DECISION APPROACH FOR WORKLOAD DISTRIBUTION

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ABSTRACT

This paper deals with decisions of workload temporal distribution in scheduling discrete and diversified productions.

A new way of formulating the scheduling problem is proposed, from which some concepts and tools are presented:

The notion of Time Resource Interval objects, TRIs, allows the management of some technical (time and resource) aspects at the different levels of a hierarchical structuration of the set of decisions taken in the workshop, from "load distribution" ones, to "effective realization of the operations" others.

Constraint-based reasoning handles different TRIs corresponding to a given kind of decisions. It helps to highlight the bounds or limits to be respected while deciding, to remain consistent according to an initial set of constraints, issued for example from an upper level of decisions.

Decisions of load temporal distribution consist in readjustments of some time constraints on a set of planned operations, by taking into account the (or some more detailed) constraints on the resource(s) on which they have been planned, as finite capacity one, and/or minimal profitability.

The analysis on temporal proximities of the planned operations, involves some particular structurations of the time axis into successive time intervals: those structurations are associated with sets of temporal bounds, and are called adjacent decompositions of time axis.

Such a decomposition introduces some specific TRIs, associated with load constraints (coming from the planned operations), and resource constraints (coming from limited quantities of resource, or profitability concerns). By respecting the given time and resource constraints, they can "exchange" some quantities of load according to communicating vessels processes.

Those phenomena have been modeled as bounded flows in a temporal network, and offer new flexible curves of load with finite capacities, to help the decision.

KEYWORDS: Workload distribution, decision aid, constraint-based reasoning, adjacent decompositions of time intervals, capacity.
I INTRODUCTION

In manufacturing workshops, decisions at different levels consist, in part, in determining two types of characteristics for each task, or operation, for every product to be manufactured:

- allocation in time scale, taking into account the time utilization constraints
- allocation on one (or several) resource(s), taking into account the resource utilization constraints.

Thus, the decisions result from constraints-oriented reasonings, where the constraints are related to time and resource bounds.

This work is based on a decomposition of the system of decisions into different centers of decision, characterized by a specific level of aggregation (or level of details) of the different concepts of time, resource and work, manipulated while reasoning and deciding. More particularly, it concerns the decisions of workload distribution, consisting in insuring consistency between some constraints of workload which can be considered as aggregation of time and resource consumption constraints associated with a set of planned operations, and some aggregate constraints of resource capacity and profitability.

From theoretical point of view, works have essentially pointed out problems of scheduling, concerning temporal distribution and allocation on the resources of discrete operations. Such decisions take into account relatively detailed constraints, associated with "time-phased representations of the workload" and instantaneous availability of the resources.

Few approaches consider the importance of more aggregate constraints of type "non-time-phased workload" and resource capacity to evaluate and predict the workshop behaviour (Bertrand 1983, Bertrand et al. 1990), without being confronted to the high complexity of more detailed approaches.

Nevertheless, from a practical point of view, some studies of work psychology (Valax et al. 1988), have revealed real needs of tools which aid such types of aggregate decisions, generally made by the workshop supervisor, and that can be considered as intermediate steps between planning and scheduling. In existing softwares, such a load distribution is generally achieved by using load curves which show the workload level evolution along the time (Chassang and Tron 1983). The fact that the operations are not accurately time-located is taken into account by considering either earliest or latest load curves. But the decider is confronted to the high difficulty of making aggregate decisions with the aid of non-time-phased representations of the workload which do not offer him the possibility of directly reasoning on the aggregate flexibility, as free load one, or free capacity one. He often has to use a detailed time-phased information on discrete operations proposed in load curves or in a calculated scheduling to exploit the time and resource scopes.
In order to overcome that difficulty, new loads curves are proposed which formally show *workload flexibility* coming from time flexibility on the operations. The way those curves can be modified by exploiting the workload flexibility is presented: such modifications can be used to help the workload readjustments, which take into account some capacity and profitability resource constraints.

