Linear flux observers for induction motors with quadratic Lyapunov certificates

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Abstract

We propose a full order Linear Time-Varying (LTV) Luenberger observer for the rotor flux estimation of an induction motor. Introducing a suitable reduced-order Linear-Time-Invariant (LTI) system that is always observable and controllable, we show that any arbitrary LTI design and its quadratic Lyapunov certificates can be lifted to the higher-order original LTV dynamics obtaining the same certificates. As a result, we show that arbitrary global uniform exponential bounds can be imposed on the estimation error, regardless of the rotor speed. Then applying a suitable order reduction technique, we build a reduced observer providing the same guarantees. We also establish interesting connections between these reduced observers and existing non-adaptive observers. We also show applications of our results when wanting to minimize the effects of measurement noise on the estimates.

Index Terms

Induction motor, flux observer, linear time-varying systems, Lyapunov techniques.

I. Introduction

This paper deals with the rotor flux estimation in Induction Motors (IMs). The literature presents several works regarding the design of observers for this purpose [1], [2], [3], [4]. The main challenge is that the IM dynamics is linear time-varying, as it depends linearly on the rotation speed, so that standard LTI techniques cannot be used. A typical approach is then a robust control one, by fixing a possible range of speed and designing the observer gain in order to guarantee stability when the speed is constant in this fixed set, i.e. the eigenvalues of the dynamic matrix have negative real part for all physically acceptable values of the rotor speed [5]. Then a rule of thumb is to fix the convergence rate of the error dynamics larger than the fastest state variable of the IM model [3], [2] to obtain time scale separation.

A different solution to overcome this problem is to use a time-varying gains in order to eliminate the effect of the speed in the error dynamics. One important first work that uses time-varying gain for flux estimation in IM is [1]. Other examples are [6] that deals with the full-order flux observer design in encoderless IM drives and the gain is chosen time-varying as a function of the rotor speed. Moreover, for the same observer, [7] shows how to choose the gain using the linearized model, but it is also showed that the linearized model reveals potential instability problems. A reduced order version is also presented in [8]. In [9] it is shown that the observer gain selection is extremely critical for the good behavior of the system, and a framework is developed, in which the properties of any gain selection can be easily assessed. Four candidate gain selections are considered, two of which do not use the rotor speed in their equations (inherently encoderless schemes). The stability of these schemes is analyzed in [10] for different working conditions. A Lyapunov-based analysis of the stability of rotor flux observers for IM is proposed in [11] and [12], where it is shown how the observer gain is chosen such that the time derivative of the Lyapunov function, developed for the proposed observer, is negative and independent of the speed. Moreover [12] carries out an analysis of the behavior when the speed signal is not correct. It is useful to underline that all these implementations, that use time-varying gains, suffer the problem that the speed signal, together with the model parameters, are not perfectly known, moreover the measured speed is also noisy and this leads to a further noise superimposed to the estimated variables.

Regarding the sensitivity to parameter variations, two main approaches can be found. In the first one, assuming nominal values for the motor parameters, the observer gain is designed in order to obtain satisfactory error dynamics and steady-state behaviour also in the presence of motor parameter variations [13], [14], [5]. In [15] the equations of the parameter sensitivity of the rotor flux estimation are derived for a full-order flux observer, and based on the parameter sensitivity analysis, practical methods of designing robust observer gains are proposed. The same holds for [14], where the sensitivity analysis makes it possible to establish design criteria for a rotor flux reduced-order observer for the IM. The second approach considers online adaptation of the model parameters [2], [16], [17]. In particular the parameters adapted are those that vary during normal operation, i.e. stator and rotor resistances. For example in [2] a flux observer for an induction motor with an adaptive scheme is proposed. The parameters identified adaptively are stator and rotor resistance, which vary with the motor temperature, and the stability is proved by Lyapunov's theorem. Furthermore, robustness of the induction motor drive system with the proposed flux observer is shown. Recently in [17] a novel local adaptive flux observer for induction motors is presented which, under persistency of excitation, estimates the motor fluxes and identifies on-line the rotor resistance. Other works consider a more

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complex model, i.e. saturation effects, in order to have a more accurate estimate. For example [18] considers a nonlinear dynamic model of the IM that includes the effects of magnetic saturation and designs an observer–controller strategy that achieves semi-global exponential rotor position tracking. The flux estimate in this case is embedded in the control strategy.

Despite the large number of different flux observers presented in literature, the approach followed in this paper is new. Here, a full order LTV Luenberger observer is proposed, with an output injection. The observer gain is selected by extending to the four-dimensional current-flux state-space a smaller design carried out on a representative two-dimensional LTI dynamics where standard LTI design techniques can be used. Our main results show that the properties of the four-dimensional LTV error dynamics coincide with those of the simpler two-dimensional LTI error system in terms of quadratic time-invariant Lyapunov certificates. This is particularly important because no bound on the rotor speed is required for the stability of the error dynamics. Using this main result, several selections of the observer gain are proposed, some of them leading, after a suitable observer order reduction, to existing approaches. Among other things, we show that even if the estimation error exhibits a peculiar time-varying transient, the upper bound on its norm is a time-invariant function and the rate of convergence can be freely chosen. In addition, a suitable disturbance attenuation problem is solved, designing the observer gain to reduce as much as possible the $\mathcal{L}_2$ gain between an unmeasured disturbance acting on the current measurement and the estimation error, subject to a prescribed desired convergence rate for the error dynamics.

