_u(\cdot) \) is the unconstrained response, then

- whenever \( x_e(0) = 0 \) and \( u_u(\cdot) \equiv \text{sat}(u_u(\cdot)) \), then \( x(\cdot) \equiv x_u(\cdot) \);
- The system is GAS (and LES)
  - The performance is retained as much as possible
The proposed solution

**Assumption U**: The unconstrained closed-loop system is GAS and LES

▷ Select the following function

\begin{equation}
\gamma((q, \dot{q}), (q^*, \dot{q}^*)) := \\
\frac{\partial V}{\partial q}(q) - \frac{\partial V}{\partial q}(q^*) - K_G \text{sat}(K_Q(q - q^*)) - K_0(\dot{q} - \dot{q}^*)
\end{equation}

where

- $K_G, K_Q, K_0$ are positive definite; $K_G, K_Q$ diagonal
- $\sup_{q \in \mathbb{R}^n} \left| \frac{\partial V}{\partial q_i}(q) \right| + k_{Gi} m_i < m_i$, $i = 1, \ldots, n$

**Main Theorem**: Under Assumption U, the plant (♣) controlled by (☹) and augmented with the anti-windup compensator (◊) and the selection (♣) satisfies Property GOAL

▷ Generalized results: relax Assumption U and select a different (♣)
Understanding the anti-windup solution (block diagram)

**Main Theorem:** Under Assumption U, the plant (♠) controlled by (♡) and augmented with the anti-windup compensator (♦) and the selection (♣) satisfies Property GOAL.
Sketch of the proof

▷ Change of coordinates: \((x, x_c, x_e) \rightarrow (e, x_c, x) := (x - x_e, x_c, x)\) the dynamics assume the cascade structure

\[
\begin{align*}
\text{Virtual Plant} & \quad \dot{e} = f(e, y_c) \\
\text{Unconstrained controller} & \quad \begin{cases} 
\dot{x}_c = g(x_c, e, r) \\
y_c = k(x_c, e, r)
\end{cases} \\
\text{Actual plant} & \quad \dot{x} = f(x, \text{sat}(\gamma(x, e) + y_c))
\end{align*}
\]

▷ Exploit the following property:

1. \(\gamma(x, x) = 0, \forall x \in \mathbb{R}^{2n};\)
2. the point \(x = x^*\) is GAS+LES for the system \(\dot{x} = f(x, \text{sat}(\gamma(x, x^*) + u^*))\);
3. for all exponentially vanishing functions \(\varepsilon_1(t), \varepsilon_2(t)\), the trajectories of

\[
\dot{x} = f(x, \gamma(x, \text{sat}(x^* + \varepsilon_1(t)) + u^* + \varepsilon_2(t)))
\]

remain bounded.
Sketch of the proof (cont’d) - items 2 and 3

- Introduce the modified potential energy

\[
V_d(q, q^*) := -V(q) + \sum_{i=1}^{n} \int_{0}^{q_i - q_i^*} \kappa_{gi} \sigma_i(\kappa_{qi}s) \, ds
\]

- Observe that

\[
\gamma(x, x^*) = -\left( \frac{\partial V_d}{\partial q}(q, q^*) - \frac{\partial V_d}{\partial q}(q^*, q^*) \right) - K_0(\dot{q} - \dot{q}^*)
\]

- GAS and item 3: use \( H_0(q, \dot{q}) := T(q, \dot{q}) + V(q) + V_d(q, q^*) \) for which

\[
\dot{H}_0 \leq \dot{q}^T \left( \text{sat}(\gamma(x, x^* + \varepsilon_1) + u^* + \varepsilon_2) + \frac{\partial V_d}{\partial q}(q, q^*) \right) \\
\leq -\beta(|\dot{q}|) + |\dot{q}| |\delta(\varepsilon)|
\]

- LES: use \( H_1(q, \dot{q}) := H_0(q, \dot{q}) + \varepsilon \dot{q}^T \mathbf{I}(q)(q - q^*) \)
Anti-windup design for fully actuated rigid robots

Plant
\[ I(q)\ddot{q} + C(q, \dot{q})\dot{q} + R(q)\dot{q} + h(q) = \text{sat}(u) \]  \hspace{1cm} (♠)