The concept of *Time Resource Interval*, TRI, introduced in Section II, allows the manipulation of both time and resource aspects, and is quite suitable to a hierarchical approach of the set of decisions to be taken in the workshop. This approach tends to conciliate a global consistency according to the objectives, and the necessary autonomy for the different decision centers.

After the description of particular TRIs used in this work, the *neighbourhood property* is defined in Section III, which introduces some *adjacent decompositions* of the time horizon associated with a given temporal distribution of the planned operations: the elements of such decompositions can be considered as "communicating vessels".

Section IV shows how the comparison between different aggregate characteristics of load and resource of the communicating vessels may involve the necessity, or only the possibility of some decisions of load/capacity readjustments, which have been modeled as flows circulating in a network, and are presented as levels to be decided in the vessels.

In Section V, one example is described.

II PROBLEM STATEMENT

II-1 Definition of a Time Resource Interval (Erschler et al. 1991)

A TRI J is defined by the following characteristics, more or less well known, to be calculated, or decided, depending on the kind of problem to be studied:

-**Instantaneous characteristics:**
  * time bounds: CJ, FJ (starting, finishing times)
  * resource intensity functions: defined for any instant t of the temporal interval [CJ,FJ] and any resource k associated with TRI J. In the case where those functions are constant in time, J is said to be *uniform*.

-**Integral characteristics:**
  * duration DJ=FJ-CJ
  * integration over time of resource intensity:
- the resource **capacity** or maximal amount of resource available on a given time interval, and the resource **profitability**, or minimal amount of resource to be used during a given time interval;
- the resource consumption, or **load** (/workload), is the amount of resource required for the realization of an operation, or a set of operations.

Such integral characteristics are expressed in time \( \times \) resource units (for example: hour \( \times \) machine).

### II-2 Particular TRIs of the problem

The studied decisions concern a set of planned operations, constrained by earliest starting and latest finishing times, sharing some common resource. In order to simplify the presentation, the planned operations will be supposed to share a single type of resource (**eg**, a set of given type of machines); the different TRIs are supposed to be uniform, and intensities to be equal to one: those assumptions will allow a simplification of the notations and graphics, but generalizations can easily be done.

This work takes place in a hierarchical approach, which supposes a structuration of the different decisions of the production system, in successive levels; at each level, decisions are made and transmitted in terms of objectives to be followed, or constraints to be respected by the lower levels. At a given level, there is a manipulation of specific kinds of TRIs, that are considered as "suppliers" (of time and resource) for their "consumers" TRIs manipulated by a lower level.

**-The tasks:** they are TRIs associated with effective realization of the operations.

* \( t_i \) = effective starting time of operation \( i \).
* \( D_i \) = processing time, or duration of operation \( i \).
* resource consumption, or load is the quantity of resource allocated to the realization of operation \( i \). As a result, the load associated with operation \( i \) is equal to \( D_i \).

**- The windows:** they are TRIs associated with the planned operations.

* \( C_i \) = earliest starting time of operation \( i \).
* \( F_i \) = latest finishing time of operation \( i \).

Those temporal characteristics are supposed to have been specified by the upper level of decisions (most often the planning service). They are considered as **constraints**, or necessary conditions to be respected by any consistent decision made in the workshop.

* resource consumption is the same as the load associated with the corresponding task \( i \).
Due to the upper assumptions, window i and task i can be drawn as blocks, such as any "time and resource consumer" task i must remain in its corresponding "time and resource supplier" window i.

III ADJACENT DECOMPOSITIONS OF THE TIME HORIZON

The complexity in scheduling, or reasoning on a set of planned operations, can be decreased by decomposition approaches (Chambers et al. 1991, Chu et al. 1992). In the case of temporal decomposition, the attempt consists in displaying time intervals associated with subsets of planned operations that "belong" to a common time area (Portmann 1987).

But most often, difficulty lies on the closed temporal overlapping of the planned operations; it is not possible to find "independent" time intervals from which could be elaborated classes of "neighbours in time operations", without any overlapping between elements of two different classes: the global problem cannot be considered as the conjunction of independent sub-problems on successive time intervals.

