Hybrid Discrete-Continuous Optimization for the Frequency Assignment Problem in Satellite Communication System

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Abstract: This paper studies static Frequency Assignment Problem (FAP) in a satellite communication system involving a satellite and a number of users located in a service area. The objective is to maximise the number of users that the system can serve while maintaining the signal to interference plus noise ratio of each user under a predefined threshold. Traditionally, interference is binary and fixed. In this paper, the interference is cumulative and variable depending on how the frequency is assigned. To solve the problem, we work on both discrete and continuous optimizations. Integer linear programming formulations and greedy algorithms are proposed for solving the discrete frequency allocation problem. The solution is further improved by beam moving algorithm which involves continuous adjustment of satellite beams and deals with non-linear change of interference.

Keywords: Optimization problem, Combinatorial mathematics, Operation research, Integer linear programming, greedy algorithms

1. INTRODUCTION

With the continuing increase in demand, satellite communication technology continuously evolves and move towards greater capacity, higher flexibility, and better service to the end-users. Spatial Division Multiple Access (SDMA) appears to be an alternative to achieve these requirements simultaneously, see Liberti and Rappaport (1999). The technology employs antenna arrays and multi-dimensional non-linear signal processing techniques to provide significant increases in capacity and quality of many wireless communication systems. The technology is not restricted to any particular modulation format or air-interface protocol, and is compatible with all currently deployed air-interfaces, see Roy (1997) and Roy (1998).

An SDMA satellite equips with antennas that transmit signals to numerous zones on the earth's surface. The antennas are highly directional, allowing the same frequency to be reused in other surface zones where the frequency separation is sufficiently large. To support a large number of users, frequency selection should be carefully performed. The frequency allocation strategy thus plays an important role in the system performance. This class of problem is well-known as Frequency Allocation Problem (FAP).

The satellite communication system that we study in this paper aims at establishing bi-directional communications to stationary user terminals located in a service area. We propose Integer Linear Programming (ILP) formulations and greedy algorithm for solving the problem and then use beam moving algorithm to improve the solutions.

The paper is organised as follow: Section 2 provides the description of the telecommunication system. In Section 3, we describe ILP formulation, greedy algorithm, and beam moving algorithm. Section 4 presents the experimental results while conclusions are given in Section 5.

2. SYSTEM DESCRIPTION

In general, a satellite communications system consists of a satellite, a gateway, and a number of users within a service area. The satellite provides bi-directional communication links towards users and acts as a repeater between them and the gateway, the node that connects the satellite system to the terrestrial network. In this study, we consider only the satellite, the users, and communication links between them.

To simulate the system, actual parameters are used in conjunction with a number of randomly generated user's positions distributed over the service area which is defined by a set of geographic coordinates. The satellite utilizes SDMA technology to form beams and center them over the users. The user's perceived antenna gain, as shown in Fig. 1, is determined by the radiation pattern of the antenna and the distance between the user and the satellite, see

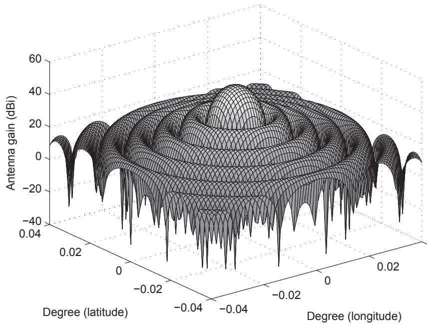

Fig. 1. An example of antenna diagram

Houssin et al. (2011). By centering the beam over the user, maximum antenna gain is achieved.

