Prediction of Uncertain Test Outcomes on Resistive Networks

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***Abstract*—In the automotive domain, the use of complex electronic control unit (ECU) in order to monitor functions such as the injection or the ABS has been widely developed during these last years. When such a function fails, the concerned ECU is able to reliably detect the faulty electronic circuit. Then, the task of the garage mechanic consists in localizing and replacing the faulty component of this circuit. This work is done by traversing a diagnosis tree composed of test sequences and whose leaves represent the different possible repairs to operate. Nowadays, these diagnosis trees are hand-made by experts and, because of the increasing complexity of these circuits, errors are not unusual in them.**

The software application AGENDA (for Automatic GENeration of DiAgnosis trees) uses algorithms that allow to generate automatically these diagnosis trees from the design data supplied by the automotive manufacturer.

Different models of the circuit to diagnose are built from these data and the specific knowledge given by the expert by means of a dedicated interface. First, this paper details how the different possible faults of this circuit as the different possible tests that can be performed are anticipated. Then, the way the values corresponding to the tests with occurrence of one of the faults of the circuit are computed is presented.

I. Introduction

In the automotive domain, the use of electronic systems to control several functions is widely spread. These functions span diverse automotive areas such as engine control (fuel injection or ignition), breaking and driving (ABS, suspensions), security (air-bags, seat-belt), or comfort (air conditioning, heating system). As shown schematically in figure 1, these electronic systems are composed of a voltage supply (battery, fuses and relays), sensors (potentiometer, temperature sensor) and actuators (electric valves) linked to electronic control units (ECUs for short) by a wire harness.

The main task of the ECU is to elaborate and send control signals to the actuators, taking into account the signals received by the sensors. Moreover, an ECU is equipped with an self-diagnosis function which reliably detects which of the functional electric circuits that it controls are failing. The failed electric circuits are associated with fault codes memorized in the ECU. However, the ECU is not able to localize precisely the faulty components within the functional circuit.

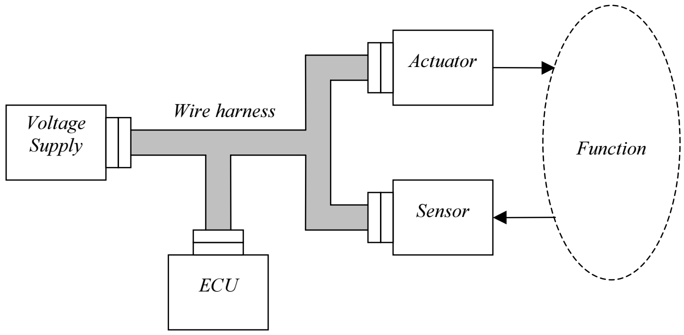

Fig. 1. Electronic systems for function control

Due to the increased use of such electronic systems, the garage diagnosis task is not limited to mechanical repairs, but also involve electronic ones. Therefore, car manufacturers have developed diagnostic tools for fault detection and isolation to help garage mechanics in the diagnostic task of electronic systems.

First, the diagnostic tools are able to read the fault codes stored in the memory of the ECU to identify the corresponding failed electric circuit. Then, in order to accurately localize the faulty components, diagnostic tests are proposed. The sequencing of these tests and their descriptions are displayed by the diagnostic tools in the form of decision trees (also called *diagnosis trees*,*test trees* or *troubleshooting trees*).

Currently, diagnosis trees are built by human experts. This task is time consuming and laborious as the complexity of electric circuits and mechatronic components increases. Consequently, errors are not unusual in the resulting diagnosis trees. Hence, it is imperative to reduce human intervention in the generation of diagnosis trees to reduce the cost of maintenance.

The method that we propose fully automate the decision tree generation process(see [1] and [2]) in two steps:

1. A behavioral model of the circuit is obtained from the design knowledge. Then, a set of possible system faults and a set of tests are anticipated from this

model. The possible outcomes of a given test when the circuit is in a given faulty state are obtained using a prediction algorithm that makes use of symbolic and interval computations. A “cross-table” is generated from the results of the prediction algorithm.

2. The use of a search algorithm coupled with an original heuristic evaluation function allows us to generate the optimal diagnosis tree from the “cross-table” and a topological model of the circuit. The topological model is used during the search algorithm to evaluate the dynamic test costs, which take into account the current topology of the mechanical system.

This paper focuses on the first step of the method and presents the prediction procedure which allows us to build the ”cross-table” from the optimization of the symbolic expressions relative to the possible (fault/test) pairs.

Given the frequent use of such circuits in the automotive domain, the procedure has been devised for Resistive Network circuits supplied by One Voltage Source (RNOVS).

The paper is organized as follows. Section 2 presents respectively the single faults set and the tests set that are anticipated for a given RNOVS. The behavioral model corresponding to this type of electric circuits is built according to a classical component-oriented approach based on a structural model and a library of basic components behavioral models. The considered faults impact on the model parameters and they are represented by bounded uncertain values,i.e. interval values.

Section 3 explains how the formal matrix expression of the system corresponding to a (fault/test) pair is built and how the symbolic expressions of the test is then obtained. Mathematical characteristics of these test symbolic expressions are also presented in this section. A prediction algorithm which organizes the way the (fault/test) pairs have to be evaluated and which minimizes the number of matrix constructions and test formal expression evaluations is proposed.

Given a (fault/test) pair, section 4 details the way the corresponding symbolic expression is optimized on the domain defined by the parameter bounded uncertain values relative to the given fault. The min and max values define the interval domain values.

Finally, section 5 discusses the general method and outlines several interesting directions for future investigation.

II. Example

The example that has been chosen to illustrate the different steps of the prediction procedure throughout the paper is the throttle valve potentiometer electric circuit (see [3] and [4]). This electric circuit provides information to the injection ECU about the position of a valve, called throttle valve, which controls the air flow into the carburettor. The throttle valve is directly activated by

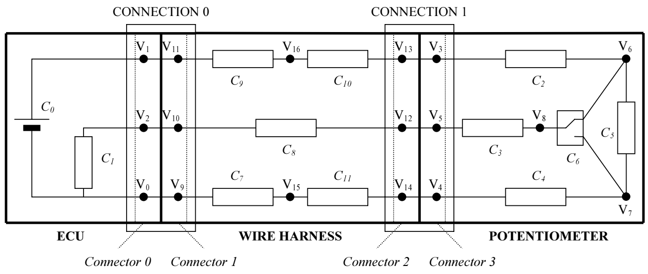

Fig. 2. Throttle valve potentiometer

the car driver with the accelerator pedal. Then, a potentiometer connected to the injection ECU transforms the valve position into an intensity signal I_s from the cursor pin of the potentiometer to the corresponding pin of the injection calculator.