Unconstrained controller
\[
\begin{align*}
\dot{x}_c &= g(x_c, u_c, r) \\
y_c &= k(x_c, u_c, r)
\end{align*}
\]  \hspace{1cm} (♡)

Anti – windup augmentation
\[ \dot{x}_e = f(x, \text{sat}(u)) - f(x - x_e, y_c) \]  \hspace{1cm} (♢)

Stabilizing law
\[ v_1 = h(q - q_e) - h(q) + K_G \text{sat}(K_Q q_e) + K_0 \dot{q}_e \]  \hspace{1cm} (♣)

Connection equations
\[ u = \text{sat}(y_c + v_1), \quad u_c = x - x_e \]

Requirements:
1. The unconstrained closed-loop system is GAS+LES
2. \( \sup_{q \in \mathbb{R}^n} |h_i(q)| + k_{Gi} m_i < m_i, \ i = 1, \ldots, n \)

\( K_G, K_Q, K_0 \) are free design parameters
Example 1: SCARA revisited

Anti-windup compensation parameters:

\[ K_0 = \begin{bmatrix}
75 & 0 & 0 & 0 \\
0 & 45 & 0 & 0 \\
0 & 0 & 20 & 0 \\
0 & 0 & 0 & 40 \\
\end{bmatrix} \]

\[ K_G = \begin{bmatrix}
0.9 & 0 & 0 & 0 \\
0 & 0.9 & 0 & 0 \\
0 & 0 & 0.4 & 0 \\
0 & 0 & 0 & 0.9 \\
\end{bmatrix} \]

\[ K_Q = \begin{bmatrix}
500 & 0 & 0 & 0 \\
0 & 250 & 0 & 0 \\
0 & 0 & 500 & 0 \\
0 & 0 & 0 & 500 \\
\end{bmatrix} \]

Responses of perturbed dynamics (1.2 mass uncertainty + 1 Kg load) to

- Small signals: \( r = [3 \text{ deg}, -2 \text{ deg}, 2 \text{ cm}, 4 \text{ deg}] \)
- Medium signals: \( r = 2 \times [3 \text{ deg}, -2 \text{ deg}, 2 \text{ cm}, 4 \text{ deg}] \)
- Large signals: \( r = 50 \times [3 \text{ deg}, -2 \text{ deg}, 2 \text{ cm}, 4 \text{ deg}] \)
SCARA: small signals

The reference is \( r = [3\, \text{deg}, -2\, \text{deg}, 2\, \text{cm}, 4\, \text{deg}] \)

All the trajectories coincide: linear decoupled responses
> The reference is \( r = [6 \, \text{deg}, -4 \, \text{deg}, 4 \, \text{cm}, 8 \, \text{deg}] \)

> Stability is recovered, performance is almost fully preserved
SCARA: large signals (perturbed)

▷ The reference is \( r = [150 \text{ deg}, -100 \text{ deg}, 1 \text{ m}, 200 \text{ deg}] \)

▷ Stability is retained, performance is partially lost
Example 2: PUMA

▷ Computed torque controller gains:
\[ K_p = \text{diag}(800, 1100, 720, 300, 250, 100) \]
\[ K_i = \text{diag}(10, 10, 4, 10, 9, 6) \]
\[ K_d = \text{diag}(50, 55, 40, 37.5, 30, 25) \]

▷ Anti-windup compensation parameters:
\[ K_0 = \text{diag}(80, 50, 40, 7, 4, 4) \]
\[ K_G = \text{diag}(0.99, 0.25, 0.85, 0.99, 0.99, 0.99) \]
\[ K_Q = \text{diag}(500, 800, 45, 10, 10, 10) \]

▷ Responses of perturbed dynamics (1.2 mass uncertainty + 0.5 Kg load) to

- Medium signals: \( r = [5 \ deg, 5 \ deg, -5 \ deg, 5 \ deg, 5 \ deg, 5 \ deg] \)
- Large signals: \( r = 20 \times [5 \ deg, 5 \ deg, -5 \ deg, 5 \ deg, 5 \ deg, 5 \ deg] \)
PUMA: medium signals (perturbed)

The reference is $r = [5 \text{ deg}, 5 \text{ deg}, -5 \text{ deg}, 5 \text{ deg}, 5 \text{ deg}, 5 \text{ deg}]$