Nevertheless, as it is shown below, it is always possible to generate decompositions of the time horizon such as the overlapping phenomena occur at the most between two successive time intervals, minimizing in that way the couplings and simplifying the "recomposition" problem from the decomposed sub-problems.

Such structurations decompose the time horizon into successive time intervals $[T_j, T_{j+1}]$, called $T_j$. $T_{j+1}$ vessels, which possess the following fundamental neighbourhood property:

\[ \forall \ i, \exists \ j \text{ such as } T_{j-1} \leq C_i < F_i \leq T_{j+1} \]

That means that any window i does not overlap more than two successive time intervals of the structuration: and so, any sliding of a task i inside its window i is done between at the most two successive time intervals.

This definition is generic, and the work presented in this paper does not suppose any a priori on the elaboration of such decompositions. But the following algorithm proposes an example of elaboration from the choice of a temporal bound $T_0$: at each step n, the class "context$_{n-1}$" of neighbour planned operations is released, which conditions the choice of a pair of temporal boundaries $(T_{-n}, T_n)$. The set $\{T_{-n}, T_{-(n-1)}, \ldots, T_{-1}, T_0, T_1, \ldots, T_n\}$ involves an adjacent decomposition of the time horizon in successive vessels $T_j, T_{j+1}$.

Step 1:
1) context\(_0(T_0)\) is the set of windows \(i\) such as \(C_i < T_0 < F_i\) (first class of "neighbours operations")

2) choice of a temporal bound \(T_1\) such as \(T_1 \geq \text{MAX}\{F_i\}\) and a temporal bound \(T_{-1}\) such as \(T_{-1} \leq \text{MIN}\{C_i\}\) for any window \(i\) belonging to context\(_0(T_0)\).

**Step n:**
1) context\(_{n-1}(T_0)\) is the set of windows \(i\) such as \(T_{-(n-1)} < F_i \leq T_{-(n-2)}\) or such as \(T_{n-2} \leq C_i < T_{n-1}\)

2) choice of \(T_n\) such as \(T_n \geq \text{MAX}\{F_i\}\) and \(T_{-n}\) such as \(T_{-n} \leq \text{MIN}\{C_i\}\) for any window \(i\) belonging to context\(_{n-1}(T_0)\).

An example is proposed in Figure 1:

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**Properties**

1 - An updating means a decrease in the width of the window. Any consistent (ie, respecting the initial time constraints \(C_i\) and \(F_i\)) updating of a date \(C_i\) (increasing) or \(F_i\) (decreasing), does not imperil the neighbourhood property of an adjacent decomposition of the time horizon: once narrowed, a window \(i\) continues to overlap at the most two successive vessels. The temporal structuring is stable according to consistent windows updatings.

2 - Given a set \(C^* = \{T_0, ..., T_j, T_{j+1}, ...\}\) associated with an adjacent decomposition of the time horizon, any subset \(C\) of \(C^*\) is also associated with such a type of structuring (ie, the neighbourhood property is maintained). The time intervals issued from \(C\) are equal to intervals issued from \(C^*\), or aggregations of them: therefore, the decomposition associated with \(C^*\) can be considered as a disaggregation of the one associated with \(C\).

3 - The previous property involves the definition of an ordering relation, \(<\), between sets of temporal boundaries associated with the different adjacent decompositions of a given problem:

\[ C^* < C \iff C \text{ is included in } C^*. \]
The previous relation introduces a partial ordering in the set of adjacent time decompositions, in which the smallest elements are the most disaggregated, or detailed ones.

In the previous example, it can easily be shown that the adjacent decomposition obtained from the choice of an initial $T_0$, with, at any step $n$:

$$T_n = \text{MAX}\{F_i\}$$
$$T_{-n} = \text{MIN}\{C_i\}$$

for any window $i$ belonging to context$_{n-1}(T_0)$, is a smallest element.