This circuit is hence composed of a potentiometer linked by a wire harness to the injection ECU. During a diagnosis session, only two positions of the accelerator pedal are used : foot fully up or foot fully down. Consequently, the throttle valve potentiometer is represented as a two positions switch corresponding to these two accelerator pedal positions as shown in figure 2.

The potentiometer resistors C_2, C_3 and C_4 are associated with parameter values, respectively, R_2, R_3 and R_4 representing their resistance. These parameters have the same nominal value interval $[350, 650]$ Ohms. The nominal value interval $[960, 1440]$ Ohms is assigned to the parameter R_5 of resistor C_5. The resistor C_1 is associated with R_1 and its nominal values interval $[3.999 \times 10^6, 4.001 \times 10^6]$ Ohms. The electromotive power parameter U_0 of the voltage supply (component C_0 is associated with the nominal values interval $[4.9, 5.1]$ Volts. All the wires C_7, C_8, C_9, C_{10} and C_{11} have the same nominal values interval $[0.001, 0.001]$ Ohms of their parameters, respetively, R_7, R_8, R_9, R_{10} and R_{11}.

A *connector* represents the available pins of a component for connection, i.e. those where you can plug. By instance, for the component potentiometer, V_3, V_5 and V_4 belong to the $Connector3$ whereas V_6, V_7 and V_8 do not. A *connection* is a virtual component composed by two components *connector*. This kind of component is used to deal with structure faults.

III. Diagnosis problem formulation

This section describes how the set F of system faults and the set S of tests that can be performed on the system are anticipated from the behavioral model of the system to diagnose.

These two sets F and S are ground elements of the Test Sequencing Problem instance whose solution is the optimal diagnosis tree of the system.

The generation of these two sets F and S is more specifically detailed for electric circuits and illustrated on the throttle valve potentiometer example.

A. System fault set

This subsection presents the general definitions concerning the set of the possible faults within a classical component-oriented modeling approach (see [5]).

Let Ψ be the system to diagnose, defined as the set of its n_Ψ individual components ψ_i, $i \in \{1, ..., n_\Psi\}$. For each component ψ_i, let Φ^i be the set of the n^i_Φ possible component fault modes ϕ^i_j, $j \in \{1, ..., n^i_\Phi\}$. Let also $\Phi^i_{\neg AB}$ and Φ^i_{AB} be the set of the $n^i_{\neg AB}$ fault-free modes and the set of the n^i_{AB} faulty modes, respectively, such that $\Phi^i_{\neg AB} \cup \Phi^i_{AB} = \Phi^i$ and $\Phi^i_{\neg AB} \cap \Phi^i_{AB} = \emptyset$.

A.1. Definitions: A fault mode of the system Ψ, also called system fault, is defined as an n_Ψ component vector which associates with each component ψ_i one of its n^i_Φ fault modes ϕ^i_j, $j \in \{1, ..., n^i_{\Phi_i}\}$. Consequently, the set F of the faults which may occur in the system Ψ is composed of $n_F = \prod_{i=1}^{n_\Psi} n^i_\Phi$ elements, called f_k, $k \in \{1, ..., n_F\}$.

According to the number of the component fault modes which are faulty among the n_Ψ ones that define a fault, these n_F faults are classified into the two following fault classes.

- *Pure Single Faults*
  The Pure Single Faults (PSF) are defined as faults whose origin is at most one faulty component. They include the possible corresponding cascaded multiple faults. They also include the $\prod_{i=1}^{n_\Psi} n^i_{\neg AB}$ fault-free system modes, called empty faults.
- *Pure Multiple Faults*
  The Pure Multiple Faults (PMF) denote the faults whose independent origins (i.e. not cascaded multiple faults) refer to a set of at least two faulty components.

A.2. Fault set in a resistive network: Adopting the single fault hypothesis, the proposed diagnosis method selects a fault set F composed of the possible PSFs of the system. Then, the following five types of faults are distinguished according to their initial faulty component.

- *Empty fault*
  The elementary entities, connectors and connections of the system are all in one of theirs fault-free modes.
- *Parameter fault*
  A parameter of a component of the system is in one of its faulty modes. This mode corresponds to a non-nominal interval value assigned to one of the parameters of this elementary component.
- *Switch fault*
  One switch of the system is in one of its faulty modes. This mode given for one position corresponds to normal mode of another position (i.e. the switch is stuck at one of its possible positions).
- *Connector fault*
  One connector of the system is in one of its faulty modes. This mode corresponds to a short circuit between two neighboring pins of this connector.
- *Connection fault*
  One connection of the system is in one of its faulty modes. This mode corresponds to an open circuit between two opposite pins of this connection.

The uncertainties of the values that can take the system parameters are represented by value intervals (see [6], [7], [8], [9] and [10] for more about diagnosis of uncertain systems).

A.3. Example: In the throttle valve potentiometer example, at most 34 faults f_i, $i \in \{0, ..., 33\}$ may be considered where the fault f_0 corresponds to the empty fault.

For some of these 34 faults f_i such that $i \in \{0, ..., 33\}$, figure 3 gives the concerned faulty elementary entity and its precise faulty mode.

B. Test set

This section presents the general definitions concerning the set of tests that can be anticipated from a classical component-oriented model of the system.

Let Ψ be the system to diagnose, defined as the set of its n_Ψ individual components ψ_i, $i \in \{1, ..., n_\Psi\}$. Let also X be the set of the n_X system variable states x_i, $i \in \{1, ..., n_X\}$ defined by the structural model of the system Ψ.

For each component ψ_i, let U^i be the set of the n^i_U possible component normal behavioral modes u^i_j, $j \in \{1, ..., n^i_U\}$.

