Robust control of a bimorph mirror for adaptive optics system

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Abstract : This paper aims at applying robust control technics for an adaptive optics system including a dynamic model of the deformable mirror. The dynamic model of the mirror is a modification of the usual plate equation. An H_∞ controller is designed in an infinite dimensional setting. Due to the multivariable nature of the control problem involved in adaptive optics system, a significant improvement is expected with respect to traditional single input single output methods. A potential application concerns adative optics system with very large telescopes.

Keywords : robust control, bimorph mirror, adaptive optics, partial differential equations.

AMS Classification : 35B37, 93B36, 93C20, 93D09.

1 Introduction

The technological developments of the eighties have made possible the use of adaptive optics systems to actively correct the wave front distortions affecting an incident wavefront propagating through a turbulent medium. A particularly interesting application of this technique is in the field of astronomical ground-based imaging.

Making its way through the earth's atmosphere to a telescope, beams of light from some remote star are disturbed by atmospheric turbulence. An initially plane wave front is therefore distorted by random spatial and temporal perturbations induced by turbulence in various layers of the atmosphere. This phenomenon produces aberrated images whose precise analysis is not possible. The idea behind adaptive optics systems is to generate a corrected wavefront as close as possible to the genuine incident plane wavefront thanks to a deformable mirror. An adaptive optics system is principally based on a wavefront sensor measuring the resulting distortion of the wave front after correction by the deformable mirror. Based on these measured signals, a control law is computed in

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order to reshape a deformable mirror through piezoelectric actuators. This deformable mirror is made of a swing mirror dedicated to the correction of the tilts of the wavefront in two dimensions and a deformable mirror that is part of the control-loop for the correction of higher-order modes of aberrated wave-front. Different types of sensors (curvature sensor, pyramid wave-front sensor) may be used to estimate the distortions affecting the incoming wave-front but the most common encountered in existing application is the Shack-Hartmann sensor. For additional details on basic principles of adaptive optics, the interested reader may have a look at reference [18].

This paper is devoted to the design of specific control laws for an adaptive optics system formed by a bimorph mirror and a Shack-Hartmann sensor (see Figure 1). Most often, the existing adaptive optics systems use static simple models and very basic control algorithms only based on physical insights. Here, our goal is to consider the design of an adaptive optics system from a modern automatic control point of view [7]. This means first that dynamics of the different elements involved in the control-loop have to be taken into account. In particular, a specific dynamic model for the deformable mirror is proposed for control purpose [15].

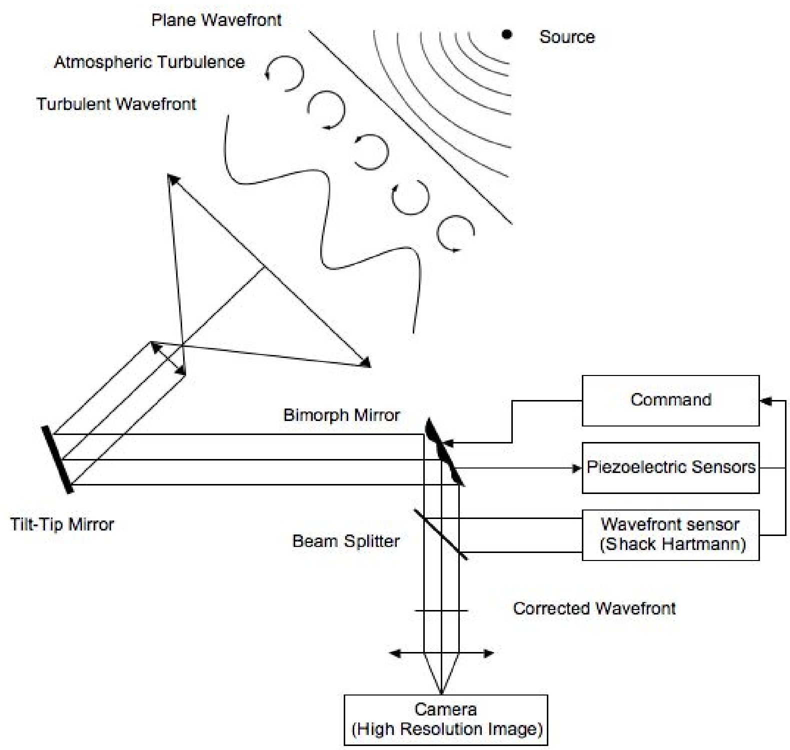

Figure 1: An adaptive optics system

Moreover, in the literature, only finite dimensional models are considered, that fits well the situation where the number of actuators and sensors is small. Here, we deal with a partial differential equation model corresponding to an infinite dimensional setting. A potential application indeed concerns adaptive optics involved in the so called *very large telescopes*, equipped with a large distribution of sensors and actuators.