**Remark**

The effectiveness of such adjacent decompositions of the time horizon is strongly dependent on the "width" of the windows of the operations. If for example a planned operation exists with a window which ranges from zero to the end of the planning horizon $T_N$ it is not possible to distinguish more than one time interval $[0,T_N]$. Thus, this paper is based on the assumption that the width of the windows is restricted to some value which is much smaller than the planning horizon, supposing a quite homogeneity between the different processing times $D_i$, and the time slacks allocated by the planning.

If it occurs that one or more windows, associated for example with fabrications of prototypes, is much wider than the mean one, then a possibility consists in taking such windows out of the problem for a while, and making a decision of workload distribution with the remaining planned operations, which lets enough freedom of capacity to add the load corresponding to the previous and momentarily forgotten windows.

**IV TEMPORAL DISTRIBUTION OF LOAD**

**IV-1 Constraints associated with the $T_j,T_{j+1}$ vessels**

The previous $T_j,T_{j+1}$ vessels can be considered as TRIs, with some local resource constraints: for example, a limited set of machines, or a minimal quantity of work to be insured for the workers teams, ... Moreover, the load quantities induced on these TRIs by the set of planned operations, can be considered as external constraints, issued from an upper level of decisions (most often the planning service).

**- local constraints on resource availabilities:**

* $Q(T_j,T_{j+1})$ = maximal amount of resource, available between $T_j$ and $T_{j+1}$. It corresponds to traditional "capacity".
* \( R(T_j, T_{j+1}) \) = minimal amount of resource to be used during \([T_j, T_{j+1}]\). It may correspond to the notion of "profitability".

- **load, or resource consumption characteristics**: they are the quantities of resource required for the realization of the planned operations.

* **Maximal load** = maximal amount of resource consumption of all "consumer" windows \( i \) during \([T_j, T_{j+1}]\). It has been shown (Thuriot and Valax 1989) that:

\[
W_{\text{MAX}}(T_j, T_{j+1}) = \sum_i (\max(0, \min(F_i - T_j, D_i, T_{j+1} - C_i))).
\]

It is obtained by distributing all tasks \( i \) the most inside \([T_j, T_{j+1}]\), as shown in Figure 2:

------------- INSERT HERE FIGURE 2 -------------

* **Minimal load** = compulsory or minimal amount of resource consumption during \([T_j, T_{j+1}]\). It has been shown (Erschler et al. 1991) that:

\[
W_{\text{MIN}}(T_j, T_{j+1}) = \sum_i (\max(0, \min(C_i + D_i - T_j, T_{j+1} - F_i + D_i, D_i))).
\]

It is obtained by distributing tasks \( i \) the most outside of \([T_j, T_{j+1}]\), as shown in Figure 2':

------------- INSERT HERE FIGURE 2' -------------

**IV-2 Decisions**

The decisions studied here concern readjustments of the different \( W_{\text{MAX}} \) and/or \( W_{\text{MIN}} \), necessary to become compatible with the different \( Q \) and/or \( R \). It is assumed that the upper level of decisions has been robust enough, and that there is no case where \( W_{\text{MAX}} < R \), nor \( W_{\text{MIN}} > Q \).

But some decisions remain necessary when \( W_{\text{MAX}} > Q \), and/or \( W_{\text{MIN}} < R \).

In such cases, decreasings of \( W_{\text{MAX}} \), and/or increasings of \( W_{\text{MIN}} \), have to be done. Those decisions on load quantities could then be disaggregated through \( C_i \) increasings and \( F_i \) decreasings; these disaggregation problems are not presented here, but a solution is proposed for the example in Section V. Afterwards, we introduce the notions of **Rejected** (WR) and **Entered** load quantities (WE).

* **Overload risk**
When $WMAX(T_j, T_j+1) > Q(T_j, T_j+1)$, one has to decide a decreasing of $WMAX(T_j, T_j+1)$, by an amount $WR(T_j, T_j+1) = WMAX(T_j, T_j+1) - Q(T_j, T_j+1)$.