B.1. Definitions: The configuration of the system Ψ is defined as an n_Ψ component vector which associates with each component ψ_i, one of its n^i_U corresponding possible normal modes u^i_j, $j \in \{1, ..., n^i_{U_i}\}$. Consequently, the set E of the possible configurations of the system Ψ is composed of $n_E = \prod_{i=1}^{n_\Psi} n^i_U$ elements, called e_k, $k \in \{1, ..., n_E\}$.

A test is defined as a pair composed of a measurement description based on a subset of the system state variable set X and a subset of the possible system configuration set E. The test then consists of measuring each system state variables of X for any of the system configurations of E. These are all equivalent, i.e. give the same test outcome.

B.2. Test set in a resistive network: For an electric circuit, the system configuration is defined by the switches configuration and the connections configuration.

Let Ψ the electric circuit to diagnose, composed of n_{Cx} connections Cx_i, $i \in \{1, ..., n_{Cx}\}$ and of n_{Sw} switches Sw_i, $i \in \{1, ..., n_{Sw}\}$.

f_i	Faulty elementary component	Faulty mode
f_0	$\emptyset$	$\emptyset$
f_1	C_0	Short-Circuit
f_2	C_0	Open-Circuit
f_{18}	Switch C_6	Stuck at position 1
f_{19}	Switch C_6	Stuck at position 2
f_{24}	*Connector 2*	Short-Circuit ($V_{13} - V_{12}$)
f_{25}	*Connector 2*	Short-circuit ($V_{12} - V_{14}$)
f_{28}	CONNECTION 0	Open-circuit ($V_1 - V_{11}$)
f_{29}	CONNECTION 0	Open-circuit ($V_2 - V_{10}$)
f_{30}	CONNECTION 0	Open-circuit ($V_0 - V_9$)

Fig. 3. Fault description

For each switch Sw_i, let W^i be the set of its n_W^i possible normal mode,i.e. the different positions w_j^i, $j \in \{1, ..., n_W^i\}$.

Then, a switch configuration of the system Ψ is defined as a n_{Sw} component vector which associates with each switch Sw_i one of its n_W^i normal mode w_j^i, $j \in \{1, ..., n_W^i\}$. Consequently, the set G of the possible switch configurations of the system Ψ is composed of $n_G = \prod_{i=1}^{n_{Sw}} n_W^i$ elements, called g_k, $k \in \{1, ..., n_G\}$.

Each connection Cx_i of the system Ψ may be in connected or disconnected mode. Then, a connection configuration of the system Ψ is defined as a n_{Cx} component vector which associates with each connection Cx_i, its connected mode or its disconnected mode.

Consequently, the considered set Q of the connection configurations of the system Ψ is composed of $n_Q = 1 + 2^{n_{Cx}-1}$ elements, called q_k, $k \in \{0, ..., n_Q - 1\}$. By convention, q_0 denotes the system connection configuration such that all the system connections are connected.

For electric circuits from the automotive domain, the three following kinds of tests are considered.

- *Potential test*
  A potential test U_{V_A,V_0} has to be performed between an accessible system potential point V_A and V_0, the ground of the system in its connection configuration q_0 and one of its switch configurations g_j, $j \in \{1, ..., n_G\}$.
- *Equivalent resistance test*
  An equivalent resistance test $R_{V_A - V_B}$ has to be performed between two accessible system potential points V_A and V_B of the system in one of its connection configurations q_i, $i \in \{1, ..., n_Q\}$ and one of its switch configurations g_j, $j \in \{1, ..., n_G\}$.
- *Perceptible test*
  These tests do not require a measurement tool. The test output is perceptible to the eye or to the ear. A perceptible test S_{X_S} is concerned with a perceptible state variable X_S of the system in its connection configuration q_0 and one of its switch configurations g_j, $j \in \{1, ..., n_G\}$.

Among all the possible tests, some are redundant or not relevant and are filtered away. The remaining tests are called "useful tests".

B.3. Example: Consider the throttle valve potentiometer circuit example, Figure 4 describes the set G of 2 possible switch configurations, called g_i, $i \in \{1, 2\}$.

g_i	Switches	Switch position
g_1	Switch C_6	Position 1
g_2	Switch C_6	Position 2

Fig. 4. Switch configuration description

Figure 5 describes the set Q of 3 possible connection configurations, called q_i, $i \in \{0, 1, 2\}$, of the throttle valve potentiometer circuit.

q_i	Connections	Mode
q_0	CONNECTION 0	Connected
	CONNECTION 1	Connected
q_1	CONNECTION 0	Disconnected
	CONNECTION 1	Connected
q_2	CONNECTION 0	Disconnected
	CONNECTION 1	Disconnected

Fig. 5. Connection configuration description

With regard to the 34 considered faults of the fault set F, at most 36 useful tests s_i, $i \in \{0, ..., 35\}$ are considered.

For some of the 36 tests s_i such that $i \in \{0, ..., 35\}$, figure 6 gives the system state variables subset that have to be measured, the subset of equivalent possible system switch configurations for this measurement and the subset of equivalent possible system connection configurations for this measurement.

IV. Prediction procedure

Consider a RNOVS. For this system, let F be the set of the n_F considered faults f_i, $i \in \{1, ..., n_F\}$ and S the set of the n_S considered tests s_j, $j \in \{1, ..., n_S\}$.

The aim of the prediction procedure is to provide the symbolic expressions corresponding to the outcome of any test of the test set S in the occurrence of any fault of the fault set F.

A. Symbolic matrix expression of the system

First of all, the prediction elaborates the symbolic matrix expression of the system behavioral model corresponding to a given pair (f_i, s_j) for any $i \in \{1, ..., n_F\}$ and $j \in \{1, ..., n_S\}$.

This subsection details how this is achieved. The symbolic matrix construction algorithm is illustrated with different pairs (f_i, s_j) on the throttle valve potentiometer example.

A.1. Matrix construction algorithm: First, the equations corresponding to the voltage supply behavior are written. If the considered test s_j has to be performed in connection configuration q_0 then the algorithm starts from the voltage source of the system; otherwise, it starts from the pins where the equivalent resistance measurement tool has to be placed according to s_j.