In reference [12], a thin elastic plates model of a deformable bimorph mirror is derived thanks to the two-scale convergence analyzed in [1]. This model is based on a periodic distribution of embedded piezoelectric patches that may be used as sensors or actuators. The idea is then to elaborate a robust control strategy based on modern control tools developed during the last years and extended to the control of distributed parameter systems [21].

In this article, the control problem is recast in an H_∞ control setting. The first motivation is that H_∞ control theory is now well established in the infinite dimensional case and H_∞ controller provides intrinsic properties of robustness while optimizing on the worst-case performance. Another motivation is the multivariable nature of the control problem involved in adaptive optics system design [7]. Current adaptive optics control systems use decoupling modal control to rewrite the original problem as many decoupled single input single output control problems. We therefore propose to take advantage of the standard H_∞ control configuration leading to a natural mutivariable setting. Because H_∞ control framework may easily handle a multivariable dynamic model of the bimorph deformable mirror in the synthesis process, the obtained robust controller is expected to outperform usual static control approaches of the literature.

The outline of the paper is the following. First, the adaptive optics control system is described. A particular emphasis has been put on the exposition of the two dimensions bimorph mirror model. The third section is dedicated to the robust H_∞ control setting in the infinite dimension case and its formulation in our particular case.

2 The Model

2.1 Adaptive optics model

We are concerned with the control of the Bimorph Mirror which is composed of a purely elastic and reflective plate in contact with a piezoelectric plate, equipped with piezoelectric actuators (in order to deform the shape of the mirror) and piezoelectric sensors (to measure the effective deformation). A Shack-Hartmann sensor then analyzes the resulting phase Φ_{res} of the wavefront, after reflection in the deformable mirror of the turbulent phase Φ_{tur}.

In this kind of application, different types of disturbances have to be faced: w_{mod} represents unstructured uncertainty (neglected dynamics) affecting the model, w_{piezo} and w_{SH} are noise signals respectively attached to piezoelectric and Shack-Hartmann's sensors. Finally, Φ_{tur} is the turbulent phase of the wavefront introduced by the atmospheric perturbation.

We denote by e the transverse displacement of the mirror, λ the wavelength. The corrected phase produced by e is given by $\Phi_{cor} = \frac{4\pi}{\lambda} e$ leading to a resulting phase Φ_{res}:

$$\Phi_{res} = -\frac{4\pi}{\lambda} e + \Phi_{tur} \tag{1}$$

Noting D the interaction matrix between the Shack-Hartmann sensor and the resulting phase it analyzes [13], [18], the optic sensor's output, computed with the Shack-Hartmann sensor, is given by:

$$y_{SH} = -\frac{4\pi}{\lambda} De + D\Phi_{tur} + w_{SH}. \tag{2}$$

Finally, we note that the control input is the voltage φ_c applied to the piezoelectric actuators and the corresponding piezoelectric output is the voltage y_{pe} measured with the piezoelectric inclusions used as sensors. Indeed, in comparison with many other devices (see [13] for instance), where the only information used to compute the voltage φ_c comes from the wavefront analyzer, we consider here the possibility of measuring the deflection of the mirror through a layer of piezoelectric sensors (see Figure 1).

The goal of the adaptive optics control system is to minimize the resulting phase of the wavefront using Shack-Hartmann measurements.

