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Frequency assignment in a SDMA satellite communication system with beam decentring feature

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Abstract In satellite communication, Spatial Division Multiple Access (SDMA) has become one of the most promising techniques that can accommodate continuing increase in the number of users and traffic demands. The technology is based on radio resource sharing that separates communication channels in space. It relies on adaptive and dynamic beam-forming technology and well-designed algorithms for resource allocation among which frequency assignment is considered. This paper studies static Frequency Assignment Problem (FAP) in a satellite communication system involving a satellite and a number of users located in a service area. The objective is to maximize the number of users that the system can serve while maintaining the signal to interference plus noise ratio of each user under a predefined threshold.

Traditionally, interference is treated as fixed (binary interferences or fixed minimal required separation between frequencies) . In this paper, the interference is cumulative and variable. To solve the problem, we work on both discrete and continuous optimizations. Integer linear programming formulations and greedy algorithms are proposed for solving the discrete frequency assignment problem. The solution is further improved by beam decentring algorithm which involves continuous adjustment of satellite beams and deals with non-linear change of interference.

Keywords SDMA system · Frequency assignment problem · Integer programming · Greedy algorithm · Non-linear optimization

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1 Introduction

Satellite communications have revolutionised the world we live in. Fixed and mobile telephone services, television broadcast, internet access, and a large number of applications have changed the way people all over the globe interact. With the continuing increase in traffic demand, satellite communication technology continuously evolves and move towards greater capacity, higher flexibility, and better service to the end-users. Spatial Division Multiple Access (SDMA) appears to be an alternative to achieve these requirements simultaneously [14]. The technology employs antenna arrays and multi-dimensional non-linear signal processing techniques to provide significant increases in capacity and quality of many wireless communication systems [21]. The technology is not restricted to any particular modulation format or air-interface protocol, and is compatible with all currently deployed air-interfaces [20].

An SDMA satellite equips with multi-spot-beam antenna [6] that transmit signals to numerous zones on the Earth's surface. The antennas are highly directional, allowing the same frequency to be reused in other surface zones where the frequency separation is sufficiently large. To support a large number of users, frequency selection should be performed carefully. The frequency assignment strategy thus plays an important role in the system performance. This class of problem is well-known as Frequency Assignment Problem (FAP) [9], [13].

The satellite communication system that we study in this paper aims at establishing bi-directional communications to stationary user terminals located in a service area. We propose Integer Linear Programming (ILP) formulations and greedy algorithm for solving the problem and then we use beam decentring algorithm to improve the solutions.

The paper is organised as follows: Section 2 provides the description of the telecommunication system; in Section 3, we describe ILP formulation, greedy algorithm and beam decentring method based on non linear programming. Section 4 presents the experimental results while conclusions are given in Section 5.

2 System description

In general, a satellite communications system consists of a satellite, a gateway, and a number of users within a service area. The satellite provides bi-directional communication links towards users and acts as a relay point between them and a gateway, the node that connects the satellite system to the terrestrial network. In this study, we consider only the satellite, the users, and communication links between them, see Figure 1.

To simulate the system, actual parameters are used in conjunction with randomly generated and uniformly distributed user positions. Satellite antenna uses SDMA technology to form dedicated beams and center them over the users. Satellite's antenna gain (simplified) is determined by radiation pattern of the antenna and distance between each user and the satellite [10] , *i.e.*,

$$G_{Sat}(u,v,u_0,v_0) = G_1 \cdot G_2(u,v,u_0,v_0) \cdot G_3(u,v), \tag{1}$$

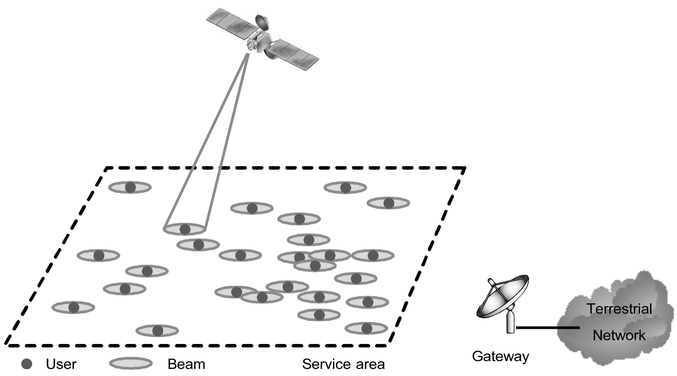


Fig. 1 A satellite communications system.