While there are several existing approaches not requiring speed measurements, we present an ideas in the simplified context of known speed of rotation, so that we may clearly illustrate the advantages of the proposed approach in a short ad focused note with clear connection to the existing literature. Nevertheless, the elegant results proved allow for extensions to the case where the speed is observed within a separation principle type of approach that we regard as future work.

The paper is organized as follows. Section 2 describes the model of the IM and its properties. In Section 3 our main result is given, and some practical tools to design the observer gain are given. In Section 4 the reduced order version of the proposed observer is derived. Finally Section 5 presents some simulation results, which show the effectiveness of the proposed approach.

Notation: $A \otimes B$ denotes the Kronecker product between matrices A and B. Given a square matrix A, $\mathrm{He}(A) = A + A^T$.

Table I
STATES, INPUT AND PARAMETERS OF THE IM.

u_{sd}, u_{sq} (i_{sd}, i_{sq})	direct and quadrature stator voltage (current) components in a fixed $d-q$ reference frame;
ψ_{rd}, ψ_{rq}	direct and quadrature rotor flux components in a fixed $d-q$ reference frame;
$L_s(L_r)[L_m]$	stator (rotor) [magnetizing] inductance;
$R_s(R_r)$	stator (rotor) resistance;
ω_{re}	electrical angular rotor speed;
p	pole pairs;
j_z	inertia moment;

II. LTV MODEL OF THE INDUCTION MOTOR

Using the space-vector direct and quadrature components, (see [4, Equation (1.26)]), the LTV equations of the electromagnetic part of an induction motor can be written, using the Kronecker product, in the following compact state-space form:

$$\begin{aligned} \dot{x} &= \overbrace{\left[\bar{A} \otimes I + \Omega(t) \otimes J\right]}^{:=A(t)} x + \overbrace{\left(\bar{B} \otimes I\right)}^{:=B} u, \\ y &= \overbrace{\left(\bar{C} \otimes I\right)}^{:=C} x, \end{aligned} \tag{1}$$

where $I = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$, and x, u and y are respectively the state, the input and the output vectors defined, according to the notation in Table I, as:

$$x = \left[\, i_{sd} \;\; i_{sq} \;\; \psi_{rd} \;\; \psi_{rq} \,\right]^T ; \;\; y = \left[\, i_{sd} \;\; i_{sq} \,\right]^T ; \;\; u = \left[\, u_{sd} \;\; u_{sq} \,\right]^T . \tag{2}$$

Matrices $\bar{A}$, $\Omega(t)$, $\bar{B}$ and $\bar{C}$ in (1) are defined as:

$$\left(\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & \end{array}\right) = \left(\begin{array}{cc|c} -\gamma & \alpha\beta & \frac{1}{\sigma L_s} \\ \alpha L_m & -\alpha & 0 \\ \hline 1 & 0 & \end{array}\right), \tag{3a}$$

$$\Omega(t) = \begin{bmatrix} 0 & -\beta\omega_{re}(t) \\ 0 & \omega_{re}(t) \end{bmatrix}, \tag{3b}$$

where the magnetizing inductance L_m is a fraction of $L_r L_s$ inversely proportional to the total leakage factor σ. Parameter σ, together with α, β and γ appearing in (3), is defined as (see [2] for details):

$$\sigma = 1 - \frac{L_m^2}{L_s L_r} \in (0,1), \quad \alpha = \frac{R_r}{L_r} > 0,$$
$$\beta = \frac{L_m}{\sigma L_s L_r} > 0, \quad \gamma = \frac{R_s}{\sigma L_s} + \beta\alpha L_m > 0. \tag{4}$$

Matrix $\Omega(t)$ in (3b) encapsulates the time dependence of the LTV model (1) and it contains the rotor speed $\omega_{re}(t)$ at time t.

The results of this paper are based on the following two dimensional LTI dynamics, naturally arising from the four-dimensional model (1), and satisfying the result given next.

$$\begin{aligned} \dot{\bar{x}} =& \bar{A}\bar{x} + \bar{B}u, \\ \bar{y} =& \bar{C}\bar{x}. \end{aligned} \tag{5}$$

Lemma 1: For any value of the physical parameters satisfying (4), matrix $\bar{A}$ is Hurwitz and triple ($\bar{A}$, $\bar{B}$, $\bar{C}$) is controllable and observable.