2.2 Two dimensions bimorph mirror model

To obtain the model of a bimorph mirror as proposed in the previous section, we consider three layers. One is purely elastic, the second one is equipped with piezoelectric inclusions used as actuators, the third one is equipped with piezoelectric inclusions used as sensors. The heterogeneities are periodically distributed. In reference [12], the authors derive the partial differential equation model presented below by making the period tends to 0. Therefore, we consider the following dynamical model of the mirror

$$\rho\, \partial_{tt}^2 e(x,t) + Q_1 \Delta^2 e(x,t) + Q_2 e(x,t) = \widetilde{d}_{31} \Delta\varphi_c(x,t) + bw_{mod}(x,t), \tag{3}$$

whereas the voltage y_{pe} computed by the piezoelectric sensors is given by

$$y_{pe}(x,t) = \tilde{e}_{31} \Delta e(x,t) + dw_{pe}(x,t). \tag{4}$$

This model is completed with the initial conditions:

$$e(x, t=0) = e_0(x) \tag{5}$$
$$\partial_t e(x, t=0) = e_1(x) \tag{6}$$

and some boundary conditions, e.g. the free case

$$\Delta e(x,t) = 0,\ \nabla\Delta e(x,t) \cdot \nu(x) = 0\ ,\ \forall x \in \partial\Omega\ . \tag{7}$$

where the following notations were used:

- x is the spatial coordinate, t is time, Ω is a disc of radius a and ν is the normal unit outward vector on $\partial\Omega$,
- e is the transverse displacement of the mirror;
- φ_c is the voltage applied to the inclusions of the actuator layer;
- y_{pe} is the voltage measured with the inclusions of the sensor layer;
- ρ is the surface density, Q_1 is the stiffness coefficient of the mirror, and Q_2 is a correction coefficient;
- $\tilde{e}_{31}$ and $\widetilde{d}_{31}$ are proportional to the piezoelectric tensor coefficient d_{31} (for more physical details, see [11] or [16]).
- b and d are two coefficients in $\mathcal{L}(L^2(\Omega))$;
- w_{mod} and w_{pe} are two unknown perturbations modelling the model errors of the plate equation and the measurement noise of the piezoelectric output.

3 Robust Control Results

3.1 H_∞ control with measurement-feedback

We recall in this subsection a simplified version of the main result we apply on the partial differential equations model. It relies on H_∞-control with measurement-feedback. We explain in the next subsection how this theorem can be applied to the model we consider, following the reasoning presented in [22].

For a survey of the H_∞-control theory with state-feedback for the infinite-dimensional case, see [5] or [2]. In [20], the author deals with dynamic measurement-feedback for the same class of linear infinite-dimensional systems and using also the state-space approach. The main result is a generalization of the finite-dimensional regular H_∞-control problem (see for instance [6] and [19]). In particular, the solution will be given in terms of the solvability of two coupled Riccati equations.

Let A be the infinitesimal generator of a C_0-semigroup $T(.)$ on a real separable Hilbert space X and let $B_1 \in \mathcal{L}(W,X)$, $B_2 \in \mathcal{L}(U,X)$, $C_1 \in \mathcal{L}(X,Z)$, $D_{12} \in \mathcal{L}(U,Z)$, $C_2 \in \mathcal{L}(X,Y)$ and $D_{21} \in \mathcal{L}(W,Y)$, where U, W, Z and Y are also real separable Hilbert spaces. We consider the system given by, $\forall t \geq 0$,

$$\left\{ \begin{array}{rcl} x(t) & = & T(t)x_0 + \displaystyle\int_0^t T(t-s)\left(B_1 w(s) + B_2 u(s)\right) ds, \\ z(t) & = & C_1 x(t) + D_{12} u(t), \\ y(t) & = & C_2 x(t) + D_{21} w(t), \end{array} \right. \tag{8}$$

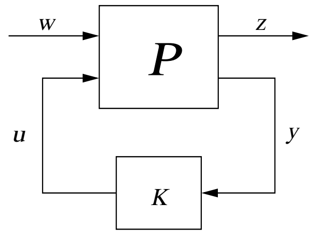

Figure 2: Closed-loop system

where $x(t) \in X$ is the state of the system, x_0 is the initial condition, $u(t) \in U$ is the control input, $w(t) \in W$ is the disturbance input, $y(t) \in Y$ is the measured output and $z(t) \in Z$ is the controlled output.