where

$$G_1 = \eta \left(\frac{\pi D}{\lambda}\right)^2, \tag{2}$$

$$G_2(u,v,u_0,v_0) = \left(\frac{2J_1\left(\frac{\pi D}{\lambda}\sqrt{(u-u_0)^2+(v-v_0)^2}\right)}{\frac{\pi D}{\lambda}\sqrt{(u-u_0)^2+(v-v_0)^2}}\right)^2, \tag{3}$$

and

$$G_3(u,v) = \left(\frac{2J_1\left(\frac{\pi d}{\lambda}\sqrt{u^2+v^2}\right)}{\frac{\pi d}{\lambda}\sqrt{u^2+v^2}}\right)^2. \tag{4}$$

$J_1(\cdot)$ represents the Bessel function of the first kind while u,v and u_0,v_0 are Cartesian coordinates of the user and the beam center. η, D, d and λ are antenna efficiency, antenna diameter, diameter of the antenna's primary source and carrier wavelength, respectively. The corresponding antenna diagram is shown in Figure 2. The antenna is very directional in that the gain is very high at the center and diminishes rapidly when moving out. We call each concentration of antenna gain as a satellite beam. By centring the beam over the user, it gets the maximum gain.

The objective of the study is to serve as many users as possible. A user is considered served if it is assigned with a frequency and satisfies the link budget constraint having the user's signal (C) to interference (I) plus noise (N) ratio (SINR) no less than the required signal to noise ratio, as below:

$$\frac{C}{N+I} \geq \left(\frac{C}{N}\right)_{Required}. \tag{5}$$

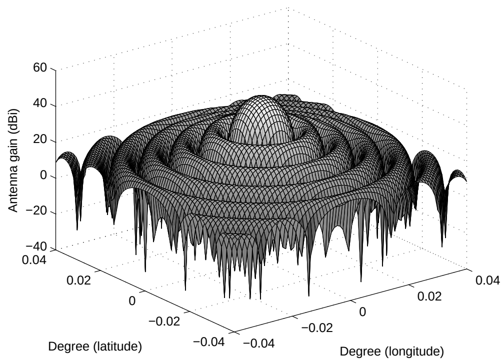


Fig. 2 Example of an antenna diagram.

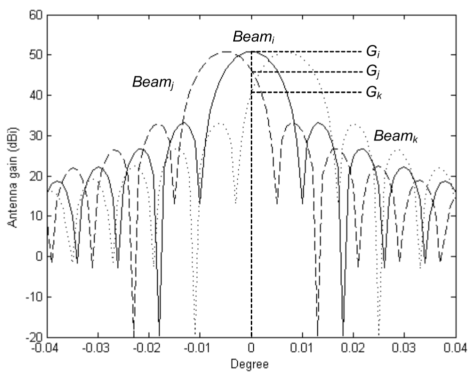


Fig. 3 Cross sections of three satellite beams.

Figure 3 shows cross sections ($Y = 0$) of three satellite beams associated to and centered at users i, j, k located at three different positions. Let's assume uniform receivers, transmitter output power and propagation loss, we can consider the received signal power from the perceived antenna gain. G_i denotes the corresponding antenna gain from $Beam_i$ at position $(0,0)$. It can be seen that, at this position, there exist also G_j and G_k from $Beam_j$ and $Beam_k$. Interference occurs if these users share the same frequency (i.e. co-channel interference). The interference is cumulative in that the total interference at user i is the sum of the interferences from user j and k. Note that the interference is more critical in the uplink (from users to the satellite).

The SINR of a user i considers both interference and noise and is defined by $\left(\frac{C}{N+I}\right)_i^{-1} = A + \left(\frac{C}{N}\right)_i^{-1} + \left(\frac{C}{I}\right)_i^{-1}$ where A is a system constant,

$$\begin{aligned}\left(\frac{C}{N}\right)_i &= \frac{(EiRPTerm)_i/(RS)_i}{L_{Atmo} \cdot L_{FSL}} \cdot \frac{G_{Sat(Beam_i \to i)}}{(T_A + T_{Rep}) \cdot k} \\ &= (K_1)_i \cdot \frac{G_{Sat(Beam_i \to i)}}{K_2}\end{aligned} \tag{6}$$

and

$$\left(\frac{C}{I}\right)_i = \frac{(K_1)_i \cdot G_{Sat(Beam_i \to i)}}{\sum_{j \in Interf} (K_1)_j \cdot G_{Sat(Beam_i \to j)}}. \tag{7}$$

Terms K_1 and K_2 represent technical parameters which are terminal's effective isotropic radiated power (EiRPTerm), symbol rate (RS), atmospheric loss (L_{Atmo}), free space loss (L_{FSL}), antenna equivalent temperature ($T_A + T_{Rep}$), and the Boltzmann constant (k). Users could have different values of EiRPTerm, symbol rates and losses; nonetheless, we keep them as constants in this study. Thus, one has

$$\left(\frac{C}{I}\right)_i = \frac{G_{Sat(Beam_i \to i)}}{\sum_{j \in Interf} G_{Sat(Beam_i \to j)}}. \tag{8}$$

$G_{Sat(Beam_i \to i)}$ and $G_{Sat(Beam_i \to j)}$ are antenna gains of $Beam_i$ regarding to the user i and the interferer j.