Proof. To show that $\bar{A}$ is Hurwitz, consider its characteristic polynomial: $P_{\bar{A}}(s) = s^2 + (\alpha+\gamma)s + \alpha(\gamma - \alpha\beta L_m)$, which has roots with negative real part because from (4) we have $\gamma - \alpha\beta L_m = \frac{R_s}{\sigma L_s} > 0$. Moreover we may compute the determinant of the controllability and observability matrices $\bar{R}$ and $\bar{O}$ for (5), which correspond to $\det(\bar{R}) = \frac{L_r}{\sigma L_s}\alpha\beta > 0$ and $\det(\bar{O}) = \alpha\beta > 0$. □

III. Main results

In this section we exploit the symmetries in the structure of model (1), representation (5) and its properties of Lemma 1, to construct a full order dynamic observer having the same rate of convergence of the error dynamics as the one induced by a linear Luenberger gain $\bar{L}$ designed for dynamics (5), as characterized next (note that from Lemma 1, there is no restriction in Assumption 1, because pair $(\bar{C}, \bar{A})$ is observable):

Assumption 1: Gain $\bar{L} \in \mathbb{R}^2$ is such that matrix $\bar{A} - \bar{L}\bar{C}$ is Hurwitz.

Based on any gain $\bar{L}$ satisfying Assumption 1, and a suitably selected scalar ρ, consider the following LTV observer dynamics:

$$\begin{aligned} \dot{\hat{x}} = \left[\bar{A} \otimes I + \Omega(t) \otimes J\right] \hat{x} + \left(\bar{B} \otimes I\right) u \\ + \left[\bar{L} \otimes I + \begin{bmatrix} 0 \\ \rho\omega_{re}(t) \end{bmatrix} \otimes J\right] (y - \hat{y}). \end{aligned} \tag{6}$$

The following theorem establishes that the LTV observer (6) is associated with uniformly globally exponentially stable error dynamics inheriting the convergence properties induced by $\bar{L}$ on the LTI closed loop $\bar{A} - \bar{L}\bar{C}$ of Assumption 1.

Theorem 1: Consider any gain $\bar{L}$ satisfying Assumption 1 and any pair[1] of positive definite matrices $\bar{P} = \left[\begin{smallmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{smallmatrix}\right]$, $\bar{Q}$ such that:

$$\text{He}\left(\bar{P}(\bar{A} - \bar{L}\bar{C})\right) = -\bar{Q} < 0. \tag{7}$$

Then, selecting $\rho = \frac{\beta p_{11} - p_{12}}{p_{22}}$, and denoting the estimation error as $e = x - \hat{x}$, the quadratic positive definite Lyapunov function $V(e) = \frac{1}{2} e^T \left(\bar{P} \otimes I\right) e$ satisfies:

$$\dot{V}(e) = \langle \nabla V(e), \dot{e} \rangle = -e^T \left(\bar{Q} \otimes I\right) e, \tag{8}$$

along all solutions to (1), (6).

Proof. Consider the error dynamics that, using (1) and (6), can be conveniently written as:

$$\dot{e} = \left((\bar{A} - \bar{L}\bar{C}) \otimes I + \begin{bmatrix} 0 & -\beta\omega_{re} \\ -\rho\omega_{re} & \omega_{re} \end{bmatrix} \otimes J\right) e. \tag{9}$$

Computing $\dot{V}(e)$, we get,

$$\begin{aligned} \dot{V}(e) =& e^T (\bar{P} \otimes I) \left((\bar{A} - \bar{L}\bar{C}) \otimes I\right) e \\ &+ e^T (\bar{P} \otimes I) \left(\begin{bmatrix} 0 & -\beta\omega_{re} \\ -\rho\omega_{re} & \omega_{re} \end{bmatrix} \otimes J\right) e \\ =& e^T \left(\bar{P}(\bar{A} - \bar{L}\bar{C}) \otimes I\right) e \\ &+ \underbrace{e^T \left(\bar{P} \begin{bmatrix} 0 & -\beta\omega_{re} \\ -\rho\omega_{re} & \omega_{re} \end{bmatrix} \otimes J\right) e}_{=0} \\ =& e^T \left(\bar{P}(\bar{A} - \bar{L}\bar{C}) \otimes I\right) e = e^T \left(\bar{Q} \otimes I\right) e, \end{aligned} \tag{10}$$

[1]Note that infinitely many such pairs exist because $\bar{A} - \bar{L}\bar{C}$ is Hurwitz by design.

as to be proven.

The term $e^T\left(\left(\bar{P}\left[\begin{smallmatrix}0 & -\beta\omega_{re}\\ -\rho\omega_{re} & \omega_{re}\end{smallmatrix}\right]\right)\otimes J\right)e$ in (10) is equal to zero because, with $\rho=\frac{\beta p_{11}-p_{12}}{p_{22}}$, it is easy to verify that $\left[\begin{smallmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{smallmatrix}\right]\left[\begin{smallmatrix}0 & -\beta\omega_{re}\\ -\rho\omega_{re} & \omega_{re}\end{smallmatrix}\right]$ is symmetric, which straightforwardly implies that $\left(\bar{P}\left[\begin{smallmatrix}0 & -\beta\omega_{re}\\ -\rho\omega_{re} & \omega_{re}\end{smallmatrix}\right]\right)\otimes J$ is skew-symmetric. □