The aim is to find a dynamic measurement-feedback controller that exponentially stabilizes this system and ensures that the influence of w on z is smaller than some specific bound. The dynamic measurement-feedback is assume to have the following form:

Let M be the infinitesimal generator of a C_0-semigroup $V(.)$ on a real separable Hilbert space H, $N \in \mathcal{L}(Y,H)$, $L \in \mathcal{L}(H,U)$, $R \in \mathcal{L}(Y,U)$. The controller, which we denote by $\mathcal{K}$, is given by $\forall t \geq 0$,

$$\left\{ \begin{array}{rcl} p(t) & = & V(t)p_0 + \displaystyle\int_0^t V(t-s)Ny(s)\,ds, \\ u(t) & = & Lp(t) + Ry(t), \end{array} \right. \tag{9}$$

where $p_0 \in H$ is the initial condition.
With this controller, the closed-loop system can easily be derived and with $x_0 = 0$ and $p_0 = 0$, it defines a bounded linear map $S_{\mathcal{K}}$ from $L^2(0,\infty;W)$ to $L^2(0,\infty;W)$ such that $z(t) = (S_{\mathcal{K}}w)(t)$. Its bound is denoted $||S_K||_\infty$.

We make the following assumptions:

- There exists $\varepsilon > 0$ such that for all $(\omega, x, u) \in \mathbb{R} \times D(A) \times U$ satisfying $i\omega x = Ax + B_2 u$, there holds

$$\|C_1 x + D_{12} u\|_Z^2 \quad \geq \quad \varepsilon \|x\|_X^2, \tag{10}$$
$$D_{12}^*[C_1 \quad D_{12}] \quad = \quad [0 \quad I]. \tag{11}$$

- There exists $\varepsilon > 0$ such that for all $(\omega, x, u) \in \mathbb{R} \times D(A^*) \times Y$ satisfying $i\omega x = A^* x + C_2^* u$, there holds

$$\|B_1^* x + D_{21}^* y\|_W^2 \quad \geq \quad \varepsilon \|x\|_X^2, \tag{12}$$
$$D_{21}[B_1^* \quad D_{21}^*] \quad = \quad [0 \quad I]. \tag{13}$$

Theorem 1. [20] *Let $\gamma > 0$ and assume that the assumptions* $(10)-(13)$ *hold. There exists an exponentially stabilizing dynamic output-feedback controller $\mathcal{K}$ of the form* (9) *with $\|S_{\mathcal{K}}\|_\infty < \gamma$ if and only if there exist two nonnegative definite operators P, $Q \in \mathcal{L}(X)$ satisfying the three conditions*

(i) $\forall x \in D(A)$, $Px \in D(A^*)$,

$$\big(A^*P + PA + P(\gamma^{-2}B_1B_1^* - B_2B_2^*)P + C_1^*C_1\big)x = 0 \tag{14}$$

and $A + (\gamma^{-2}B_1B_1^* - B_2B_2^*)P$ generates an exponentially stable semigroup,
(ii) $\forall x \in D(A^*)$, $Px \in D(A)$,

$$\big(AQ + QA^* + Q(\gamma^{-2}C_1^*C_1 - C_2^*C_2)Q + B_1B_1^*\big)x = 0 \tag{15}$$

and $A^* + (\gamma^{-2}C_1^*C_1 - C_2^*C_2)Q$ generates an exponentially stable semigroup,
(iii)

$$r_\sigma(PQ) < \gamma^2,$$

where $r_\sigma(PQ)$ stands for the spectral radius of PQ. In this case, the controller $\mathcal{K}$ given by (9) *with $H = X$ and*

$$\begin{array}{rcl} M &=& A + (\gamma^{-2}B_1B_1^* - B_2B_2^*)P - Q(I - \gamma^{-2}PQ)^{-1}C_2^*C_2 \\ N &=& -Q(I - \gamma^{-2}PQ)^{-1}C_2^* \\ L &=& B_2^*P \\ R &=& 0 \end{array} \tag{16}$$

is exponentially stabilizing and guarantees that $\|S_{\mathcal{K}}\|_\infty < \gamma$. Finally, if the solutions to the Riccati equations exists, then they are unique.

Remark: Assumptions (10) and (12) are the infinite-dimensional analogue of the weakest assumptions under which the regular finite-dimensional version of the H_∞-problem has been solved (see [6]). Moreover, (11) and (13) can be replaced by the assumption that $D_{12}^*D_{12}$ and $D_{21}D_{21}^*$ are coercive leading therefore to more involved Riccati equations (see [22] for details). Finally, if the operators defining the system are not bounded, there is also a proof of a similar result in [21] where the controller parameterization is slightly different.

3.2 Application of infinite-dimensional setting to adaptive optics

A linear infinite-dimensional model derived from the partial differential equation model presented in Section 2.2 will be used in the sequel. In order to fit in the formalism presented in the previous subsection, the following notations are introduced:

- the state vector $x = (e, e')$ where e is the transverse displacement of the plate.