Let $B = \left(\frac{C}{N}\right)_i^{-1}$ and $D = \left(\frac{C}{N}\right)_{Required}$. The cumulative interference constraint for user i can be written in a linear form as

$$\sum_{j \in Interf} \delta_{ij} \leq \alpha_i, \tag{9}$$

where

$$\delta_{ij} = D \cdot G_{Sat(Beam_i \to j)}, \tag{10}$$

$$\alpha_i = G_{Sat(Beam_i \to i)} \cdot (1 - AD - BD). \tag{11}$$

The term α_i can be perceived as an acceptable interference threshold for the user i while δ_{ij} as an interference coefficient from user j towards user i.

Figure 4 shows an example of frequency assignment for 5 users with their beams centered on them. Four users can be allocated with the Color 1 or 2 as shown next to the user. Color 0 means that the user cannot be assigned a frequency. The corresponding α_i and δ_{ij} are shown in Table 1.

Table 1 α and δ of the users in the given example.

i	$\alpha_i \times 10^{19}$			$\delta_{ij} \times 10^{19}$		
1	9.10	0	1.27	115.86	12.29	0.04
2	8.08	1.14	0	1.07	0.63	86.58
3	9.31	118.30	1.21	0	56.73	0
4	9.64	12.93	0.73	58.47	0	0.67
5	8.05	0.03	86.29	0	0.57	0

If we assign a color to the unassigned user, the cumulative interference will surpass the acceptable interference threshold (the difference becomes negative) as shown in the Table 2 with Color Set 2 and 3. These allocations are not allowed.

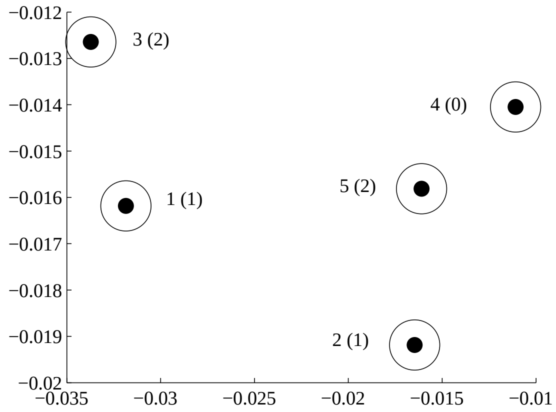


Fig. 4 A frequency assignment example.

Table 2 Cumulative interference constraints of the users in different color sets.

i	Color set 1	$\alpha_i - \sum_{j \in Interf} \delta_{ij}$	Color set 2	$\alpha_i - \sum_{j \in Interf} \delta_{ij}$	Color set 3	$\alpha_i - \sum_{j \in Interf} \delta_{ij}$
1	1	7.83	1	-4.46	1	7.83
2	1	6.94	1	6.31	2	6.94
3	2	9.31	2	9.31	2	-47.42
4	**0**	-	**1**	-4.03	**2**	-49.50
5	2	8.04	2	8.04	2	7.47

3 Modelling and solving frequency assignment problem

3.1 Literature review

Several strategies for the optimization of satellite resource management have been investigated [6]. Apart from the traffic demand, there are other system variations that have a strong impact on the adopted resource management techniques. These include changes in the link quality due to weather conditions, mobility, jamming, and other factors [6]. The resource management techniques thus encompass one or combinations of frequency, time channels, transmitted power, access methods, power allocation, and call admission control.

Frequency assignment problem (FAP) is common in many different types of wireless communication networks and there have been a lot of research on this topic. Interested readers are referred to the FAP web site[1] for a digest and a survey of frequency assignment literature. To which category a frequency assignment problem belongs is determined by its objective function. Five common objective functions are Maximum Service FAP, Minimum Blocking FAP, Minimum Order FAP, Minimum Span FAP and Minimum Interference FAP. Our study is based on the latter. A different approach is proposed recently, [3] suggests new concepts of frequency use and

[1] `http://fap.zib.de/`

allocation that consider a certain measure of fairness in the allocation of resource. This involves mathematical disciplines such as social choice theory / social welfare theory and axiomatic theory.