The matrix construction algorithm follows recursively the elementary entities linked by the system structural model. Their respective behavior, described by Ohm's law equations, is selected from the elementary entity behavioral model library according to the considered pair (f_i, s_j) and reported in the symbolic matrix. If a circuit node (i.e., a potential point having more than two neighbors) is crossed, then the Kirchhoff's law equation relative to this potential point is written in the symbolic matrix.

During its construction, the symbolic matrix expression keeps the same structure $A \times X = B$ as shown in figure 7.

$$\left(\begin{array}{c|c|c} A_{1,1} & A_{1,2} & A_{1,3} \\ \hline A_{2,1} & A_{2,2} & A_{2,3} \\ \hline A_{3,1} & A_{3,2} & A_{3,3} \end{array}\right) \times \left(\begin{array}{c} X_1 \\ \hline X_2 \\ \hline X_3 \end{array}\right) = \left(\begin{array}{c} B_1 \\ \hline B_2 \\ \hline B_3 \end{array}\right)$$

Fig. 7. Symbolic matrix expression model

The square matrix A is decomposed into 9 blocks, denoted $A_{i,j}$, $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3\}$. $A_{1,2}$, $A_{1,3}$, $A_{3,1}$ and $A_{3,2}$ are null matrices and $A_{1,1}$, an identity (2×2) matrix.

The vector X is decomposed into 3 sub-vectors, X_i with $i \in \{1, 2, 3\}$. X_1 is a 2 component vector such that the first component corresponds to the ground of the system and the second one to the supply pin. X_2 corresponds to the other system potential points that are circuit nodes or involved in test s_j. X_3 corresponds to the different intensities of the system.

The vector B is decomposed into 3 sub-vectors, called B_i with $i \in \{1, 2, 3\}$. B_1 is a 2 components vector such that the first component is 0 and the second one U_{Supply} (i.e., electromotive power value of the system voltage supply). B_2 and B_3 are null sub-vectors.

According to this decomposition of the symbolic matrix expression, the Kirchhoff's law equations are identified in the last lines of the matrix expression ($A_{3,3}$ in A, X_3 in X and B_3 in B). In the same way, the Ohm's law equations ($A_{2,1}$, $A_{2,2}$ and $A_{2,3}$ in A, X_1, X_2 and X_3 in X and B_2 in B) are identified between the first lines which describe the voltage supply behavior ($A_{1,1}$ in A, X_1 in X and B_1 in B) and the last lines which correspond to the Kirchhoff's law equations as seen just before. This recursive traverse of the system stops when the supplied circuit is totally described.

In order to reduce the complexity of the matrix expression corresponding to the pair (f_i, s_j), branches (i.e., serial resistors) of the circuit are automatically identified and reduced by the matrix construction algorithm. Actually, for each identified circuit branch, only one intensity and one equivalent resistive parameter are considered rather than the set of equivalent intensities and the set of resistive parameters corresponding to the serial resistors which constitute this circuit branch, respectively.

The set of performed circuit branch reductions must be such that only the ground and supply pins of the voltage supply, circuit nodes and potential points that are involved in s_j appear in the final symbolic matrix corresponding to the pair (f_i, s_j).

A.2. Example: For the throttle valve potentiometer example, the symbolic matrices corresponding to the three following pairs (f_i, s_j) are presented.

- *Test s_2 and fault f_0*
  Figure 8 represents the symbolic matrix expression of the circuit corresponding to the test s_2 defined by the measurement U_{V_{12},V_0}, for the system connection configuration q_0 and the system switch configuration g_1, when the system is normal.

  According to the branch reductions, the symbolic matrix is simplified by considering the equivalent resistive parameters R_a, R_b, R_c and R_d for the serial resistive parameters $R_2 + R_9 + R_{10}$, R_3, $R_1 + R_8$ and $R_4 + R_5 + R_7 + R_{11}$, respectively.

  Then, the four corresponding branch intensities I_0 from V_1 to V_6, I_1 from V_6 to V_{12}, I_2 from V_{12} to V_0 and I_2 from V_6 to V_0 via V_{14} are defined.

  It follows that only the potential points V_0, V_1, V_6 and V_{12} appear in the resulting symbolic matrix expression.

	Measurements	Switch configuration	Connection configuration
s_2	U_{V_{12},V_0}	g_1	q_0
s_{26}	U_{V_{12},V_0}	g_2	q_0
s_6	R_{V_2,V_0}	g_1, g_2	q_1, q_2
s_7	$R_{V_{11},V_{10}}$	g_1	q_1
s_{11}	$R_{V_{11},V_{13}}$	g_1, g_2	q_2

Fig. 6. Test description

$$\left(\begin{array}{cc|cc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & -1 & 1 & 0 & R_a & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & R_b & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & R_c & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & R_d \\ \hline 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right) \times \left(\begin{array}{c} V_0 \\ V_1 \\ \hline V_6 \\ V_{12} \\ \hline I_0 \\ I_1 \\ I_2 \\ I_3 \end{array}\right) = \left(\begin{array}{c} 0 \\ U_0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \end{array}\right)$$

Fig. 8. Test s_2 and fault f_0

- *Test s_{11} and fault f_0*
Figure 9 represents the symbolic matrix expression of the circuit corresponding to the test s_{11} defined by the measurement $R_{V_{11},V_{13}}$, for the system connection configuration q_2 and the system switch configuration g_1 or g_2, when the system is normal.

According to the branch reductions, the symbolic matrix is simplified by considering the equivalent resistive parameter R_a rather than the serial resistive parameters $R_9 + R_{10}$.

Then, the corresponding branch intensity I_0 from V_{13} to V_{11} is defined.

It follows that only the potential points V_{11} and V_{13} appear in the resulting symbolic matrix expression where U_{tool} represents the voltage source supplied by the measurement tool used to perform an equivalent resistance measurement.

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \hline 1 & -1 & R_a \end{array}\right) \times \left(\begin{array}{c} V_{11} \\ V_{13} \\ \hline I_0 \end{array}\right) = \left(\begin{array}{c} 0 \\ U_{tool} \\ \hline 0 \end{array}\right)$$

Fig. 9. Test s_{11} and fault f_0

- *Test s_2 and fault f_{24}*
Figure 10 represents the symbolic matrix expression of the circuit corresponding to the test s_2 defined by the measurement U_{V_{12},V_0}, for the system connection configuration q_0 and the system switch configuration g_1, with occurrence of the connector fault f_{24}.