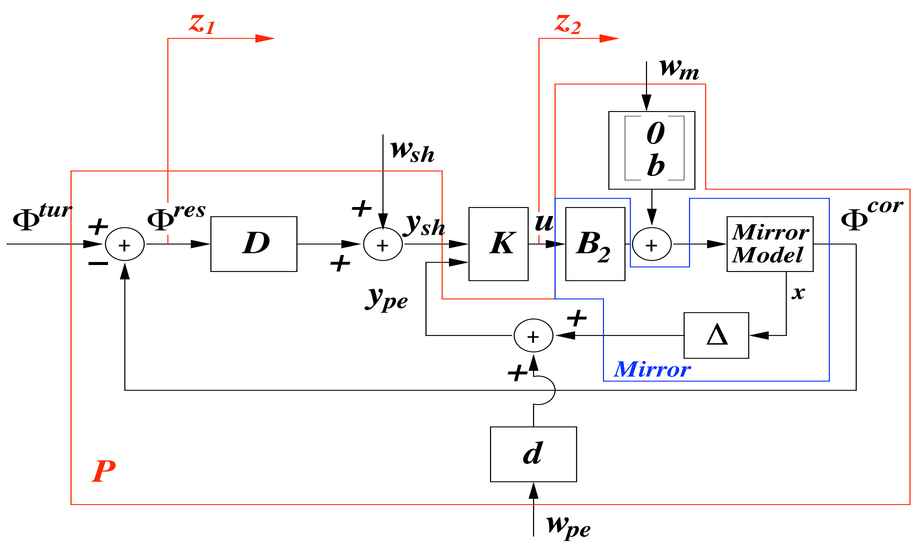

Figure 3: Standard model for adaptive optics system control loop

- the exogenous disturbance inputs vector $w = (w_{mod}, w_{SH}, \Phi_{tur}, w_{pe})$ gathers the different types of perturbations (uncertainty affecting dynamics of the model, noise vectors of the wavefront analyzer and of piezoelectric sensors, the atmospheric turbulence phase)

- the control inputs vector u is defined as the voltage φ_c applied to piezoelectric patches.

- the measurement outputs vector $y = (y_{pe}, y_{SH})$ is composed with the piezoelectric and the wavefront analyzer measured outputs.

- the controlled outputs vector $z = (\Phi_{res}, u)$ contains an optical part (the resulting phase of the wavefront after correction by the deformable mirror and comparison with the turbulence phase) and the control input vector u.

The block diagram of the complete adaptive optics system is sketched in Figure 3.

Therefore, the operators defining the standard form:

$$\begin{cases} x' = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w \end{cases} \tag{17}$$

are built from the optic variables computed with (1)-(2) and the model of the mirror (3)-(4) i.e.

$$A = \begin{pmatrix} 0 & I \\ -\dfrac{Q_1}{\rho}\Delta^2 - \dfrac{Q_2}{\rho} I & 0 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ \dfrac{\tilde{d}_{31}\Delta}{\rho} \end{pmatrix},$$

$$C_1 = \begin{pmatrix} -\dfrac{4\pi}{\lambda} I & 0 \\ 0 & 0 \end{pmatrix}, \quad D_{11} = \begin{pmatrix} 0 & 0 & D & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad D_{12} = \begin{pmatrix} 0 \\ I \end{pmatrix},$$

$$C_2 = \begin{pmatrix} \tilde{e}_{31}\Delta & 0 \\ -\dfrac{4\pi}{\lambda} D & 0 \end{pmatrix}, \quad D_{21} = \begin{pmatrix} 0 & 0 & 0 & d \\ 0 & I & D & 0 \end{pmatrix}.$$

The appropriate functional spaces associated to the infinite-dimensional model are now precisely defined. With the boundary (7), we consider the state space

$$X = H^2_{bc}(\Omega) \times L^2(\Omega) = \left\{ e \in H^2(\Omega), \Delta e|_{\partial\Omega} = 0 \ , \ \nabla\Delta e \cdot \nu|_{\partial\Omega} = 0 \right\} \times L^2(\Omega)$$

and the output and input spaces $W = \left(L^2(\Omega)\right)^4$, $U = H^2(\Omega) \cap H^1_0(\Omega)$ and $Y = Z = \left(L^2(\Omega)\right)^2$.