This can be done by disaggregated decisions of some $Ci$ increasings, and/or some $Fi$ decreasings, as shown in Figure 3:

* $WRL$, as "Rejected to the Left", is the load quantity corresponding to the decreasing of the latest finishing times of some windows whose $[Ci, Fi]$ overlaps $[T_j-1, T_j]$ and $[T_j, T_j+1]$.

* $WRR$, as "Rejected to the Right", is the load quantity corresponding to the increasing of the earliest starting times of some windows whose $[Ci, Fi]$ overlaps $[T_j, T_j+1]$ and $[T_j+1, T_j+2]$.

Because of the neighbourhood property of adjacent decompositions, this load amount $WR$ is "discharged" on the adjacent vessels $T_j-1, T_j$ and $T_j+1, T_j+2$. This phenomenon can be seen as a communicating vessels process, as it is presented in Figure 4:

* Underload risk

When $WMIN(T_j, T_j+1) < R(T_j, T_j+1)$, one has to decide an increasing of $WMIN(T_j, T_j+1)$, by an amount $WE(T_j, T_j+1) = WER(T_j, T_j+1) + WEL(T_j, T_j+1)$.

* $WEL$, as "Entered by the Left", is the load quantity corresponding to the increasing of the earliest starting times of some windows whose $[Ci, Fi]$ overlaps $[T_j-1, T_j]$ and $[T_j, T_j+1]$.

* $WER$, as "Entered by the Right", is the load quantity corresponding to the decreasing of the latest finishing times of some windows whose $[Ci, Fi]$ overlaps $[T_j, T_j+1]$ and $[T_j+1, T_j+2]$.
The neighbourhood property of adjacent decompositions involves a discharging of the adjacent vessels \( T_{j-1}.T_j \) and \( T_{j+1}.T_{j+2} \), as it is presented in Figure 5:

\[ \text{------------------- INSERT HERE FIGURE 5 -------------------} \]

That leads to introduce new kinds of load curves, that give at the same time, or on the same screen, informations about minimal and maximal loads or resource consumptions, and capacity and profitability curves, associated with a given adjacent decomposition of the time horizon. This is shown in Figure 6:

\[ \text{------------------- INSERT HERE FIGURE 6 -------------------} \]

Grey surfaces are delimited by levels \( \text{WMAX}(T_j,T_{j+1})/T_{j+1}-T_j \) and \( \text{WMIN}(T_j,T_{j+1})/T_{j+1}-T_j \).

Thick lines represent levels in \( Q(T_j,T_{j+1})/T_{j+1}-T_j \) and \( R(T_j,T_{j+1})/T_{j+1}-T_j \).

* The grey surfaces represent the quantity of load which is the difference between \( \text{WMAX} \) and \( \text{WMIN} \), ie, the spaces of admissible quantities of load, when taking into account only the temporal constraints on the windows, or planned operations.

* Grey surface over thick line "Q" means \( \text{WMAX} > Q \): there is an overload risk. In this case, \( \text{WMAX} \) will have to be decreased.
   \( \text{WMAX} < Q \) means a slack of capacity or free capacity, equal to the surface delimited by the concerned interval \([T_j,T_{j+1}]\), and its levels in \( \text{WMAX} \) and \( Q \). For example, this slack could be used for the realization of some additional load during \([T_j,T_{j+1}]\); or else, it may signify the possibility to destinate a part of the resource to maintenance: both of such eventualities could be adopted without imperilling the respect of the capacity constraint.

* Grey surface under thick line "R" means \( \text{WMIN} < R \): there is an underload risk. In this case, \( \text{WMIN} \) will have to be increased.

In the case when \( \text{WMIN} > R \), the quantity \((\text{WMIN}-R)\) can be considered as a slack of load or free load, equal to the surface delimited by the concerned \([T_j,T_{j+1}]\), and its levels in \( \text{WMIN} \) and \( R \). This slack could be used for the making profitable of some additional resource during \([T_j,T_{j+1}]\); or else, it may signify the possibility to take a part of load out of the production: both of such eventualities could be adopted without imperilling the respect of the profitability constraint.
In fact, a white surface within the surface delimited by thick lines "Q" and "R" points out some "external flexibility", related to the given initial problem: possibility to add some more load or to take out of the production a part of the resource in the case of slack of capacity, and to keep busy some more resource, or to take out of the production a part of the load in the case of slack of load.