Most approaches dealing with MI-FAP consider interference constraints involving only two users and requiring a minimum separation between frequencies , *i.e.*, constraints of the form $|f_j - f_i| \geq \varepsilon_{ij}$ with $\varepsilon_{ij} \geq 0$. Because of the strong links between graph coloring and frequency assignment with binary interference constraints, most methods found in the literature are inspired by coloring algorithms. The graph coloring algorithms are well known to be NP-hard, thus, consequently the FAP. Among the proposed methods, the constructive (greedy) algorithms are widely used since they are simple and fast. In this category, we find the generalisation of DSATUR procedure [4]. [15] proposes a hybrid method combining a problem specific crossover and a Tabu search procedure while the interference is formulated by np directed graphs. Other more sophisticated algorithms, such as local search, metaheuristics, ILP, and constraint programming approaches, are frequently encountered [1].

One of the difficulties appearing in the telecommunication system considered in this study lies in the explicit consideration of cumulative interference constraints. It is not so straightforward to adapt the graph coloring problem in this context.

In terms of graph coloring, deciding whether a given coloring is feasible or not cannot be made any more by checking pairwise user colors or assignments. Instead, for a given user, the cumulative interferences of the users assigned to the same color (frequency) has to be computed. The coloring is feasible if this cumulative interference remains under a threshold.

In the literature, only a few approaches explicitly take into account this cumulative interference, see [5], [16], [2], [18], [8] and [7]. According to Aardal et al. [1], cumulative interference is ignored in most models where only interference between pairs of connections or antennae is measured.

Reference [2] presents an algorithm for resource allocation in multi-spot satellite network to obtain a quasi-optimal time/frequency plan for a set of terminals with a known geometric configuration under interference constraints. The study is based on spatial distribution of satellite spots and model interference based on geographical zones in that the users within the same zone exhibit the same radio propagation condition. Our study is based on dedicated spot-to-user concept and model interference based on each user's radio propagation property.

Note that there are other research branches utilizing SDMA technology. These concern channel access methods over WLAN or cellular network systems, for example, [11] and [19].

3.2 Integer linear programming

Taking account of hypotheses and simplifications presented in Section 2, the FAP is similar to coloring problems and thus formalized as the corresponding combinatorial optimization problems. Each user has to be assigned a color representing the frequency.

Let n denotes the number of users, $U = \{1,\ldots,n\}$ a set of users, and C the number of colors (frequencies). Binary decision variables x_{ic} are defined for $i \in \{1,\ldots,n\}$ and $c \in \{1,\ldots,C\}$ in that $x_{ic} = 1$ if color c is allocated to users i and $x_{ic} = 0$ otherwise. The problem can be represented by the following ILP:

$$\max \sum_{i=1}^{n} \sum_{c=1}^{C} x_{ic}, \tag{12}$$

$$\sum_{c=1}^{C} x_{ic} \leq 1 \quad i = 1,\ldots,n, \tag{13}$$

$$\sum_{j=1}^{n} \delta_{ij} x_{jc} \leq \alpha_i + M_i(1 - x_{ic}) \quad i = 1,\ldots,n \quad c = 1,\ldots,C, \tag{14}$$

$$x_{ic} \in \{0,1\} \quad i = 1,\ldots,n \quad c = 1,\ldots,C. \tag{15}$$

Objective (12) maximizes the number of accepted users while Constraints (13) restrict that at most one color has to be selected for each user. Constraints (14) are the cumulative interference constraints. The constant M_i has to be large enough to withdraw these constraints if i is not assigned a color c ($x_{ic} = 0$). More precisely, we set $M_i = \sum_{j=1}^{n} \delta_{ij} - \alpha_i$.

3.3 Greedy algorithm

Solving the ILP formulations provides optimal solutions only for small instances of (12)-(15). For large-sized instances, a heuristic approach is necessary. We propose greedy algorithms to solve this problem. The principle of the greedy algorithm is, at first, to consider the users sequentially according to a given criterion named *user priority rule*. Secondly, either the selected user is assigned a color or rejected according to a second criterion, the *frequency priority rule*.