According to the branch reductions, the symbolic matrix is simplified by considering the equivalent resistive parameters R_a, R_b, R_c, R_d, R_e and R_f for the serial resistive parameters $R_9 + R_{10}$, R_2, $R_{Connector}$ (i.e., resistive parameter of the short circuit between the pins V_{12} and V_{13}), R_3, $R_1 + R_8$ and $R_4 + R_5 + R_7 + R_{11}$, respectively.

Then, the six corresponding branch intensities I_0 from V_1 to V_{13}, I_1 from V_{13} to V_6, I_2 from V_{13} to V_6, I_3 from V_{13} to V_6, I_4 from V_{13} to V_6, and I_5 from V_6 to V_0 are defined.

It follows that only the potential points V_0, V_1, V_6, V_{12} and V_{13} appear in the resulting symbolic matrix expression.

B. Symbolic expression of the test

Once the symbolic matrix expression is obtained, the prediction algorithm applies a sequence of matrix resolutions in order to obtain the symbolic expression corresponding to the test s_j when fault f_i occurs.

The matrix resolution method used by the method is the Cramer's rule. Actually, Gaussian elimination, LU decomposition or Crout elimination both have an excellent $0(n^3)$ performance. However, problems arise when these are applied to symbolic systems for which the elimination approach is among the worst possible algorithms. Firstly, the result is calculated as a rational, requiring costly symbolic division and GCD (Greatest Common Denominator) operations. Secondly, a large amount of unnecessary calculations are performed for factors which cancel exactly afterwards. This phenomenon is called intermediate expression swell and must be considered the primary cost factor in solving symbolic systems (see [11]). Moreover, the matrix obtained from Ohm's law and Kirchhoff's rules are always sparse; this reduces the large complexity of the

$$
\left(\begin{array}{cc|ccc|cccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & -1 & 0 & 1 & 0 & R_a & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & R_b & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & R_c & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & R_d & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & R_e & 0 \\
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_f \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0
\end{array}\right)
\times
\left(\begin{array}{c}
V_0 \\ V_1 \\ \hline V_6 \\ V_{13} \\ V_{12} \\ \hline I_0 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5
\end{array}\right)
=
\left(\begin{array}{c}
0 \\ U_0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0
\end{array}\right)
$$

Fig. 10. Test s_2 and fault f_{24}

$$s_2 = U_{V_{12},V_0} = \frac{R_c.R_d.U_0}{R_a.R_b + R_a.R_c + R_a.R_d + R_b.R_d + R_c.R_d} \tag{1}$$

$$s_{11} = R_{V_{11},V_{13}} = \frac{U_{tool}}{I_0} = \frac{U_{tool}}{\left(\frac{U_{tool}}{R_a}\right)} = R_a \tag{2}$$

$$s_2 = U_{V_{12},V_0} = \frac{R_e.(R_d.R_f + R_b.R_f + R_c.R_f + R_b.R_d).U_0}{\begin{array}{c} R_e.(R_d.R_f + R_b.R_f + R_c.R_f + R_b.R_d + R_a.R_b + R_b.R_c + R_a.R_d + R_d.R_c)+ \\ R_a.R_b.R_f + R_b.R_c.R_f + R_a.R_d.R_f + R_c.R_d.R_f+ \\ R_a.R_c.R_f + R_a.R_b.R_d + R_b.R_c.R_d + R_a.R_c.R_d \end{array}} \tag{3}$$

Cramer's resolution method. Finally, as it will be seen later, a particular symbolic expression form is expected from this resolution process. This particular symbolic expression form is directly obtained from the Cramer's method whereas the result obtained from an elimination method would require simplification algorithms (which have a large complexity) to be changed into the expected specific symbolic expression form.

B.1. Symbolic matrix resolution: This subsection briefly presents the Cramer's matrix resolution method. As previously seen, let $A \times X = B$ be the matrix system to be solved. Let n be the number of columns of A (A is a square matrix) composed of elements $a_{i,j}$ with $i, j \in \{1, ..., n\}$. Consequently, n represents also the number of components of the vectors X and B, called x_i and b_i, $i \in \{1, ..., n\}$, respectively.

According to the Cramer's matrix resolution method, any component x_i can be computed as shown in Figure 11 where det denotes the determinant operator of a matrix.

$$x_i = \frac{det\begin{pmatrix} a_{1,1} & \cdots & a_{1,i-1} & b_1 & a_{1,i+1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,i-1} & b_n & a_{n,i+1} & \cdots & a_{n,n} \end{pmatrix}}{det\begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix}}$$

Fig. 11. Cramer's matrix resolution method

B.2. Example: For the throttle valve potentiometer example, the three test symbolic expressions corresponding to the three pairs (f_i, s_j) studied in the previous section are given by equation (1), (2), (3), respectively.

V. Global Optimization

In this section, the main results that the symbolic expressions obtained in section IV. have the specific form of multi-variable homographic functions are presented. Their mathematical properties are outlined and an optimization algorithm (see [12] for a theoretical background about optimization methods) is used to find the minimum and the maximum of this type of function on a strictly positive multi-variable domain (each parameter may take only strictly positive values in the actual problem) (see [13] and more recently [14] for a theoretical background about interval arithmetic).

A. Description of the specific symbolic expression

This subsection characterizes the test symbolic expression specific form as multi-variable homographic symbolic functions.