* When WMAX and WMIN, once updated, become consistent according to Q and R, their difference represents the "internal flexibility", meaning the level of freedom on the amount of work to be done on the planned operations during the concerned \([T_j, T_{j+1}]\).

### IV-3 Model

The variables of decision

\[
\begin{align*}
WE(T_j, T_{j+1}) &= WEL(T_j, T_{j+1}) + WER(T_j, T_{j+1}) \\
WR(T_j, T_{j+1}) &= WRL(T_j, T_{j+1}) + WRR(T_j, T_{j+1}),
\end{align*}
\]

can be considered as bounded flows, circulating in a temporal network (Ford and Fulkerson 1962), as represented in Figure 7:

----------------- INSERT HERE FIGURE 7 -----------------

The necessary conditions for the consistency of the decisions according to the planned dates \(C_i\) and \(F_i\), and to the local constraints of resource capacity and profitability, are insured by the respect of maximal and minimal bounds of the different arcs of the network:

- **arcs supporting flows of type \(WR(T_j, T_{j+1})\):**
  * lower bound \(= \text{SUP}(0, \text{WMAX}(T_j, T_{j+1}) - Q(T_j, T_{j+1}))\)
  * upper bound \(= \text{WMAX}(T_j, T_{j+1}) - \text{SUP}(\text{WMIN}(T_j, T_{j+1}), R(T_j, T_{j+1}))\)

- **arcs supporting flows of type \(WE(T_j, T_{j+1})\):**
  * lower bound \(= \text{SUP}(0, R(T_j, T_{j+1}) - \text{WMIN}(T_j, T_{j+1}))\)
  * upper bound \(= \text{INF}(\text{WMAX}(T_j, T_{j+1}), Q(T_j, T_{j+1})) - \text{WMIN}(T_j, T_{j+1})\)

- **flows of type \(\text{WMIN}(T_j, T_{j+1})\) and \(\text{WMAX}(T_j, T_{j+1})\)** are fixed to their given values calculated as seen in IV-1.

- **the existence of arcs supporting flows of type \(W(T_j, T_{j+1})\)** with minimal bound equal to 0 and infinite maximal bound, expresses the feasibility and the level of robustness of aggregate decisions of load
"readjustments", relatively to disaggregated decisions of dates updatings: once decided, $W(T_j, T_{j+1})$ represents the allowed variation of the load on $T_j$. $T_{j+1}$ vessel, _ie_, the internal flexibility presented in IV.2.

### IV-4 Remark

Let us note:

- $W_{LEFT}(T_j, T_{j+1}) = W_{EL}(T_j, T_{j+1}) + W_{RL}(T_j, T_{j+1})$: load quantity moved by the "right hand" of $[T_j, T_{j+1}]$.
- $W_{RIGHT}(T_j, T_{j+1}) = W_{ER}(T_j, T_{j+1}) + W_{RR}(T_j, T_{j+1})$: load quantity moved by the "left hand" of $[T_j, T_{j+1}]$.

It can be shown that:

- $W_{LEFTMAX}(T_j, T_{j+1}) = W_{RIGHTMAX}(T_{j-1}, T_j)$
- $W_{LEFTMAX}(T_j, T_{j+1}) + W_{RIGHTMAX}(T_j, T_{j+1}) = W_{MAX}(T_j, T_{j+1}) - W_{MIN}(T_j, T_{j+1})$

When associated with the "communicating vessel" processes seen in IV-2, those properties allow a graphical representation of the considered decisions, as shown in Figure 8:

--- INSERT HERE FIGURE 8 ---

* The vertical line in grey surface ($W_{MAX}(T_j, T_{j+1}) - W_{MIN}(T_j, T_{j+1})$) delimits the two sub-surfaces $W_{LEFTMAX}(T_j, T_{j+1})$ and $W_{RIGHTMAX}(T_j, T_{j+1})$: it can be considered as a constraint to be respected when deciding the load quantities to be moved, issued from the temporal distribution of the windows.