Let Q denotes a set of users that have not been assigned a color yet. Initially we have $Q = U$. At each step of the greedy algorithm, a user i is removed from Q and is either rejected or assigned a color. The principle of the greedy algorithm is summarized in *Algorithm 1*, where F_i denotes the color allocated to user i if $1 \leq F_i \leq C$ and $F_i = 0$ indicates that user i is rejected.

```
Input: n, C, α, δ
Output: F
F_i ← 0, ∀i = 1,...,n ;
for q = 1 to n do
    i ← SelectUser(m, C, α, δ, F) ;
    F_i ← SelectColor(i, n, C, α, δ, F) ;
end
```

Algorithm 1: Greedy algorithm

For the user priority rule (*SelectUser* function), we may use the frequency margin, where the margin $M(i,c)$ of a user $i \in Q$ for a color c is given by $M(i,c) = \alpha_i -$

$\sum_{j\in U\setminus Q\cup\{i\},F_j=c}\delta_{ij}$. This margin corresponds to the positive or negative slack of the cumulative interference constraint for user i if it is assigned a color c.

As a preliminary result, we observed that the user priority rule aimed at selecting first the most constrained users in terms of available colors while it is well known that, with this environment, the DSATUR algorithm for standard graph coloring problem gives bad results. We thus consider a kind of hybrid reverse DSATUR rule by alternately selecting the user having the largest number of available colors and the user having maximum interference with the previously assigned user. In fact, we tested two following user priority rules:

- Lexicographic: the user with the smallest number is selected,
- Hybrid: the user having the largest number of available colors is selected. A color c is available for user $i \in Q$ if $M(i,c) \geq 0$ and if for all users $j \in U \setminus Q$ that have already been assigned color c, $M(j,c) \geq 0$. In case of a tie, we select the user having the largest total margin for all its available colors. Let i denotes the selected user with this rule. For the next iteration, we select the user having maximum interference with i, *i.e.* the user j maximizing $\delta_{ij} + \delta_{ji}$ and we alternate the two rules.

For the frequency selection (*SelectColor* function), we tested two following frequency priority rule:

- Lexicographic: the smallest available frequency is selected,
- Most used: the most used available frequency is selected. In case of a tie, we select the color c that maximizes the sum of margins $M(j,c)$ for all users $j \in Q$.

The proposed greedy algorithms run in $O(n^2C)$ time.

3.4 Beam decentring algorithm

To further improve the results from the ILP and greedy algorithm, we propose a subsequent non-linear local optimization, called beam decentring algorithm. This algorithm exploits the benefit of SDMA technology by moving a number of satellite beams from their center positions.

In fact the δ_{ij} and α_i in Equation (10) and (11) can be written as functions of user position and beam position which are

$$\delta_{ij} = D \cdot G_{Sat}(User\_u_i, User\_v_i, Beam\_u_j, Beam\_v_j), \tag{16}$$

$$\alpha_i = G_{Sat}(User\_u_i, User\_v_i, Beam\_u_i, Beam\_v_i) \cdot (1 - AD - BD). \tag{17}$$

The terms D and $(1-AD-BD)$ are constant. We will keep the user position fixed but alter the beam position; as a result, both δ_{ij} and α_i changes. Nonetheless, the change is non-linear as of the non-linear antenna gain shown previously in Figure 2.

Beam decentring algorithm (refer to *Algorithm 2*) takes the output solutions from either ILP or greedy algorithm as its input, identifies the rejected users, and, for each rejected user, moves the most k interfering beams and tries to reassign the user a color (frequency).

Let i denotes an unassigned user, the beam decentring algorithm selects (Step 5) a color c, *i.e.*, sets $x_{ic} = 1$, and identifies (Steps 6-7) a set of interferers S containing all users j having $x_{jc} = 1, \forall j \in S$ (unassigned user included). Let $K \subseteq S$ consists of a set of users whose beams will be moved. The parameter k defines the number of strongest interferers to the unassigned user i that are included in the set K. The parameter $UTVAR \in (0,1)$, if set to 1, tells the algorithm to replace the least interferer in the set K with i thus including i in the move.

$MAXINEG$ parameter provides a maximum negative margin from the required signal to noise ratio. It is based on the fact that the closer the unassigned user's signal to interference plus noise ratio is to the required signal to noise ratio, the more the possibility the algorithm has to search for a solution. Before the algorithm tries to move beams, the unassigned user is tested (Steps 8-9) with this margin (*LinkBudget* function). If failed, the remaining colors are tried or the user is rejected.