We will actually prove that A is the infinitesimal generator of a C_0-semigroup on the real separable Hilbert space X. Indeed, consider the unbounded linear operator

$$A_1 : \mathcal{D}(A_1) \to X, \quad A_1 \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} = \begin{pmatrix} x^1 \\ -\Delta^2 x^0 \end{pmatrix}$$

where $\mathcal{D}(A_1) = \left\{ x^0 \in H^4(\Omega), \Delta x^0|_{\partial\Omega} = 0 \ , \ \nabla\Delta x^0 \cdot \nu|_{\partial\Omega} = 0 \right\} \times H^2(\Omega)$.
A simple calculation gives that A_1 is dissipative :

$$\left\langle A_1 \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}, \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} \right\rangle_X \leq 0$$

Moreover, one can easily check that A_1 is also self-adjoint and onto.
Therefore, from Lumer-Phillips' Theorem (see [17], p15), A_1 generates a continuous semigroup of linear contractions acting on X. And finally, since A is the sum of A_1 and of a linear operator bounded on X, from reference [10] (for instance), A is the infinitesimal generator of a C_0-semigroup on X.

In addition, B_1, B_2, C_1, D_{11}, D_{12}, C_2 and D_{21} are bounded operators (mainly thanks to Poincaré's inequality) well defined in the corresponding spaces allowing us to apply Theorem 1.

Nevertheless, we want to precise how a transformation of this measurement-feedback problem satisfies appropriate hypothesis (not precisely the one of Theorem 1) to build a controller from the solutions of coupled Riccati equations. Indeed, one can see that Theorem 1 does not take into account the presence of a direct feedthrough from the disturbance w to the controlled output z, that brings an operator D_{11} in the system. Taking $D_{11} = 0$ is actually a classical simplification in robust control literature (see [20], [22]). Nevertheless, it is possible to eliminate D_{11} (see [21, pages 165-173]), and to compute a controller using Theorem 1 and then deduce the right controller from a "backward" transformation. Furthermore, there exist numerical algorithms to take such situations directly into account.

4 A truncated model for numerical design

4.1 Truncation

This section deals with the two dimensions in space situation. The corresponding finite dimensional model can be presented as :

$$\left\{ \begin{array}{l} x'_N = A_N x_N + B_{1N} w_N + B_{2N} u_N \\ z_N = C_{1N} x_N + D_{11N} w_N + D_{12N} u_N \\ y_N = C_{2N} x_N + D_{21N} w_N \end{array} \right. \tag{18}$$

where the operators of system (17) have been replaced by real-valued matrices computed on a truncated basis of the $2N$ first eigenfunctions precisely defined below. $x_N \in \mathbb{R}^{4N}$ is the state vector, $w_N \in \mathbb{R}^{8N}$ is the exogenous perturbation vector, $u_N \in \mathbb{R}^{2N}$ is the control vector, $z_N \in \mathbb{R}^{4N}$ is the controlled output vector and $y_N \in \mathbb{R}^{8N}$ is the measured output vector. The matrices $A_N \in \mathcal{M}_{4N\times 4N}$, $B_{1N} \in \mathcal{M}_{4N\times 4N}$, $B_{2N} \in \mathcal{M}_{4N\times 2N}$, $C_{1N} \in \mathcal{M}_{4N\times 4N}$, $D_{11N} \in \mathcal{M}_{4N\times 8N}$, $D_{12N} \in \mathcal{M}_{4N\times 2N}$, $C_{2N} \in \mathcal{M}_{4N\times 4N}$, $D_{21N} \in \mathcal{M}_{4N\times 8N}$ are of appropriate dimensions.

In order to compute these objects, we consider the case of a circular bimorph mirror which is free at all the boundary (this is the case of the mirror considered in Section 4.2 below). The radius is denoted by a. The eigenvectors of the operator $-\frac{Q_1}{\rho}\Delta^2 - \frac{Q_2}{\rho} I$ are given by, for all $(k, j) \in \mathbb{N}^2$,

$$\begin{aligned} L_{kj}(r, \theta) &= J_k\left(\tfrac{\lambda_{kj} r}{a}\right)\cos(k\theta) \\ M_{kj}(r, \theta) &= K_k\left(\tfrac{\lambda_{kj} r}{a}\right)\cos(k\theta) \end{aligned}$$

where (r, θ) is the polar coordinates of $x \in \Omega$, J_k (respectively K_k) is the Bessel function (resp. modified Bessel function) of first kind and order k, and λ_{kj} is a dimensionless coefficient depending of the boundary conditions (see e.g. [3]).

