A review of the book "Functional analysis, calculus of variations and optimal control" by Francis Clarke, Graduate Texts in Mathematics 264, Springer, London, 2013.

This book deals with functional analysis (part I), optimization (part II), calculus of variations (part III) and optimal control (part IV). This outline is motivated by the observation that functional analysis was developed in the late 19th century to consolidate the mathematical foundations of the calculus of variations, a branch of mechanics and physics initiated in the late 17th century and that consists of minimizing an integral functional, like an energy, a distance, or the area of a surface, subject to constraints. Optimization, and especially nonsmooth analysis, as well as optimal control, can then be seen as natural extensions of these developments, mainly carried out in the 1960s, and largely contributing to the practical solutions of problems coming e.g. from the aerospace industry.

The author, F. Clarke, is a key contributor to the field of optimization and optimal control theory. As a former student of R. T. Rockafellar, he pioneered the development of nonsmooth analysis in the mid 1970s. Since then, through years of teaching and research experience, he has produced and collected a considerable amount of technical material, and this book can be seen as a culmination of his efforts to disseminate basic, as well as more advanced, concepts of nonsmooth analysis.

Convexity plays a prominent role in the book, and especially in Part I, where convex sets and functions are introduced quite early (in Chapters 2 and 4, before Banach, Lebesgue and Hilbert spaces), thereby contrasting with many other textbooks of functional analysis. Variational principles, and conditions of extremality, can also be found everywhere in the book. For example, J. E. Littlewood's famous three principles of analysis (namely, every measurable function is nearly continuous, every convergent sequence of functions is nearly uniformly convergent, and every measurable set is nearly a finite union of intervals) are completed in Section 5.2 by I. Ekeland's variational principle which states, roughly speaking, that every function that is bounded below nearly attains a minimum.

The book culminates with the extended (nonsmooth hybrid) maximum principle for optimal control, a significant outgrowth of the original result from Pontryagin and followers, which formulates necessary conditions for optimality of a trajectory. The result is stated in Chapter 22, and the technically difficult proof (based on nonsmooth analysis) is relegated to the end of the book in the final Chapter 25. Sufficient conditions for optimality, and the use of verification functions and the connection with value functions solving Hamilton-Jacobi partial differential equations, are nicely surveyed in Chapter 19.

The author's preference for mathematical tools of nonsmooth analysis is pervasive throughout the book. Sometimes, one may regret that alternative routes are not followed, let alone sketched. For example, weak-star topology is addressed in Section 3.3, but measures (as dual objects to continuous functions) are never defined. They are only indirectly mentioned on page 53 as pointwise evaluation functionals. Similarly, the use of measures as relaxed controls is only very briefly mentioned in a footnote on page 477, in the context of convexified differential inclusions, whereas measure-valued controls, and more generally controls in dual Banach spaces, are (to this reviewer's taste) a key technical ingredient in modern optimal control, pioneered by L. C. Young in the first half of the 20th century.

In summary, this book is a monumental work collecting an impressive amount of technical material ranging from historical results of calculus of variations to modern advances in nonsmooth analysis and optimization. As such, the book as a whole cannot be considered as a textbook: its scope is too broad, and some developments (especially in Part IV) seem to be too technical. However, on pages ix-x the author suggests various roadmaps for course adoption, at various levels, based on his own teaching experience. Moreover, the author's lively writing style very often provides additional insight and intuition behind the key technical concepts. This is why, in this reviewer's opinion, this book is a central, and unprecedented reference in the field of optimization and optimal control.

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