In Step 10, the algorithm continuously moves the beams of users in the set K from their center positions $(u_0^{(k)}, v_0^{(k)})$ and in each step evaluates if the new positions pass the link budget constraints (*Algorithm 3*). The problem we aim to solve can be represented as:

$$\min \sum_{k \in K} \| (u_0^{(k)} - u_k)^2 + (v_0^{(k)} - v_k)^2 \|^2, \tag{18}$$

subject to

$$\left(\frac{C}{N+I}\right)(u_k, v_k, u_0^{(k)}, v_0^{(k)}) \geq \left(\frac{C}{N}\right)_{Required} \qquad \forall k \in K. \tag{19}$$

When a beam is moved from its center, the associated user will obtain lower antenna gain and hence lower SINR. Any move that violates the link budget constraints (Equation 19) is rejected. Nonetheless, this move could benefit the unassigned user by reducing its tentative interference level. For a selected color c, the beam decentring algorithm minimizes the total squared distance of the moves of interferers' beams (Equation 18), maintains their interference constraints' validity, and reduces the tentative interference of the unassigned user i to the level that the reassignment is valid.

If a suitable move could not be found within a number of iterations defined by *MAXITER* each of the remaining colors is tried. If all colors have been tried and there is no possible solution, the user i is rejected and the algorithm moves to next unassigned users.

Figure 5 shows a result of beam decentring algorithm applied to the example presented previously in Section 2. It can be seen that the beam of the two interferers and the unassigned users are moved. This yields a reassignment of Color 1.

3.5 Closed-loop implementation

The ILP solver or the greedy algorithm would have more possibility to find the optimal solution or provide a better feasible solution if an initial feasible solution is given. Consider an iteration as a combination of ILP - Beam decentring algorithm

```
Input: C, User_u_i, User_v_i, Channel, α, δ, N, k, MAXINEG, UTVAR
Output: Channel, Beam_u_i, Beam_v_i
Beam_u_i ← User_u_i, ∀i = 1,...,n ;
Beam_v_i ← User_v_i, ∀i = 1,...,n ;
for i = 1 to n do
    if Channel_i = 0 then
        for color = 1 to C do
            u ← [User_u_j : Channel_j = color; User_v_j : Channel_j = color], ∀j = 1,...,n ;
            b ← [Beam_u_j : Channel_j = color; Beam_v_j : Channel_j = color], ∀j = 1,...,n ;
            ineg ← LinkBudget(u,b) ;
            if min(ineg) > MAXINEG then
                bool, bsol ← BeamMove(i,u,b,k,UTVAR) ;
                if bool = 1 then
                    Beam_u, Beam_v ← bsol ;
                    Channel_i ← color ;
                else
                    Channel_i ← 0 ;
                end
            end
        end
    end
end
```

Algorithm 2: Beam decentring algorithm

```
Input: i, u, b, k, UTVAR
Output: bool, bsol
d ← distance(b,i) ;
sort d ;
if UTVAR = 1 then
    x_0 ← [b_j; b_i], ∀j = 1,...,k−1 (according to ordering index d) ;
else
    x_0 ← [b_j], ∀j = 1,...,k (according to ordering index d) ;
end
while Iteration < MAXITER do
    solve (18) and (19) starting with x_0 ;
    Iteration ← Iteration + 1 ;
    if LinkBudget(u,x_0) > 0 then
        bool ← 1 ;
        Break ;
    end
end
bsol ← [b; x_0] ;
```

Algorithm 3: BeamMove function

or Greedy algorithm - Beam decentring algorithm. We propose the closed-loop implementation in that, in the next iteration of ILP or greedy algorithm, the frequency assignment result from beam decentring algorithm is used as an initial solution and the moved beam positions are used for recalculating the α_i and δ_{ij} values.

The ILP starts with the initial solution, continues to improve the solution, and by the given CPU time, outputs the best found solution. We implemented two variations of greedy algorithm. The first variation (Greedy 1) considers both the frequency assignment result and the updated α_i and δ_{ij} values and works further on the unassigned

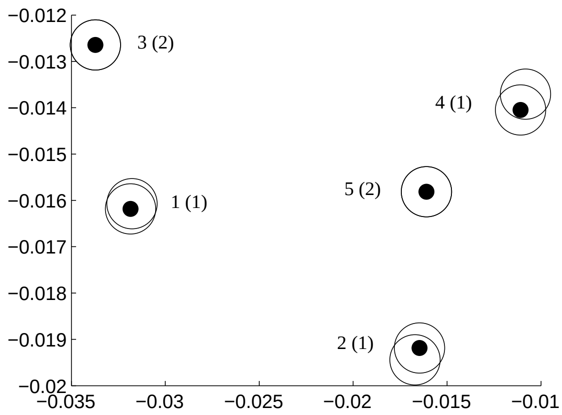


Fig. 5 An example on beam decentring.

users. The second variation (Greedy 2) only considers the updated α_i and δ_{ij} values and restarts the frequency assignment from scratch.