The result in Theorem 1 is appealing because it exploits the structure of the 4-dimensional LTV dynamics (1) to design an observer inducing a quadratically stable error dynamics. Moreover this design can be performed by looking at a lower dimensional LTI dynamical system joining the useful properties established in Lemma 1. Clearly, the error variables depend on ω_{re} and exhibit a peculiar time-varying transient, but the upper bound on their norm is a simple exponential function (see the solid and dashed curves in our example studies of Figure 1. Note that once a gain $\bar{L}$ satisfying Assumption 1 has been chosen, one can compute $\alpha_l=-\max_i\left(\mathrm{Re}\{\lambda_i(\bar{A}-\bar{L}\bar{C})\}\right)$ and then solve the convex optimization:

$$\begin{aligned}\min_{k,\bar{P}=\bar{P}^T} k,\quad &\text{subject to:}\\ &\mathrm{He}\left(\bar{P}(\bar{A}-\bar{L}\bar{C})\right)\leq -2\alpha_l\bar{P},\\ &I\leq\bar{P}\leq kI,\end{aligned}\tag{11}$$

to obtain the tight bound:

$$|e(t)|\leq\sqrt{k}e^{-\alpha_l t}|e(0)|,\tag{12}$$

which follows from straightforward Lyapunov results obtained from Theorem 1 applied with $\bar{Q}=2\alpha_l\bar{P}$.

An additional way to exploit the results of Theorem 1 is to solve a suitable disturbance attenuation problem, for example designing gain $\bar{L}$ to reduce as much as possible the $\mathcal{L}_2$ gain between an unmeasured disturbance d acting on the current measurement and the estimation error e, subject to a prescribed desired convergence rate $\alpha_l>0$ for the error dynamics. In this case, Theorem 1 can be applied for the special case $\beta p_{11}-p_{12}=0$ (implying $\rho=0$), which leads to the following error dynamics (see (1) and (6)):

$$\dot{e}=\left((\bar{A}-\bar{L}\bar{C})\otimes I+\Omega(t)\otimes J\right)e+\left(\bar{L}\otimes I\right)d.\tag{13}$$

Using the well known LMI derivations associated with the bounded real lemma (see, e.g., [19, Corollary 7.5] for the case $\mu=1$), an upper bound μ on the $\mathcal{L}_2$ gain from d to e for dynamics (13) can be minimized by solving the following LMI optimization:

$$\begin{aligned}\min_{\mu,\bar{P},\bar{X}}\mu,\quad &\text{subject to:}\\ &\bar{P}=\left[\begin{smallmatrix}p_{11} & \beta p_{11}\\ \beta p_{11} & p_{22}\end{smallmatrix}\right]>0,\\ &\mathrm{He}\left[\begin{smallmatrix}\bar{P}\bar{A}-\bar{X}\bar{C} & -\bar{X} & 0\\ 0 & -\mu I & 0\\ I & 0 & -\mu I\end{smallmatrix}\right]<0,\\ &\mathrm{He}\left(\bar{P}\bar{A}-\bar{X}\bar{C}\right)\leq -2\alpha_l\bar{P},\end{aligned}\tag{14}$$

and then selecting $\bar{L}=\bar{P}^{-1}\bar{X}$.

In addition to the above numerical solutions, a few relevant explicit selections of $\bar{L}$ ensuring Assumption 1 can also be given. We provide below three relevant cases corresponding to different trade-offs between convergence speed and computational complexity.

Proposition 1: Consider model (1) with parameters satisfying (4), the following holds:

1 (Open-loop observer) Selection:

$$\bar{L}=\begin{bmatrix}0 & 0\end{bmatrix}^T,\tag{15}$$

satisfies Assumption 1 and selection $\bar{P}=\left[\begin{smallmatrix}\sigma L_s L_r & 0\\ 0 & 1\end{smallmatrix}\right]>0$ and $\bar{Q}=2\left[\begin{smallmatrix}\gamma\sigma L_s L_r & -\alpha L_m\\ -\alpha L_m & \alpha\end{smallmatrix}\right]>0$ satisfies (7).

2 (Speed of convergence α) Selection:

$$\bar{L}=\begin{bmatrix}\alpha-\gamma & \alpha L_m\end{bmatrix}^T,\tag{16}$$

satisfies Assumption 1 and assigns both eigenvalues of $\bar{A}-\bar{L}\bar{C}$ in $-\alpha$. Moreover selection $\bar{P}=\left[\begin{smallmatrix}\frac{1}{\alpha\beta} & 0\\ 0 & \frac{\beta}{\alpha}\end{smallmatrix}\right]>0$ and $\bar{Q}=\left[\begin{smallmatrix}\frac{2}{\beta} & -1\\ -1 & 2\beta\end{smallmatrix}\right]>0$ satisfies (7).