The objective of the study is to serve as many users as possible. A user is considered served if it is assigned with a frequency and satisfies the link budget constraint (1) with the user's signal to interference plus noise ratio (SINR) no less than the required signal to noise ratio.

$$\frac{C}{N+I} \geq \left(\frac{C}{N}\right)_{Required} \tag{1}$$

The SINR considers both interference and noise and is given by $\left(\frac{C}{N+I}\right)_i^{-1} = A + \left(\frac{C}{N}\right)_i^{-1} + \left(\frac{C}{I}\right)_i^{-1}$ where A is a system constant,

$$\begin{aligned}\left(\frac{C}{N}\right)_i &= \frac{(EiRPTerm)_i \,/\, (RS)_i}{L_{Atmo} \cdot L_{FSL}} \cdot \frac{G_{Sat(Beam_i \to i)}}{(T_A + T_{Rep}) \cdot k} \\ &= (K_1)_i \cdot \frac{G_{Sat(Beam_i \to i)}}{K_2}\end{aligned} \tag{2}$$

and

$$\left(\frac{C}{I}\right)_i = \frac{(K_1)_i \cdot G_{Sat(Beam_i \to i)}}{\sum_{j \in Interf} (K_1)_j \cdot G_{Sat(Beam_j \to i)}}. \tag{3}$$

The terms K_1 and K_2 represent technical parameters which are the terminal's equivalent isotropic radiation power (EiRP), the symbol rate (RS), the atmospheric loss (L_{Atmo}), the free space loss (L_{FSL}), the antenna equivalent temperature ($T_A + T_{Rep}$), and the Boltzmann constant (k). Users could have different values of EiRPs, symbol rates and losses; nonetheless, we keep them as constants in this study. Thus

$$\left(\frac{C}{I}\right)_i = \frac{G_{Sat(Beam_i \to i)}}{\sum_{j \in Interf} G_{Sat(Beam_j \to i)}}. \tag{4}$$

$G_{Sat(Beam_i \to i)}$ and $G_{Sat(Beam_j \to i)}$ are user i's antenna gain (regarding to its beam and position) and the interferer j 's antenna gain at user i's position.

Let $B = \left(\frac{C}{N}\right)_i^{-1}$, and $D = \left(\frac{C}{N}\right)_{Required}$. The cumulative interference constraint for user i can be written in a linear form as

$$\sum_{j \in Interf} \delta_{ij} \leq \alpha_i \tag{5}$$

where

$$\delta_{ij} = D \cdot G_{Sat(Beam_j \to i)}, \tag{6}$$

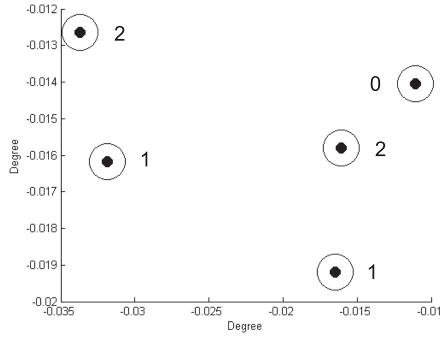

Fig. 2. A frequency allocation for 5 users

$$\alpha_i = G_{Sat(Beam_i \to i)} \cdot (1 - AD - BD). \tag{7}$$

The term α_i can be perceived as an acceptable interference threshold for the user i while δ_{ij} as an interference coefficient from users j towards the user i.

Fig. 2 shows an example of frequency allocation for 5 users distributed in a service area with dedicated beams centered on them. The X and Y axes correspond to longitude and latitude in degree. The beam size is not to scale. Four users can be allocated with the Color 1 or 2 as shown next to the user. Color 0 means that the user can not be assigned a frequency. The corresponding α_i and δ_{ij} are shown in Table 1.

Table 1. Alpha and Delta of the users in the given example

i	$\alpha_i \times 10^{19}$	$\delta_{ij} \times 10^{19}$				
1	9.10	0	1.27	115.86	12.29	0.04
2	8.08	1.14	0	1.07	0.63	86.58
3	9.31	118.30	1.21	0	56.73	0
4	9.64	12.93	0.73	58.47	0	0.67
5	8.05	0.03	86.29	0	0.57	0

If we assign a color to the fourth user, the cumulative interference will surpass the acceptable interference threshold (negative constraints) as shown in the Table 2 with Color set 2. These allocations are not allowed.