4 Computational experiments and results

The ILP formulation has been solved using IBM/ILOG CPLEX 12.2 [12]. The greedy algorithm has been coded in C++. We tested the proposed algorithms with $C = 8$; increasing stepwise the number of users by 20 from 20 to 200 users with 100 instances each. The user positions are randomly generated and uniformly distributed over the defined service area. All data are available for download on this website: `homepages.laas.fr/lhoussin/FAP/SDMA_Sat_FAP.htm`.

The results were obtained on a 2.7GHz Intel Core i5 machine with 4GB RAM. The CPU times for the ILP resolutions have been limited to 60s, 120s, and 180s after which the best integer solution is obtained. The CPU times for the greedy algorithm were negligible while the beam decentring was performed with the maximum of 40 iterations with no limitation on the calculation time.

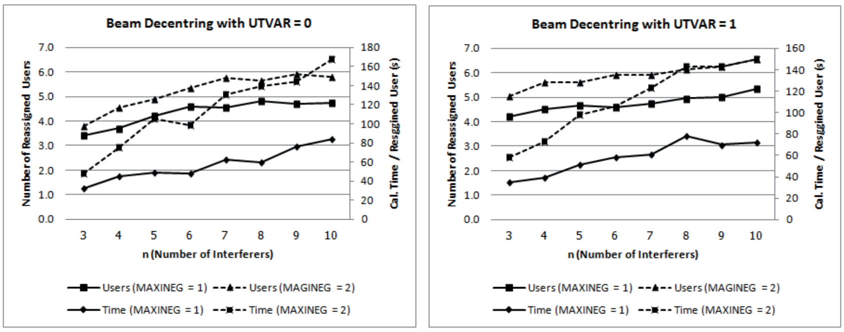
The beam decentring algorithm is coded in Matlab [17]. The function *fmincon* with active-set algorithm is used for computing the minimum of the non-linear program defined by equations (18) and (19)

We first present a comparison of the greedy algorithms. Table 3 reports the average number of accepted users over 1,000 instances. The results of the greedy algorithms are very close. It was difficult to give better results than the simple lexicographic rules. The algorithm that uses Hybrid and Most used rules gives the best result.Therefore, we use it as the baseline for performance comparison with the results from ILP and beam decentring.

We tested 36 configurations of *k-MAXINEG-UTVAR* for the beam decentring algorithm over 20 instances of 200 users. Test results are provided in Figure 6. It can

Table 3 Average number of accepted users over 1,000 instances.

Lexicographic (user + frequency)	85.30
Lexicographic (user) + Most used (frequency)	85.31
Hybrid (user) + Most used (frequency)	85.63



(a) (b)

Fig. 6 Average number of reassigned users and calculation time per reassigned user for different beam decentring configurations over 20 instances of 200 users with, (a) UTVAR=0 and (b) UTVAR=1.

Table 4 Number of optima provided by ILPs.

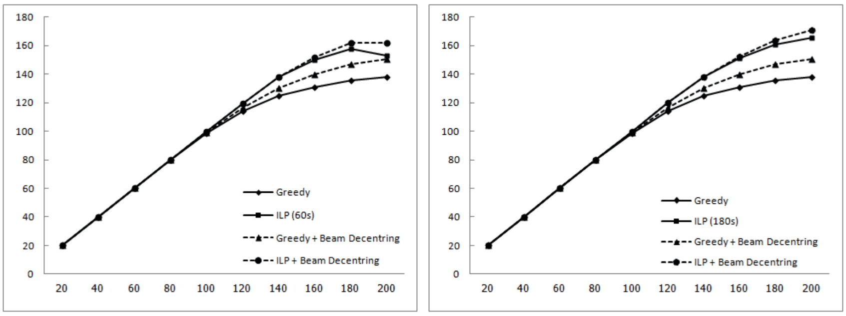
n	20	40	60	80	100	120	140	160	180	200
ILP60s	100	100	100	100	100	97	54	0	0	0
ILP120s	100	100	100	100	100	98	61	0	0	0
ILP180s	100	100	100	100	100	100	67	0	0	0

be seen that increasing any of k (from 3 to 10) or *MAXINEG* (from 1 to 2) or enabling *UTVAR* (0 or 1) yields higher number of reassigned users, at an expense of longer calculation time. Both configuration 7-2-0 and 6-2-1 provide good performances with acceptable calculation times. We choose configuration 7-2-0 for improving the results from the ILP and greedy algorithm through beam decentring.