Definition 1 (Form 1 function) *Let X be a set of n variables $\{x_1, ..., x_n\}$. A form 1 function F on X is a sum of m products of variables x_i of degree d_{ij} equal to zero or one.*

$$F(X) = \sum_{i=1}^{m} c_i \times \prod_{j=1}^{n} x_j^{d_{ij}}$$

$$\begin{cases} c_i \in \mathbf{R} & \forall i \in \{1...m\} \\ d_{ij} \in \{0,1\} & \forall i \in \{1...m\} \quad \forall j \in \{1...n\} \end{cases} \tag{4}$$

Definition 2 (Form 2 function) *Let X be a set of n variables $\{x_1, ..., x_n\}$. A form 2 function F on X is a sum of m products of a same number $L \in \mathbf{N}^*$ of variables x_i of degree d_{ij} equal to one. Moreover, the c_i coefficients belong to $\{-1, 0, +1\}$.*

$$F(X) = \sum_{i=1}^{m} c_i \times \prod_{j=1}^{n} x_j^{d_{ij}}$$

$$\begin{cases} c_i \in \{-1,0,+1\} & \forall i \in \{1...m\} \\ d_{ij} \in \{0,1\} & \forall i \in \{1...m\} \quad \forall j \in \{1...n\} \\ \sum_{j=1}^{n} d_{ij} = L & \forall i \in \{1...m\} \quad L \in \mathbf{N}^* \end{cases} \tag{5}$$

Proposition 1 (Form 1 function) *Let Σ be a linear matrix system defined by $A \times X = B$ composed of n linear equations. X is the set of the n variables $\{x_1, ..., x_n\}$ involved in Σ and P the set of the m parameters $\{p_1, ..., p_m\}$ involved in Σ such that one parameter has one and only one occurrence in A. Then, the expression of any variable x_i of the system as a function of the elements of P is a fraction of form 1 functions on P.*

Proposition 2 (Form 2 function) *Let Σ be a linear matrix system defined by $A \times X = B$ composed of n linear equations obtained from the Ohm's laws and the Kirchhoff's laws of a resistive net supplied by one voltage supply. X is the set of the n variables $\{x_1, ..., x_n\}$ involved in Σ and P the set of the m parameters $\{p_1, ..., p_m\}$ involved in Σ such that one parameter has one and only one occurrence in A. Then, the expression of any variable x_i of the system Σ as a function of the elements of P is a fraction of form 2 functions on P.*

The determinant of any matrix A representing a resistive net composed of k nodes and b branches supplied by one voltage source, is a sum of positive or negative terms $\prod_{i=1}^{n} p_i^{d_i}$, $\sum_{i=1}^{n} d_i = L$ and $L = b - k$ among the C_L^b possible ones. The terms which appear in the determinant expression depend on the structure of the resistive net.

B. *Mathematical properties of fractions of form 2 functions*

Since the possible system parameter values are represented by value intervals, this section studies the mathematical properties of fractions of form 2 functions on the parallelotop defined by the intervals corresponding to each of the system parameters.

Theorem 1 (Optimum) *Let $X \in (\mathbf{R}^{+*})^{\mathbf{n}}$ be a n independent components vector $(x_1, ..., x_n)^T$, Δ be a parallelotop defined by*

$$\Delta = \left\{X = (x_1, ..., x_n)^T; 0 < a_i \leq x_i \leq b_i, \forall i = \{1, ..., n\}\right\}$$

$Y \in \mathbf{R}^*$ and F be a fraction of form 2 functions on Δ such that

$$\begin{cases} F : \Delta \longrightarrow \mathbf{R}^* \\ Y = F(X) \end{cases}$$

The minimum Y^- and the maximum Y^+ values of F on Δ are respectively obtained for the vectors X^- and X^+ of Δ such that $X^- = (x_1^-, ..., x_n^-)^T$ and $X^+ = (x_1^+, ..., x_n^+)^T$ where $x_i^s \in \{a_i, b_i\}$ $\forall s \in \{-, +\}$ and $\forall i \in \{1, ..., n\}$.

Proof 1 (Optimum) *On the domain Δ, for each variable x_i such that $i \in \{1, ..., n\}$, the function F restricted to the variable x_i such that the fixed value associated with each variable x_k with $k \neq i$ belongs to $[a_k, b_k]$, can be written as the homographic function shown in equation 6 where $A, B, C, D \in \mathbf{R}$.*

$$F_i(x_i) = \frac{A.x_i + B}{C.x_i + D} \tag{6}$$

Then, the partial derivative of F according to the variable x_i is written as shown in equation 7.

$$\frac{\partial F}{\partial x_i} = F_i'(x_i) = \frac{A.D - B.C}{(C.x_i + D)^2} \tag{7}$$

The sign of the denominator expression $(C.x_i + D)^2$ is always strictly positive and the sign of the numerator $A.D - B.C$ is always constant.

Consequently, the sign of $\frac{\partial F}{\partial x_i}$ is always constant on the domain Δ. So, the maximum value Y^+, respectively the minimum value Y^-, of F is necessarily obtained for a vector X^+, respectively X^-, composed of extreme values a_i or b_i, for each variable x_i.

□

C. *Algorithms*

C.1. Presentation: Starting from a (fault/test) pair (f, s), let F be the formal expression relative to the (f, s) pair, function of n parameters x_i, $i \in \{1, ..., n\}$. Let Δ be the domain defined by the set of $[a_i, b_i]$, $i \in \{1, ..., n\}$ values intervals affected respectively to the x_i variables according to the fault f. Then, the optimization algorithm allows to find the interval values corresponding to the possible results of the test s, knowing that the system is in fault f.

Let F^- and F^+ be the current functions corresponding respectively to the minimum and the maximum of F on the Δ domain. At the beginning of the algorithm, $F^- = F$ and $F^+ = F$. At end of the algorithm, $F^- = Y^-$ and $F^+ = Y^+$. This algorithm has a recursive structure such that, at each iteration of the algorithm, F^+ has to be maximized and F^- has to be minimized.

According to the theorem 1, one can easily imagine a simple algorithm which consists in enumerating all the combinations generated by the 2^n vectors $X = (x_1, ..., x_n)$ such that only extreme values a_i or b_i of the corresponding value interval are affected to each component x_i. Consequently, this $O(2^n)$ complexity is the worst one can expect for this optimization problem.