Straightforward computations yield (see e.g. [8]), for each $(k, j) \in \mathbb{N}^2$ and for $(r, \theta) \in \Omega$,

$$\Delta L_{kj}(r, \theta) = -\left(\tfrac{\lambda_{kj} r}{a}\right)^2 L_{kj}(r, \theta) \ , \tag{19}$$

$$\Delta M_{kj}(r, \theta) = \left(\tfrac{\lambda_{kj} r}{a}\right)^2 M_{kj}(r, \theta) \ . \tag{20}$$

The sequence of functions L_{kj} and M_{kj} might be ordered by following the order of λ_{kj}. Using this order $(\lambda_n)_{n\geq 1}$, these functions are re-labelled by L_1, M_1, L_2, M_2... and therefore,

$$\forall x \in X,\ x = \sum_{n\in\mathbb{N},\ n\geq 1} \alpha_n L_n(r,\theta) + \beta_n M_n(r,\theta)$$

where $(\alpha_n)_{n\in\mathbb{N},\ n\geq 1}$, and $(\beta_n)_{n\in\mathbb{N},\ n\geq 1}$ are two sequences of real numbers satisfying $\sum_{n\in\mathbb{N},\ n\geq 1} \alpha_n^2 + \beta_n^2 < \infty$.

Given $N \in \mathbb{N}$, we compute A_N, B_{1N}, B_{2N}, C_{1N}, C_{2N}, D_{11N}, D_{12N} and D_{21N} using the truncated basis $\{L_0, M_0, L_1, M_1, \ldots, L_N, M_N\}$. As claimed in [13], the matrix D is close to a diagonal matrix $D = \mathrm{diag}_n(D_n)$, where $(D_n)_{n\in\mathbb{N},\ n\geq 1}$ is a sequence of real numbers. We assume analogous assumptions for b and d, i.e. $b = \mathrm{diag}_n(b_n)$, and $d = \mathrm{diag}_n(d_n)$ where $(b_n)_{n\in\mathbb{N},\ n\geq 1}$, and $(d_n)_{n\in\mathbb{N},\ n\geq 1}$ are two sequences of real numbers. In particular, this gives with (19) and (20)

$$A_N = \mathrm{diag}_n\left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_n^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_n^2 & 0 \end{bmatrix}\right)$$

$$B_{1N} = \mathrm{diag}_n\left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_n & 0 & 0 & 0 \end{bmatrix}\right)$$

$$B_{2N} = \mathrm{diag}_n\left(\begin{bmatrix} 0 & 0 \\ -\frac{\tilde{d}_{31}}{\rho}\left(\frac{\lambda_n}{a}\right)^2 & 0 \\ 0 & 0 \\ 0 & \frac{\tilde{d}_{31}}{\rho}\left(\frac{\lambda_n}{a}\right)^2 \end{bmatrix}\right)$$

$$C_{1N} = \mathrm{diag}_n\left(\begin{bmatrix} -\frac{4\pi}{\lambda} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{4\pi}{\lambda} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right)$$

$$C_{2N} = \mathrm{diag}_n\left(\begin{bmatrix} -\tilde{e}_{31}\left(\frac{\lambda_n}{a}\right)^2 & 0 & 0 & 0 \\ -\frac{4\pi}{\lambda} D_n & 0 & 0 & 0 \\ 0 & 0 & \tilde{e}_{31}(\frac{\lambda_n}{a})^2 & 0 \\ 0 & 0 & -\frac{4\pi}{\lambda} D_n & 0 \end{bmatrix}\right)$$

$$D_{11N} = \mathrm{diag}_n \left(\begin{bmatrix} 0 & 0 & D_n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right)$$

$$D_{12N} = \mathrm{diag}_n \left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$D_{21N} = \mathrm{diag}_n \left(\begin{bmatrix} 0 & 0 & 0 & d_n & 0 & 0 & 0 & 0 \\ 0 & 1 & D_n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_n \\ 0 & 0 & 0 & 0 & 0 & 1 & D_n & 0 \end{bmatrix} \right)$$

where $\omega_n^2 = \frac{Q_1}{\rho} \left(\frac{\lambda_n}{a} \right)^4 + \frac{Q_2}{\rho}$.