Table 2. Cumulative interference constraints of the users given the assigned colors

i	Color set 1	Constraints *	Color set 2	Constraints *
1	1	7.83	1	-4.46
2	1	6.94	1	6.31
3	2	9.31	2	9.31
4	0	-	1	-4.03
5	2	8.04	2	8.04

* $(\alpha_i - \sum_{j \in Interf} \delta_{ij})$

3. MODELING AND SOLVING FREQUENCY ALLOCATION PROBLEM

3.1 FAP literature review

Several strategies for the optimization of satellite resource management have been investigated. Apart from the traffic demand, there are other system variations that have a strong impact on the adopted resource management techniques. These include changes in the link quality due to weather conditions, mobility, jamming, and other factors, see Giambene (2007). The resource management techniques thus encompass one or combinations of frequency, time channels, transmitted power, access methods, power allocation, and call admission control.

Frequency allocation problem (FAP) is common in many different types of wireless communication networks and there have been a lot of research on this topic. Interested readers are referred to the FAP web site http://fap.zib.de/ for a digest and a survey of frequency assignment literature. To which category a frequency assignment problem belongs is determined by its objective function. Five common objective functions are Maximum Service FAP, Minimum Blocking FAP, Minimum Order FAP, Minimum Span FAP and Minimum Interference FAP. Our study is based on the latter.

Most approaches dealing with MI-FAP consider binary interference constraints, *i.e.* involving only two users. Because of the strong links between graph coloring and frequency allocation with binary interference constraints, most methods found in the literature are inspired by coloring algorithms. The graph coloring algorithms are well known to be NP-hard, thus, consequently the FAP. Among the proposed methods, the constructive (greedy) algorithms are widely used since they are simple and fast. In this category, we find the generalisation of DSATUR procedure, see Brélaz (1979). Other more sophisticated algorithms, such as local search, metaheuristics, ILP, and constraint programming approaches, are frequently encountered, see Aardal et al. (2003).

One of the difficulties in this study lies in the explicit consideration of non-binary interference constraints. In terms of graph coloring, deciding whether a given coloring is feasible or not cannot be made any more by checking pairwise user colors or assignments. Instead, for a given user, the cumulative interferences of the users assigned to the same color (frequency) has to be computed. The coloring is feasible if this cumulative interference remains under a threshold. In the literature, only a few approaches explicitly take into account of such interferences, see Dunkin et al. (1998), Mannino and Sassano (2003), Alouf et al. (2005) and Palpant et al. (2008).

Alouf et al. (2005) presents an algorithm for resource allocation in multi-spot satellite network to obtain a quasi-optimal time/frequency plan for a set of terminals with a known geometric configuration under interference constraints. The study is based on spatial distribution of satellite spots and model interference based on geographical zones in that the users within the same zone exhibit the same radio propagation condition. Our study is based on dedicated spot-to-user concept and model interference based on each user's radio propagation property.

Note that there are other research branches regarding SDMA technology. These concern channel access methods over WLAN or cellular network systems.

3.2 Integer linear programming

Taking account of hypotheses and simplifications presented in Section 2, the FAP is similar to coloring problems and thus formalised as the corresponding combinatorial optimization problems. Each user has to be assigned a color, representing a frequency.

Let n denotes the number of users, $U = \{1, \ldots, n\}$ a set of users, and C the number of colors. Binary decision variables x_{ic} are defined for $i \in \{1, \ldots, n\}$ and $c \in \{1, \ldots, C\}$ in that $x_{ic} = 1$ if color c is allocated to users i and $x_{ic} = 0$ otherwise. The problem can be represented by the following ILP:

$$\max \sum_{i=1}^{n} \sum_{c=1}^{C} x_{ic}, \tag{8}$$

$$\sum_{c=1}^{C} x_{ic} \leq 1 \quad i = 1, \ldots, n, \tag{9}$$

$$\sum_{j=1}^{n} \delta_{ij} x_{jc} \leq \alpha_i + M_i(1 - x_{ic}) \; i = 1, \ldots, n, c = 1, \ldots, C, \tag{10}$$

$$x_{ic} \in \{0, 1\} \quad i = 1, \ldots, n \quad c = 1, \ldots, C. \tag{11}$$

Objective (8) maximises the number of accepted users while Constraints (9) restrict that at most one color has to be selected for each user. Constraints (10) are the cumulative interference constraints. The constant M_i has to be large enough to withdraw these constraints if i is not assigned a color c ($x_{ic} = 0$). More precisely, we set $M_i = \sum_{j=1}^{n} \delta_{ij} - \alpha_i$.