3 (Arbitrary speed of convergence $(\alpha+\eta)$). Given any scalar $\eta>0$, selection:

$$\bar{L}=\begin{bmatrix}\alpha-\gamma+2\eta\\ \alpha L_m+\frac{\eta}{\beta}\left(1+2\frac{\eta}{\alpha}\right)\end{bmatrix},\tag{17}$$

satisfies Assumption 1. Moreover, selection:

$$\bar{P}=\begin{bmatrix}\frac{\eta}{\alpha}\left(1+2\frac{\eta}{\alpha}\right) & -\frac{\beta}{\alpha}\eta\\ -\frac{\beta}{\alpha}\eta & \beta^2\end{bmatrix},\tag{18}$$

is positive definite and satisfies (7) with $\bar{Q} = 2(\alpha+\eta)\bar{P}$.

Proof. With selection (15), Assumption 1 holds because $\bar{A}$ is Hurwitz from Lemma 1. Moreover, for the suggested selection of $\bar{P}$, one may easily verify that $\mathrm{He}\left(\bar{P}\bar{A}\right) = -\bar{Q}$. Note that $\bar{Q}$ is positive definite because, using expressions (4), its upper left entry is clearly positive and its determinant satisfies $\alpha\gamma\sigma L_s L_r - \alpha^2 L_m^2 = 4R_r R_s > 0$.

With selection (16), one can easily verify that the eigenvalues of $\bar{A} - \bar{L}\bar{C} = \left[\begin{smallmatrix} -\alpha & \alpha\beta \\ 0 & -\alpha \end{smallmatrix}\right]$ are $(-\alpha, -\alpha)$. For the suggested selection of $\bar{P}$, one may verify that $\mathrm{He}\left(\bar{P}(\bar{A} - \bar{L}\bar{C})\right) = -\bar{Q}$, and it is positive definite because, using expressions (4), its diagonal terms are clearly positive and its determinant is $\det(\bar{Q}) = 3 > 0$.

With selection (17) it is easy to verify that $\bar{A} - \bar{L}\bar{C} = \left[\begin{smallmatrix} -(\alpha+2\eta) & \alpha\beta \\ -\frac{\eta}{\beta}\left(1+2\frac{\eta}{\alpha}\right) & -\alpha \end{smallmatrix}\right]$ is Hurwitz, therefore Assumption 1 is satisfied. Moreover with selection (18) we have that $\bar{P}$ is positive definite because $\det(\bar{P}) = \beta^2 \frac{\eta}{\alpha}\left(1 + \frac{\eta}{\alpha}\right) > 0$, and selecting $\bar{L}$ as in (17), then we get:

$$
\begin{aligned}
&\mathrm{He}\left(\bar{P}(\bar{A} - \bar{L}\bar{C})\right) \\
&= \mathrm{He}\begin{bmatrix} -(\alpha+\eta)\frac{\eta}{\alpha}\left(1+2\frac{\eta}{\alpha}\right) & 2(\alpha+\eta)\frac{\beta}{\alpha}\eta \\ 0 & -(\alpha+\eta)\beta^2 \end{bmatrix} \\
&= -2(\alpha+\eta)\begin{bmatrix} \frac{\eta}{\alpha}\left(1+2\frac{\eta}{\alpha}\right) & -\frac{\beta}{\alpha}\eta \\ -\frac{\beta}{\alpha}\eta & \beta^2 \end{bmatrix} = -2(\alpha+\eta)\bar{P}.
\end{aligned} \tag{19}
$$

Expression (19) shows that (7) is satisfied with $\bar{Q} = 2(\alpha+\eta)\bar{P}$, and this guarantees that the speed of convergence of the error dynamics for (1), (3), (6) is $\alpha+\eta$, as to be proven. □

It is useful to note that the tuning rules given in Proposition 1 or in (14) are new, and they represent a rigorous useful tool for designing a rotor flux observer with well defined certificates and in an optimal way.

IV. Reduced order observer

If Assumption 1 holds, then a reduced order version of observer (6) can be obtained by virtue of the quadratic Lyapunov function established in Theorem 1. In particular, using the result in [20, Lemma 3.1], given any gain $\bar{L}$ and a pair of matrices:

$$
\bar{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} > 0, \quad \bar{Q} = \begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix} > 0, \tag{20}
$$

satisfying (7), partitioning:

$$
\begin{aligned}
A(t) &= \left[\frac{A_i(t)}{A_\psi(t)}\right] = \left[\frac{-\gamma I \qquad \alpha\beta I - \beta\omega_{re}(t)J}{\alpha L_m I \qquad -\alpha I + \omega_{re}(t)J}\right], \\
x &= \left[\frac{i_s}{\psi_r}\right] = \left[\frac{\left[\begin{smallmatrix} i_{sd} & i_{sq} \end{smallmatrix}\right]^T}{\left[\begin{smallmatrix} \psi_{rd} & \psi_{rq} \end{smallmatrix}\right]^T}\right],
\end{aligned} \tag{21}
$$

we introduce the reduced order observer:

$$
\begin{aligned}
\dot{\hat{\phi}} =& A_\psi(t)\begin{bmatrix} y \\ \hat{\phi} - p_{22}^{-1}p_{12}y \end{bmatrix} + \\
&+ p_{22}^{-1}p_{12}\left(A_i(t)\begin{bmatrix} y \\ \hat{\phi} - p_{22}^{-1}p_{12}y \end{bmatrix} + \frac{1}{\sigma L_s}u\right), \\
\hat{\psi}_r =& \hat{\phi} - p_{22}^{-1}p_{12}y,
\end{aligned} \tag{22}
$$

with $\hat{\phi} \in \mathbb{R}^2$. Then the next lemma is established by directly applying the results in [20, Lemma 3.1] and [20, Remark 3.2] (where our variable $\hat{\phi}$ is denoted by ζ).