The performance loss is mitigated
PUMA: large signals (perturbed)

▷ The reference is $r = [100, 100, -100, 100, 100, 100]$ deg

▷ Stability is recovered, performance is partially lost
Performance improvements via nonlinear selections of $v_1$

- Replace the simple PD-type law

$$v_{1old} = h(q - q_e) - h(q) + K_G \text{sat}(K_Q q_e) + K_0 \dot{q}_e$$

- Observe that the plant dynamics are given by

$$I(q) \ddot{q} + C(q, \dot{q}) \dot{q} + R(q) \dot{q} + h(q) = \text{sat}(y_c + v_1)$$

large over- and under-shoots of $y_{ce}$ (outside the $\text{sat}(\cdot)$ limits) may badly affect the plant response ⇒ Cancel them out by selecting

$$v_1 = \text{sat}(y_c) - y_c + v_{1old}$$

- Further improvement ⇒ allow a nonlinear derivative term

$$v_{1new} = \text{sat}(y_c) - y_c + h(q - q_e) - h(q) + K_G \text{sat}(K_Q q_e) + K_d(q_e, \dot{q}_e) \dot{q}_e$$

**Theorem:** if $K_d(\cdot, \cdot)$ is diagonal (and regular) and $K_d(\cdot, \cdot) > K_0 > 0$ then

*Main theorem* holds with $v_{1new}$
\[ q_e = q - q_{unconstr} \implies \text{want to drive it to zero fast and smooth (no overshoots)} \]

\[ v_{1\text{new}} = \text{sat}(y_c) - y_c + h(q - q_e) - h(q) + K_G \text{sat}(K_Q q_e) + K_d(q_e, \dot{q}_e) \dot{q}_e \]

\[ \dot{q} \]

- Derivative term: \( K_d(q_e, \dot{q}_e) \dot{q}_e \) (recall \( K_d(q_e, \dot{q}_e) > 0 \))
  - Breaking torque if \( q_e \dot{q}_e < 0 \) (needed close to 0)
  - Accelerating torque if \( q_e \dot{q}_e > 0 \) (needed, e.g., when \( y_c \neq \text{sat}(y_c) \))

\[ k_{di}(q_e, \dot{q}_e) = \begin{cases} 
\gamma_E(q_e) k_0, & \text{if } q_e \dot{q}_e < 0 \\
0, & \text{if } q_e \dot{q}_e \geq 0,
\end{cases} \]

where \( \gamma_E(q_e) := \frac{\text{sat}(k_{qi} q_e)}{k_{qi} q_e} \) is the “equivalent gain” at the \( i \)-th input

\[ \implies \text{Note that } K_d(q_e, \dot{q}_e) \dot{q}_e \text{ is Lipschitz!} \]
SCARA: large signals (linear $v_1$)

▷ The reference is $r = [150 \, \text{deg}, -100 \, \text{deg}, 1 \, \text{m}, 200 \, \text{deg}]$

▷ Stability is retained, performance is partially lost
SCARA: large signals (nonlinear $v_1$)

- The reference is $r = [150 \ deg, -100 \ deg, 1 \ m, 200 \ deg]$

- Performance is dramatically improved (input authority is largely exploited)
PUMA: large signals (linear $v_1$)

- The reference is $r = [100, 100, -100, 100, 100, 100] \text{ deg}$

- Stability is recovered, performance is partially lost
PUMA: large signals (nonlinear $v_1$)

▷ The reference is $r = [100, 100, -100, 100, 100, 100] \text{ deg}$

▷ Performance is dramatically improved (input authority is largely exploited)
Lending a helping hand (and fading away)

- The anti-windup action should not be too invasive
  - the unconstrained controller is always preferred
  - the anti-windup compensator should only act when needed
  - after “fixing” things up, it should fade away
A tracking simulation

- SCARA robot following a reference imposing a circular motion

The anti-windup compensator gives up a little on output performance but keeps the closed-loop well behaved (saturated controller has no clue!)
Concluding remarks

▶ Summary:
  ● anti-windup construction for E-L systems
  ● successful validated by simulations on fully actuated rigid robot arms

▶ Ramifications:
  ● build robots with smaller actuators
  ● improve the performance of existing robots

▶ Future work:
  ● experimental validation
  ● bounded (or even more sophisticated) control of satellites