4.2 Preliminary numerical experiments

In this subsection, preliminary numerical experiments are proposed. Our goal is to show that for a moderate size of the truncated state vector, an H_∞ dynamical output feedback may be computed using the standard MATLAB©routine hinfsyn. To get more realistic results, we consider the experimental device of the project SESAME of the Observatoire de Paris, France. This experimentation uses a bimorph mirror with a distribution of 31 piezoelectric actuators and without any sensor inclusions. The piezoelectric inclusions are PZT patches (see the physical properties in [9] e.g.). The wavefront analyzer computes 56 output variables. We use the following physical constants:

- radius of the mirror: $a = 82 \times 10^{-3}$ m
- stiffness coefficients: $Q_1 = 84$ Nm and $Q_2 = 9.74 \times 10^8$ Nm^{-3}
- surfacic density: $\rho = 16.3$ kg.m^{-2}
- piezoelectric modulus: $\tilde{d}_{31} = -0.0044$ NV^{-1}
- Vector of coefficients λ_n: $\Lambda = \begin{bmatrix} 5.253 & 9.084 & 12.23 & 20.52 \end{bmatrix}$

The numerous numerical experiments conducted with those data have shown that by choosing $b = \mu_1 \mathbf{1}_N$, $D = \mu_2 \mathbf{1}_N$, $d = \mu_3 \mathbf{1}_N$ and playing on μ_i, $\forall\, i = 1, 2, 3$ allows to reduce the closed-loop H_∞ performance level to $\gamma_\infty = 1$ no matter the dimension of the state vector is. As is shown by the map of the poles and zeros of the closed-loop at figure 4, the H_∞ controller slightly shifts the open-loop poles to internally stabilize the interconnection. Of course, this result cannot be considered as satisfying from the optics adaptive point of view. A more complete and elaborate control strategy has to be defined in order to be sufficiently efficient.

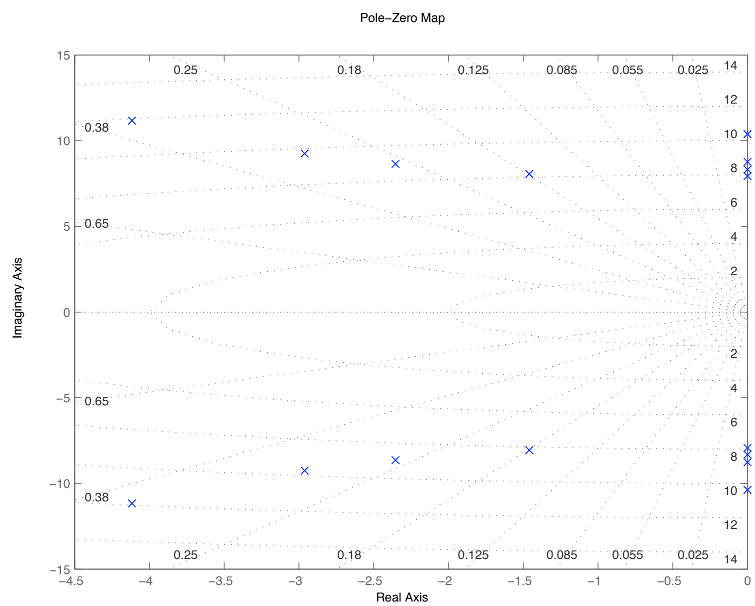

Figure 4: Poles-zeros location of the closed-loop system

5 Conclusion

In this paper, a new framework to deal with the problem of adaptive optics is proposed. It is mainly based on an infinite-dimensional model of the deformable mirror associated with the definition of a standard model on which robust control techniques may be applied. The preliminary numerical experiments show the applicability of the proposed approach. Of course, many points have to be tackled. The first one is related to the modelling of the dynamics of the atmospheric turbulence and to the modelling of the Shack-Hartman sensor (possibly including measurement delay). Another important issue concerns the order of truncation needed to adequately represent the dynamics of the deformable mirror. Once a realistic and tractable model of the adaptive optics control loop is available, it remains to define precise control objectives and the related robust control strategy. All these points will be addressed in future works.

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