* The decisions will consist in choosing sub-surfaces: $W_{EL}$ and $W_{RL}$ of $W_{LEFTMAX}$, and $W_{ER}$ and $W_{RR}$ of $W_{RIGHTMAX}$, in order to decrease $W_{MAX}$ and to increase $W_{MIN}$. The communicating vessel process is visualized by a moving of the chosen sub-surfaces into adjacent vessels, and a resulting changing of their levels of load.

* The graphical constraint consisting in specifying sub-surfaces $W_{EL}$ and $W_{RL}$ (respectively $W_{ER}$ and $W_{RR}$) such as $W_{EL} + W_{RL} \leq W_{LEFTMAX}$ (respectively $W_{ER} + W_{RR} \leq W_{RIGHTMAX}$), insures the respecting of minimal bound equal to 0 for the arc supporting $W(T_j, T_{j+1})$, _ie_, the feasibility of the decisions, relatively to times $(C_i, F_i)$ updatings.

### V Example

Let us consider the example of 17 planned operations to be realized by a given resource, and note $i(C_i, F_i, D_i)$ each planned operation $i$; $C_i$ is the earliest starting time, $F_i$ the latest finishing time, and $D_i$ the processing time of operation $i$: 
A(0,6,4), B(3,9,4), C(0,6,4), D(4,8,2), E(2,7,2), F(2,4,2), G(6,14,6), H(8,14,4) I(16,22,4), J(14,20,4), K(16,24,4), L(14,20,4), M(22,28,2), N(24,28,2), O(30,36,2), P(30,36,4), Q(28,32,2).

The curves of local constraints on the resource capacity and profitability are presented in Figure 9, on which are proposed as well the informations on the temporal characteristics of the planned operations.

R is supposed to be constant, equal to eight hours \( \times \) resource per periode of eight hours. Q is supposed to be periodic, successively equal to twenty and sixteen hours \( \times \) resource per periode of eight hours.

----------------- INSERT HERE FIGURE 9 -----------------

One can easily show that the set \{C1,C2,C3,C4\} is associated with an adjacent decomposition of the time horizon: the neighbourhood property is verified.

The comparison between curves of resource capacity and profitability, and curves of load is presented in Figure 10:

----------------- INSERT HERE FIGURE 10 -----------------

The grey surfaces represent the quantity of load which is the difference between WMAX and WMIN, ie, the admissible variations of quantities of load, when taking into account the temporal constraints on the planned operations.

It appears that there exist:
- an overload risk on temporal intervals \([2,10]\) and \([10,18]\)
- an underload risk on temporal intervals \([18,26]\) and \([26,34]\).

According to the graphical possibilities seen in IV (Figure 8), an example of decisions of load updatings, in terms of "surfaces exchangings", is proposed in Figure 11, in which the numbered surfaces are the decided ones, and arrowed surfaces represent the consequences of the decisions (ie, the moved surfaces); the numbers correspond to the numerical values of the moved surfaces, in hour \( \times \) resource units; the vertical line between Tj and Tj+1 delimits WLEFTMAX(Tj,Tj+1) and WRIGHTMAX(Tj,Tj+1), and constrains in that way the choices of the different surfaces.
Finally, the updated curves of load are presented in Figure 12, in which the grey surfaces represent the internal flexibility on the load, remaining after the decisions of load distribution (WMAX decreasings and WMIN increasings), \textit{ie}, when taking into account the temporal constraints on the planned operations, \textit{and the constraints on resource capacity and profitability}.

\textbf{Remark:} As seen in IV-2 (Figure 3), those aggregated decisions of load quantities updatings, have to be disaggregated by \textit{Ci} increasings, and/or \textit{Fi} decreasings.