3.3 Greedy algorithm

Solving the ILP formulations provides optimal solutions only for small problems. For large-sized problems, a heuristic approach is necessary. We propose greedy algorithms to solve this problem. The principle of the greedy algorithm is, at first, to consider the users sequentially according to a given criterion named (user priority rule). Secondly, either the selected user is assigned a color or rejected according to a second criterion (frequency priority rule).

Let Q denotes a set of users that have not been assigned a color yet. Initially we have $Q = U$. At each step of the greedy algorithm, a user i is removed from Q and is either rejected or assigned a color.

For the user priority rule, we may use the frequency margin, where the margin $M(i, c)$ of a user $i \in Q$ for a color c is given by $M(i, c) = \alpha_i - \sum_{j \in U \setminus Q \cup \{i\}, F_j = c} \delta_{ij}$. This margin corresponds to the positive or negative slack of the cumulative interference constraint for user i if it is assigned a color c.

As a preliminary result, we observed that the user priority rule aimed at selecting first the most constrained users in terms of available colors while it is well known that, with this environment, the DSATUR algorithm for standard graph coloring problem gives bad results. We thus consider a kind of hybrid reverse DSATUR rule by alternately selecting the user having the largest number of available colors and the user having maximum interference with the previously assigned user. In fact, we tested two following user priority rules:

- Lexicographic: the user with the smallest number is selected,

- Hybrid: the user having the largest number of available colors is selected. A color c is available for user $i \in Q$ if $M(i,c) \geq 0$ and if for all users $j \in U \setminus Q$ that have already been assigned color c, $M(j,c) \geq 0$. In case of a tie, we select the user having the largest total margin for all its available colors. Let i denotes the selected user with this rule. For the next iteration, we select the user having maximum interference with i, i.e. the user j maximising $\delta_{ij} + \delta_{ji}$ and we alternate the two rules.

For the frequency selection, we tested two following frequency priority rule:

- Lexicographic: the smallest available frequency is selected,
- Most used: the most used available frequency is selected. In case of a tie, we select the color c that maximises the sum of margins $M(j,c)$ for all users $j \in Q$.

The proposed greedy algorithms run in $O(n^2C)$ time.

3.4 Beam moving algorithm

To further improve the results from the ILP and greedy algorithm, we propose a subsequent non-linear local optimization, called beam moving algorithm. This algorithm exploits the benefit of SDMA technology by moving a number of satellite beams from their center positions.

In fact the δ_{ij} and α_i in (6) and (7) can be written as functions of user position (u,v) and beam position which are

$$\delta_{ij} = D \cdot G_{Sat}(u_i, v_i, Beam\_u_j, Beam\_v_j), \quad (12)$$

$$\alpha_i = G_{Sat}(u_i, v_i, Beam\_u_i, Beam\_v_i) \cdot (1 - AD - BD). \quad (13)$$

The terms D and $(1-AD-BD)$ are constant. We will keep the user position fixed but alter the beam position; as a result, both δ_{ij} and α_i changes. Nonetheless, the change is non-linear according to the antenna gain shown previously in Fig. 1.

Beam moving algorithm takes the output solutions from either ILP or greedy algorithm as its input, identifies the rejected users, and, for each rejected user, moves the most k interfering beams and tries to reassign the user a color.