Lemma 2: If matrices $(\bar{P}, \bar{Q})$ in (20), and gain $\bar{L}$ satisfy (7), then, defining the estimation error $e_\psi = \psi_r - \hat{\psi}_r$ and function $V_\psi(e_\psi) = \frac{1}{2}e_\psi^T(p_{22} \otimes I)e_\psi$, we have:

$$
\dot{V}_\psi(e_\psi) = -\alpha\left(1 - \beta p_{22}^{-1}p_{12}\right)V_\psi(e_\psi), \tag{23}
$$

along dynamics (1), (3), (22).

Under the two special selections in (15) and (17), the reduced order observer (22) coincides with the two solutions proposed in [4, equation (3.8)] and [4, equation (3.33)] respectively, thereby establishing an interesting link between our results and those in [4]. To see this, consider (17) and fix $\eta = \beta\alpha c$ in (18) for some $c > 0$. Then term $p_{22}^{-1}p_{12}$ in (20) becomes $p_{22}^{-1}p_{12} = -c$, which can be used in (22), to obtain:

$$
\begin{aligned}
\dot{\hat{\phi}} &= A_\psi(t)\begin{bmatrix} y \\ \hat{\phi} + cy \end{bmatrix} - cA_i(t)\begin{bmatrix} y \\ \hat{\phi} + cy \end{bmatrix} - \frac{c}{\sigma L_s}u, \\
\hat{\psi}_r &= \hat{\phi} + cy.
\end{aligned} \tag{24}
$$

Considering relations (21) and (2), observer (24) becomes:

$$\begin{aligned}\dot{\hat{\phi}} &= \Bigg(\begin{bmatrix} -\alpha(1+\beta c) & \alpha L_m + c(\gamma - \alpha - \alpha\beta) \end{bmatrix} \otimes I \\ &\quad + \omega_{re}(t) \begin{bmatrix} (1+\beta c) & c(1+\beta c) \end{bmatrix} \otimes J \Bigg) \begin{bmatrix} \hat{\phi} \\ i_s \end{bmatrix} - \frac{c}{\sigma L_s} u, \\ \hat{\psi}_r &= \hat{\phi} + c i_s, \end{aligned} \tag{25}$$

which coincides with [4, Equation (3.33)].

Consider now selection (15). We established in Proposition 1 that a possible choice for $\bar{P}$ is $\bar{P} = \left[\begin{smallmatrix} \sigma L_s L_r & 0 \\ 0 & 1 \end{smallmatrix} \right]$, that yields $p_{22}^{-1} p_{12} = 0$. Then the reduced order observer (22) simplifies to:

$$\begin{aligned}\dot{\hat{\phi}} &= \Bigg(\begin{bmatrix} -\alpha & \alpha L_m \end{bmatrix} \otimes I + \begin{bmatrix} \omega_{re}(t) & 0 \end{bmatrix} \otimes J \Bigg) \begin{bmatrix} \hat{\phi} \\ i_s \end{bmatrix}, \\ \hat{\psi}_r &= \hat{\phi}, \end{aligned} \tag{26}$$

which coincides with the dynamics of observer [4, Eq. (3.8)].

Remark 1: It is emphasized that although the reduced order observers developed in this section may be desirable due to their lower computational complexity (2 states instead of 4), it is often desirable to adopt a full order strictly proper observer so that high frequency measurement noise is suitably filtered out in the feedback signal. This is accomplished, for example, by designing $\bar{L}$ according to optimization (14) as illustrated by the simulations of Figure 1.

V. Simulation and experimental results

The proposed Luenberger observer has been evaluated by means of simulations carried out in MATLAB®-Simulink®. The Simulink®model includes observer (6) and the IM model (1), coupled with the following mechanical equation:

$$\frac{d\omega_{re}}{dt} = \frac{1}{j_z} \left(\frac{L_m}{L_r} \psi_r^T J \, i_s - t_l \right), \tag{27}$$

where t_l is the load torque. For both the IM model and the observer, the parameters in Table II have been used, that correspond to an experimental 0.75–kW IM.

Table II
Nominal parameters of the IM.