For example,
* \textit{WRL}(10,18) = 2 (=\textit{WER}(2,10)), can be disaggregated by:
  a decreasing of \textit{FG} of one hour: \textit{FG}=13
  and a decreasing of \textit{FH} of one hour: \textit{FH}=13.

  * \textit{WEL}(10,18) = 2 (=\textit{WRR}(2,10)), can be disaggregated by:
    an increasing of \textit{CG} of one hour: \textit{CG}=7
    and an increasing of \textit{CH} of one hour: \textit{CH}=9.

But other possibilities could lead to \textit{G}(6,12,6) and \textit{H}(10,14,4)
or else to \textit{G}(8,14,6) and \textit{H}(8,12,4).

Such disaggregated decisions of windows narrowings remain to be studied. As there may exist more than one way of disaggregating a given quantity of load, further researchs will consist in designing some pertinent indicators to help the choice of the windows, and the width of their narrowings.

\textbf{VI CONCLUSION}

The new kind of load curves proposed in section IV offers different informations:

- Critical time intervals of a planning horizon, which risk to be under- and/or overloaded are directly visualized.
- The different levels of the successive vessels represent the constraints, or necessary conditions for a decision, in order to remain consistent according to the upper level of decisions which specifies the temporal characteristics of the planned operations, and to respect more local resource constraints.

- The graphical constraints consisting in choosing sub-surfaces WER and WRR (respectively WEL and WRL) such as their sum must "remain" inside the given surface WRIGHTMAX (respectively WLEFTMAX), insures the feasibility of the decisions according to the lower level of decisions which realizes temporal bounds updatings on the planned operations.

- Consequences of the decisions taken about a time interval, or vessel, over its neighbours, are graphically represented by the communicating vessels process.

All those informations are independent of any decision strategy, and do not need any detailed schedule generation, unlike traditional load curves with limited capacities do (Orlicky 1975): the maximal autonomy is left with the decider.

Nevertheless, the use of the "bounded flows of load in a temporal network" model, and the possibility to associate circulating costs to the arcs, will allow the exploration of quite a large diversity of optimized strategies: soonest strategy, latest one, particular time intervals privilegins which need for example to be the most underloaded, etc. Such computed strategies could then be proposed in a menu, and used as reference when deciding.

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Figure 1
\[ W_{\text{MAX}}(T_j, T_{j+1}) \]

Figure 2
\[ t = \text{WMIN}(T_j, T_{j+1}) \]

**Figure 2**

sum of surfaces

\[ = \text{WMIN}(T_j, T_{j+1}) \]
Figure 3
$\text{surface} = \text{sum of surfaces}$

Figure 4
\[ W_{\text{MAX}}(T_{j-1}, T_j) / T_j - T_{j-1} \]
\[ R(T_j, T_{j+1}) / T_{j+1} - T_j \]
\[ W_{\text{MIN}}(T_j, T_{j+1}) / T_{j+1} - T_j \]

\[ W_{\text{MAX}}(T_{j+1}, T_{j+2}) / T_{j+2} - T_{j+1} \]

Figure 5
Remark: the curves correspond in fact in middle quantities between two successive $T_j$ and $T_{j+1}$.
For example, $Q$ is to be taken as $Q(T_j,T_{j+1})/T_{j+1}-T_j$

*Figure 6*
Figure 7
\[ \text{effect: } \text{WMIN}(T_0, T_1) \text{ and } \text{WMIN}(T_2, T_3) \text{ increasing} \]

Decision: \( \text{WMAX}(T_1, T_2) \text{ decreasing} \)

\( \text{WRL}(T_1, T_2) \text{ and } \text{WRR}(T_1, T_2) \)

\( \text{WER}(T_0, T_1) \text{ and } \text{WEL}(T_2, T_3) \)

Figure 8
capacity and profitability resource levels

$$R(T_j, T_{j+1}) / T_{j+1} - T$$

$$Q(T_j, T_{j+1}) / T_{j+1} - T$$

Figure 9
Figure 10
Figure 12

* Published in *Production Planning & Control*, 1994, Vol.5, No.6, pp.533-542.