Let i denotes an unassigned user, the beam moving algorithm selects a color c, i.e. sets $x_{ic} = 1$, and identifies a set of interferers S containing all users j having $x_{jc} = 1, \forall j \in S$ (unassigned user included). Let $K \subseteq S$ consists of a set of users whose beams will be moved. The parameter k defines the number of strongest interferers to the unassigned user i that are included in the set K. The parameter $UTVAR \in (0,1)$, if set to 1, tells the algorithm to replace the least interferer in the set K with i thus including the user i in the move.

MAXINEG parameter provides a maximum negative margin from the required signal to noise ratio. It is based on the fact that the closer the unassigned user's signal to interference plus noise ratio is to the required signal to noise ratio, the more the possibility the algorithm has to search for a solution. Before the algorithm tries to move beams, the unassigned user is tested with this margin. If failed, the remaining colors are tried or the user is rejected.

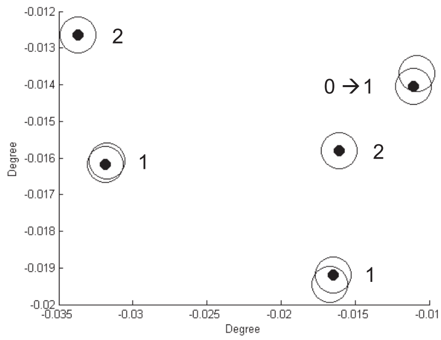

Fig. 3. An example on beam moving

The algorithm continuously moves the beams of users in the set K from their center positions $(u_0^{(k)}, v_0^{(k)})$ and in each move evaluates if the new positions pass the link budget constraints. The problem can be represented as:

$$\min \sum_{k \in K} \parallel (u_0^{(k)} - u_k)^2 + (v_0^{(k)} - v_k)^2 \parallel^2, \quad (14)$$

subject to

$$\left(\frac{C}{N+I}\right)(u_k, v_k, u_0^{(k)}, v_0^{(k)}) \geq \left(\frac{C}{N}\right)_{Required} \quad \forall k \in K. (15)$$

When a beam is moved from its center, the associated user will obtain lower antenna gain and hence lower SINR. Any move that violates the link budget constraints (15) is rejected. Nonetheless, this move could benefit the unassigned user by reducing its tentative interference level. For a selected color c, the beam moving algorithm minimizes the total move distance of the interferers' beams (14), maintains their interference constraints' validity, and reduces the tentative interference of the unassigned user i to the level that the reassignment is valid.

If a decent move could not be found within a number of iterations defined by *MAXITER* each of the remaining colors is tried. If all colors have been tried and there is no possible solution, the user i is rejected and the algorithm moves to next unassigned users.

Fig. 3 shows a result of beam moving algorithm applied to the example presented previously in Section 2. It can be seen that the beam of the two interferers and the unassigned users are moved. This yields a reassignment of Color 1.

3.5 Closed-loop implementation

The ILP solver or the greedy algorithm would have more possibility to find the optimal solution or provide a better feasible solution if an initial feasible solution is given. Consider an iteration as a combination of ILP - beam moving or greedy - beam moving. We propose the closed-loop implementation in that, in the next iteration of ILP or greedy algorithm, the frequency allocation result from beam moving algorithm is used as an initial solution and the moved beam positions are used for recalculating the α_i and δ_{ij} values.

The ILP starts with the initial solution, continues to improve the solution, and by the given CPU time, outputs the best found solution. We implemented two variations for greedy algorithm. The first variation (Greedy 1) considers both the frequency allocation result and the updated α_i and δ_{ij} and works further on the unassigned users. The second variation (Greedy 2) only considers the updated α_i and δ_{ij} and restarts the frequency allocation from scratch.

4. COMPUTATIONAL EXPERIMENTS AND RESULTS

The ILP formulation has been solved using IBM/ILOG CPLEX 12.2, see Cpl (2010). The greedy algorithm has been coded in C++. We tested the proposed algorithms with $C = 8$; increasing stepwise the number of users by 20 from 20 to 200 users with 100 instances each. Real system parameters are used in conjunction with randomly generated and uniformly distributed user positions. The results were obtained on a 2.7GHz Intel Core i5 machine with 4GB RAM. The CPU times for the ILP resolutions have been limited to 60s, 120s, and 180s after which the best integer solution is obtained. The CPU times for the greedy algorithm were negligible while the beam moving was performed with the maximum of 40 iterations with no limitation on the calculation time.