C.2. Recursive partial derivative study algorithm: By studying the partial derivative, one can easily reduce this combinational method and, so, the algorithm complexity. Actually, knowing that the values belonging to $[a_i, b_i]$ that can take the variables x_i are always strictly positive, it is often possible to find some components x_k such that the partial derivative according to one of these variables has a constant sign whatever the possible values affected to the $n-1$ other variables.
If the partial derivative according to the x_k variable is shown to be always positive (i.e. $\frac{\partial F}{\partial x_k} > 0$) then :

$$\begin{cases} x_k^- = a_k \text{ and } F^- = F(x_1, ..., x_{k-1}, a_k, x_{k+1}, ..., x_n) \\ x_k^+ = b_k \text{ and } F^+ = F(x_1, ..., x_{k-1}, b_k, x_{k+1}, ..., x_n) \end{cases}$$

If the partial derivative according to the x_k variable is shown to be always negative (i.e. $\frac{\partial F}{\partial x_k} < 0$) then :

$$\begin{cases} x_k^- = b_k \text{ and } F^- = F(x_1, ..., x_{k-1}, b_k, x_{k+1}, ..., x_n) \\ x_k^+ = a_k \text{ and } F^+ = F(x_1, ..., x_{k-1}, a_k, x_{k+1}, ..., x_n) \end{cases}$$

While F^- or F^+ are modified during the current iteration the study of partial derivative algorithm continues else it stops.

C.3. Branch and bound-like algorithm: For this algorithm, only the F^+ maximization problem is studied, considering that the F^- minimization treatment is similar.

The main idea of this algorithm is to build a binary tree where the root is F^+.

The branch and bound-like algorithm consists in making two assumptions at each node of the binary tree allowing to obtain the two children of this node. These two assumptions are $x_k = a_k$ and $x_k = b_k$ such that x_k is one of the not already evaluated components. A node is said to be non consistent if one of the assumptions $x_k = a_k$ or $x_k = b_k$ that have been made on the path from the root to this node is not consistent with the evaluated gradient value for the x_k component respectively $\frac{\partial F^+}{\partial x_k} > 0$ or $\frac{\partial F^+}{\partial x_k} < 0$.

If a node is proven non consistent, it is pruned. If a node is consistent and if it remains components that have not been evaluated, then, the signs of each gradient components are evaluated according to the previous recursive partial derivative study algorithm.

This branch and bound-like algorithm allows to obtain all the local maxima of F on the Δ domain. Then, by comparing these local maximum values during the previous algorithm, it is easy to deduce the Y^+ value and its relative X^+ vector corresponding to the global maximum of F on the Δ domain.

	s_1 (V)	s_{21} (Ω)
f_6	$[4.9, 5.1]$	∞
f_{19}	$[4.8997966, 5.0999774]$	$[1660, 2740]$

Fig. 12. Prediction table

C.4. Conclusion: Let n^+, respectively n^-, be the current numbers of variables which are not already evaluated in F^+ maximization, respectively F^- minimization, problem. At the beginning, n^+, respectively n^-, is initialized at n. First, the recursive partial derivative study algorithm is applied on F^+, respectively F^-. If $n^+ \neq 0$, respectively $n^- \neq 0$, the branch and bound-like algorithm is executed.

It has been proven that the average complexity of the branch and bound-like algorithm on $n \in \mathbf{N}^{+*}$ variables is always lower than $O(2^n)$ (see [15]).

For the throttle valve potentiometer example, a sample of the resulting prediction table is given in figure 12 for the 4 (fault/test) pairs previously proposed.

VI. Conclusion

This paper presents a prediction method whose input consists of a set of behavioral models, nominal as well as faulty, of the electric circuit to diagnose and a topological model of the mechanical system in which the electric circuit is embedded. The system behavioral model is directly built from first principles and cabling diagrams supplied by the automotive manufacturer and a library of basic linear electric components. The parameter ranges in nominal and faulty conditions are represented by interval values, which calls for a dedicated optimization method. Actually, only electric circuits that are equivalent to resistive networks supplied by one voltage source are considered in this work. The topological model is built from the assembly tree of the concerned mechanical system assuming that the way to disassemble this mechanical system is the opposite of the way to assemble it.

From the system behavioral model, a set of possible tests and a set of available faults are generated (see section III.). In the set of faults, only pure single faults (PSF) are considered. In the set of available tests, potential and equivalent resistance measurement tests and perceptible tests (i.e., light or noise) are considered.

This work proves that the symbolic expression of any of the considered tests in the occurrence of any of the considered faults is a function of the parameters of the system of a specific form (see section IV.). The uncertainty of the parameter values being expressed as intervals, an optimization process for this specific symbolic expression form is proposed. It is then possible to predict the exact resulting interval value of a given test when a given fault occurs (see section V.). The obtained predictions summarized in a "cross-table" are the input of an algorithm to determine the optimal diagnosis tree [2] [15].

To perform on more complex electric circuit, the proposed method has to be able to take into account more than one voltage supply and non linear electric components. At the moment, non linear components are approximated by piecewise linear models, which is not always satisfactory. The consideration of dynamic components is also an issue for further work. Basic concepts and ideas can be taken from [16].

Another important issue is the automatic test anticipation step. The current procedure does not succeed in producing a reasonable sized test set, which impacts significantly on the algorithm complexity. Ideas from system diagnosability analysis [17] could be a basis for an improved method.

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VII. Appendix

Proof of Proposition 1 [Form 1 function] Considering the Cramer's matrix resolution method (see Figure 11) to solve the system Σ according to the x_i variable, if each of the m parameters implied in S has one and only one occurrence in Σ. Then, the determinant of any square sub-matrix only includes $c_i \times \prod_{j=1}^{n} p_j^{d_{ij}}$ terms, $c_i \in \mathbf{R}$ and $d_{ij} \in \{0, 1\}$. Consequently, the expression of any variable x_i of the system Σ as a function of the elements of P is a fraction of form 1 functions on P.

□

Proof of Proposition 2 [Form 2 function] In a Ohm's laws equation, potential identifiers are associated with to -1 or $+1$ coefficients whereas the intensity identifier is associated with a parameter identifier. In Kirchhoff's laws equation, intensity identifiers are also associated with a -1 or $+1$ coefficients. Consequently, the matrix representation of the system Σ is only constituted by -1, 0 or $+1$ coefficients and the parameter identifiers.

Let us consider the block decomposition of the matrix A proposed in figure 7 defined by k nodes (i.e., potential points that have more than two neighbors) and b branches (i.e., serial resistors from one node to another). For each branch, one intensity and one resistive parameter are characterized. The branch resistive parameters, called p'_i, $i \in \{1, ..., b\}$, are equivalent to sums of resistive parameters from the set $\{p_1, ..., p_m\}$. Actually, each resistive parameter occurs just once in the symbolic expression of only one branch resistive parameter.