$L_s(L_r)[L_m]$	0.241 (0.257) [0.225] H;
$R_s(R_r)$	15.6 (4.42) Ω;
p	2;
j_z	$4.4 \cdot 10^{-3}$ N·m·s^2;

Figures 1.(a) and 1.(b) show flux and current errors during a test where the current measurement, the input voltage and the speed measurement used in dynamics (6) are corrupted by zero mean white noise. The observer gains have been tuned fixing a convergence rate equal to $\alpha_l = \alpha$ by means of selection (17). These figures confirm our main result. In fact, the error variables, represented by the solid curves, depend on ω_{re}, and thus exhibit a peculiar oscillatory transient due to the time-varying nature of the error dynamics. Conversely the upper bound on their norm, represented by the dashed curves, is a simple exponential function. In particular these dashed curves represent bounds (12) with $\sqrt{k} = 68.1$ which has been computed by solving (11) with $\alpha_l = \alpha$. The same test has been carried out tuning the observer parameters with the $\mathcal{L}_2$ gain optimization (14) (with $\alpha_l = \alpha$) in order to reduce the noise effects, obtaining the gain $\bar{L} = \left[\begin{smallmatrix} -11.98 \\ 1.89 \end{smallmatrix} \right]$. The results shown in Figures 1.(c) and 1.(d) highlight a significantly improved disturbance rejection at the steady state (see the lower curves). Also for this case the bound (12) is represented by the dashed curves, where $\sqrt{k} = 6.67$ has been computed by solving (11) with $\alpha_l = \alpha$. Finally Figure 1(e) shows the behavior of the reduced order observer arising from the full order one of Figures 1.(a) and 1.(b) reduced following equation (22). A comparison between Figures 1.(a), 1.(b) and 1(e) reveals the advantages of a full order solution, as commented in Remark 1. Indeed the lower plots of Figures 1.(a) and 1.(b), compared to the two lowermost plots of Figure 1(e), show that the noise is more effectively filtered out by the full order solution.

Figures 2 and 3 show the experimental results obtained when the IM is operated by means of the field oriented control technique, and the feedback is performed using the proposed full-order observer. In particular the figures show the waveforms of the mechanical speed, the current estimation versus measurement mismatch, and the magnitude of the rotor flux when the IM is fluxed at zero speed and a reference speed of 80 rad/s is given at 0.5 s. The results confirm the same considerations given for the simulation results, showing the effectiveness of the proposed approach.