Figure 7 displays, for each algorithm and number of users, the average number of accepted users in the computed frequency assignment plans. The number of optima provided by ILPs is given in Table 4. The greedy algorithm performs as good as the other two ILPs at up to 120 users (ILP can solve to optima for all or almost all of 100 instances up to this point). For 140-200 users, the performance gap becomes larger as the number of user increases. Performance degradation is found in ILP60s at 200 user instances, contrast to that of ILP180s. This signifies that, though not reaching the optima, the ILP needs more time for a larger instance to provide a better results.

Table 5 presents lower bounds and upper bounds for ILP180s. Large gaps signify that the ILP formulation yields poor relaxations.

Beam decentring gives performance improvement for both greedy algorithm and ILP. Significant improvements can be seen in the greedy algorithm case. It could provide comparable results at 200 users compared to ILP60s. Nonetheless, the algo-



(a) (b)

Fig. 7 Average number of accepted users before and after beam decentring for Greedy algorithm and (a) ILP 60s or and (b) ILP 180s.

Table 5 Average upper and lower bounds for ILP180s.

n	LB	UB	$\%(UB-LB)/UB$		
			min.	avg.	max.
120	119.79	119.81	0.00	0.02	1.67
140	138.17	139.18	0.00	0.71	3.76
160	151.07	158.21	1.25	4.46	7.50
180	160.69	177.19	5.06	9.25	13.22
200	165.22	194.36	9.33	14.90	23.59

Table 6 Average calculation time (s) performed by Beam decentring algorithm.

n	20	40	60	80	100	120	140	160	180	200
Greedy	-	-	-	9.19	22.85	67.60	241.67	570.69	1017.28	1542.53
ILP60s	-	-	-	-	-	13.57	29.65	125.26	365.21	1032.01
ILP180s	-	-	-	-	-	-	28.40	114.91	272.85	622.00

rithm's calculation time is high, see Table 6, it could be strongly reduced by using a compiled code (c++ or Fortran) with a call to Ipopt library[2] for example.

The results for closed-loop simulations are shown in Table 7. Greedy 1 continuously improves the solutions over the iterations and approaches saturation after Iteration 3. Degraded performance is found for Greedy 2 in ILP Iteration 2 and 3. These are caused by restarting frequency assignment from scratch. For both ILPs, small improvement can be seen in the second iteration but no improvement in the third. ILPs converge to the saturation faster than Greedy algorithms.

5 Conclusion and perspective

In this paper we have developed integer linear programming formulation, greedy algorithms and non-linear continuous algorithms for Frequency Assignment Problems involving cumulative interference. The greedy algorithm, though simple, but is very

[2] `http://www.coin-or.org/projects/Ipopt.xml`

Table 7 Average percentage of accepted users over 100 instances of 200 users.

	Iteration 1		Iteration 2		Iteration 3	
	ILP	BD *	ILP	BD	ILP	BD
Greedy 1	69.15	75.29	76.05	76.05	76.20	76.20
Greedy 2	69.15	75.29	70.27	71.71	70.94	72.37
ILP 60s	76.53	81.05	81.58	81.84	81.84	-
ILP 180s	82.66	85.49	85.53	85.53	85.53	-
No. ** (Greedy 1)	-	100	73	24	24	1
No. ** (Greedy 2)	-	100	7	93	19	93
No. ** (60s)	-	100	14	13	0	-
No. ** (180s)	-	100	4	3	0	-

* (Beam decentring), ** (Number of improved solutions)

fast and efficient enough to provide comparable results to ILP up to a certain number of users. The beam decentring algorithm, utilising SDMA benefits, offers performance improvement for both ILP and greedy algorithm; the latter gains significant improvement. Closed-loop implementation provides further improvement yet marginal. To improve these results, an integrated approach where frequency assignment and beam position are determined simultaneously and not sequentially, could be proposed. This yields highly complex mixed non-linear integer programming formulations. As a short term follow-up, the closed loop implementation solves the integrated problem as a hill-climbing method. More improvements could be reached by allowing temporary decrease of the objective functions via metaheuristic framework such as tabu search. Better upper bound techniques could also be helpful to stop the search earlier.

We have considered frequency assignment problems based on single frequency over a total period of time. We can further generalize the problem in both domains in that a user could occupy more than one frequency over a fraction of time. The problem with frequency demand of cardinality n but fixed in time could be treated as 1-dimensional bin packing problem with additional constraints on cumulative interference between different bins. Further generalization on time gives rise to 2-dimensional bin packing problem with cumulative interference constraints between different bins based on overlapping of $frequency \times time$.

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