Then, as shown in Figure 13, the sub-matrices $A_{1,2}$, $A_{1,3}$, $A_{3,1}$ and $A_{3,2}$ are null sub-matrices and $A_{1,1}$ is a (2×2) identity matrix.

The sub-matrices $A_{2,1}$ and $A_{2,2}$ correspond to Ohm's laws equations and are such that there are only one -1 and one $+1$ on each of their lines.

In the same way, $A_{3,3}$ corresponds to the Kirchhoff's laws equations and is such that there are at most one -1 and one $+1$ on each of its columns. Actually, each branch intensity starts from and arrives at one node, the supply pin or the ground pin.

The structure of the electric circuit is described in the Ohm's law equations by the sub-matrix $A_{2,2}$ and in the Kirchhoff's law equations by the sub-matrix $A_{3,3}$. Then, in our resistive net case, it can be observed that $A_{2,2}^T = A_{3,3}$.

Now, let us consider the computation of the determinant of this matrix according to its k last lines corresponding to the Kirchhoff's law equations. First of all, the determinant of the matrix A is equivalent to the determinant of the sub-matrix A^0 composed of the blocks $A_{2,2}$, $A_{2,3}$, $A_{3,2}$ and $A_{3,3}$.

In Figure 14, this determinant is computed according to the non null coefficients (i.e., -1 or $+1$) of each line of the sub-matrix $A_{3,3}$ since the sub-matrix $A_{3,2}$ is null.

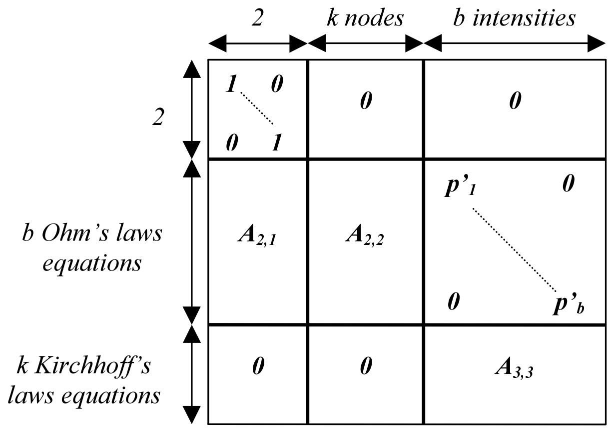

Fig. 13. General structure of the matrix A

The determinant of a matrix form A^{i+1} corresponding to the recursive step i of the determinant computation, is equivalent to a sum of determinant of matrix form A^i associated with -1 or $+1$ coefficients. Whatever the step i, in A^i, the block $A_{2,2}$ keeps the same and is equivalent to the concatenation of the blocks A_1^i and A_2^i.

Consequently, the determinant of A^0 is equivalent to a sum of determinant of matrix form A^k associated, -1 or $+1$ coefficients. The determinant of the matrix form A^k is a product of $L = b - k$ parameters multiplied by the determinant of the square $(k \times k)$ block A_2^k.

Any block A_2^k is only composed of coefficient -1, 0 or $+1$. Moreover, in each of its line, there are at most one -1 and one $+1$, the other coefficients of the line being null (i.e., these are the properties of an incidence matrix).

Let us prove, by recurrence according to the size k of the matrix A_2^k, the following proposition : $\forall k \in \mathbf{N}^*$, $det(A_2^k)$ is -1, 0 or $+1$. In the literature, this type of matrix is called totally unimodular matrix.

It is evident for $k = 1$ since the only term of the matrix A_2^1 is -1, 0 or $+1$. Now, for any $k \in \mathbf{N}^*$, if the determinant calculus is developed according to one of the lines of matrix A_2^1 then one of the following three alternatives occurs.

- If all the terms of one line are null, then, $det(A_2^k) = 0$
- If only one of the terms of one line is non-null (i.e., -1 or $+1$), then, $det(A_2^k) = \pm det(A_2^{k-1})$
- If all the lines have two non-null coefficients (i.e., -1 and $+1$), then the sum of the terms of each line is null and hence, $det(A_2^k) = 0$.

According to the Cramer's resolution method (see Figure 11), the denominator of the fraction is obtained directly from the determinant calculus of the matrix A whereas the numerator is obtained from the A matrix in which the j^{th} column corresponding to the variable to evaluate is replaced by the second member vector

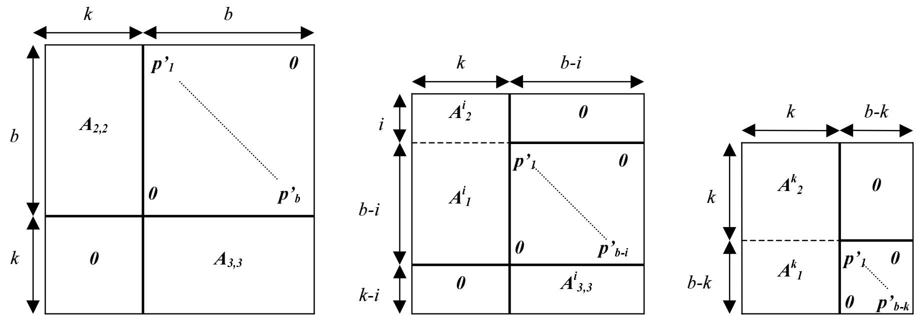

Fig. 14. Computation of the determinant of the matrix A

B. If U_0 represents the electromotive power parameter of the voltage supply, then the numerator has the form $(-1)^j \times U_0 \times det(A_j)$ where A_j is the A matrix without the 2^{nd} line and the j^{th} column. The previous proof holds for matrix $A^{'}$ as well as for the matrix A.

However, the L values of the denominator L_{den} and the numerator L_{num} can be different. Actually, in the case where the variable to evaluate is an intensity, the corresponding column among the sub-matrix composed of $A_{1,3}$, $A_{2,3}$ and $A_{3,3}$ disappears and $L_{num} = L_{den} - 1$.

□