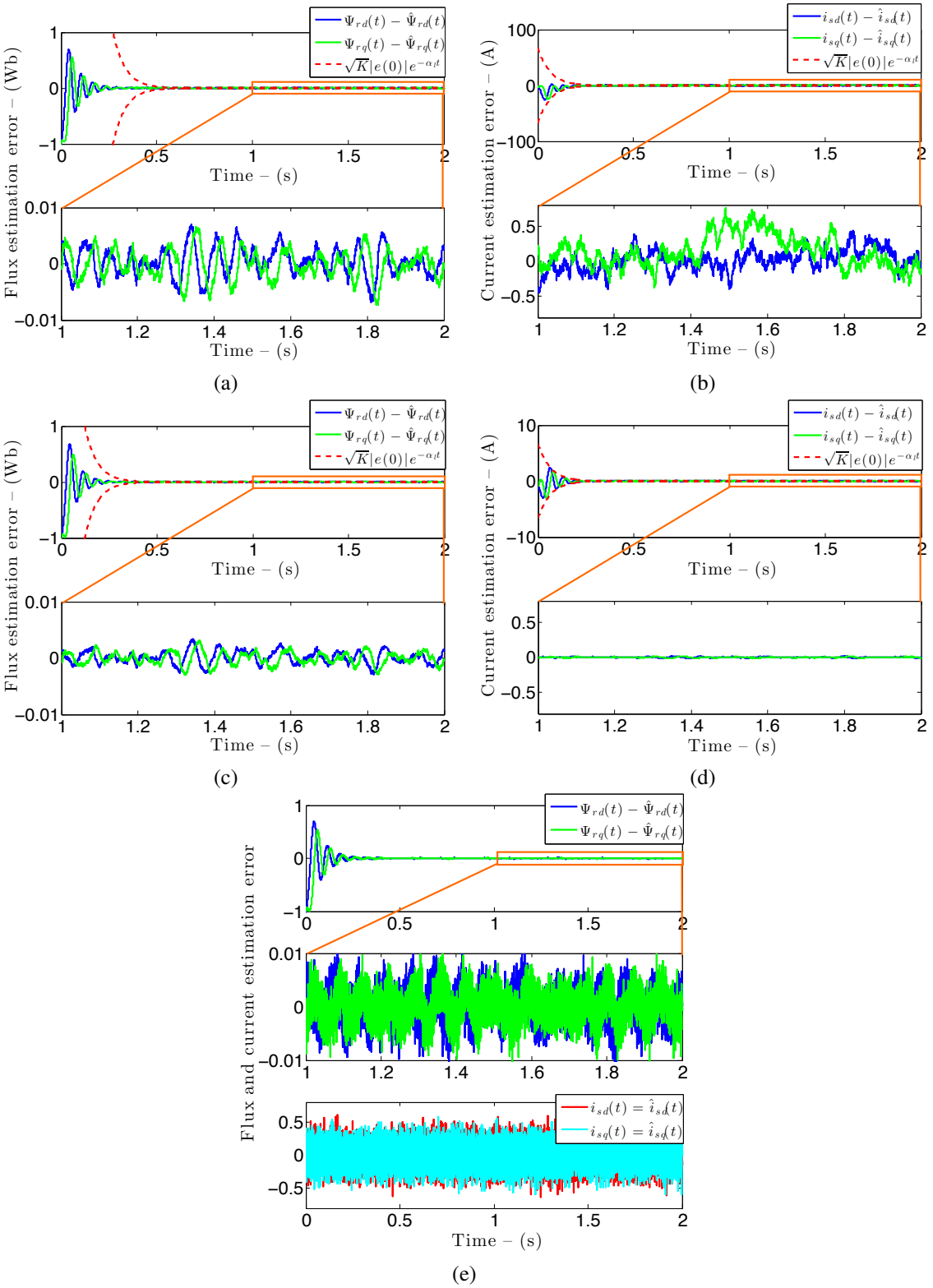

Figure 1. In (a) and (b) the flux and current errors are shown with observer gains computed from (17) without $\mathcal{L}_2$ optimization, while in (c) and (d) the flux and current errors are shown with observer gains computed by means of $\mathcal{L}_2$ optimization (14). In (c) the flux error for the reduced order observer (22) (top two plots), and noise affecting the current measurement (lowest plot) are shown.

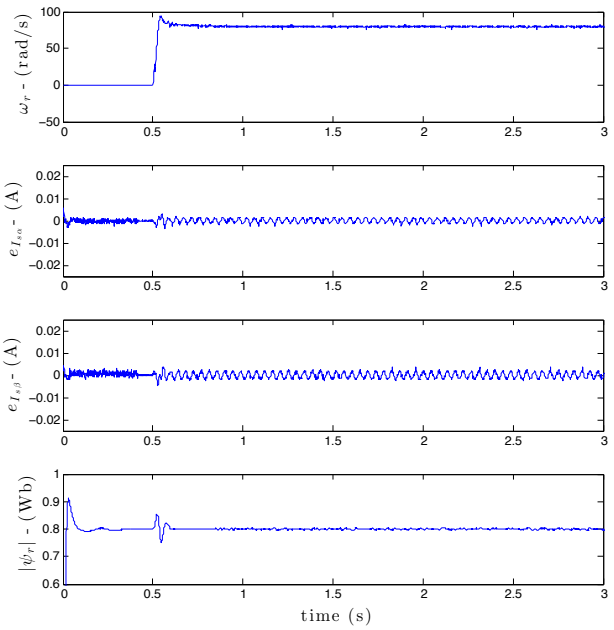

Figure 2. Experimental results obtained by means of $\mathcal{L}_2$ optimization (14).

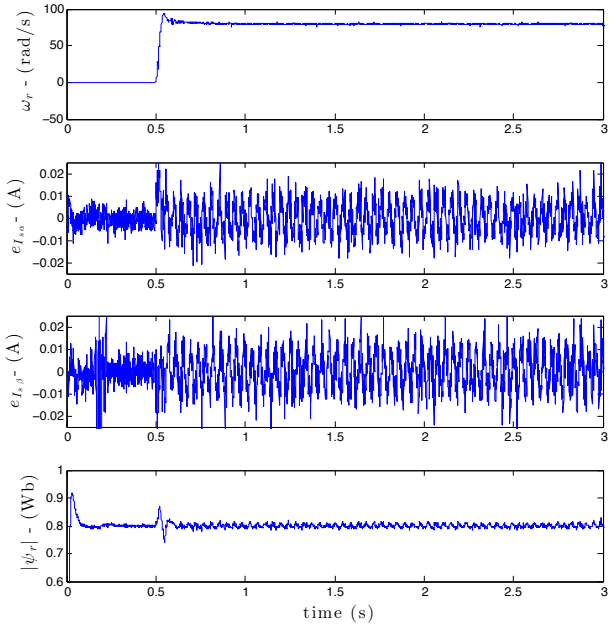

Figure 3. Experimental results obtained with observer gains computed from (17) without $\mathcal{L}_2$ optimization.

VI. Conclusion

In this paper a full order linear time-varying Luenberger observer for the rotor flux estimation of an IM has been proposed. A compact representation of the linear time-varying dynamics of the IM has been proposed, and arbitrary global uniform exponential bounds have been derived on the estimation error, regardless of the rotor speed. Then a reduced observer has been built applying a suitable order reduction technique and providing the same guarantees. Simulation results and experiments confirm the presented results. Despite the restrictive assumption of measuring the speed of rotation, novel and well defined tuning rules for flux observers for induction motor have been given and these rules imply well-defined Lyapunov certificates. This represents a general and useful tool for practitioners of this field who want to design quickly a rotor flux observer with well defined certificates and in an optimal way.

As future developments, we want to take advantage from the intuitions exploited in this paper, in order to implement an encoderless control scheme where the speed is not measured, but it is estimated as well.

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