The beam moving algorithm is coded in Matlab, see MATLAB (2008). The function $fmincon$ with active-set algorithm is used for computing the minimum move distance according to the given non-linear constraints.

We first present a comparison of the greedy algorithms. Table 3 reports the average number of accepted users over 1,000 instances. The results of the greedy algorithms are very close. It was difficult to give better results than the simple lexicographic rules. The algorithm that uses Hybrid and Most used rules gives the best result. As of this, we use it as the baseline for performance comparison with the results from ILP and beam moving.

Table 3. Average number of accepted users over 1,000 instances

Lexicographic (user + frequency)	93.83
Lexicographic (user) + Most used (frequency)	93.84
Hybrid (user) + Most used (frequency)	94.19

We tested 32 configurations of *k-MAXINEG-UTVAR* for the beam moving algorithm over 20 instances of 200 users. Test results are provided in Fig. 4 and 5. It can be seen that increasing any of k (from 3 to 10) or *MAXINEG* (from 1 to 2) or enabling *UTVAR* (0 or 1) yields higher number of reassigned users, at an expense of longer calculation time. Both configuration 7-2-0 and 6-2-1 provide good performances with acceptable calculation times. We choose configuration 7-2-0 for improving the results from the ILP and greedy algorithm through beam moving.

Fig. 6 display, for each algorithm and number of users, the average number of accepted users in the computed frequency allocation plans. The number of optima provided by ILPs is given in Table 4. The greedy algorithm performs as good as the other two ILPs at up to 120 users (ILP can solve to optima for all or almost all of 100 instances up to this point). For 140-200 users, the performance gap becomes larger as the number of user

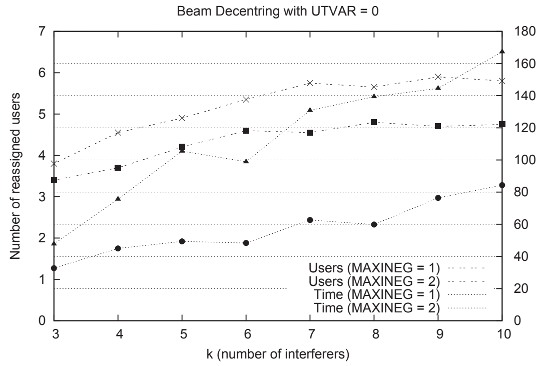

Fig. 4. Average number of reassigned users and calculation time for different beam moving configurations with UTVAR=0

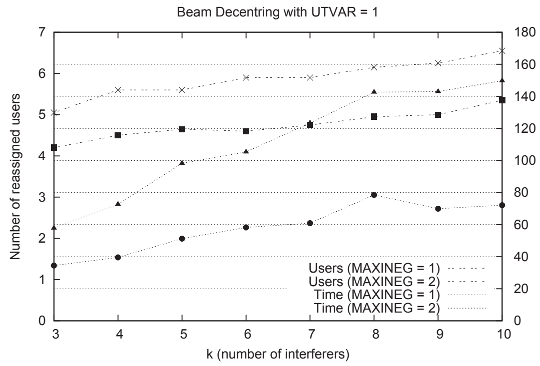

Fig. 5. Average number of reassigned users and calculation time for different beam moving configurations with UTVAR=1

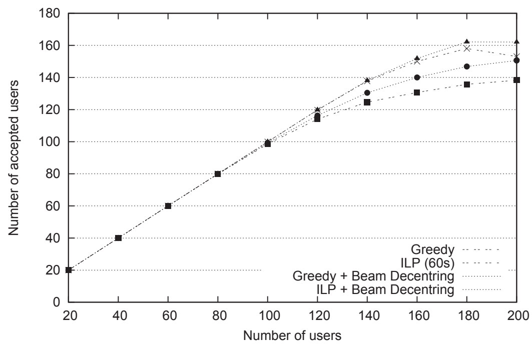

Fig. 6. Average number of accepted users before and after beam moving for Greedy algorithm and ILP 60s

increases. Performance degradation is found in ILP60s at 200 user instances. This signifies that, though not reaching the optima, the ILP needs more time for a larger instance to provide a better results.

Table 4. Number of optima provided by ILPs

n	20	40	60	80	100	120	140	160
ILP60s	100	100	100	100	100	97	54	0
ILP120s	100	100	100	100	100	98	61	0
ILP180s	100	100	100	100	100	100	67	0

Table 5 presents lower bounds and upper bounds for ILP180s. Large gaps signify that the ILP formulation yields poor relaxations.

Beam moving gives performance improvement for both greedy algorithm and ILP. Significant improvements can be seen in the greedy algorithm case. It could provide comparable results at 200 users compared to ILP60s. Nonetheless, the algorithm's calculation time is high, see Table 6.

The results for closed-loop simulations are shown in Table 7. Greedy 1 continuously improves the solutions over the iterations and approaches saturation after Iteration 3. Degraded performance is found for Greedy 2 in ILP Iteration 2 and 3. These are caused by restarting frequency allocation from scratch. For both ILPs, small improvement can be seen in the second iteration but no improvement in the third. ILPs converge to the saturation faster than Greedy algorithms.

5. CONCLUSION

In this paper we have developed an integer linear programming formulation and greedy algorithms for solving FAP which involves cumulative interference. The greedy algorithm is simple, fast and efficient enough to provide comparable results to the ILP at up to a certain number of users. To improve the solutions, a non-linear continuous algorithm a.k.a. beam moving algorithm is implemented. The algorithm yields higher number of accepted users for both ILP and greedy algorithm while significant improvement is found in the latter case. Closed-loop implementation provides marginal solution improvement.

To further improve these results, an integrated approach where frequency allocation and beam position are determined simultaneously and not sequentially, could be proposed. This yields highly complex mixed non-linear integer programming formulations. As a short term follow-up, the closed loop implementation solves the integrated problem as a hill-climbing method. More improvements could be reached by allowing temporary decrease of the objective functions via metaheuristic framework such as tabu search. Better upper bound techniques could also be helpful stop the search earlier.

Table 5. Average upper and lower bounds for ILP180s.

n	LB	UB	$\%(UB-LB)/UB$ min.	avg.	max.
120	119.79	119.81	0.00	0.02	1.67
140	138.17	139.18	0.00	0.71	3.76
160	151.07	158.21	1.25	4.46	7.50
180	160.69	177.19	5.06	9.25	13.22
200	165.22	194.36	9.33	14.90	23.59

Table 6. Average calculation time (s) performed by beam moving algorithm

n	80	100	120	140	160	180	200
Greedy	9.2	22.9	67.6	241.7	570.7	1017.3	1542.5
ILP60s	-	-	13.6	29.7	125.3	365.2	1032.0
ILP180s	-	-	-	28.4	114.9	272.9	622.0

Table 7. Average percentage of accepted users over 100 instances of 200 users

	Iteration 1		Iteration 2		Iteration 3	
	ILP	BM *	ILP	BM	ILP	BM
Greedy 1	69.15	75.29	76.05	76.05	76.20	76.20
Greedy 2	69.15	75.29	70.27	71.71	70.94	72.37
ILP 60s	76.53	81.05	81.58	81.84	81.84	-
ILP 180s	82.66	85.49	85.53	85.53	85.53	-
# ** Greedy 1	-	100	73	24	24	1
# ** Greedy 2	-	100	7	93	19	93
# ** 60s	-	100	14	13	0	-
# ** 180s	-	100	4	3	0	-

* (Beam moving), ** (Number of improved solutions)

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